The Modeling of Form Drag in a Porous Medium Saturated by a Power-Law Fluid

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The alternative ways of modeling form drag in a porous medium saturated by a power-law fluid in current usage are discussed. It is argued that the best alternative is to use the same expression as that used in the case of a Newtonian fluid, but with a modified Forchheimer coefficient. [DOI: 10.1115/1.3180809]

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1 Introduction and Discussion

In the experience of the author as a reviewer, there is currently a considerable amount of uncertainty about how the Forchheimer form drag term should be modeled in the case where a porous medium is saturated by a power-law fluid. Whereas many authors [1–6] used a term in u^2, where u denotes the velocity, some authors [7,8] used a term in u^{2n}, where n is the index that appears in the Darcy drag term as u^n. This note is an attempt to reduce the amount of that uncertainty.

A popular form of the momentum equation, as expressed by Chen and Hadim [9], who cited Shenoy and co-worker [3,4], is

$$\frac{\rho}{\varepsilon}(\mathbf{v}\cdot\nabla)\left(\frac{\mathbf{v}}{\varepsilon}\right) = -\nabla p - \frac{\mu^*}{K^*}|\mathbf{v}|^{n-1}\mathbf{v} - \frac{\rho C_F}{\sqrt{K}}|\mathbf{v}|\mathbf{v} + \frac{\mu^*}{\varepsilon}\nabla\left[\frac{1}{\varepsilon^{n-1}}\left(\frac{\Delta:\Delta}{2}\right)^{(n-1)/2}\Delta\right] \tag{1}$$

where ρ is the fluid density, ε is the porosity, $\mathbf{v}$ is the Darcy velocity, p is the pressure, μ^* is the consistency index, K^* is the modified permeability, K is the permeability, C_F is a Forchheimer drag coefficient, and Δ is the rate of deformation tensor. Here, we are concerned about the third term on the right hand side of Eq. (1). The authors who use such an expression typically then adopt the Ergun formula

$$C_F = \frac{1.75}{\sqrt{150}\varepsilon^{3/2}} \tag{2}$$

This formula was originally derived for a packed bed of spheres. In their numerical simulation, Inoue and Nakayama [6] obtained a similar result but with the coefficient 1.75 replaced with a value of about 0.5. These authors commented that Ergun's functional form should hold, whether or not the fluid is Newtonian. Shenoy [5], who was cited in Ref. [10], made a similar remark. He wrote that the Forchheimer term should be left unchanged because it is independent of the viscosity.

In principle one should be able to decide between the alternatives by performing some averaging over a representative elementary volume, but in practice, there is difficulty. The surface integrals that arise in this procedure need to be found, or estimated somehow, in order to achieve closure. In their investigation, Hayes et al. [11] achieved the closure by assuming that the Forchheimer expression took the same form for a power-law fluid as for a Newtonian fluid. Thus, they did not provide any independent information on which one can decide between alternative formulations.

The argument used by Shenoy and his colleagues is very plausible. If the nonlinear drag is indeed the form drag, as Joseph et al. [12,13] argued, and if the expression for the form drag is indeed independent of the viscosity, then it immediately follows on dimensional grounds that the expression must be quadratic in the velocity. However, the situation is not as clear cut as it seems at first sight.

It was pointed out by Nield [14] that, because the viscosity acts throughout the fluid and not just at the solid boundaries, viscous dissipation is a process that occurs throughout the space occupied by fluid. As a result, the total drag is influenced by what happens throughout the fluid. Although the Forchheimer form drag term does not explicitly involve the viscosity as a factor in the expression, it does arise from the action of the viscosity, mediated by the inertial effects affecting the distribution of pressure that also contributes to the stress at the solid boundaries. On this argument one should thus expect that the non-Newtonian aspects of the fluid (dilatancy or pseudoplasticity) would affect the form drag. The question now becomes: What is the best way of modeling the non-Newtonian effects?

The author suggests that the following stepwise approach is useful. One can retain the form of the expression for the form drag but allow the Forchheimer coefficient C_F to become a function of the power-law index n. A further option is to go another step and suppose that C_F is a weak function of the velocity u. If one takes that option, then a reasonable additional hypothesis is that C_F is of the form

$$C_F = C_{F0}(1 + \delta u^{2(n-1)}) \tag{3}$$

where δ is a constant small in comparison with unity, so that then

$$C_F u^2 = C_{F0}u^2 + \delta C_{F0}u^{2n} \tag{4}$$

Thus the net effect is that the expression for the form drag is now the sum of two terms: the first as used in Refs. [1–6] and the second as used in Refs. [7,8]. The results from Ref. [6] (indicating a reduction in the value of the coefficient in C_F from 1.75 to about 0.5) suggest that the sign of δ will be negative.

It is clear that further progress on this problem is dependent on more experimental work being carried out. Until that is done, the author recommends that the simple quadratic expression for the form drag be used as the default option in theoretical work on convection in porous media, with the understanding that the coefficient is not necessarily given by the Ergun formula.

Nomenclature

C_F = Forchheimer coefficient
K = permeability
K^* = modified permeability
p = pressure
u = magnitude of the Darcy velocity
$\mathbf{v}$ = Darcy velocity

Greek Symbols

Δ = rate of deformation tensor
δ = constant defined in Eq. (3)
ε = porosity
μ = fluid viscosity
μ^* = consistency index
ρ = fluid density

References

[1] Nakayama, A., and Shenoy, A. V., 1992, "A Unified Similarity Transformation for Darcy and Non-Darcy Forced-, Free- and Mixed-Convection Heat Transfer

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in Non-Newtonian Inelastic Fluid-Saturated Porous Media," Chem. Eng. J., **50**, pp. 33–45.

[2] Nakayama, A., and Shenoy, A. V., 1993, "Combined Forced and Free-Convection Heat-Transfer in Power-Law Fluid-Saturated Porous Media," Appl. Sci. Res., **50**, pp. 83–85.

[3] Nakayama, A., and Shenoy, A. V., 1993, "Non-Darcy Forced Convective Heat Transfer in a Channel Embedded in a Non-Newtonian Inelastic Fluid-Saturated Porous Medium," Can. J. Chem. Eng., **71**, pp. 168–173.

[4] Shenoy, A. V., 1993, "Darcy–Forchheimer Natural, Forced and Mixed Convection Heat Transfer in Non-Newtonian Power-Law Fluid-Saturated Porous Media," Transp. Porous Media, **11**, pp. 219–241.

[5] Shenoy, A. V., 1994, "Non-Newtonian Fluid Heat Transfer in Porous Media," Adv. Heat Transfer, **24**, pp. 101–190.

[6] Inoue, M., and Nakayama, A., 1998, "Numerical Modeling of Non-Newtonian Fluid Flow in a Porous Medium Using a Three-Dimensional Periodic Array," ASME J. Fluids Eng., **120**, pp. 131–135.

[7] Kumari, M., and Jayanthi, S., 2004, "Non-Darcy Non-Newtonian Free Convection Flow Over a Horizontal Cylinder in a Saturated Porous Medium," Int. Commun. Heat Mass Transfer, **31**, pp. 1219–1226.

[8] Kumari, M., and Nath, G., 2004, "Non-Darcy Mixed Convection in Power-Law Fluids Along a Non-Isothermal Horizontal Surface in a Porous Medium," Int. J. Eng. Sci., **42**, pp. 353–369.

[9] Chen, G., and Hadim, H. A., 1998, "Numerical Study of Non-Darcy Forced Convection in a Packed Bed Saturated With a Power-Law Fluid," J. Porous Media, **1**, pp. 147–157.

[10] Nield, D. A., and Bejan, A., 2006, *Convection in Porous Media*, 3rd ed., Springer, New York.

[11] Hayes, R. E., Afacan, A., Boulanger, B., and Shenoy, A. V., 1996, "Modelling the Flow of Power Law Fluids in a Packed Bed Using a Volume-Averaged Equation of Motion," Transp. Porous Media, **23**, pp. 175–196.

[12] Joseph, D. D., Nield, D. A., and Papanicolaou, G., 1982, "Nonlinear Equation Governing Flow in a Saturated Porous Medium," Water Resour. Res., **18**, pp. 1049–1052.

[13] Joseph, D., Nield, D., and Papanicolaou, G., 1982, "Correction to ′Nonlinear Equation Governing Flow in a Saturated Porous Medium' by Joseph et al.," Water Resour. Res., **19**, p. 591.

[14] Nield, D. A., 2000, "Resolution of a Paradox Involving Viscous Dissipation and Nonlinear Drag in a Porous Medium," Transp. Porous Media, **41**, pp. 349–357.