Predicting Temperature Profiles in a Flowing Well

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Summary. A simple model suitable for hand calculations is presented to predict temperature profiles in two-phase flowing wells. The model, developed with measured temperature data from 392 wells, assumes that the heat transfer within the wellbore is steady-state. Comparisons between the model's predictions and field data indicate that the model is highly accurate within its range of application.

Introduction

Predicting accurate temperature profiles in flowing wells can greatly improve the design of production facilities in petroleum engineering. Temperature profiles help calculate accurate two-phase-flow pressure-drop predictions, which in turn can improve an artificial-lift system design. Gas-lift design can be enhanced by more accurate prediction of temperature at valve depth. In this way, the valve's dome pressure can be set more accurately, thereby improving the predictability of valve throughput.

Existing temperature correlations are often inaccurate because they do not consider the effects of different fluids in the annulus and the cooling and heating of the fluid resulting from phase change. Rigorous theoretical models are often complex and inconvenient. They depend on many variables and require information about fluid composition. This paper describes two methods for predicting the temperature profile in a flowing well. The first is a model derived from the steady-state energy equation that considers the heat-transfer mechanisms found in a wellbore. The second is a simplified version of the model intended for hand calculations. An extensive data bank of temperature profiles from 392 wells was used in its development.

Literature Review

One of the earliest works on predicting temperature profiles in a flowing well was presented by Kirkpatrick.[1] He presented a simple flowing-temperature-gradient chart that can be used to predict gas-lift valve temperatures at the injection depth.

Much of the classic work in this area was developed by Ramey,[2] who presented approximate methods for predicting the temperature of either a single-phase incompressible liquid or a single-phase ideal gas flowing in injection and production wells. Satter[3] later improved Ramey's method by considering phase changes that occur within steam-injection projects. Shiu and Beggs[4] simplified Ramey's method by correlating for a specific coefficient in Ramey's equation.

Willhite[4] gave a detailed discussion of the overall heat-transfer mechanism in an injection well, and Coulter and Bardon[6] developed a method for predicting temperatures in gas transmission lines.

Complex theoretical models, such as those by Zelić[7] and modified Ramey's methods,[8] can be used to predict temperature profiles in flowing wells. All these methods require additional information about the fluid mixture composition. In addition, these methods are computationally complex and require the use of a computer. Such models are ideal for predicting temperature profiles associated with more difficult problems—e.g., a well flowing retrograde condensates.

The model developed in this paper is based on the Coulter-Bardon equation and incorporates Ramey's and Willhite's heat-transfer mechanisms in a wellbore.

Model Development

The model is derived from the total-energy-balance equation. Assuming steady-state conditions and no work done by or to the flowing fluid, the total-energy equation reduces to

$$\frac{dh}{dL}=\frac{dQ}{dL}-\frac{v}{Jg_c}\frac{dv}{dL}-\frac{g\sin\theta}{g_cJ}. \quad (1)$$

The sign convention used in this equation is displayed for the wellbore system shown in **Fig. 1**.

Ramey[2] and Willhite[5] discussed the radial transfer of heat between the fluid and the earth in detail. Over the differential element dL shown in **Fig. 2**, the radial heat transfer from the fluid to the cement/earth interface can be described by

$$\frac{dQ}{dL}=-\frac{2\pi r_{ti}U}{w_t}(T_f-T_h), \quad (2)$$

where U=overall heat-transfer coefficient between the fluid and the cement/earth interface. The radial heat transfer from the cement/earth interface to the surrounding earth is

$$\frac{dQ}{dL}=-\frac{2\pi k_e}{w_t f(t)}(T_h-T_e). \quad (3)$$

Combining Eqs. 2 and 3 gives the equation for the radial heat transfer between the flowing fluid and the surrounding earth:

$$\frac{dQ}{dL}=-\frac{2\pi}{w_t}\left[\frac{r_{ti}Uk_e}{k_e+f(t)r_{ti}U}\right](T_f-T_e). \quad (4)$$

Using basic thermodynamic principles, we can expand the specific enthalpy term of Eq. 1 to

$$\frac{dh}{dL}=\left(\frac{\partial h}{\partial p}\right)_{T_f}\frac{dp}{dL}+\left(\frac{\partial h}{\partial T_f}\right)_p\frac{dT_f}{dL}, \quad (5)$$

where $(\partial h/\partial T_f)_p=C_{pm}$ (6)

and $(\partial h/\partial p)_{T_f}=-\mu C_{pm}$. (7)

Therefore, the overall enthalpy change in a flowing fluid is

$$\frac{dh}{dL}=-\mu C_{pm}\frac{dp}{dL}+C_{pm}\frac{dT_f}{dL}. \quad (8)$$

Finally, combining Eqs. 1, 4, and 8 yields the steady-state ordinary differential equation for the temperature of the flowing fluid:

$$\frac{dT_f}{dL}=-A\left[(T_f-T_e)+\frac{g}{g_c}\frac{\sin\theta}{JC_{pm}A}-\frac{\mu}{A}\frac{dp}{dL}+\frac{v dv}{g_cJC_{pm}A}\right], \quad (9)$$

$$\text{where } A=\frac{2\pi}{C_{pm}w_t}\left[\frac{r_{ti}Uk_e}{k_e+f(t)r_{ti}U}\right]. \quad (10)$$

To use Eq. 9, it is necessary to estimate U, k_e, and $f(t)$. Evaluation of the overall heat-transfer coefficient is a difficult and critical step in finding an accurate solution. Willhite[5] and Bird *et al.*[9] provided a detailed method for calculating the overall heat-transfer coefficient in terms of natural convection, conduction, and radiation. The following assumptions are made to simplify the calculation of U.

1. Because steel has a high thermal conductivity, the thermal resistance of the pipe and casing are negligible compared with the thermal resistance of the material in the tubing/casing annulus.

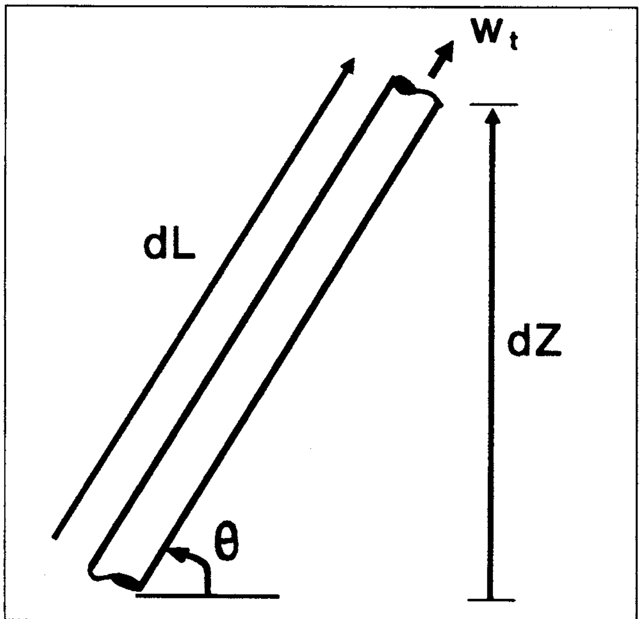

Fig. 1—Sign convention for the flowing well.

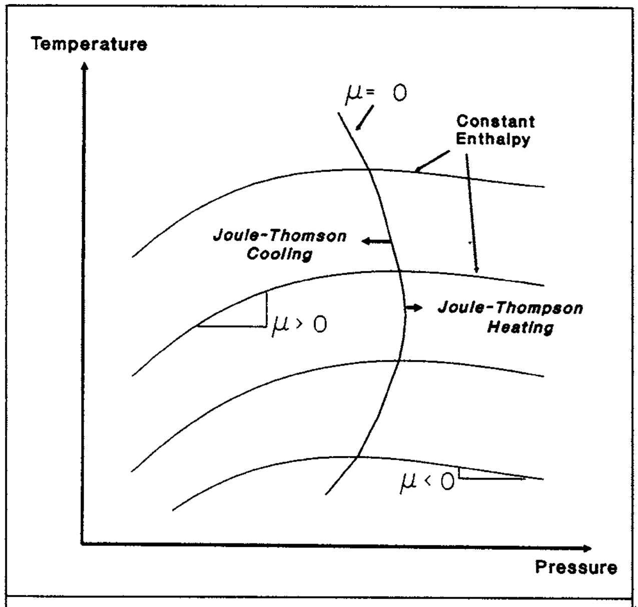

Fig. 3—Enthalpy curves for a typical hydrocarbon mixture illustrating the Joule-Thomson effect.

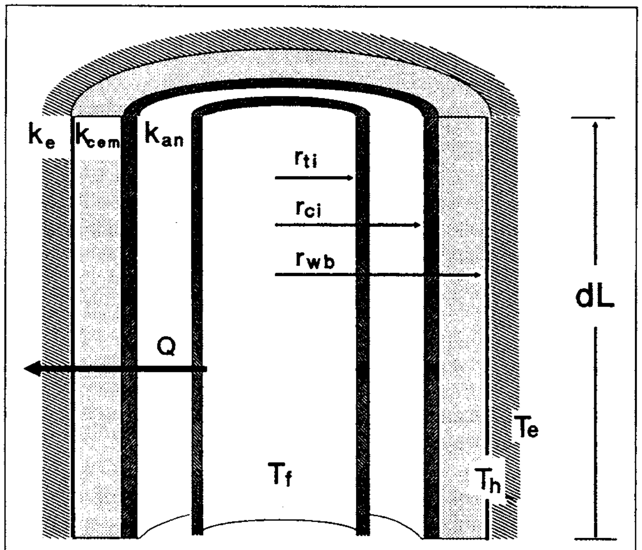

Fig. 2—Radial heat-transfer system.

2. The radiation and convection coefficients are negligible and can be ignored.

Under these assumptions, the equation developed by Willhite reduces to

$$U=\left[r_{ti}\frac{\ln(r_{ci}/r_{to})}{k_{an}}+r_{ti}\frac{\ln(r_{wb}/r_{co})}{k_{cem}}\right]^{-1} \quad \text{(11)}$$

Eq. 11 shows that the material present in the tubing/casing annulus (gas, oil, water, or any combination) is important in determining the heat transfer in a wellbore system.

Ramey[2] suggests values for both the transient-time function and the thermal conductivity of the earth. The transient-time function can be approximated by the long-time solution to the radial heat-conduction problem for an infinitely long cylinder. Under these assumptions, the equation for $f(t)$ becomes:

$$f(t)=\ln[2\sqrt{\alpha t}/(r_{wb}/12)]-0.290. \quad \text{(12)}$$

Ramey also suggests that, for most geographical areas, the value of the thermal conductivity of the earth is approximately 1.4 Btu/hr-ft-°F.

The Joule-Thomson coefficient, μ, in Eq. 9 determines the amount of heating or cooling caused by pressure changes within a fluid flowing up the well. Using basic thermodynamic principles, we can determine the Joule-Thomson coefficient from an equation of state for the particular fluid mixture:

$$\mu=\left(\frac{\partial T}{\partial p}\right)_h=\frac{1}{C_{pm}}\left[T\left(\frac{\partial V}{\partial T}\right)_p-V\right]. \quad \text{(13)}$$

To illustrate the qualitative behavior of the Joule-Thomson effect for a typical hydrocarbon mixture, a graph of a family of constant-enthalpy curves is plotted vs. temperature and pressure. As shown in **Fig. 3**, temperatures and pressures where the slope of the constant-enthalpy curve is positive correspond to Joule-Thomson cooling. Conversely, when the slope is negative, Joule-Thomson heating occurs. For the typical range of temperatures, pressures, and two-phase fluid mixtures encountered in a flowing well, Joule-Thomson cooling is much more significant than Joule-Thomson heating. Joule-Thomson heating would typically occur in some gas-condensate systems and could be determined from the mixture compositon. Joule-Thomson cooling is usually associated with a higher gas component in the two-phase mixture. The general conclusion that can be drawn from these trends is that wells having relatively high liquid holdups, caused by either lower gas/liquid ratios or higher wellhead pressures, will experience little Joule-Thomson cooling or heating. However, for wells operating with low liquid holdups, Joule-Thomson cooling could be much more significant. If such operating conditions occur, a more accurate determination of the Joule-Thomson coefficient would be required relative to the specific hydrocarbon mixture being produced.

Another term that tends to be relatively small except for high gas/liquid ratios and low wellhead pressure is the kinetic-energy term. Like the Joule-Thomson term, the kinetic-energy term requires a more accurate determination when such operating conditions are present.

The ordinary differential equation (ODE) Eq. 9 represents the temperature profile model. It can be solved through standard numerical techniques and gives an accurate solution. This model would be inconvenient, however, because calculation of the Joule-Thomson coefficient would require specific information about the hydrocarbon composition, the pressure gradient, and the kinetic-energy term. These observations motivated the following simplification.

Simplified Model

Eq. 9 can be simplified under the assumptions that the Joule-Thomson and the kinetic-energy terms are usually smaller than the other terms. It is reasonable to combine these two terms into a single term called F_c, which will be correlated for. Thus, F_c is defined by

$$F_c=\mu\frac{dp}{dL}-\frac{v dv}{Jg_c C_{pm}}, \qquad (14)$$

and Eq. 9 reduces to

$$\frac{dT_f}{dL}=-A\left[(T_f-T_e)+\frac{g}{g_c}\frac{\sin\theta}{JC_{pm}A}-\frac{F_c}{A}\right]. \qquad (15)$$

To integrate the ODE exactly over a fixed-length interval, the following assumptions are made: (1) F_c is constant; (2) the relaxation distance, A, is constant; (3) the geothermal gradient, g_G, is constant; (4) the specific heat, C_{pm}, is constant; and (5) the inclination angle, θ, is constant.

The length interval under consideration could possibly be the total well depth or just a portion of the well where the above values are nearly constant. If the above conditions cannot be met over the full length of the well, then the well must be partitioned into more than one length interval.

Defining the boundary of the length interval to be at its inlet and the associated boundary conditions to be the inlet conditions (i.e., $L=L_{in}$, $T_f=T_{fin}$, and $T_e=T_{ein}$), we can integrate Eq. 15. Then solving for the fluid temperature, T_f, yields

$$T_f=T_e-\frac{g}{g_c}\frac{\sin\theta}{JC_{pm}A}+\frac{F_c}{A}+\frac{g_G\sin\theta}{A}$$
$$+e^{-A(L-L_{in})}\left(T_{fin}-T_{ein}+\frac{g}{g_c}\frac{\sin\theta}{JC_{pm}A}-\frac{F_c}{A}-\frac{g_G\sin\theta}{A}\right). \qquad (16)$$

Eq. 16 will be valid over any length interval where the five variables specified above are constant. To use Eq. 16, it is necessary to specify a value for F_c. As discussed earlier, the complexities involved in calculating F_c make it an ideal variable to correlate for.

Correlation Development

Eq. 16, along with the data base of 392 wells, is used as a basis for the development of the F_c correlation. The data base consists of two data sets—the original Shiu and Beggs[4] data and data provided by the Amerada Hess Corp. The 337 wells from the Shiu and Beggs data base include data from the Gulf of Mexico, Cook Inlet, and Lake Maracaibo. The Amerada Hess Corp. data consist of measured temperature profiles from 55 wells, 10 of which were gas-lift wells from the Louisiana gulf coast and the Williston basin. The natural flowing wells are from the Permian Basin in west Texas. A listing of the Amerada Hess Corp. data is available upon request; the Shiu and Beggs data are summarized in Ref. 10. **Table 1** summarizes the range of the variables that form the complete data base.

TABLE 1—VARIABLE RANGE OF DATA

Variable	Minimum	Maximum	Average
Oil rate, STB/D	6	5,835	1,720
Water rate, STB/D	0	1,991	236
Gas rate, Mscf/D	6	5,244	1,899
Gas/liquid ratio, scf/STB	10	6,100	1,174
Total mass flow rate, lbm/sec	0.4	24	8.4
Oil gravity, °API	15	42.4	30
Gas specific gravity	0.62	1.43	0.79
Wellhead pressure, psig	50	1,287	215
Geothermal gradient, °F/ft	0.006	0.021	0.01
Bottomhole temperature, °F	104	274	172
Depth, ft	3,205	15,780	8,250

To develop a correlation, it was assumed that all the wells in the data base were vertical wells and that all natural flowing wells have water in the annulus up to the surface. However, the gas-lift wells in the Amerada Hess Corp. data were treated as two length intervals: one below the point of gas injection and the other above it. It is assumed that these wells have water in the annulus below the point of injection and gas above.

The Amerada Hess Corp. data included some wellhead temperature measurements made within the surface equipment. Wellhead fluid temperatures are often unreliable because they can be influenced by errors in measurement procedure and by daily and seasonal temperature variations. In particular, steel is a very good conductor of heat, and variations in temperature of the surface equipment can greatly influence the wellhead temperature. Therefore, the wellhead temperature was ultimately excluded from the data base in the development of the final F_c correlation.

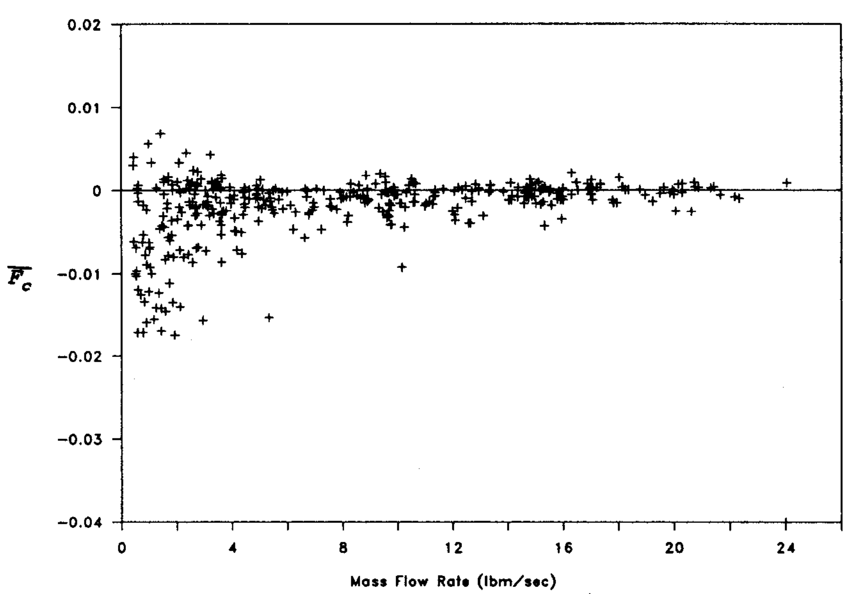

Fig. 4—$\overline{F_c}$ vs. mass flow rate.

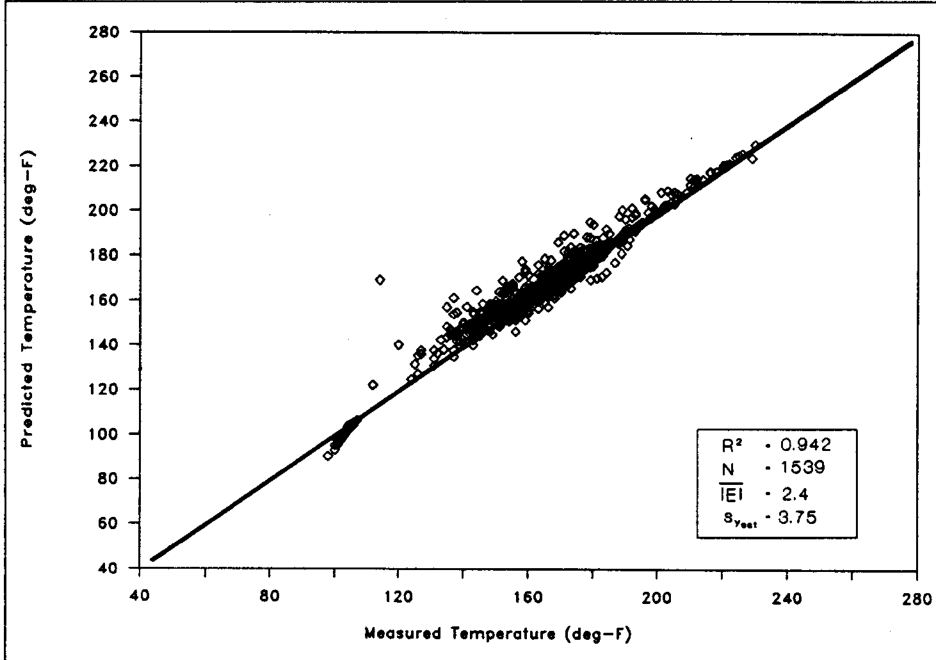

Fig. 5—Predicted vs. measured temperatures when $w_t \geq 5.0$ lbm/sec for the simplified model.

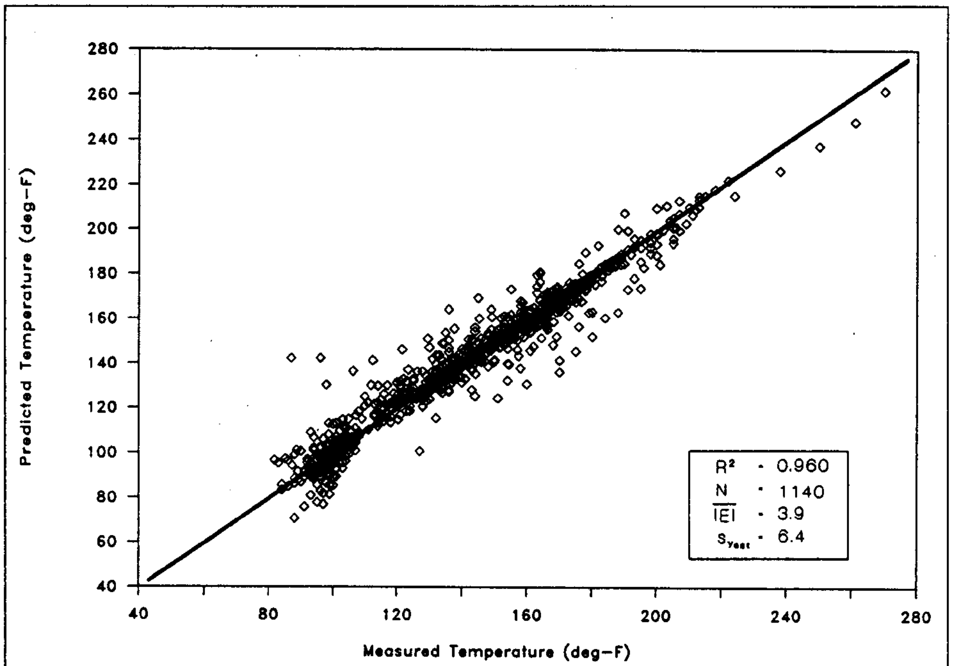

Fig. 6—Predicted vs. measured temperatures when $w_t < 5.0$ lbm/sec for the simplified model.

The correlation procedure was as follows.

1. Each well was partitioned into length intervals. In natural flowing wells, the well is one length interval. However, gas-lifted wells are partitioned into two intervals.
2. Eq. 16 was used to back calculate a value for F_c for each flowing temperature measured, excluding the surface temperature.
3. An average value of F_c, $\overline{F_c}$, is computed for each length interval.
4. A correlation was developed for the $\overline{F_c}$ values.

The purpose of correlating was to develop an equation for $\overline{F_c}$ as a function of known physical properties (independent variables) specific to a length interval. This procedure will tend to give individual wells equal weighting in the correlation rather than giving equal weighting to individual flowing temperature measurements. In addition, a single value of $\overline{F_c}$ must be used over an entire length interval.

A processed data base of $\overline{F_c}$ values vs. all possible independent physical properties was made. With the SPSS software package, multiple regression analysis was performed on the processed data set of $\overline{F_c}$ values. A number of combinations of linear and nonlinear functions were tried with various degrees of success. The following correlation for the $\overline{F_c}$ values yielded the best and most convenient overall results: $\overline{F_c}=0.0$, when $w_t \geq 5$ lbm/sec, and

$$\overline{F_c}=-2.978\times10^{-3}+1.006\times10^{-6}p_{wh}+1.906\times10^{-4}w_t -1.047\times10^{-6}R_{gL}+3.229\times10^{-5}\gamma_{API} +4.009\times10^{-3}\gamma_g-0.3551g_G, \quad (17)$$

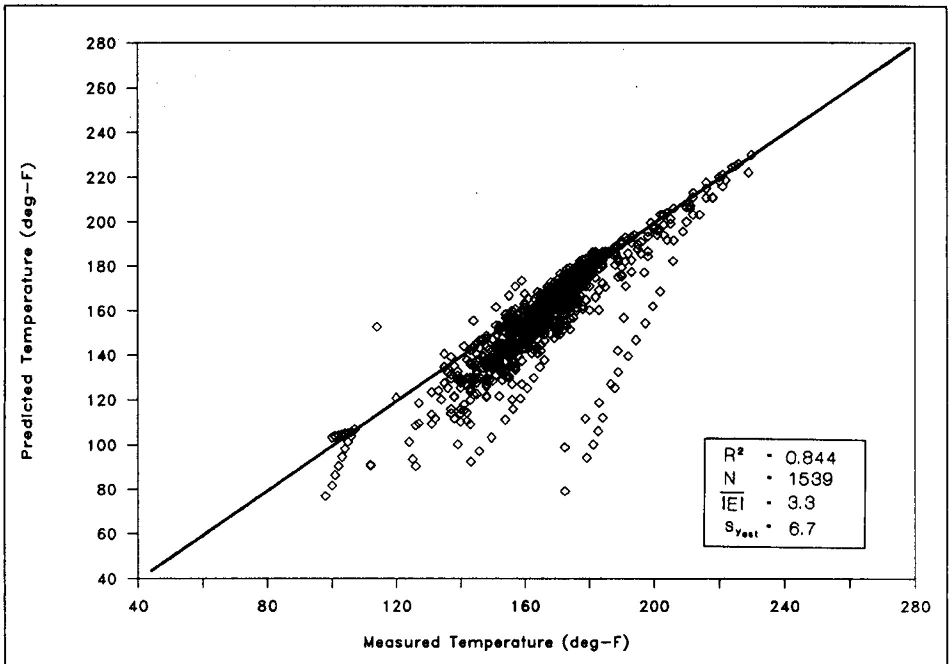

Fig. 7—Predicted vs. measured temperatures when $w_t \geq 5.0$ lbm/sec for Shiu and Beggs' method.

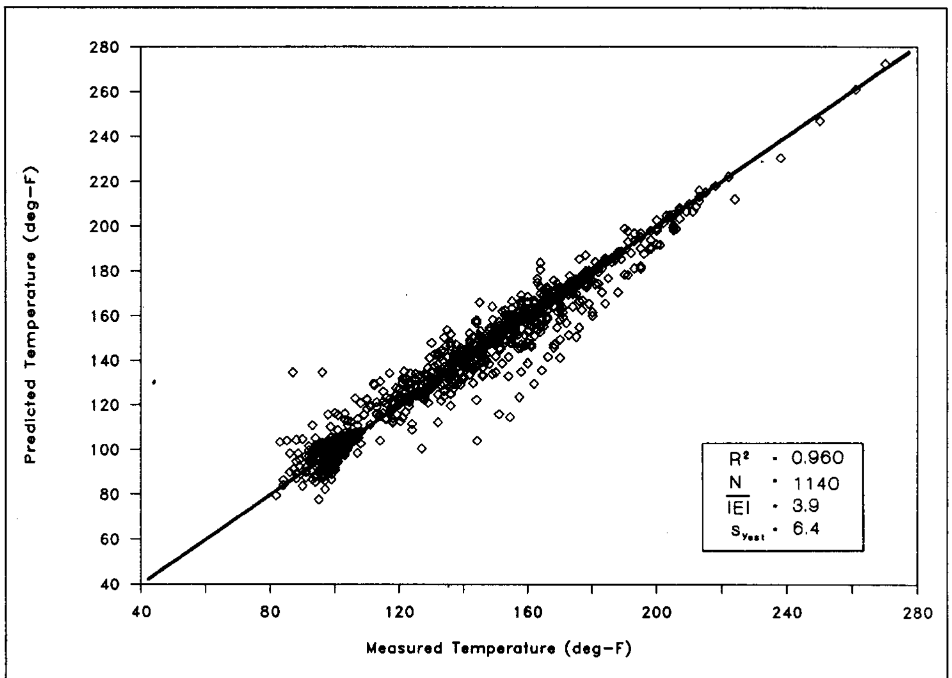

Fig. 8—Predicted vs. measured temperatures when $w_t < 5.0$ lbm/sec for Shiu and Beggs' method.

when $w_t < 5.0$ lbm/sec, where data from 12 wells with abnormal $\overline{F_c}$ values were deleted from the data set in the development of the correlation. The attempt to find the best correlation of $\overline{F_c}$ indicated the following points.

1. When the surface temperatures are excluded from the correlation for $\overline{F_c}$, the ability of the simplified model to predict the remaining flowing temperatures improved significantly.

2. As illustrated in **Fig. 4**, for mass flow rates greater than 5 lbm/sec, the value of $\overline{F_c}$ is very close to zero and the scatter in this region is very small. This is supported by the observation that both Joule-Thomson and kinetic-energy effects would tend to be less when less gas is present. The use of zero for $\overline{F_c}$ in this region is desirable because it would simplify hand calculations for the simplified model.

3. At mass flow rates less than 5 lbm/sec, which is indicative of higher gas rates, more scatter in the data was observed (Fig. 4). This indicates that, at lower mass flow rates, effects such as Joule-Thomson cooling, kinetic energy, and/or other phenomena or properties not considered can greatly influence the temperature profile in a well.

4. Radial well dimensions and oil fraction did not contribute significantly to the prediction of $\overline{F_c}$.

With a correlation for $\overline{F_c}$ in place, Eqs. 10, 16, and 17 form the components of a simplified model over a length interval.

TABLE 2—COMPARISON OF THE SIMPLIFIED MODEL AND SHIU AND BEGGS' CORRELATION

Method	Data Set (Measured vs. Predicted Temperature)	N	R^2	$\lvert\bar{E}\rvert$	s_{yest}
Simplified model	380 wells	2,679	0.968	3.1	5.3
Shiu and Beggs'	Excludes surface temperature		0.943	4.0	6.8
Simplified model	Excludes surface temperature	1,539	0.942	2.4	3.8
Shiu and Beggs'	w_t>5 lbm/sec		0.844	3.3	6.7
Simplified model	Excludes surface temperature	1,140	0.960	3.9	6.4
Shiu and Beggs'	w_t<5 lbm/sec		0.955	4.8	6.8
Simplified model	Shiu and Beggs' original data base	2,541	0.930	4.0	6.0
Shiu and Beggs'	Includes surface temperature		0.935	4.3	5.8

Simplified-Model Evaluation

The overall performance of the simplified model was evaluated by measuring its ability to predict measured flowing temperatures in a well accurately. The final data base has 2,679 temperature measurements from 380 wells, excluding wellhead temperature.

The statistical parameters used in the performance study were the coefficient of correlation, R^2; the average absolute error, $|\bar{E}|$; and the standard error of the predicted temperatures, S_{yest}.

We wish to compare the accuracy of the simplified model with that of Shiu and Beggs' correlation. Note that the simplified model is based primarily on fundamental principles, while the Shiu and Beggs procedure can be viewed as a correlation. Note also that Shiu and Beggs included wellhead temperatures. These temperatures were excluded from our data base for reasons stated earlier.

Figs. 5 and 6 show measured temperatures vs. temperatures predicted by the simplified model. **Figs. 7 and 8** show measured temperatures vs. temperatures predicted by the Shiu and Beggs correlation. **Table 2** summarizes the comparison between the simplified model and the Shiu and Beggs correlation. The overall conclusion from this comparison is that the simplified model represents an improvement over the Shiu and Beggs correlation and is accurate within its range of application.

Conclusions

1. A general model was developed from basic thermodynamic principles to predict fluid temperature profiles in two-phase flowing wells. This model requires the use of a digital computer, calculation of the Joule-Thomson coefficient, information about the hydrocarbon composition, and calculation of the pressure gradient and the kinetic-energy terms.
2. A simplified model suitable for hand calculations was proposed on the basis of the general model in which the Joule-Thomson and kinetic-energy terms were replaced with a correlation. The correlation was developed from a data base consisting of 392 two-phase flowing wells.
3. Comparison of the simplified model and measured temperature data indicates that the average absolute error is 2.4°F when the mass flow rate is greater than 5 lbm/sec, and 3.9°F otherwise.
4. The simplified model represents an improvement over the Shiu and Beggs correlation.

Nomenclature

A = coefficient, ft^{-1}
C_{pL} = specific heat of liquid, Btu/lbm-°F
C_{pm} = specific heat of mixture, Btu/lbm-°F
C_{po} = specific heat of oil, Btu/lbm-°F
C_{pw} = specific heat of water, Btu/lbm-°F
d_c = casing diameter, in.
d_t = tubing diameter, in.
d_{wb} = wellbore diameter, in.
D = depth, ft
D_{inj} = injection depth, ft
f = modified dimensionless heat conduction time function for long times for earth
$f(t)$ = dimensionless transient heat conduction time function for earth
F_c = correction factor
$\bar{F}_c$ = average correction factor for one length interval
g = acceleration of gravity, 32.2 ft/sec^2
g_c = conversion factor, 32.2 ft-lbm/sec^2-lbf
g_G = geothermal gradient, °F/ft
h = specific enthalpy, Btu/lbm
J = mechanical equivalent of heat, 778 ft-lbf/Btu
k_{an} = thermal conductivity of material in annulus, Btu/D-ft-°F
k_{ang} = thermal conductivity of gas in annulus, Btu/D-ft-°F
k_{anw} = thermal conductivity of water in annulus, Btu/D-ft-°F
k_{cem} = thermal conductivity of cement, Btu/D-ft-°F
k_e = thermal conductivity of earth, Btu/D-ft-°F
L = length of well from perforations, ft
L_{in} = length from perforation to inlet, ft
p = pressure, psi
p_{wh} = wellhead pressure, psig
q_{gf} = formation gas flow rate, scf/D
q_{ginj} = injection gas flow rate, scf/D
q_o = oil flow rate, STB/D
q_w = water flow rate, STB/D
Q = heat transfer between fluid and surrounding area, Btu/lbm
r_{ci} = inside casing radius, in.
r_{co} = outside casing radius, in.
r_{ti} = inside tubing radius, in.
r_{to} = outside tubing radius, in.
r_{wb} = wellbore radius, in.
R_{gL} = gas/liquid ratio, scf/STB
T = temperature, °F
T_{bh} = bottomhole temperature, °F
T_c = casing temperature, °F
T_e = surrounding earth temperature, °F
T_{ein} = earth temperature at inlet, °F
T_f = flowing fluid temperature, °F
T_{fin} = flowing fluid temperature at inlet, °F
T_h = cement/earth interface temperature, °F
U = overall heat transfer coefficient, Btu/D-ft^2-°F
v = fluid velocity, ft/sec
V = volume
w_t = total mass flow rate, lbm/sec
Z = height from bottom of hole, ft
Z_{in} = height from bottom of hole at inlet, ft
α = thermal diffusivity of earth, 0.04 ft^2/hr
γ_{API} = oil gravity, °API
γ_g = gas specific gravity (air=1)
γ_o = oil specific gravity
γ_w = water specific gravity
θ = angle of inclination, degrees
μ = Joule-Thomson coefficient

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Appendix—Use of the Simplified Model

The following steps make up the procedure for using the simplified model. First, partition the well into one or more length intervals where all properties of the well will be nearly constant. Then on each vertical length interval, use Eq. A-1 to calculate the flowing temperature at length L:

$$T_f = T_e - \frac{g \sin\theta}{g_c J C_{pL} A} + \frac{\overline{F_c}}{A} + \frac{g_G \sin\theta}{A} + e^{-A(L-L_{in})}\left(T_{fin} - T_{ein} + \frac{g\sin\theta}{g_c J C_{pL} A} - \frac{\overline{F_c}}{A} - \frac{g_G \sin\theta}{A}\right), \quad \text{(A-1)}$$

where *in* denotes the value of the particular quantity at the inlet of the length interval. Natural flowing wells are usually one length interval unless the annulus liquid level is known to be significantly below the surface. Gas-lifted wells are divided into two intervals, one below the point of injection and the other above the point of injection. The mass flow rate in the tubing and the thermal conductivity of the annulus material will change accordingly.

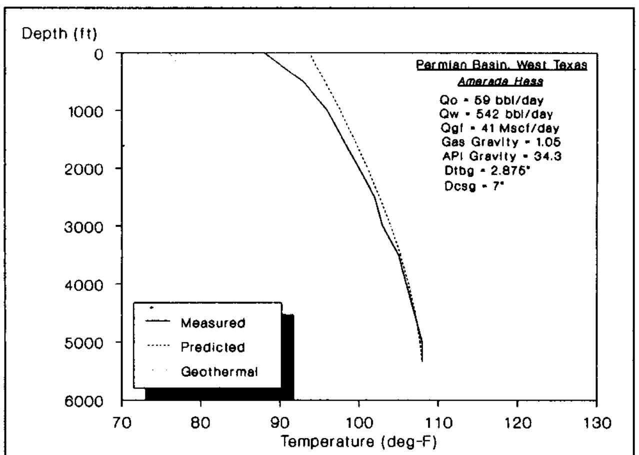


Fig. A-1—Temperature profile predicted by the simplified model for a flowing well.

The specific heat of the tubing fluid is determined as a function of the oil fraction:

$$C_{pL} = \left(\frac{q_o}{q_o+q_w}\right) C_{po} + \left(1 - \frac{q_o}{q_o+q_w}\right) C_{pw}, \quad \text{(A-2)}$$

where typical values are $C_{po}=0.485$ Btu/lbm-°F and $C_{pw}=1.0$ Btu/lbm-°F.

The dimensionless time function for long times (Eq. 13) can be further approximated by

$$f = -0.272(r_{wb}) + 3.53, \quad \text{(A-3)}$$

for wellbores ranging from 6.5 to 10 in. and for times exceeding 1 week.

The mass fluid flow rate is

$$w_t = \frac{q_g \gamma_g}{1.1309\times 10^6} + \frac{(q_w\gamma_w + q_o\gamma_o)}{246.6}. \quad \text{(A-4)}$$

The overall heat-transfer coefficient for fluid flow through a tubing is

$$U = \frac{12}{r_{ti}}\left[\frac{\ln(r_{ci}/r_{to})}{k_{an}} + \frac{\ln(r_{wb}/r_{co})}{k_{cem}}\right]^{-1}. \quad \text{(A-5)}$$

The coefficient A is

$$A = \left(\frac{2\pi}{w_t C_{pl}}\right)\left(\frac{r_{ti} U k_e}{k_e + r_{ti} Uf/12}\right)\left(\frac{1}{86{,}400\times 12}\right). \quad \text{(A-6)}$$

The necessary tubing, casing, and wellbore dimensions can be found from standard casing and tubing tables.[11]

Typical values of thermal conductivity in Btu/D-ft-°F are $k_e=33.6$, $k_{cem}=96.5$, $k_{anw}=9.192$, and $k_{ang}=0.504$. The coefficient of correction is: $\overline{F_c}=0.0$ when $w_t \geq 5.0$ lbm/sec and

$$\overline{F_c} = -2.978\times10^{-3} + 1.006\times10^{-6} p_{wh} + 1.906\times10^{-4} w_t - 1.047\times10^{-6} R_{gL} + 3.229\times10^{-5}\gamma_{API} + 4.009\times10^{-3}\gamma_g - 0.3551 g_G, \quad \text{(A-7)}$$

when $w_t < 5.0$ lbm/sec.

The temperature of the earth at height $Z(=L\sin\theta)$ is

$$T_e = T_{bh} - g_G L \sin\theta. \quad \text{(A-8)}$$

Z is calculated so that it increases from the bottom of the well where $Z=0$ ft (see Fig. 1).

Casing Flow. For casing flow, the overall heat-transfer coefficient simplifies to

$$U = \frac{12}{r_{ci}}\left[\frac{\ln(r_{wb}/r_{co})}{k_{cem}}\right]^{-1} \quad \text{(A-9)}$$

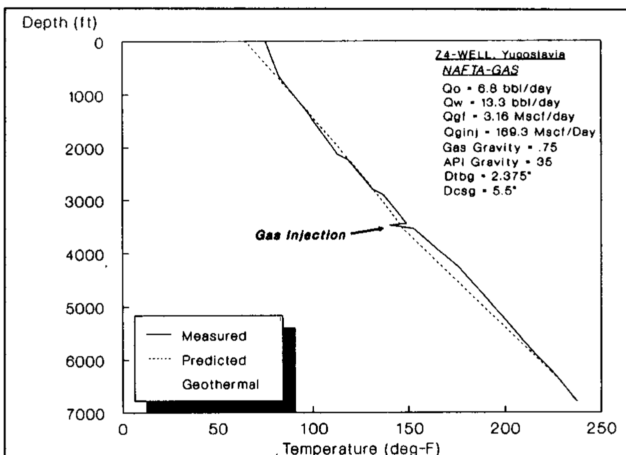
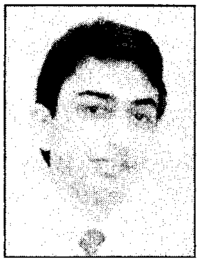
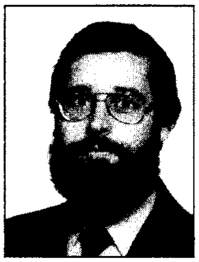
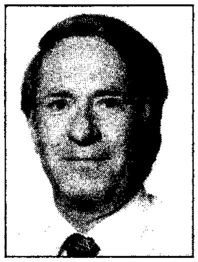


Fig. A-2—Temperature profile predicted by the simplified model for a gas-lift well.

Authors


Sagar


Doty


Schmidt

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The coefficient A is

$$A=\left(\frac{2\pi}{w_t C_{pl}}\right)\left(\frac{r_{ci}Uk_e}{k_e+r_{ci}Uf/12}\right)\left(\frac{1}{86{,}400\times 12}\right). \quad \text{(A-10)}$$

Annular Flow. For flow or gas lift through the tubing/casing annulus,

$$U=\frac{12}{(r_{ci}-r_{to})}\left[\frac{\ln(r_{wb}/r_{co})}{k_{cem}}\right]^{-1} \quad \text{(A-11)}$$

A is

$$A=\left(\frac{2\pi}{w_t C_{pl}}\right)\left[\frac{(r_{ci}-r_{to})Uk_e}{k_e+(r_{ci}-r_{to})Uf/12}\right]\left(\frac{1}{86{,}400\times 12}\right). \quad \text{(A-12)}$$

Sample Calculation. Fig. A-1 shows an example of a temperature profile calculated with the outlined procedure. The well is a natural flowing well from the Permian Basin, west Texas. Observe the excellent agreement between the predicted and measured temperature profiles.

The following example illustrates the procedure for calculating the temperature profile in a continuous gas-lift well. The example is selected from a well outside the data base used to develop the correlation. The data, provided by Nafta Gas, are from Yugoslavia.

q_o=6.8 STB/D.	q_w=13.3 STB/D.
q_{gf}=3.1 Mscf/D.	q_{ginj}=169.3 Mscf/D.
γ_g=0.75.	γ_w=1.0.
γ_{API}=35°API.	d_t=2.375 in.
d_c=5.5 in.	d_{wb}=7.5 in.
p_{wh}=174 psig.	g_G=0.0274°F/ft.
T_{bh}=237.2°F.	D=6,792 ft.
D_{inj}=3,544 ft.	θ=90°.

Because this well has gas injected at a depth of 3,544 ft, this well is divided into two length intervals, one above and one below the point of injection.

Interval 1 (Below the Point of Injection). The calculated variables are as listed below.

1. Specific heat (Eq. A-2), C_{pL}=0.826 Btu/lbm-°F.
2. Dimensionless time function (Eq. A-3), f=2.51.
3. Mass flow rate (Eq. A-4), w_t=0.08 lbm/sec.
4. Overall heat-transfer coefficient (Eq. A-5), U=143.2 Btu/lbm-°F-ft².
5. Coefficient A (Eq. A-6), A=7.9×10⁻³ ft⁻¹.
6. Correction term (Eq. A-7), $\overline{F_c}$=−0.0085.

The inlet conditions for Length Interval 1 are Z_{in}=0 ft and $T_{fin}=T_{ein}=T_{bh}$=237.2°F. For example, if the temperature of the flowing fluid at a depth of 5,000 ft is desired, then use Eq. A-8 to obtain Z=1,792 ft and T_e=188°F. From Eq. A-1, the fluid temperature at 5,000 ft is 190°F.

Interval 2 (Above the Point of Injection). The calculated variables are as follows.

1. Specific heat (Eq. A-2), C_{pL}=0.826 Btu/lbm-°F.
2. Dimensionless time function (Eq. A-3), f=2.51.
3. Mass flow rate (Eq. A-4), w_t=0.19 lbm/sec.
4. Overall heat-transfer coefficient (Eq. A-5), U=8.44 Btu/lbm-°F-ft².
5. Coefficient A (Eq. A-6), A=2.96×10⁻⁴ ft⁻¹.
6. Correction term (Eq. A-7), $\overline{F_c}$=−0.0177.

In this case, the inlet conditions for Length Interval 2 are the conditions at the point of gas injection that correspond to the exit conditions of Length Interval 1. The inlet conditions are computed with the equation developed for Length Interval 1 at a depth of 3,544 ft: Z_{in}=3,248 ft, T_{fin}=150°F, and T_{ein}=148°F.

To illustrate how to calculate a flowing temperature in Length Interval 2, if the temperature of the flowing fluid at 1,992 ft is desired, use Eq. A-8 to obtain Z=4,800 ft and T_e=106°F.

Eq. A-1 for Length Interval 2 predicts T_f=117°F at 1,992 ft.

Fig. A-2 shows the predicted and measured temperature profiles for this well. Observe that the measured surface temperature does not follow the trend. Fig. A-2 also shows the temperature kick observed in the measured temperature profile. The kick results from Joule-Thomson cooling of the gas entering the fluid through the gas-lift valve. Ideally, the temperature of the valve dome should be the kick temperature. However, the determination of the valve dome temperature resulting from the cooling caused by the gas entering the fluid is a topic for further study.

The procedure outlined can be used as is or modified to solve different problems where the temperature profile in the well would need to be known. Typical examples are monitoring the fluid temperature at the inlet of an electrical submersible pump to prevent overheating of the motor or determining temperature profiles in multizone completions.

Warnings. As with any correlation, this one often is misused and applied to cases outside the data base from which it was developed. Table 1 summarizes the variables that the simplified model can be used for with a high degree of confidence. Even though the range of the simplified model's application can be extrapolated, it must be done with caution. In particular, the simplified model's predictions for mass flow rates less than 5 lbm/sec deserve special attention. This procedure was derived from data of flowing oil wells. It is *not* to be used to calculate temperature profiles in retrograde condensate wells. To calculate the temperature profile in such wells, we recommended that the model given in Eq. 10 be used and the Joule-Thomson coefficient be calculated from the composition and the thermodynamic properties of the two-phase fluid components.

SI Metric Conversion Factors

°API	141.5/(131.5+°API)		=	g/cm³
bbl	× 1.589 873	E−01	=	m³
Btu/hr-ft-°F	× 5.678 263	E+00	=	W/m²·K
Btu/lbm-°F	× 4.184	E+03	=	J/kg·K
ft	× 3.048*	E−01	=	m
ft³	× 2.831 685	E−02	=	m²
°F	(°F−32)/1.8		=	°C
in.	× 2.54*	E+00	=	cm
lbm	× 4.535 924	E−01	=	kg
psi	× 6.894 757	E+00	=	kPa

*Conversion factor is exact.

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