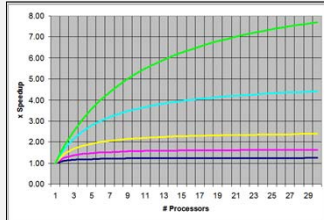


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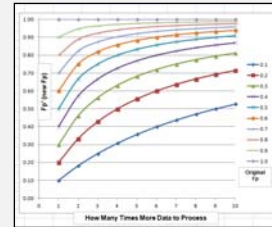
Parallel Programming: Speedups and Amdahl's law



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speedups.and.amdahls.law.pptx

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Definition of Speedup

If you are using n processors, your **Speedup _{n}** is:

$$\text{Speedup}_n = \frac{T_1}{T_n}$$

where T_1 is the execution time on **one core**
and T_n is the execution time on **n cores**.
Note that Speedup _{n} should be > 1 .

And your **Speedup Efficiency _{n}** is:

$$\text{Efficiency}_n = \frac{\text{Speedup}_n}{n}$$

which could be as high as 1., but probably never will be.



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However, Multicore is not a Free Lunch: Amdahl's Law

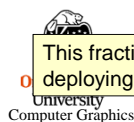
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If you put in n processors, you should get n times Speedup
(and 100% Speedup Efficiency), right? Wrong!

There are always some fraction of the total operation that is inherently sequential and cannot be parallelized no matter what you do. This includes reading data, setting up calculations, control logic, storing results, etc.

If you think of all the operations that a program needs to do as being divided between a fraction that is parallelizable and a fraction that isn't (i.e., is stuck at being sequential), then **Amdahl's Law** says:

$$Speedup_n = \frac{T_1}{T_n} = \frac{1}{\frac{F_{parallel}}{n} + F_{sequential}} = \frac{1}{\frac{F_{parallel}}{n} + (1 - F_{parallel})}$$



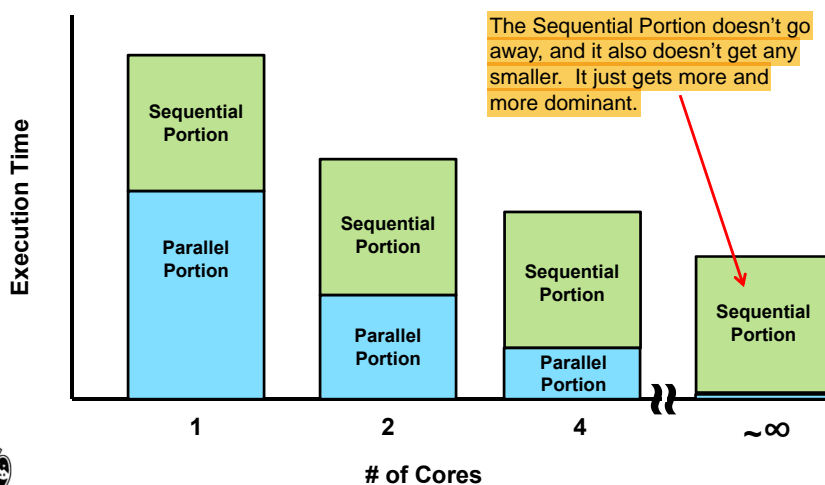
This fraction can be reduced by
deploying multiple processors.

This fraction can't.

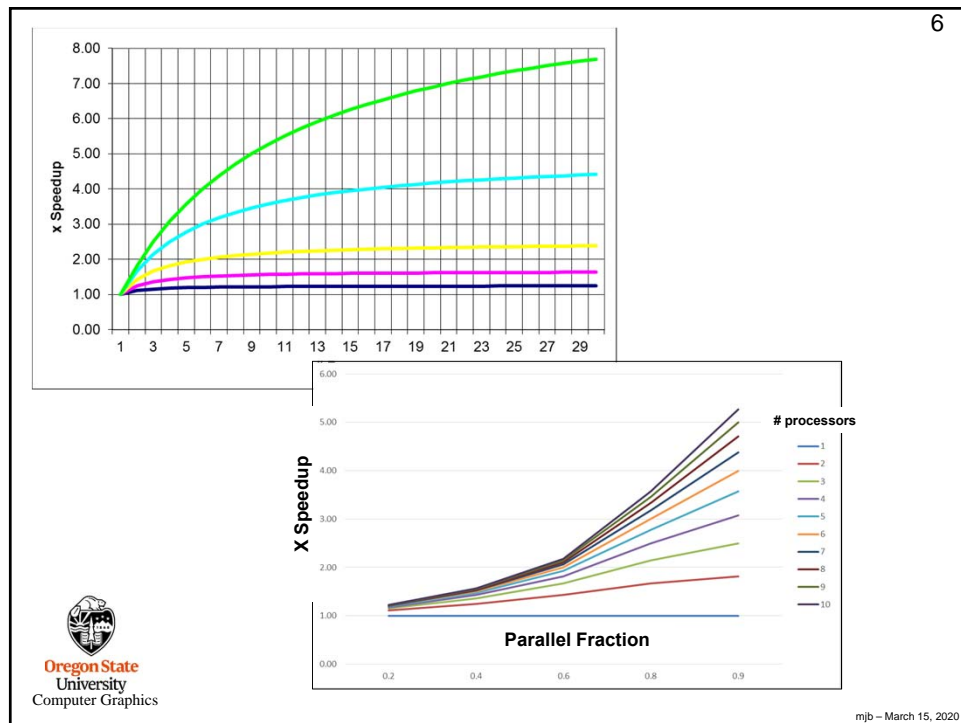
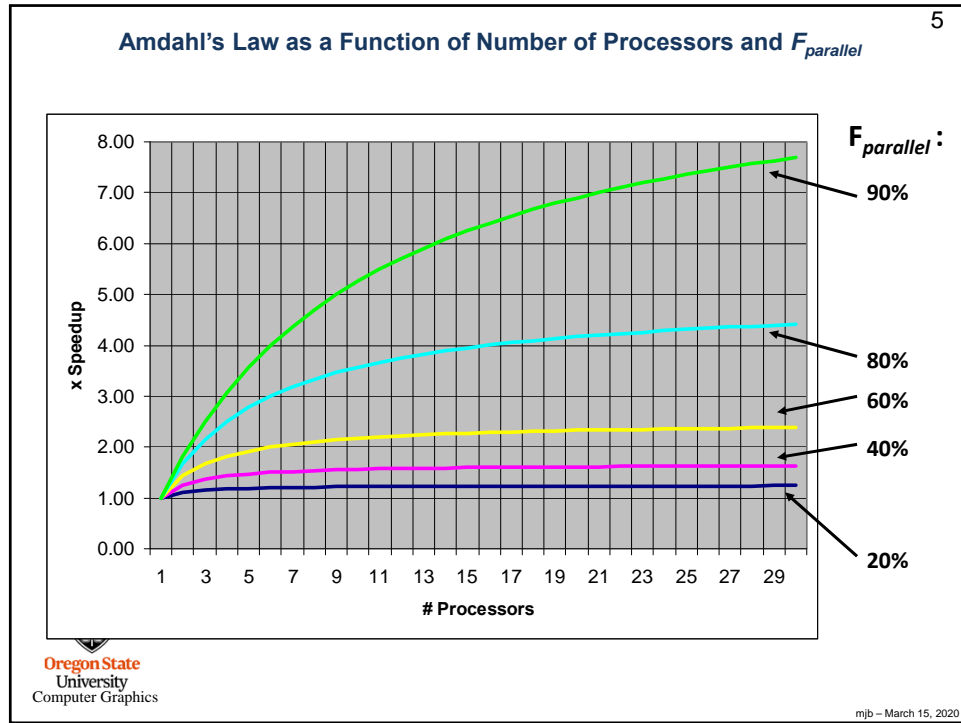
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A Visual Explanation of Amdahl's Law

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Amdahl's Law

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Note that these fractions put an upper bound on how much benefit you will get from adding more processors:

$$\max \text{Speedup} = \lim_{n \rightarrow \infty} \text{Speedup} = \frac{1}{F_{\text{sequential}}} = \frac{1}{1 - F_{\text{parallel}}}$$

Fparallel	maxSpeedup
0.00	1.00
0.10	1.11
0.20	1.25
0.30	1.43
0.40	1.67
0.50	2.00
0.60	2.50
0.70	3.33
0.80	5.00
0.90	10.00
0.95	20.00
0.99	100.00



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You can also solve for F_{parallel} using Amdahl's Law if you know your speedup and the number of processors

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Amdahl's law says:

$$S = \frac{T_1}{T_n} = \frac{1}{\frac{F}{n} + (1-F)} \Rightarrow \frac{1}{S} = \frac{F}{n} + (1-F) = 1 + \frac{F - nF}{n} \Rightarrow \frac{1}{S} - 1 = F \frac{(1-n)}{n}$$

Solving for F:

$$F = \frac{\frac{1}{S} - 1}{\frac{1-n}{n}} = \frac{\frac{T_n}{T_1} - 1}{\frac{1-n}{n}} = \frac{\frac{T_n - T_1}{T_1}}{\frac{1-n}{n}} = \frac{\frac{T_1 - T_n}{T_1}}{\frac{n-1}{n}} = \frac{n(T_1 - T_n)}{T_1(n-1)} = \frac{n}{(n-1)} \frac{T_1 - T_n}{T_1} = \frac{n}{(n-1)} \left(1 - \frac{1}{\text{Speedup}} \right)$$

Use this if you know the timing

Use this if you know the speedup

If you've got several (n,S) values, you can take the average (which is actually a least squares fit):

$$F_i = \frac{n_i}{(n_i - 1)} \frac{T_1 - T_{n_i}}{T_1}, i = 2..N$$

$$\bar{F} = \frac{\sum_{i=2}^N F_i}{N-1}$$

note that when $i=1$, $T_{n_i} = T_1$



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A More Optimistic Take on Amdahl's Law: Gustafson's Observation

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Gustafson observed that as you increase the number of processors, you have a tendency to attack larger and larger versions of the problem. He also observed that when you use the same parallel program on larger datasets, the parallel fraction, F_p , increases.

Let P be the amount of time spent on the parallel portion of an original task and S spent on the serial portion. Then

$$F_p = \frac{P}{P + S} \quad \text{or} \quad S = \frac{P - PF_p}{F_p}$$

↑ Parallel Time
 ↑ Serial Time

Without loss of generality, we can set $P=1$ so that, really, S is now a fraction of P . We now have:

$$S = \frac{1 - F_p}{F_p}$$



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We know that if we multiply the amount of data to process by N , then the amount of parallel work becomes NP . Surely the serial work must increase too, but we don't know how much. Let's say it doesn't increase at all, so that we know we are getting an upper bound answer.

In that case, the new parallel fraction is: $F'_p = \frac{P'}{P' + S} = \frac{NP}{NP + S}$

And substituting for $P (=1)$ and for S , we have:

$$F'_p = \frac{N}{N + S} = \frac{N}{N + \frac{1 - F_p}{F_p}}$$



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A More Optimistic Take on Amdahl's Law: Gustafson's Observation

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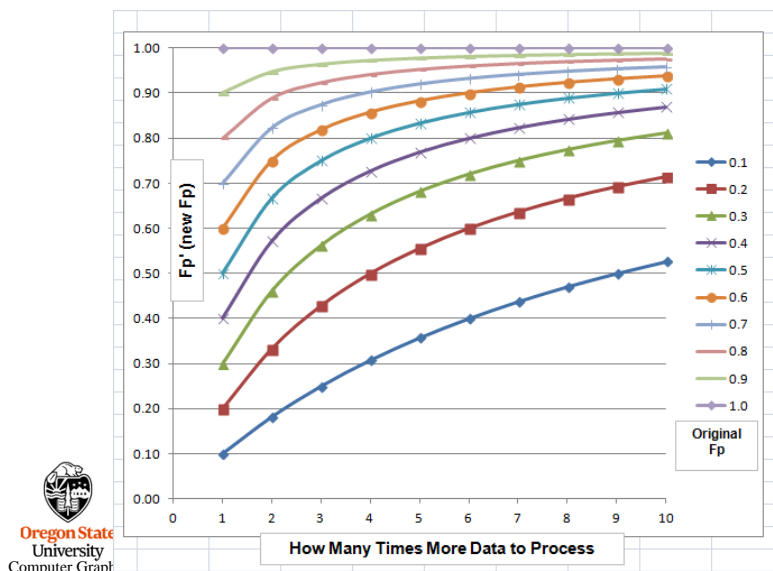
If we tabulate this, we get a table of F_p' values:

		How Many Times More Data to Process									
		1	2	3	4	5	6	7	8	9	10
Original F_p	0.1	0.10	0.18	0.25	0.31	0.36	0.40	0.44	0.47	0.50	0.53
	0.2	0.20	0.33	0.43	0.50	0.56	0.60	0.64	0.67	0.69	0.71
	0.3	0.30	0.46	0.56	0.63	0.68	0.72	0.75	0.77	0.79	0.81
	0.4	0.40	0.57	0.67	0.73	0.77	0.80	0.82	0.84	0.86	0.87
	0.5	0.50	0.67	0.75	0.80	0.83	0.86	0.88	0.89	0.90	0.91
	0.6	0.60	0.75	0.82	0.86	0.88	0.90	0.91	0.92	0.93	0.94
	0.7	0.70	0.82	0.88	0.90	0.92	0.93	0.94	0.95	0.95	0.96
	0.8	0.80	0.89	0.92	0.94	0.95	0.96	0.97	0.97	0.97	0.98
	0.9	0.90	0.95	0.96	0.97	0.98	0.98	0.98	0.99	0.99	0.99
	1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

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Or, graphing it:



A More Optimistic Take on Amdahl's Law: Gustafson's Observation

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We can also turn F_p' into a Maximum Speedup:

