$$Solving \begin{cases} a_{n+1} = 2a_n + n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1.2.1) \\ a_0 = 1 \end{cases}$$

ANS:

$$令A(X) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \cdots$$

Multiply x^n*on (1.2.1)*

$$a_{n+1}x^n = 2 \times a_n x^n + n x^n$$

Adding Sigma on the formula above

$\sum_{n=0}^{\infty} a_{n+1}x^n = \sum_{n=0}^{\infty} 2 \times a_n x^n + \sum_{n=0}^{\infty} n x^n$*.......(*)*

Considering $\frac{A(x)}{x} = \frac{a_0}{x} + a_1 + a_2 x^1 + a_3 x^2 \cdots$ *，且*$a_0 = 1$ *代入*

$$\therefore LHS\ of\,(*) = \sum_{n=0}^{\infty} a_{n+1}x^n = \frac{A(x) - 1}{x}$$

$$RHS\ of\ (*) = 2 \times \sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} n x^n$$

$$considering \sum_{n=0}^{\infty} n x^n = \sum_{n=0}^{\infty} x \times (\frac{d}{dx})x^n = x \times \frac{d}{dx}\sum_{n=0}^{\infty} x^n = x \times \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{x}{(1-x)^2}$$

$$\therefore \frac{A(x) - 1}{x} = 2A(x) + \frac{x}{(1-x)^2}$$

移項可得$A(x) = \frac{1-2x+2x^2}{(1-2x)(1-x)^2} \coloneqq \frac{A}{(1-x)^2} + \frac{B}{1-x} + \frac{C}{1-2x}$ *；要解 A,B,C 皆常數*

解得 A=(-1), B=0,C=2

$$\therefore A(x) = \sum_{n=0}^{\infty} a_n x^n = \frac{-1}{(1-x)^2} + \frac{2}{1-2x} = 2 \times \sum_{n=0}^{\infty} (2x)^n - \sum_{n=0}^{\infty} n x^n = \sum_{n=0}^{\infty} (2 \times 2^n - n) x^n$$

我能從紅色的地方推論$a_n = (2 \times 2^n - n)$*嗎*

但是看答案是$a_n = (2 \times 2^n - n - 1)$ *；*

能幫我糾錯嗎?