Naïve Bayesians

Back to Basics Series

30 Jan 2021

Goal

Developing the Bayesian muscle to solve a wide range of problems

Naïve Bayesian Philosophy

Intuitive (Visual) Understanding of the Bayesian Reasoning

Ability to model real world problems in a Bayesian Setting

Starting from Simple Probabilistic modelling

Adapting it in a a Bayesian setting
And moving towards ML models

Fluency in the Calculus of Bayesian Stats & ML model

Season 2: Back to Basics

Episode	Topic
Ep 1	Bayes Theorem
Ep 2, Ep 3	Problems with Binomial Likelihoods
Ep 4	Disease Detection
Ep 5	Naive Bayes Classification
Ep 6	Gaussian Naive Bayes Classification
Ep 7	German Tank Problem
Ep 8	Waiting Times (Continuous Distributions)

Back to Basics

		Canonical Problem	Applications
Ep 1	Bayes Theorem	There are 2 boxes from which cookies can be taken from. Box A and Box B. Box A contains 10 chocolate cookies, Box B contains 5 ginger cookies. Given that you get a chocolate cookie which box was it taken from?	The Shy Librarian Problem Naive Bayes algorithm
Ep 2 Ep 3	Problems with Binomial Likelihoods	You have 2 coins C1 and C2. p(heads for C1) = .7 & P(heads for C2) = 0.6 You flip the coin 10 times. What is the probability that the given coin you picked is C1 given you have 7 heads and 3 tails?	A/B Testing
Ep 4	Disease Detection	A particular disease affects 1% of the population. There is an imperfect test for this disease: The test gives a positive result for 90% of people who have the disease, and 5% of the people who are disease-free. Given a positive test result – what is the probability of having the disease?	COVID Tests (PCR & Antibody)! Fraud Detection
Ep 5	Naive Bayes Classification	Given these words occur in this text what's the probability it's spam?	Any Classification Problem
Ep 6	Gaussian Naive Bayes Classification	Given the weights and heights of basketball players, what's the probability that person a is a basketball player given weight = w and height = h?	

Back to Basics

		Canonical Problem	Applications
Ep 7	German Tank Problem	Suppose tanks were given a serial number based on the order in which they were manufactured. Given that you've observed a tank with serial number "10", how many tanks were actually manufactured in total?	?
Ep 8	Waiting Times (Continuous Distributions)	Suppose you need to gather 10 patients for a trial. Each signup happens at time t_i (i=1, 10). How long do you have to wait after it took you 3 weeks to accrue 2 signups?	Planning Trials Estimating Queues

Bayes Rule

Posterior Likelihood Prior

$$P(\theta_i \mid D) = \frac{P(D \mid \theta_i)\, P(\theta_i)}{\sum_{all\ j} P(D \mid \theta_j)\, P(\theta_j)}$$

Normalising Constant

Canonical Problem

Given the words “Dear Friend” occur in this email what's the probability it's spam?

$$P(\, S \mid \text{Dear Friend}\,)$$

N	S
Normal	Spam

Given the words "Dear Friend" occur in this email what's the probability it's spam?

8 Normal Emails		4 Spam Emails	
Dear:	8	Dear:	2
Friend	5	Friend:	1
Lunch:	3	Lunch:	0
Money:	1	Money:	4

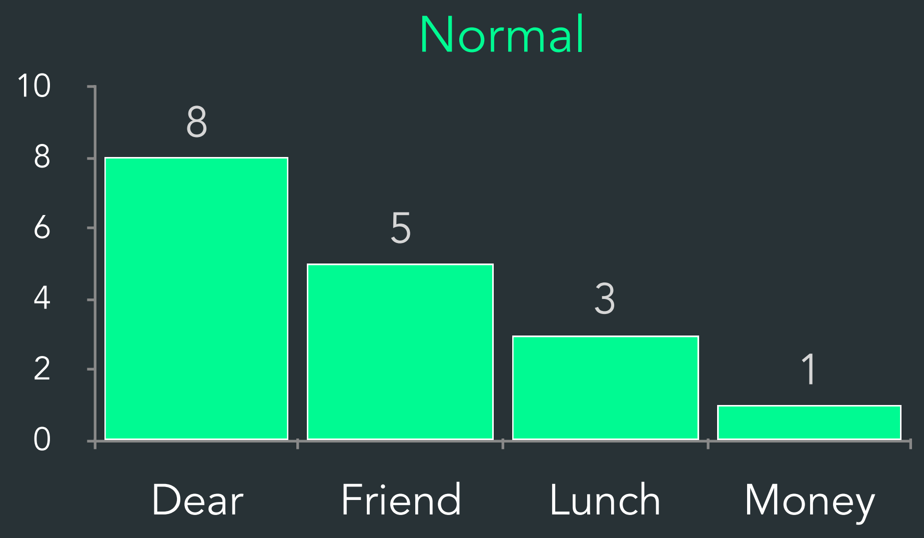
N

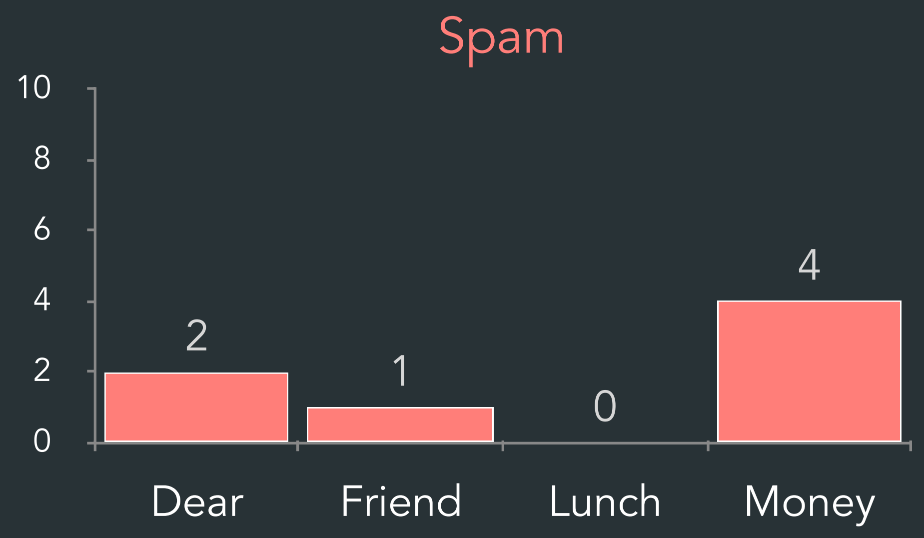
Normal

S

Spam

Given the words “Dear Friend” occur in this email, what's the probability it's spam?





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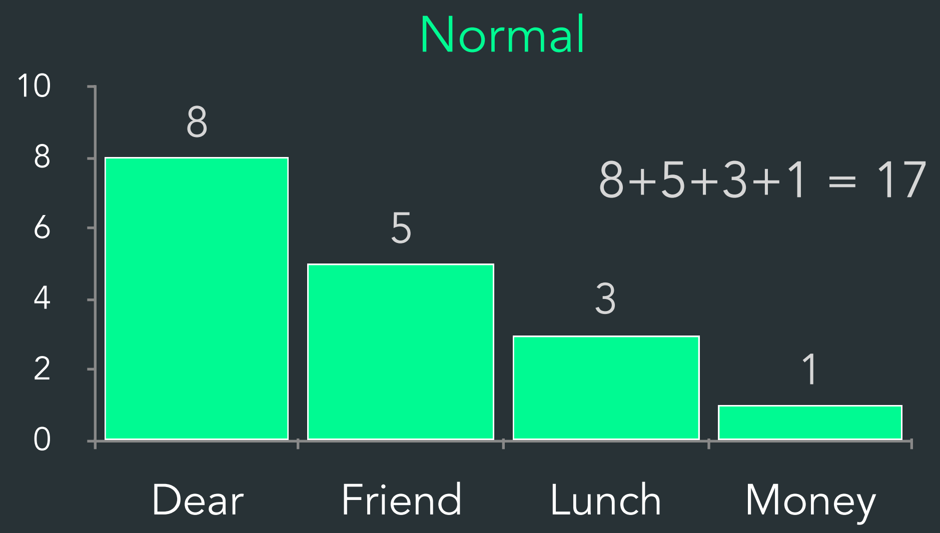

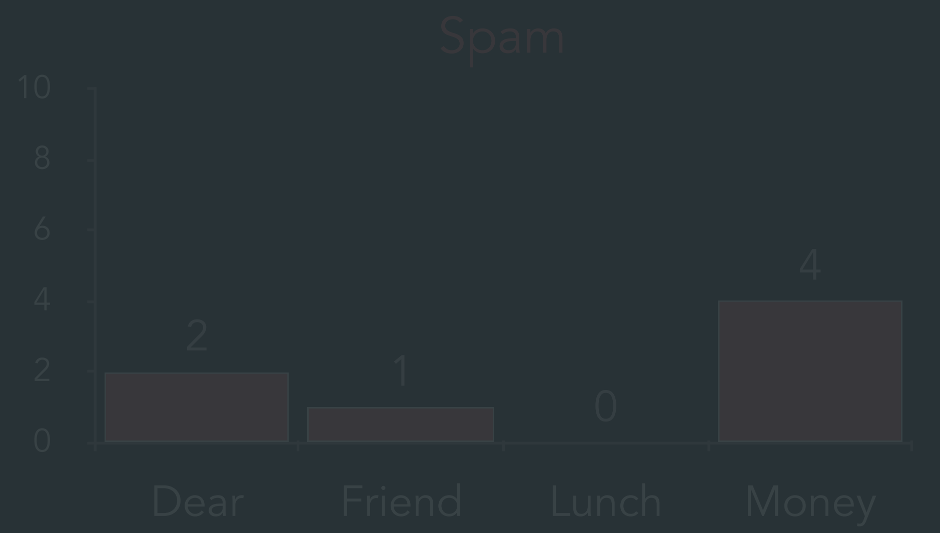

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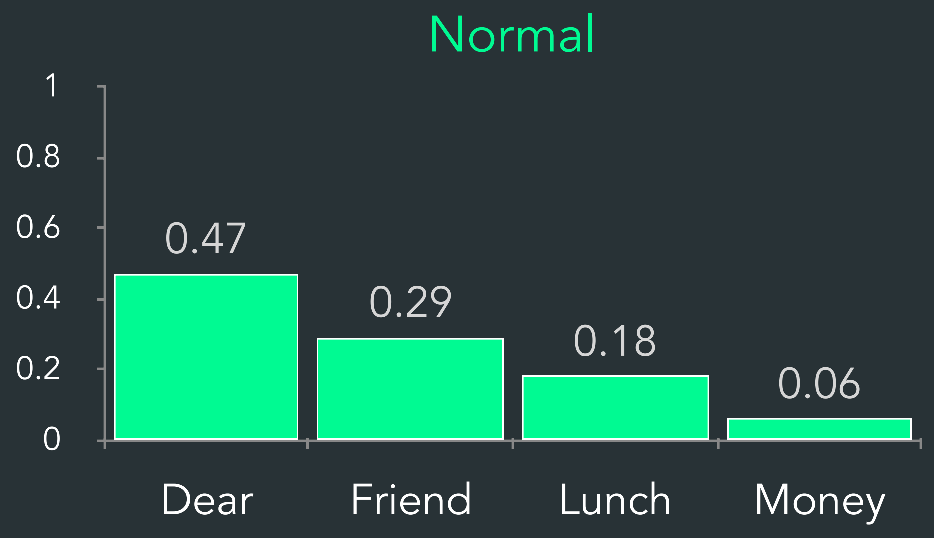

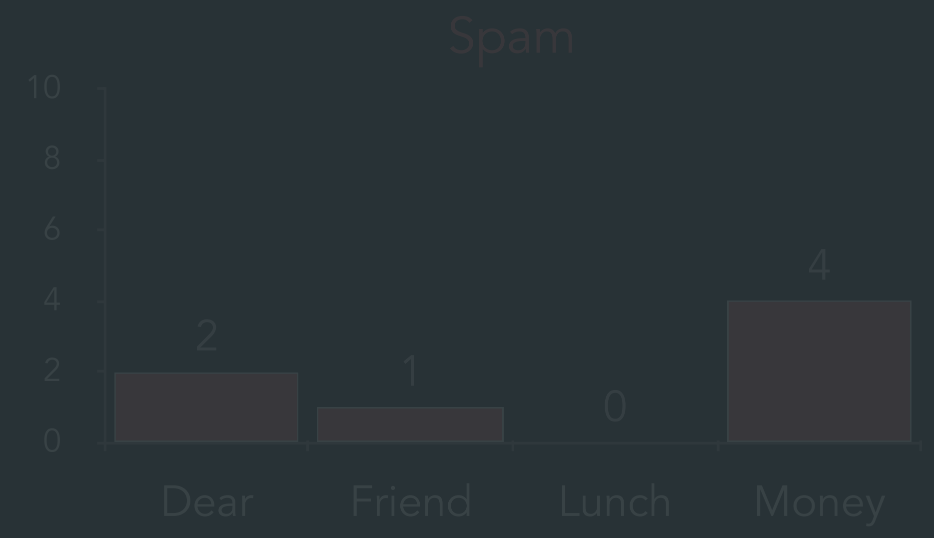

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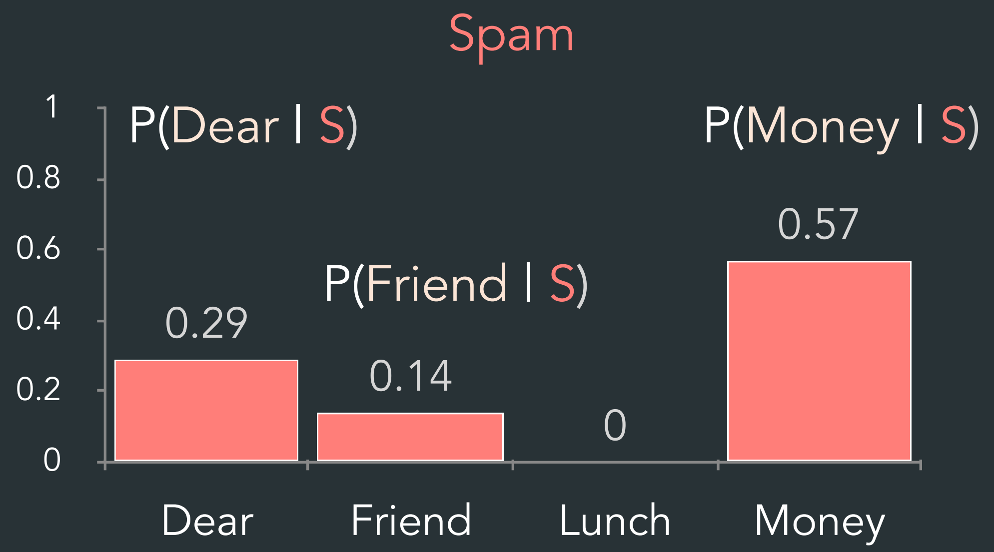
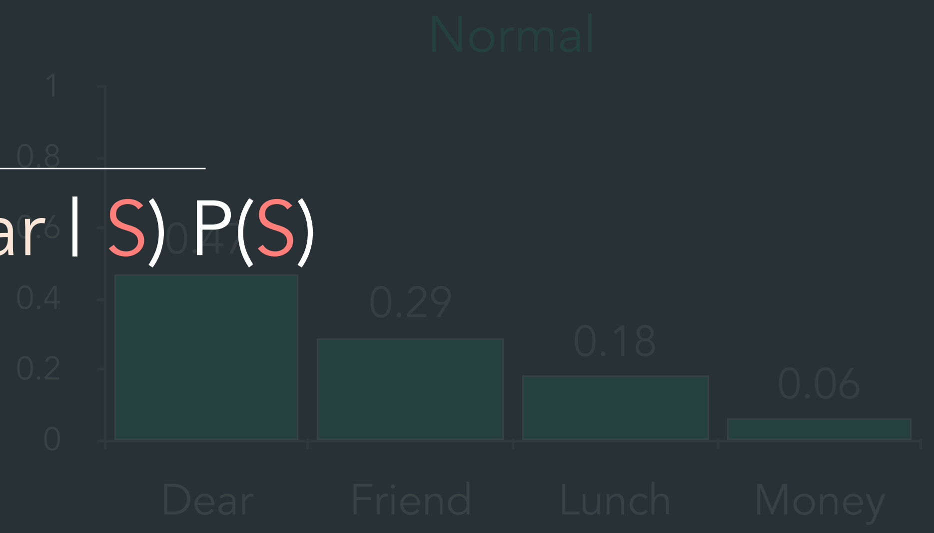
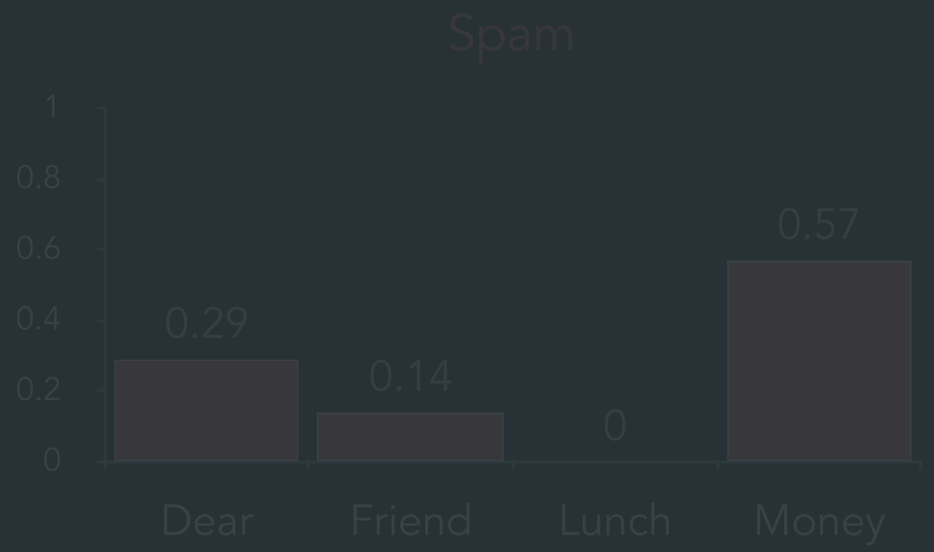


Given the words "Dear Friend" occur in this email, what's the probability it's spam?

$$P(S \mid \text{Dear}) = \frac{P(\text{Dear} \mid S)\, P(S)}{P(\text{Dear} \mid N)\, P(N) + P(\text{Dear} \mid S)\, P(S)}$$





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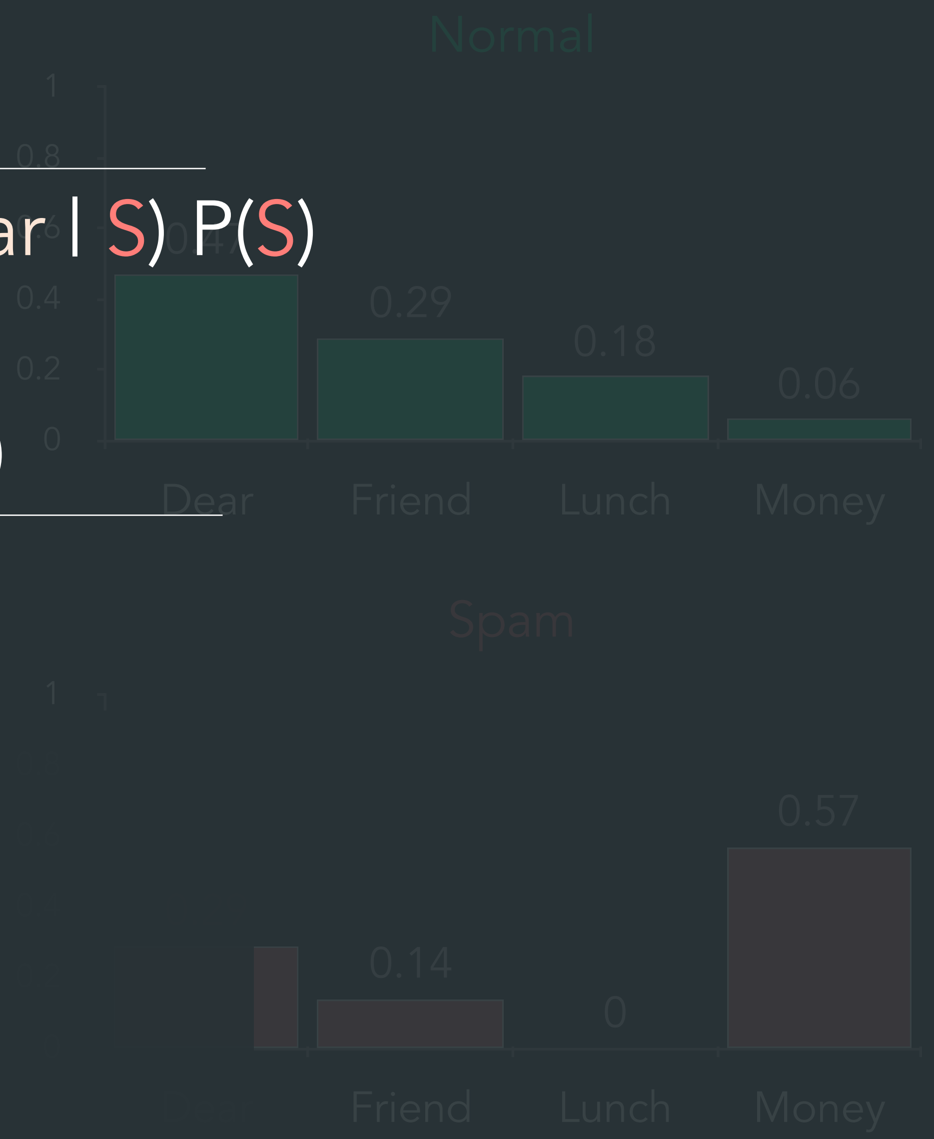
$$P(S \mid \text{Dear}) = \frac{P(\text{Dear} \mid S)\,P(S)}{P(\text{Dear})}$$

$$P(S \mid \text{Dear}) \propto P(\text{Dear} \mid S)\,P(S)$$



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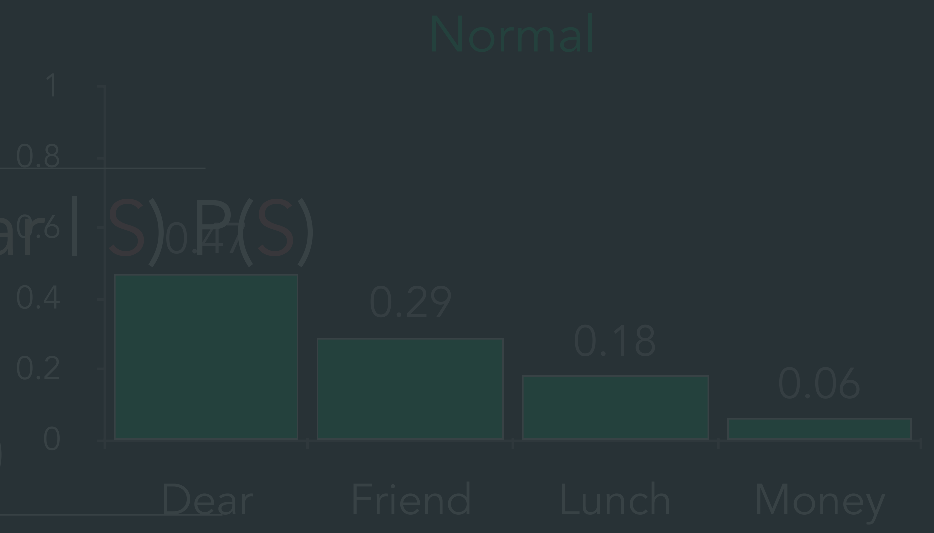
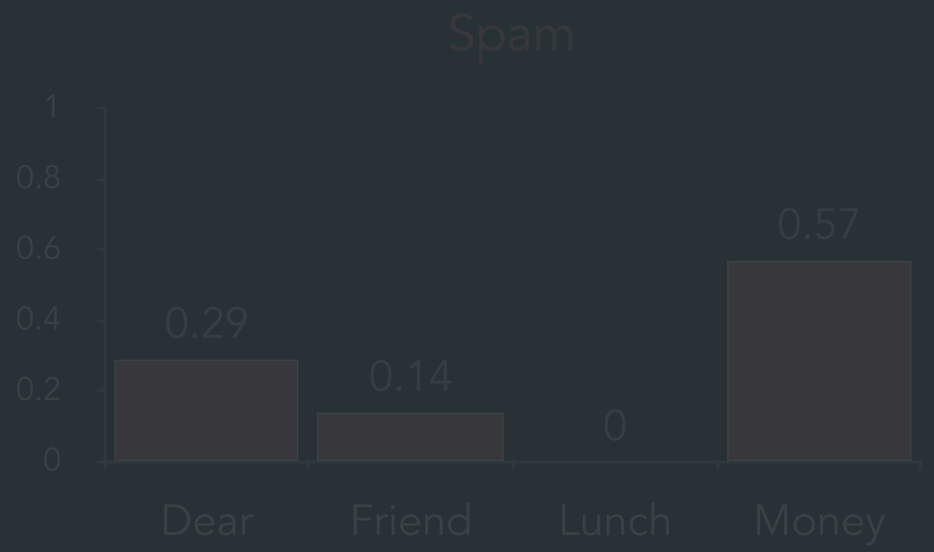




$$P(S \mid \text{Dear}) \propto P(\text{Dear} \mid S)\,P(S)$$

Proportional to

Given the words “Dear Friend” occur in this email, what's the probability it's spam?

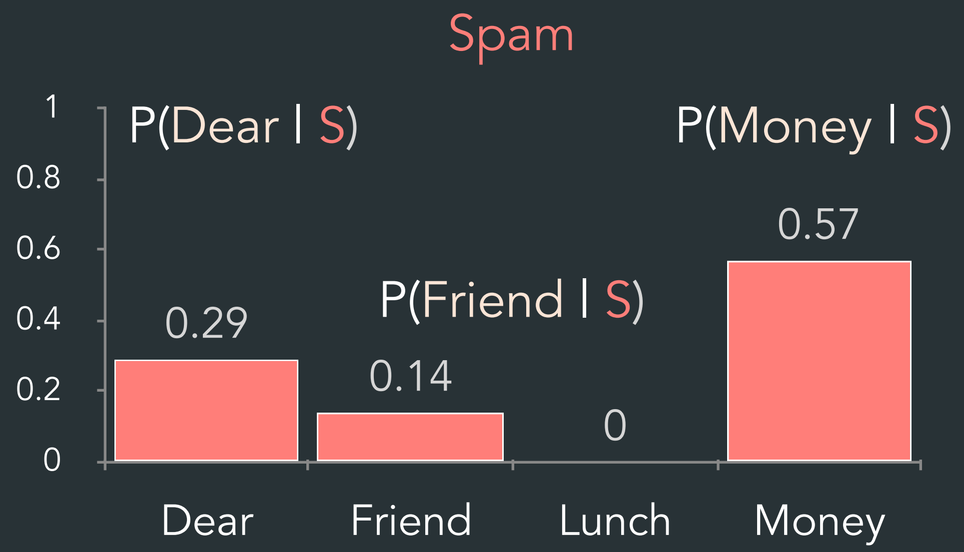
$$P(S \mid \text{Dear Friend}) \stackrel{?}{=}$$

$$P(S \mid \text{Dear})$$

or

$$P(S \mid \text{Friend})$$

or ??



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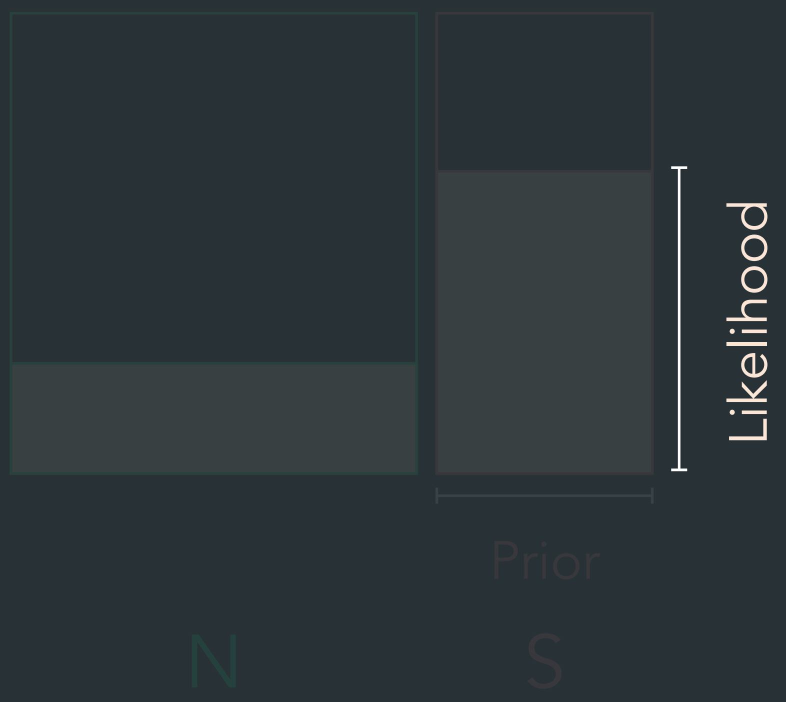
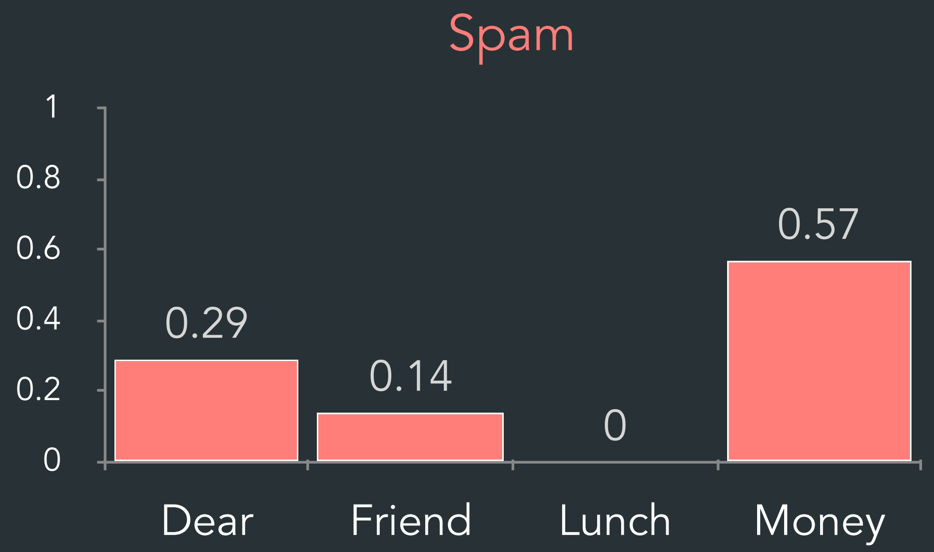


$$P(\text{Dear Friend} \mid S) \stackrel{?}{=}$$



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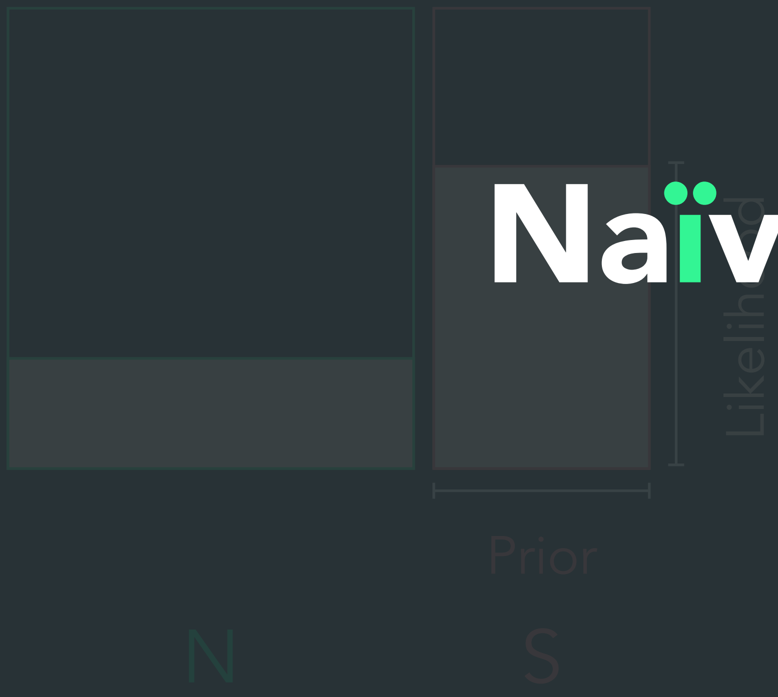


Naïve

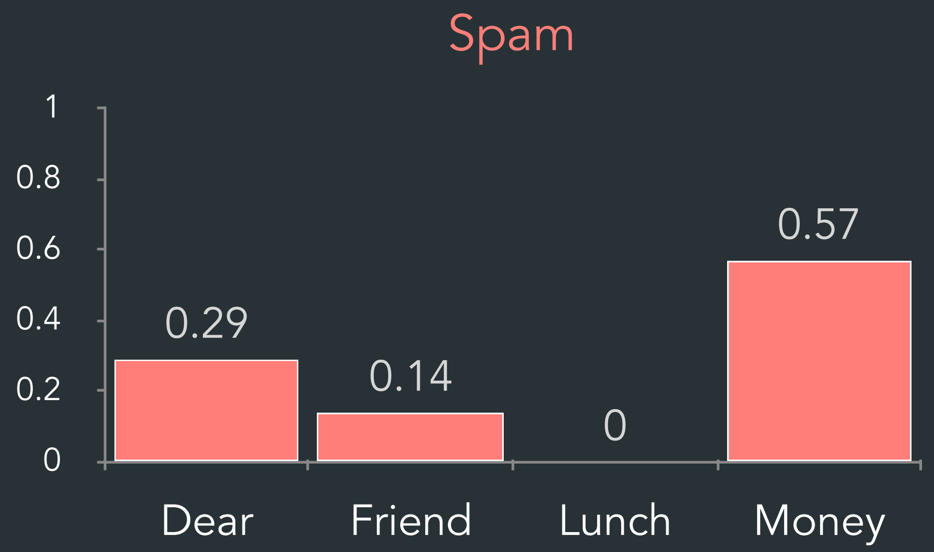
$$P(\text{ Dear Friend} \mid S) \approx P(\text{ Dear} \mid S) \times P(\text{ Friend} \mid S)$$

Given the words "Dear Friend" occur in this email, what's the probability it's spam?

$$P(S \mid \text{Dear Friend}) \propto P(\text{Dear} \mid S) \times P(\text{Friend} \mid S) \times P(S)$$

$$= (0.29) \times (0.14) \times (0.33)$$

$$= 0.0135$$



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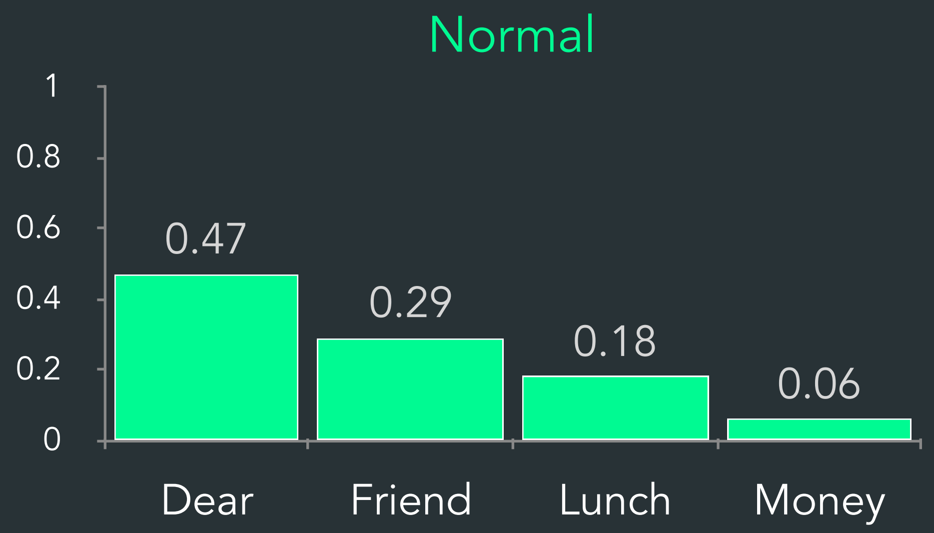
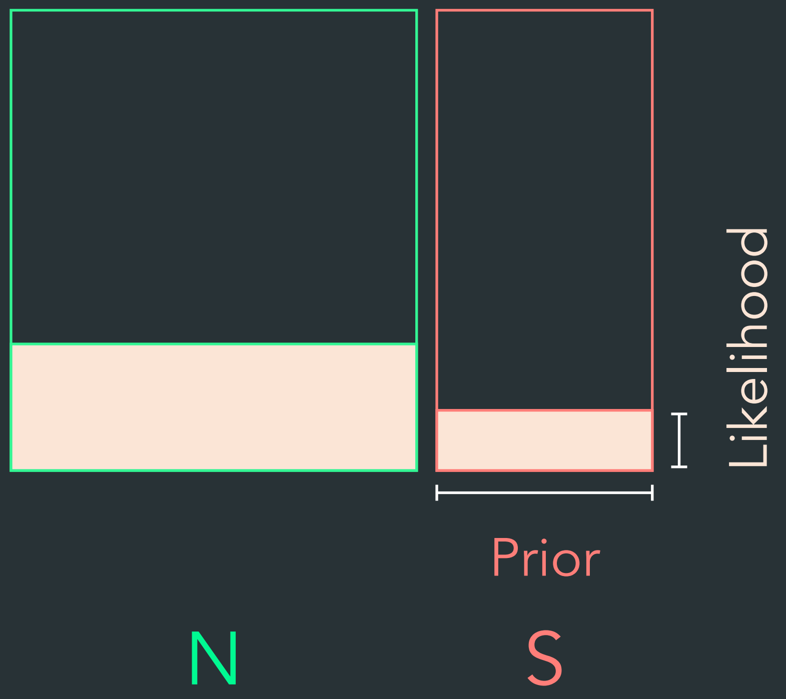


$$P(N \mid \text{Dear Friend}) \propto P(\text{Dear Friend} \mid N) \times P(N)$$

$$\approx P(\text{Dear} \mid N) \times P(\text{Friend} \mid N) \times P(N)$$

$$= (0.47) \times (0.29) \times (0.67) = 0.0909$$

Given the words “Dear Friend” occur in this email what's the probability it's spam?



$$P(S \mid \text{Dear Friend}) \propto 0.0135$$

$$P(N \mid \text{Dear Friend}) \propto 0.0909$$

$$P(N \mid \text{Dear Friend}) > P(S \mid \text{Dear Friend})$$

Classify as N

Alternative: Bayes Factor

Odds $a : b$

Probabilities: $a/(a + b)$

$S : N$

$$B = \frac{P(\text{Dear Friend} \mid S)}{P(\text{Dear Friend} \mid N)}$$

$B \times S : N$

Alternative: Bayes Factor

Odds $a : b$

Probabilities: $a/(a + b)$

$S : N$

$$B = \frac{P(\text{Dear Friend} \mid S)}{P(\text{Dear Friend} \mid N)}$$

$B \times S : N$

Alternative: Bayes Factor

Odds $a : b$ Probabilities: $a/(a + b)$

$S : N$

$$B = \frac{P(\text{Dear Friend} \mid S)}{P(\text{Dear Friend} \mid N)}$$

$B \times S : N$

Alternative: Bayes Factor

Odds $a : b$

Probabilities: $a/(a + b)$

$S : N$

$$B = \frac{P(\text{Dear Friend} \mid S)}{P(\text{Dear Friend} \mid N)} \approx \frac{P(\text{Dear} \mid S) \times P(\text{Friend} \mid S)}{P(\text{Dear} \mid N) \times P(\text{Friend} \mid N)}$$

$B \times S : N$

Alternative: Bayes Factor

Odds $a : b$

Probabilities: $a/(a + b)$

4 Spam Emails

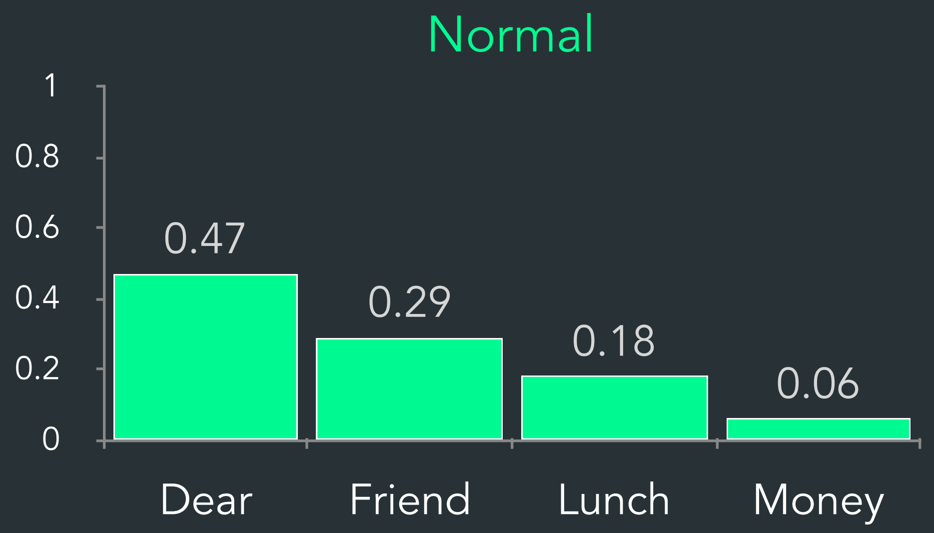
8 Normal Emails

1 : 2

B

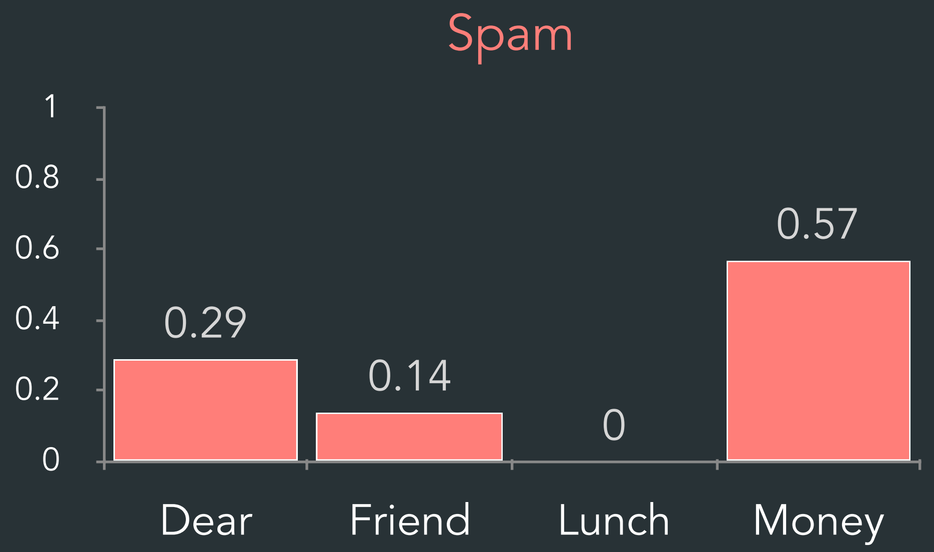
B 1 : 2

Alternative: Bayes Factor



1 : 2

$$B \approx \frac{P(\text{Dear} \mid S) \times P(\text{Friend} \mid S)}{P(\text{Dear} \mid N) \times P(\text{Friend} \mid N)}$$



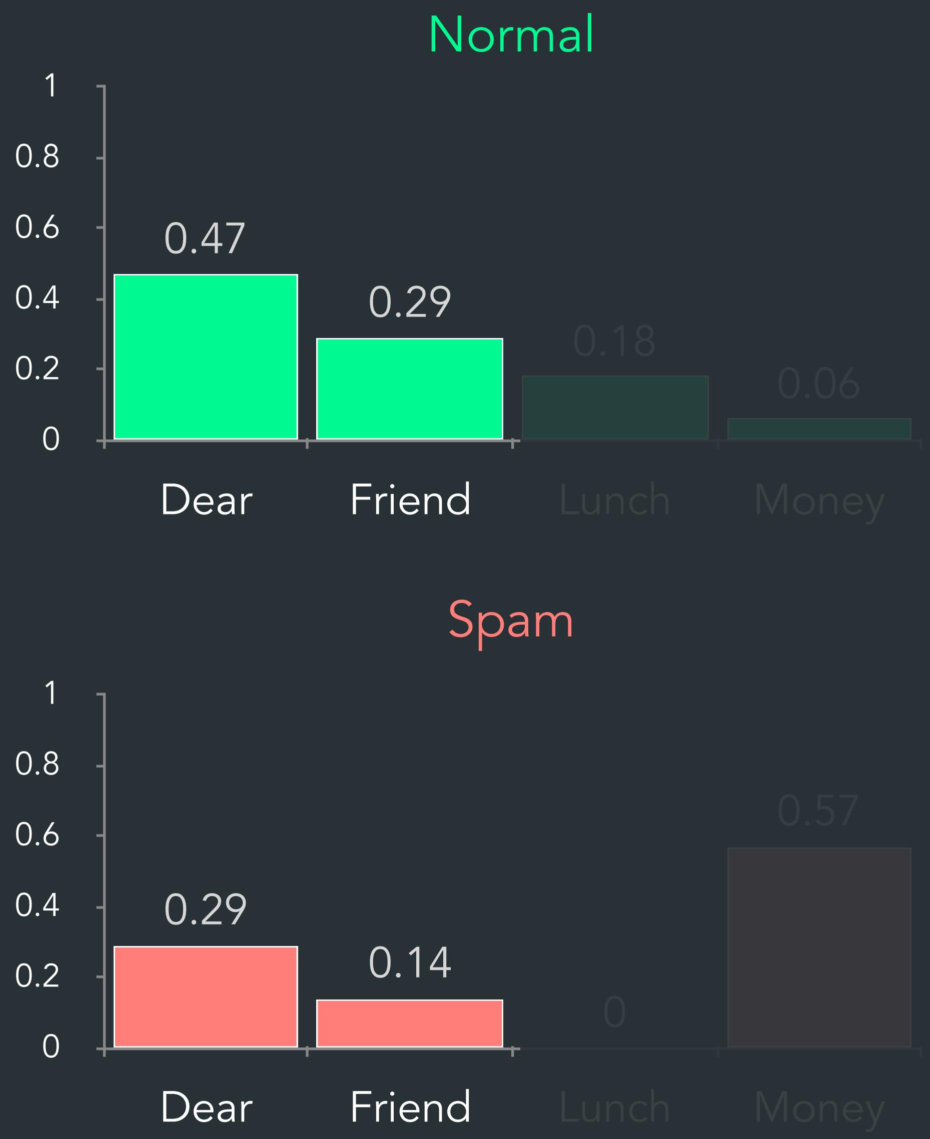
B 1 : 2

Alternative: Bayes Factor

1 : 2

$$B = \frac{(0.29) \times (0.14)}{(0.47) \times (0.29)} = 0.298$$

(0.298)1 : 2



Back to Basics

Ep 1

Bayes Theorem

Ep 4

Disease Detection

Ep 2

Ep 3

Problems with Binomial Likelihoods

Ep 5

Naive Bayes Classification

Ep 6

Gaussian Naive Bayes Classification

Ep 8

Waiting Times (Continuous Distributions)

Probabilities are Given

Probabilities are not given only data is

Discrete Data

Probabilities are derived using a known distribution

Probabilities are derived by "counting"

Recap | What is the probability that the given coin you picked is C_1?

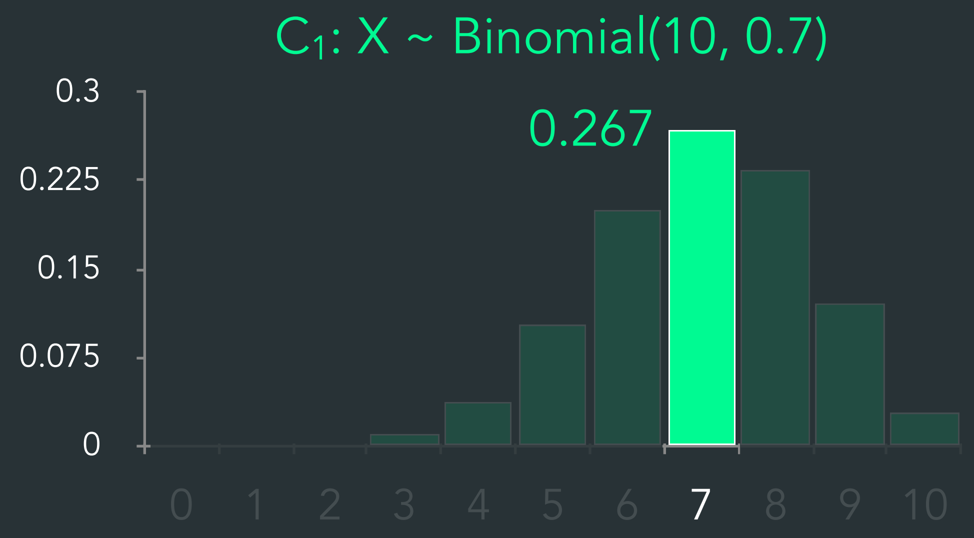
You have 2 coins C_1 and C_2.

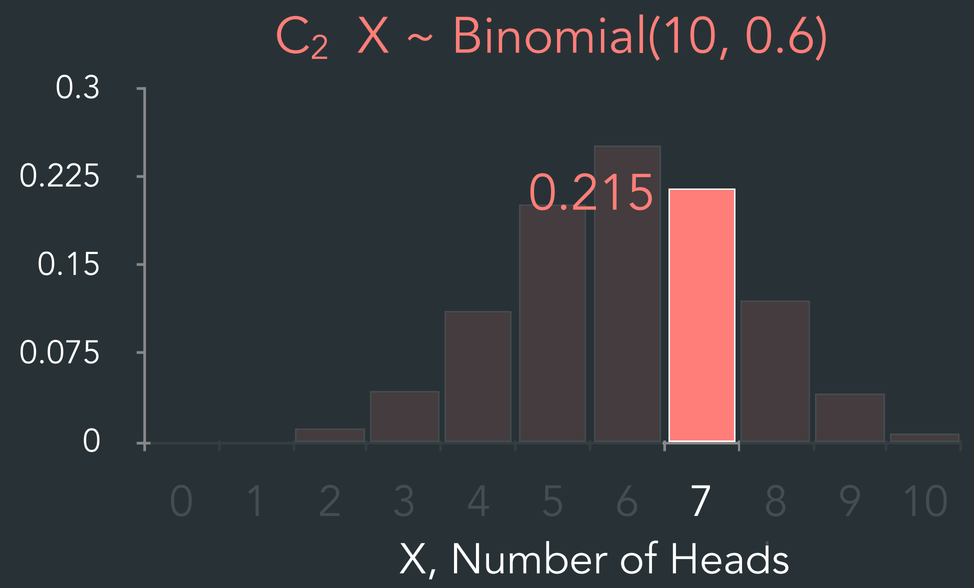
p(heads for C_1) = 0.7

p(heads for C_2) = 0.6

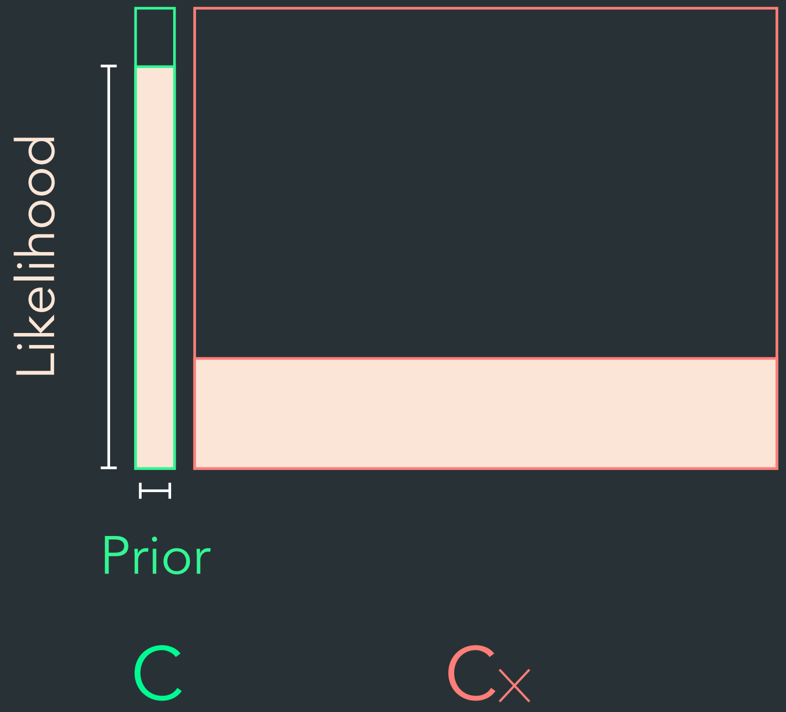
You flip one of the coins 10 times and get **7 heads** and **3 tails**

What is the probability that the given coin you picked is C_1?





Recap | Given a positive test result, what is the probability of having the disease?



$$P(C \mid +) = \frac{P(+|C)\,P(C)}{P(+|C)\,P(C) + P(+|C_x)\,P(C_x)}$$

$\boxed{P(C) = 0.01}$

$P(C_x) = 0.99$

$\boxed{P(+|C) = 0.9}$

$\boxed{P(+|C_x) = 0.05}$

Takeaways

Naive Bayes Algorithm

Bayes Factor for an Alternative Formulation

References

StatQuest: Naive Bayes, Clearly Explained
https://www.youtube.com/watch?v=O2L2Uv9pdDA