Closure to "Discussion of 'The Modeling of Viscous Dissipation in a Saturated Porous Medium' " (2009, ASME J. Heat Transfer, 131, p. 025501)

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I thank Dr. Costa for his discussion on my paper [1], and I welcome the opportunity to clarify this matter.

Costa's Energy Conservation Formulation

Costa repeated the arguments that he presented in his earlier papers [2,3], and clarified his ambiguous statement in Ref. [3] that "… the main results and conclusions apply to any natural or mixed convection problem."

The author now argues that the general claim (as clarified) by Costa is incorrect. In particular, it is argued that (1) the application of the first law of thermodynamics in the way that he has applied it is not necessary, (2) he has applied it in an invalid manner, and (3) his claim is falsified by other results.

First, the first law of thermodynamics is already incorporated in the thermal energy equation. That equation expresses the fact that energy is conserved in each elementary volume of the domain, and hence it follows that it is conserved in an enclosure that is closed to mass flow. In the case of forced convection (for which a volume fixed in space is not closed with respect to mass flow), the thermal energy equation must be supplemented by a requirement of global conservation of energy, but for natural convection in an enclosure this is not required.

Second, in the case of a laterally heated box, it is not permissible to treat the heat flux at the hot wall and the heat flux at the cold wall as independent quantities once the hot wall temperature and the cold wall temperature have been specified. One cannot specify the boundary temperature and the boundary heat flux simultaneously. To do so would be to overspecify the boundary conditions in a boundary value problem. When one is setting up the physical situation with fixed wall temperatures, the wall heat fluxes adjust accordingly as energy is exchanged with the outside.

Third, the claim is falsified by known results in the case of forced convection, which is a limiting case of mixed convection. With forced convection it is well known (e.g., Ref. [4]) that it is possible to have a situation where there is substantial viscous dissipation but negligible pressure work. If Costa's claim were correct then one would have a substantial jump in the difference between global viscous dissipation and global pressure work as soon as one added some buoyancy, no matter how small the amount.

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It appears to the present author that Costa assumed that buoyancy is a phenomenon that occurs in compressible fluids only, one intrinsically involving expansion and contraction, whereas, in fact, buoyancy results whenever there are density variations, no matter how caused. This is an additional reason why his result for natural convection in a laterally heated box does not generalize to all flows driven by buoyancy.

In the light of the above arguments it appears that the criticisms that Costa made of statements in Ref. [1] are based on a fallacious assumption. Contrary to Costa's claim, it is the scale analysis of the local energy balance (the differential equation) that is pertinent in estimating the relevant magnitude of the viscous dissipation and of the work of pressure forces. It is not possible to perform a legitimate scale analysis on a global scale.

It is the author's opinion that Costa has been misguided by some Computational fluid dynamics (CFD) calculations for the laterally heated square box—calculations based on a limited parameter range, a special geometry, and special boundary conditions. There is a need for more extensive calculations.

Addendum on Velocity Scales

The author now takes the opportunity to extend the discussion of velocity scales made in Ref. [1], where it was argued that the velocity scale for weak natural convection should be of order $U_c \mathrm{Ra}_D$, whereas that for strong natural convection it should be of order $U_c \mathrm{Ra}_D{}^{1/2}$. Here Ra_D is the Rayleigh–Darcy number and U_c is the conduction velocity scale. It is now pointed out that this is consistent with the fact that in the Horton–Rogers–Lapwood problem the Nusselt number varies as Ra_D for small supercritical values and as $\mathrm{Ra}_D{}^{1/2}$ for large values, together with the fact that the release of kinetic energy due to the buoyancy force is proportional to the product of the vertical velocity component and the temperature excess. This provides additional support for the view taken in Ref. [1] on the way in which the velocity scale affects the balance between viscous dissipation and pressure work. In that paper it was argued that (1) the magnitude of viscous dissipation compared with conduction can be estimated, a priori and independent of the consideration of pressure work, once an estimate is made for the velocity scale, and (2) there is a fundamental difference between the special case of a laterally heated box (in which, because of cancellation of the contributions from subdomains, there is to first order no net release of kinetic energy due to buoyancy) and a bottom heated box (in which there is a nonzero net release of kinetic energy due to buoyancy). This is a further reason why Costa's conclusions cannot be extended from those for a laterally heated box to all natural or mixed convection problems.

Acknowledgment

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References

[1] Nield, D. A., 2007, "The Modeling of Viscous Dissipation in a Saturated Porous Medium," ASME J. Heat Transfer, **129**, pp. 1459–1463.
[2] Costa, V. A. F., 2005, "Thermodynamics of Natural Convection in Enclosures With Viscous Dissipation," Int. J. Heat Mass Transfer, **48**, pp. 2333–2341.
[3] Costa, V. A. F., 2006, "On Natural Convection in Enclosures Filled With Fluid-Saturated Porous Media Including Viscous Dissipation," Int. J. Heat Mass Transfer, **49**, pp. 2215–2226.
[4] Nield, D. A., Kuznetsov, A. V., and Xiong, M., 2004, "Effects of Viscous Dissipation and Flow Work on Forced Convection in a Channel Filled by a Saturated Porous Medium," Transp. Porous Media, **56**, pp. 351–367.

