Naïve Bayesians

Back to Basics Series

02 Jan 2021

Goal

Developing the Bayesian muscle to solve a wide range of problems

Naïve Bayesian Philosophy

Intuitive (Visual) Understanding of the Bayesian Reasoning

Ability to model real world problems in a Bayesian Setting

Starting from Simple Probabilistic modelling

Adapting it in a a Bayesian setting
And moving towards ML models

PyMC3

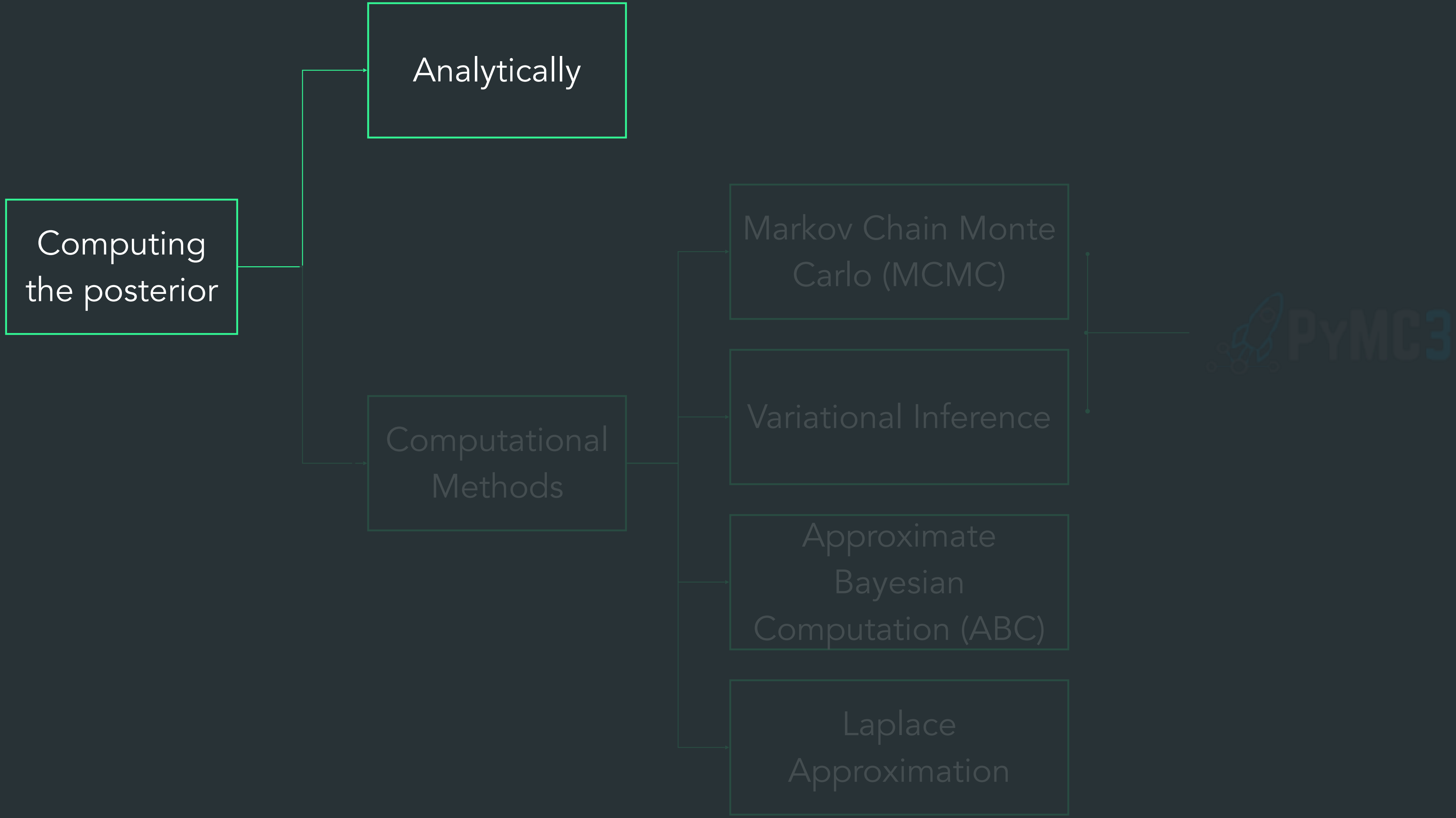
Fluency in the Calculus of Bayesian Stats & ML model

Recap of the previous seasons | Ways to get to the Posterior

- Computing the posterior
  - Analytically
  - Computational Methods
    - Markov Chain Monte Carlo (MCMC)
    - Variational Inference
    - Approximate Bayesian Computation (ABC)
    - Laplace Approximation

PyMC3

Recap of the previous seasons | Ways to get to the Posterior



Season 2: Back to Basics

Episode	Topic
Ep 1	Bayes Theorem
Ep 2, Ep 3	Problems with Binomial Likelihoods
Ep 4	Disease Detection
Ep 5	Naive Bayes Classification
Ep 6	Gaussian Naive Bayes Classification
Ep 7	German Tank Problem
Ep 8	Waiting Times (Continuous Distributions)

Back to Basics

Ep 1	Ep 4	Ep 2 / Ep 3	Ep 5	Ep 6	Ep 8	Ep 7
Bayes Theorem	Disease Detection	Problems with Binomial Likelihoods	Naive Bayes Classification	Gaussian Naive Bayes Classification	Waiting Times (Continuous Distributions)	German Tank Problem

Probabilities are Given

Probabilities are not given only data is

Back to Basics

Ep 1

Bayes Theorem

Ep 4

Disease Detection

Ep 2

Ep 3

Problems with Binomial Likelihoods

Ep 5

Naive Bayes Classification

Ep 6

Gaussian Naive Bayes Classification

Ep 8

Waiting Times (Continuous Distributions)

Ep 7

German Tank Problem

Probabilities are Given

Probabilities are not given only data is

Discrete Data

Continuous Data

Discrete Data

Back to Basics

Ep 1
Bayes Theorem

Ep 4
Disease Detection

Ep 2 Ep 3
Problems with Binomial Likelihoods

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Gaussian Naive Bayes Classification

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Waiting Times (Continuous Distributions)

Probabilities are Given

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Back to Basics

Ep 1
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Ep 2 | Ep 3
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Ep 5
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Gaussian Naive Bayes Classification

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Waiting Times (Continuous Distributions)

Probabilities are Given

Probabilities are not given only data is

Discrete Data

Continuous Data

Same Type of Data

Different Types of Data

Same Type of Data

Back to Basics

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Probabilities are Given

Probabilities are not given only data is

Discrete Data

Continuous Data

Same Type of Data

Different Types of Data

Same Type of Data

Probabilities are derived using a known distribution

Probabilities are derived by "counting"

Probabilities are derived using a known distribution

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Bayes Theorem	Disease Detection	Problems with Binomial Likelihoods	Naive Bayes Classification	Gaussian Naive Bayes Classification	Waiting Times (Continuous Distributions)	German Tank Problem
Probabilities are Given		Probabilities are not given only data is				
		Discrete Data		Continuous Data		Discrete Data
		Same Type of Data	Different Types of Data		Same Type of Data	Same Type of Data
		Probabilities are derived using a known distribution	Probabilities are derived by "counting"	Probabilities are derived using a known distribution		Probabilities are derived using a known distribution

Back to Basics

		Canonical Problem	Applications
Ep 1	Bayes Theorem	There are 2 boxes from which cookies can be taken from. Box A and Box B. Box A contains 10 chocolate cookies, Box B contains 5 ginger cookies. Given that you get a chocolate cookie which box was it taken from?	The Shy Librarian Problem Naive Bayes algorithm
Ep 2 Ep 3	Problems with Binomial Likelihoods	You have 2 coins C1 and C2. p(heads for C1) = .7 & P(heads for C2) = 0.6 You flip the coin 10 times. What is the probability that the given coin you picked is C1 given you have 7 heads and 3 tails?	A/B Testing
Ep 4	Disease Detection	A particular disease affects 1% of the population. There is an imperfect test for this disease: The test gives a positive result for 90% of people who have the disease, and 5% of the people who are disease-free. Given a positive test result – what is the probability of having the disease?	COVID Tests (PCR & Antibody)! Fraud Detection
Ep 5	Naive Bayes Classification	Given these words occur in this text what's the probability it's spam?	Any Classification Problem
Ep 6	Gaussian Naive Bayes Classification	Given the weights and heights of basketball players, what's the probability that person a is a basketball player given weight = w and height = h?	

Back to Basics

		Canonical Problem	Applications
Ep 7	German Tank Problem	Suppose tanks were given a serial number based on the order in which they were manufactured. Given that you've observed a tank with serial number "10", how many tanks were actually manufactured in total?	?
Ep 8	Waiting Times (Continuous Distributions)	Suppose you need to gather 10 patients for a trial. Each signup happens at time t_i (i=1, 10). How long do you have to wait after it took you 3 weeks to accrue 2 signups?	Planning Trials Estimating Queues

Bayes Rule

Posterior Likelihood Prior

$$P(\theta_i \mid D) = \frac{P(D \mid \theta_i)\, P(\theta_i)}{P(D)}$$

Normalising Constant

Bayes Rule

Posterior Likelihood Prior

$$P(\theta_i \mid D) = \frac{P(D \mid \theta_i)\, P(\theta_i)}{\sum_{all\ j} P(D \mid \theta_j)\, P(\theta_j)}$$

Normalising Constant

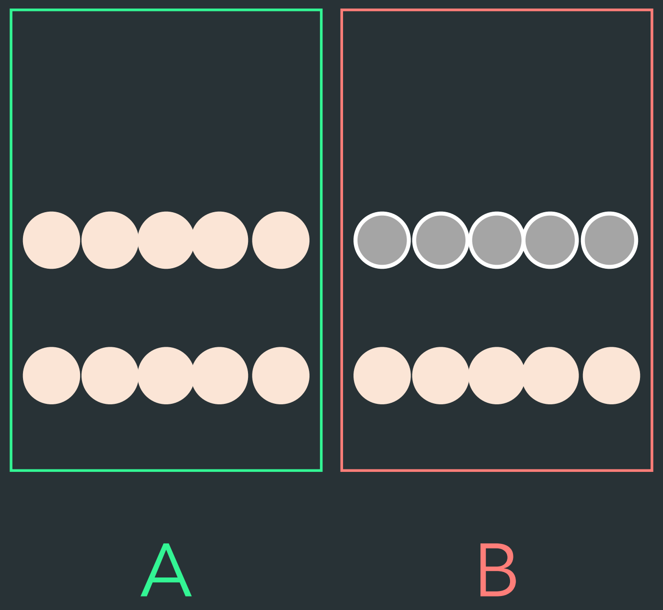
Bayes Rule

Posterior — Likelihood — Prior

$$P(\theta \mid D) = \frac{P(D \mid \theta)\, P(\theta)}{\int P(D \mid \theta')\, P(\theta')\, d\theta'}$$

Normalising Constant

What's the probability a chocolate chip cookie came from box A?



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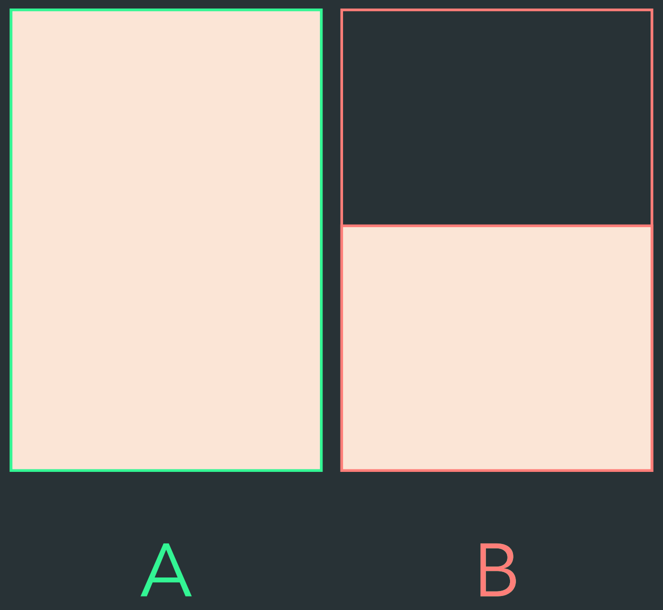

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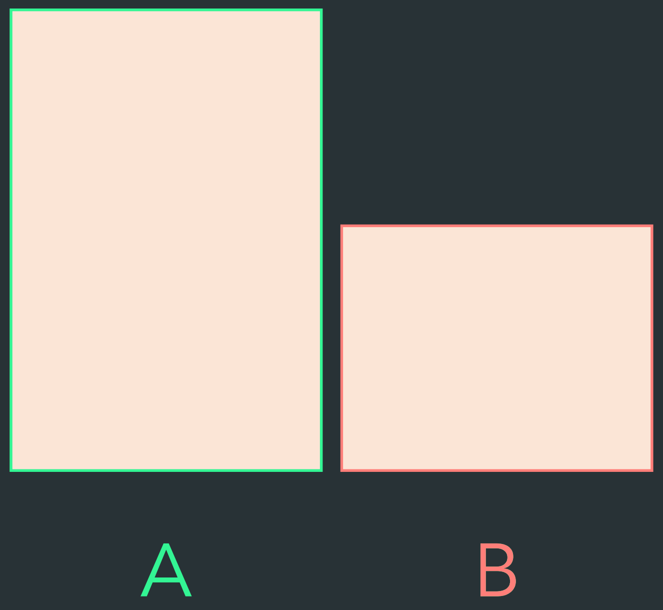


Odds of the cookie being from A vs B

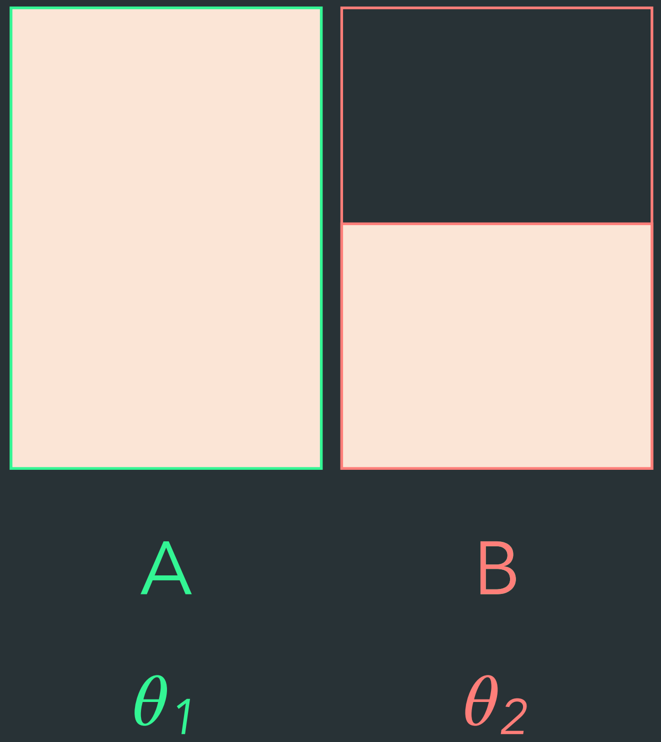
A : B

2 : 1

So the probability of it coming from A

= 2/3

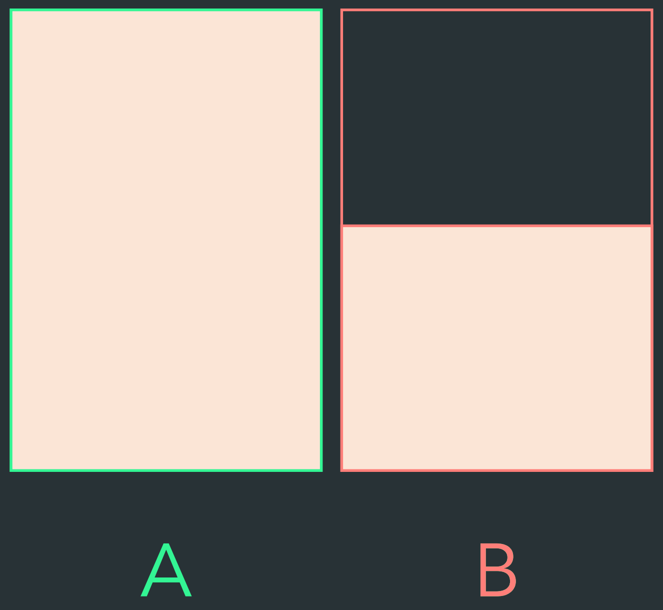
What's the probability a chocolate chip cookie came from box A?



$$P(\theta_i \mid D) = \frac{P(D \mid \theta_i)\, P(\theta_i)}{\sum_{all\ j} P(D \mid \theta_j)\, P(\theta_j)}$$

$$P(A \mid cc) = \frac{P(cc \mid A)\, P(A)}{P(cc \mid A)\, P(A) + P(cc \mid B)\, P(B)}$$

What's the probability a chocolate chip cookie came from box A?



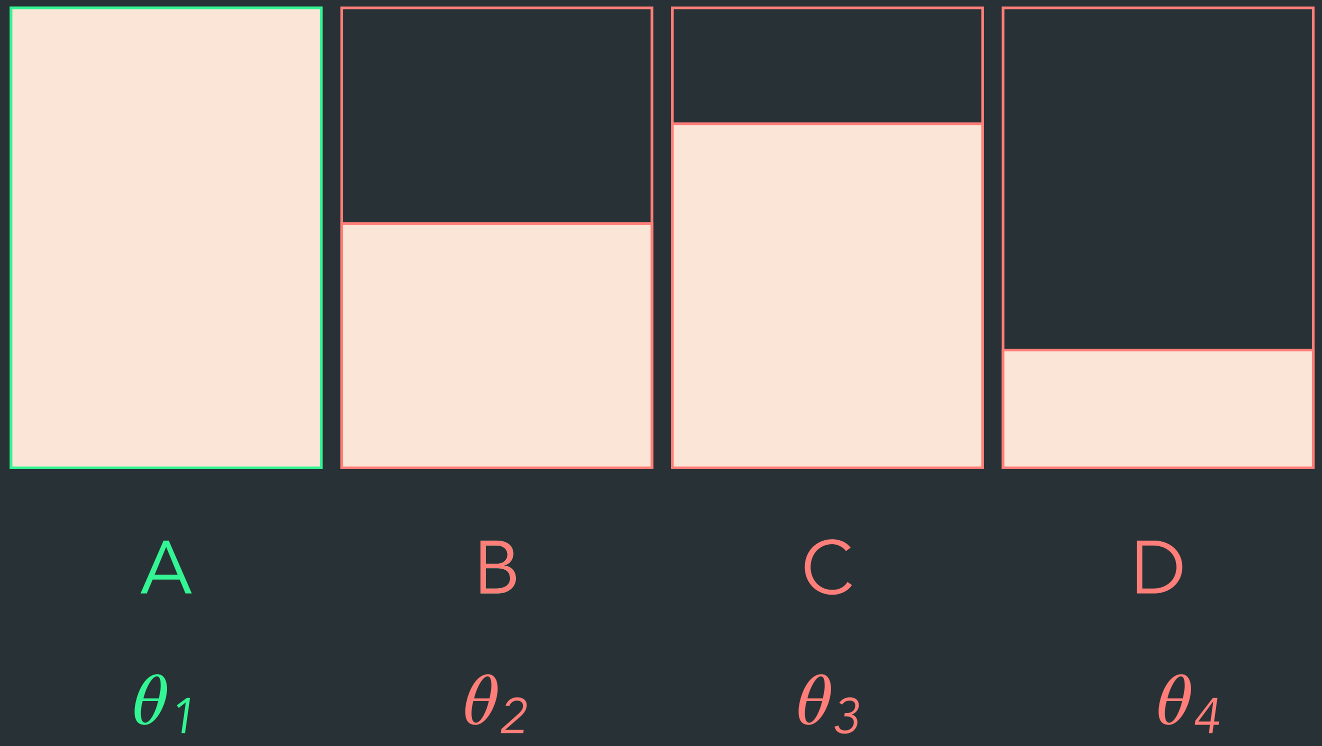
$$P(A \mid cc) = \frac{100\%\ P(A)}{100\%\ P(A) + 50\%\ P(B)}$$

$$P(A) = P(B) = 50\%$$

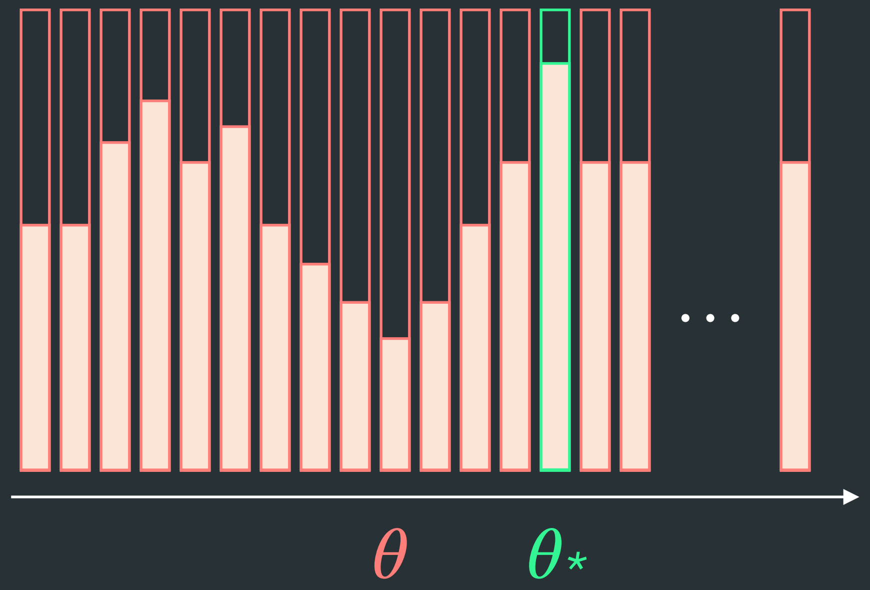
$$P(A \mid cc) = \frac{100\%}{150\%} = \frac{2}{3}$$

What if there are 3,4 or ....10 boxes?

$$P(\theta_i \mid D) = \frac{P(D \mid \theta_i)\, P(\theta_i)}{\sum_{all\ j} P(D \mid \theta_j)\, P(\theta_j)}$$

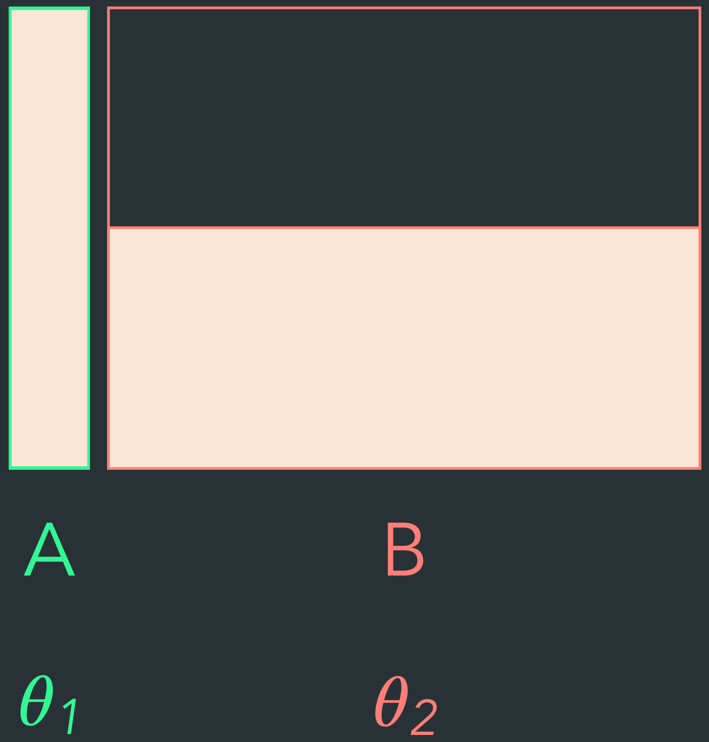


What about continuous parameters?



Moving from a discrete set of hypothesis to a continuous parameters can be thought of having an infinite number of "boxes" to choose from

Incorporating informative priors

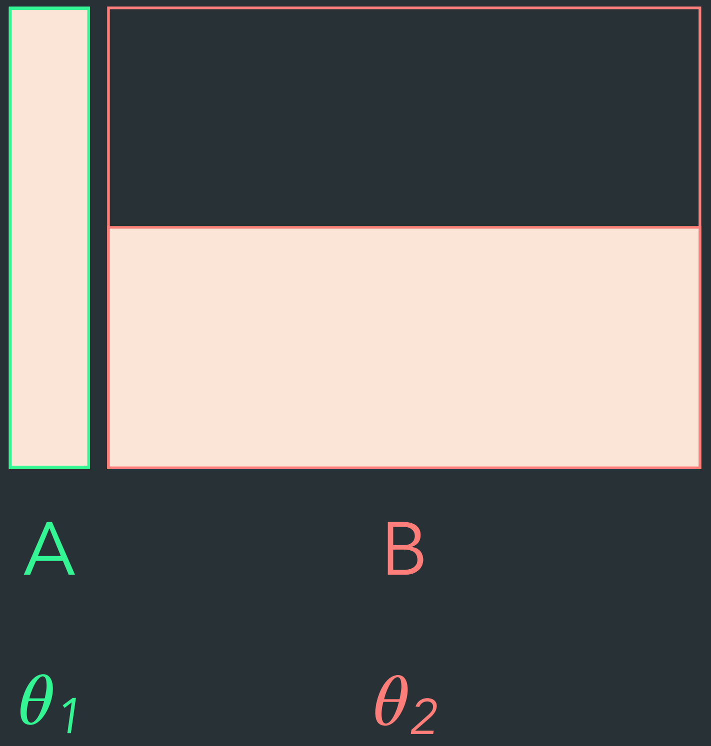


$$P(\theta_i \mid D) = \frac{P(D \mid \theta_i)\, P(\theta_i)}{\sum_{all\ j} P(D \mid \theta_j)\, P(\theta_j)}$$

$$P(A \mid cc) = \frac{P(cc \mid A)\, P(A)}{P(cc \mid A)\, P(A) + P(cc \mid B)\, P(B)}$$

$$P(A) \neq P(B)$$

Incorporating informative priors



Let

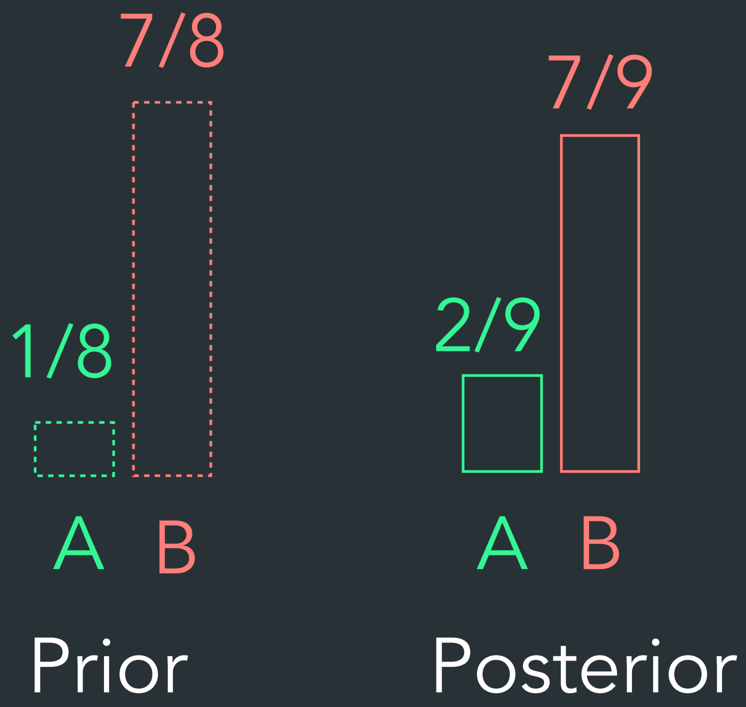
$P(A) = 1/8 \quad P(B) = 7/8$

$$P(\,A \mid cc\,) = \frac{100\%\ (1/8)}{100\%\ P(1/8) + 50\%\ (7/8)}$$

$$P(\,A \mid cc\,) = 2/9$$

$$P(\,B \mid cc\,) = 7/9$$

Incorporating informative priors



$P(A) = 1/8$

$P(B) = 7/8$

$P(A \mid cc) = 2/9$

$P(B \mid cc) = 7/9$

Takeaways

Bayesian Inference is all about finding which box did your chocolate chip cookie come from

References

1. Bayes Theorem: A Visual Introduction for Beginners by Dan Morris

2. Bayes theorem by 3Blue1Brown (YouTube)