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Some Pitfalls in the Modelling of Convective Flows in Porous Media

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Abstract. Some modelling deficiencies in various recent papers, on convective flows in porous media, are pointed out and discussed. These deficiencies are related to the Forchheimer coefficient, mixed double-diffusive convection, magnetohydrodynamic mixed convection, convection in a rarefied gas, and geophysical phenomena.

Key words: modelling, Forchheimer coefficient, double-diffusive convection, magnetohydrodynamic convection, rarefied gas, geophysical phenomena.

1. Introduction

When the author was surveying the literature on convective flows in porous media, in connection with the writing of the monograph by Nield and Bejan (1999), he came across several instances of deficiencies in the way those flows were modeled by various authors. The purpose of this note is to draw attention to these deficiencies, so that future authors do not fall into the same pits. The errors in modelling noted here range from algebraic oversights to errors of interpretation and conceptual errors. The author (Nield, 1999a,b) has elsewhere pointed out some other modelling errors, namely those related to the modelling of coriolis and magnetic effects and those related to the modelling of viscous dissipation in porous media.

2. The Relationship Between the Forchheimer Coefficient and Porosity

There is an algebraic error in the formula used to relate the Forchheimer coefficient to the porosity, that dates back 15 years and has now become widespread in the literature, and which affects papers reporting numerical results on heat transfer in a saturated porous medium. A recent example where the error is involved is the paper by Slimi *et al*. (1998). The underlying dimensional momentum equation behind their dimensionless equation takes the form

$$0 = -\nabla P - \left(\frac{\mu}{K}\right)\mathbf{v} - (\phi F\rho/K^{1/2})|\mathbf{v}|\mathbf{v}. \tag{1}$$

Here the notation of Nield and Bejan (1999) is used where possible; P is the pressure, μ the viscosity, ρ the density of the fluid, ϕ the porosity, and K the permeability of the porous medium. The Darcy velocity is denoted by $\mathbf{v}$. Slimi *et al.* (1998, p. 1115):

> The value of the Forchheimer number F, is related to the form drag caused by the porous matrix. ... The values of F and K used in all the calculations executed in this study are determined according to the following relations (Ergun, 1952):

$$K = \frac{\phi^3 d_\mathrm{p}^2}{150(1-\phi)^2}, \qquad F = \frac{1.75}{(150)^{1/2}\phi^{3/2}}. \tag{2}$$

Here d_p denotes the particle diameter. The second of these two equations is not found in Ergun (1952), and in the present context is incorrect by a factor ϕ. This extra factor arises because of the inclusion of the factor ϕ in the last term of Equation (1). The expression for K in Equation (2) and the Ergun formula together require that in Equation (1) the factor multiplying $|\mathbf{v}|\mathbf{v}$ be of the form $1.75\rho(1-\phi)/d_\mathrm{p}\phi^3$. In terms of the notation in Nield and Bejan (1999), the expression ϕF in Equation (1) should be replaced by c_F and then F replaced by c_F in Equation (2). It is a matter of choice whether or not to include the porosity as a factor in a Forchheimer coefficient, but when the choice has been made care has to be taken to check that subsidiary formulas are consistent with that choice. Slimi *et al.* (1998) used the value 0.4 for the porosity in their calculations, so the error is quite significant.

The corresponding expression for the Forchheimer coefficient used by Beckermann and Viskanta (1988) is correct. A similar expression can be found, in the correct context, in Joseph *et al.* (1982).

An expression relating the Forchheimer coefficient c_F and ϕ is very convenient, because it is often difficult to obtain an experimental value for c_F whereas the porosity is easily determined. However, the reader should appreciate that the above expression is *ad hoc*. It has been derived on the basis of the Ergun expressions for the linear and quadratic drag terms, which are appropriate for a bed of spherical solid particles but are less appropriate for other types of porous media. Empirically the Darcy drag and Forchheimer drag are independent quantities, and so one should not expect the permeability and the Forchheimer coefficient to be always correlated by a single precise algebraic expression.

3. Mixed Double-Diffusive Convection

The Abstract of the paper by Yih (1998) concludes with the statement, "Furthermore, increasing the Lewis number decreases (increases) the local heat (mass) transfer rate". The Lewis number is the ratio of the thermal diffusivity to the mass diffusivity, so this statement implies that if the mass diffusivity is held constant then increasing the thermal diffusivity leads to a decrease in the dimensional heat transfer rate. On physical grounds, the opposite should occur. Obviously something is wrong.

In fact, Yih (1998) has shown that the quantity $\mathrm{Nu}_x/(\mathrm{Pe}_x^{1/2} + \mathrm{Ra}_x^{1/2})$ decreases as the Lewis number Le increases. Here Nu_x, Pe_x and Ra_x are the local Nusselt number, Péclet number and Rayleigh number, respectively. The correct conclusion is that the nondimensional heat transfer rate, represented by Nu_x, decreases as Le increases provided that the parameters Pe_x and Ra_x are held constant. However, Pe_x and Ra_x are each proportional to the reciprocal of the thermal diffusivity, and so the general claim made in Yih (1998) is invalid. Other statements in the same paper need a similar reinterpretation.

4. Magnetohydrodynamic Mixed Convection

The summary (abstract) of Takhar and Beg (1997) contains the statement, "The heat transfer rate is, however, enhanced by the magnetic field (for positive values of the Eckert number) since the fluid is heated and temperature gradients become reduced between the fluid and the plate, with important potential applications in MHD power generators, material processing and geothermal systems containing electrically-conducting fluids". If this statement were true, then the results in the paper would indeed be of considerable technological importance.

One would expect an enhancement of heat rate to be associated with an increase rather than a reduction of temperature gradient, and the text at the bottom of page 93 is in accord with that expectation. It is clear that the abstract contains a typographical error. But, even when that error is corrected, the statement claims the opposite of what one would expect on physical grounds. The transverse magnetic field should inhibit flow in the streamwise direction, leading to a reduction in convective heat transfer. Closer examination of the paper (Figures 3 and 4) reveals that the authors have shown that the non-dimensional temperature gradient at the surface, denoted by $\theta'(\xi, 0)$, increases with the Hartmann number Ha. The authors have referred to $\theta'(\xi, 0)$ as the 'local heat transfer parameter'; but, in fact, the appropriate local heat transfer parameter is the local Nusselt number Nu_x, which is given by Equation (29) of the paper,

$$\mathrm{Nu}_x = -\theta'(\xi, 0) \times (\mathrm{Re}_x)^{1/2}, \tag{3}$$

where $\mathrm{Re}_x = U_0 x/\nu$ is the local Reynolds number. The authors have shown that Nu_x increases with Ha provided that Re_x is held constant. However, the effect of introducing an applied transverse magnetic field will be to reduce the free-stream velocity U_0 and hence to reduce Re_x. It follows that the conclusion of the authors regarding increase of heat transfer is invalid.

5. Convection in a Rarefied Gas Saturating a Porous Medium

The paper by Parthiban and Patil (1996) is concerned with the onset of convection in a rarefied gas saturating a horizontal layer of a porous medium.

The authors concluded that both the temperature jump and the velocity slip had a stabilizing effect. This is contrary to physical intuition, so one should expect to find an error in their analysis. The boundary conditions expressed by their Equations (4), (5), (17), (18) and (28) contain the error. The authors have failed to recognize that the normal derivative into the fluid is actually in the positive z-direction at the bottom boundary, but in the negative z-direction at the upper boundary. This means that, at the upper boundary, the signs of the odd-order derivatives with respect to z need to be changed.

Also, the authors have used the boundary condition (their Equation (4))

$$w = h_2(\partial w/\partial z) \quad \text{at } z = 0, d. \tag{4}$$

An equation of this form is appropriate for the tangential components of the velocity, but here w is the normal component of the velocity. At an impermeable boundary there cannot be any flow normal to the boundary, whether or not the gas is rarefied, so the correct boundary condition is

$$w = 0 \quad \text{at } z = 0, d. \tag{5}$$

Equation (18) requires a consequential change.

When correctly expressed, the boundary condition on the perturbation temperature, their Equation (17) is of the same form as the usual radiation boundary condition. With the correct hydrodynamic condition, Equation (5), taken into account, the problem on the Darcy model reduces to that treated by Wilkes (1995). The results in that paper show that, compared with the case of isothermal boundary conditions, the effect of the 'temperature jump' is indeed destabilizing, as expected on physical grounds.

6. Modelling of Geophysical Phenomena

In their introduction, Takhar and Beg (1997) write "... MHD convective flows ... [are] ... of great significance in ... geothermal zones. In the latter, for example, there occurs an interaction between the earth's geomagnetic field and the high-temperature water and steam in the geological domain, e.g. 'hot springs'.". It is the author's impression that the earth's magnetic field, the electrical conductivity of salt-laden geothermal water, and the permeability of a typical geothermal field, are all relatively small so that the MHD effect is negligible. The only publication that the author knows of, which describes a significant MHD effect in convective flow in a porous medium, is that by Bergman and Fearn (1994), who discuss convection in a mushy zone at the earth's inner–outer core boundary. Incidentally, the statement on pages 92–93 of Takhar and Beg (1997) about "... porosity $\varepsilon = 0.6$ typifying geothermal systems (see Nield and Bejan, 1992)" is certainly not true. The authors have misread Nield and Bejan (1992).

Parthiban and Patil (1996, p. 27) motivated their study involving a rarefied gas by stating that "at great depths the earth's interior attains high temperatures reaching up to 5000°C . . . ; consequently the gas is likely to be in a rarefied state". They have overlooked the fact that at the great pressures involved this will not be so. Rather, the gas will be liquefied or dissolved in another liquid.

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