Discussion: "The Modeling of Viscous Dissipation in a Saturated Porous Medium" (Nield, D. A., 2007, ASME J. Heat Transfer, 129, pp. 1459–1463)

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In a recent paper [1] modeling of viscous dissipation in fluid-saturated porous media is considered. This Comment concerns the energy conservation formulation of natural or mixed convection problems including viscous dissipation. For simplicity, sometimes only the clear fluid situation is treated, the main issues applying both to clear fluids and to fluid-saturated porous media.

Energy Conservation Formulation for Natural Convection in Enclosures

Energy conservation equation applied to any closed system gives [2]

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \tag{1}$$

Property energy is composed by the components [2]: internal energy, U, kinetic energy, $(1/2)mV^2$, and potential (gravitational) energy, mgy, that is, $E=U+(1/2)mV^2+mgy$. For a closed system operating in steady-state $dE/dt=0$ and $\dot{Q}-\dot{W}=0$. $\dot{W}$ is the mechanical power exchanged between the closed system and its surroundings, which is null in the present case. Only if a rotating shaft or electrically energized cables cross the walls of the system, or the walls are deformable or mobile it is $\dot{W}\neq 0$. In the present case it is forcedly $\dot{Q}=0$.

If the domain is a differentially heated enclosure of rigid walls, with a hot wall and a cold wall, heat crosses it from the hot wall to the cold wall, and the overall heat input $\dot{Q}_H$ equals the overall heat output $\dot{Q}_C$, that is,

$$\dot{Q} = \underbrace{\dot{Q}_H}_{>0} + \underbrace{\dot{Q}_C}_{<0} = 0 \tag{2}$$

This result holds for any prescribed temperature or heat flux at the walls of the enclosure and for any orientation of these walls. Another way to write Eq. (2) is $|\dot{Q}_H|=|\dot{Q}_C|$.

For unsteady situations it can be $|\dot{Q}_C|>|\dot{Q}_H|$ as a result of a decrease in the potential energy or a decrease in the internal energy. However, under steady-state conditions, only $|\dot{Q}_H|=|\dot{Q}_C|$ is compatible with the energy conservation principle.

If the viscous dissipation term is taken into account

$$|\dot{Q}_C| = |\dot{Q}_H| + \int_V (\text{volumetric viscous dissipation rate})dV \tag{3}$$

violating the energy conservation principle by the reasons detailed above. Thus, if the viscous dissipation is taken into account an *additional* term needs to be taken into account. The complete thermal energy conservation equation (for a clear fluid) can be obtained from Ref. [3]; this additional term is the work of pressure forces, and as $|\dot{Q}_H|=|\dot{Q}_C|$ it is

$$\int_V (\text{volumetric viscous dissipation rate})dV + \int_V (\text{volumetric rate of work of pressure forces})dV = 0 \tag{4}$$

Locally, the volumetric viscous dissipation rate can be different from the volumetric rate of work of pressure forces, and Eq. (4) applies to the overall enclosure. Viscous dissipation is always positive, and the work of pressure forces can be positive or negative depending if the fluid is contracting or expanding, respectively [3]. Viscous dissipation results from the fluid motion, in natural convection problems fluid motion results from the expansion/contraction experienced by the fluid, and both the viscous dissipation and the work of pressure forces need to be taken into account in order to have the unique consistent energy conservation formulation.

No restrictions were made concerning the orientation of the enclosure or of the enclosure walls, and it is incorrect the claim made in Ref. [1] saying that Eq. (4) applies only to a laterally heated enclosure and not to a bottom heated enclosure. It is argued in Ref. [1] that the kinetic energy released in the bottom heated enclosure comes into play, and Eq. (4) does not apply. However, the thermal energy conservation equation is obtained subtracting the kinetic energy conservation equation from the total energy conservation equation [3], and the kinetic energy effects cannot be invoked when dealing with just the thermal energy conservation equation.

Energy Conservation Formulation for Natural or Mixed Convection

In mixed convection fluid motion is partially forced and partially buoyancy induced. For the buoyancy induced flow applies the mentioned above for natural convection. In this case, however, Eq. (4) does not apply, as there are forced flow contributions for viscous dissipation.

In Sec. 5 of Ref. [1] it is argued that the sentence in Refs. [4,5] "…the main results and conclusions apply to any natural or mixed convection problem…" is incorrect. However, the main results and conclusions of Refs. [4,5] are as follows. (i) The consistent energy conservation formulation of natural or mixed convection problems needs to consider both the viscous dissipation and the work of pressure forces. (ii) The energy formulation considering only the viscous dissipation term is inconsistent and violating the energy conservation principle. (iii) Viscous dissipation results from fluid motion, and in natural convection fluid motion results from the expansion/contraction experienced by the fluid, with the associated work of pressure forces. Results, in the form of Eq. (4), which apply to closed enclosures, are a way to explain the main question and not the main result and/or conclusion of Refs. [4,5]. Natural convection heat transfer problem is used to show the essence of the problem, and extrapolations are made to what happens in mixed convection heat transfer problems, where part of the fluid motion is buoyancy induced. This is highlighted in Conclusions of Refs. [4,5]. It is thus correct the claim in Sec. 5 of Ref. [1] saying that result expressed by Eq. (4) is not valid for mixed convection problems, but this is not claimed in Refs. [4,5].

Boussinesq Approximation

The above results were obtained *without* considering any simplifying approach, and no reference was made to the Boussinesq

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approximation or to the Oberbeck–Boussinesq approximation.

Use of a simplified model results on some *contamination* of the solution, and even on some inconsistencies taking in mind the strict (exact) model and that thermodynamics sets many links between variables and properties. This is also the case when the Boussinesq or the Oberbeck–Boussinesq approximation is used to solve the natural or mixed convection problems.

One thing is to start from the consistent energy conservation formulation of the problem and to use a simplified model to solve it, and inconsistencies on the energy conservation are due to the used simplified model. A different thing is to start from an inconsistent energy conservation formulation of the problem and to use a simplified model to solve it, and try to explain inconsistencies on the energy conservation as based only on the used simplified model to solve the problem.

Scale Analysis

Conclusions are obtained in Sec. 6 of Ref. [1] concerning the relevance of the viscous dissipation and of the work of pressure forces in natural convection problems, which are presented as depending on the considered scales and on the physical situation considered (laterally heated enclosure or bottom heated enclosure).

Result expressed by Eq. (4) applies to natural convection in enclosures, no matter how they are oriented or their walls are oriented. In Sec. 6 of Ref. [1] a scale analysis is conducted over the differential thermal energy conservation equation, and thus only local conclusions can be obtained in what concerns the relative magnitude of the involved terms. However, by the reasons mentioned above, viscous dissipation and work of pressure forces are strongly linked in natural or mixed convection problems, and local and integral assessments of their relevance can lead to significantly different conclusions (description after Eq. (4)).

References

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