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Resolution of a Paradox Involving Viscous Dissipation and Nonlinear Drag in a Porous Medium

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Abstract. The modelling of viscous dissipation in a porous medium saturated by an incompressible fluid is discussed, for the case of Darcy, Forchheimer and Brinkman models. An apparent paradox relating to the effect of inertial effects on viscous dissipation is resolved, and some wider aspects of resistance to flow (concerning quadratic drag and cubic drag) in a porous medium are discussed. Criteria are given for the importance or otherwise of viscous dissipation in various situations.

Key words: viscous dissipation, Forchheimer model, Brinkman model, inertial effects, nonlinear drag, forced convection, natural convection.

1. Introduction: Review of Current Modelling of Viscous Dissipation

In most circumstances it is not necessary to allow for viscous dissipation when modeling convection in saturated porous media. (A criterion for determining whether viscous dissipation is significant or not is discussed below.) Nevertheless, despite the fact that the effect is generally negligible, recently several authors have explicitly considered the effects of viscous dissipation in problems involving convection in porous media, and they have disagreed on the way to model this effect.

For example, in their study of flow past a vertical porous plate, Takhar and Beg (1997) have modeled the viscous dissipation in the porous medium by a term $\mu(\partial u/\partial y)^2$ in their Equation (3) applicable to a unidirectional flow. Here μ is the viscosity of the fluid, y is the transverse coordinate, and u denotes the Darcy velocity, the average over a representative elementary volume (r.e.v.) of the pore velocity u_p. However, the r.e.v. average of $(\partial u_p/\partial y)^2$ is not $(\partial u/\partial y)^2$, so the term introduced by these authors is clearly incorrect.

On the other hand, Murthy and Singh (1997) have modeled the flow of an incompressible fluid in a saturated porous medium, with the effect of viscous dissipation included, by writing the momentum equation as

$$B(q)\mathbf{v} = \frac{K}{\mu}(-\nabla p + \rho \mathbf{g}), \tag{1}$$

and the heat transfer equation as

$$\mathbf{v} \cdot \nabla T = \nabla \cdot (\alpha_e \nabla T) + \left(\frac{1}{\rho c_p}\right) \mathbf{v} \cdot (-\nabla p + \rho \mathbf{g}), \tag{2}$$

where $B(q) = 1 + bK\rho q/\mu$, and so

$$\mathbf{v} \cdot \nabla T = \nabla \cdot (\alpha_e \nabla T) + \left[\frac{\nu B(q)}{K c_p}\right] \mathbf{v} \cdot \mathbf{v}. \tag{3}$$

Here $\mathbf{v}$ is the Darcy velocity, $q = |\mathbf{v}|$, K is the permeability, μ is the fluid viscosity, p is the pressure, ρ is the density, $\mathbf{g}$ is the gravitational acceleration, T is the temperature, α_e is the effective thermal diffusivity, c_p is the specific heat at constant pressure, b is a Forchheimer coefficient, and $\nu = \mu/\rho$. One may write Equation (3) as

$$\rho c_p \mathbf{v} \cdot \nabla T = \nabla \cdot (k_e \nabla T) + \left(\frac{\mu}{K} + b\rho q\right) q^2, \tag{4}$$

where $k_e = \rho c_p \alpha_e$. The last term in Equation (4) represents the ‘viscous dissipation term’ employed by Murthy and Singh (1997). For the case $b = 0$ (i.e. when Darcy’s law holds) the expression is in agreement with, for example, Equation (12.33) of Bejan (1995). However, for the case $b \neq 0$ the contribution from the Forchheimer drag term *is independent of the viscosity*, and so apparently cannot represent *viscous* dissipation. The author (Nield and Bejan, 1999) originally thought that this meant that inertial effects did not contribute to the viscous drag (here ‘drag’ is used as an abbreviation for ‘resistance to flow’), and that this anomalous result was due to a failure somehow to recognize that inertial terms modeled a reversible process whereas viscous terms involved an irreversible process.

It is thus desirable to look at the way in which the viscous dissipation enters the relevant energy equation in the standard formulations of the theory as presented, for example, in Kacaç and Yener (1987). The first law of thermodynamics leads to an equation (their Equation (1.66)) for the total energy. This reads:

$$\begin{aligned} &\rho \frac{\mathrm{D}}{\mathrm{D}t}\left[\upsilon + \frac{1}{2}(u^2 + v^2 + w^2)\right] \\ &\quad = \nabla \cdot (k\nabla T) + \frac{\partial}{\partial x}(u\sigma_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{yx} + v\sigma_{yy} + w\tau_{yz}) + \\ &\qquad + \frac{\partial}{\partial z}(u\tau_{zx} + v\tau_{zy} + w\sigma_{zz}) + \rho \mathbf{V} \cdot \mathbf{f}. \end{aligned} \tag{5}$$

Here υ denotes the internal energy of the fluid per unit mass; (u, v, w) are the components of the fluid velocity, $\mathbf{V}$; while σ_{xx}, τ_{yz}, etc., are the stress components; and $\mathbf{f}$ is the body force per unit mass and k is the thermal conductivity. From the momentum equation, a second equation (1.67) is obtained, this one involving the mechanical energy, namely,

$$\rho \frac{\mathrm{D}}{\mathrm{D}t}\left[\frac{1}{2}(u^2+v^2+w^2)\right] = u\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)+ \\ + v\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)+ \\ + w\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}\right)+\rho \mathbf{V}\cdot\mathbf{f}. \tag{6}$$

When this second equation is subtracted from the first, the result is a third equation (1.68) that, confusingly, is often referred to as ‘the energy equation’, but is better described as the thermal energy equation, namely,

$$\rho \frac{\mathrm{D}\upsilon}{\mathrm{D}t} = \nabla\cdot(k\nabla T)+\sigma_{xx}\frac{\partial u}{\partial x}+\sigma_{yy}\frac{\partial v}{\partial y}+\sigma_{zz}\frac{\partial w}{\partial z}+ \\ +\tau_{xy}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+\tau_{yz}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)+\tau_{zx}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right). \tag{7}$$

For a Newtonian fluid, the last equation can be written as

$$\rho \frac{\mathrm{D}\upsilon}{\mathrm{D}t} = \nabla\cdot(k\nabla T) - p\nabla\cdot V + \mu\Phi, \tag{8}$$

where the dissipation function Φ is defined as

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+ \\ +\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2-\frac{2}{3}(\nabla\cdot\mathbf{V})^2. \tag{9}$$

The viscous dissipation term in Equation (8) represents the conversion (actually the rate of conversion per unit volume) of mechanical energy into thermal energy. For an incompressible fluid, this term is of constant sign, and so represents an irreversible process, while the inertial term simply contributes to the mechanical energy. The power (rate of doing work) of the drag on an elemental volume (a term that appears in Equation (6)), is intrinsically different from the viscous dissipation (a term that appears in Equation (7)), but from those equations one concludes that, in the case of zero thermal conduction, zero body force and constant internal energy, these two things are equal. Furthermore, the bottom line is that when fluid is forced to flow unidirectionally down a channel, whether occupied by a solid porous matrix or not, the work done by the applied pressure difference has to be matched by the increase in thermal energy, because there is no other mechanism available to achieve the balance of total energy.

The consequences of this are discussed in Section 2. This discussion raises further matters which are discussed in Section 3. The extension from the Forchheimer model to the Brinkman–Forchheimer model is discussed in Section 4. In Section 5, scale analysis arguments are used to obtain estimates of the importance or otherwise of the viscous dissipation in various situations.

2. A Paradox and Its Resolution

Thus the Forchheimer drag does indeed contribute to the viscous dissipation, and the paradox, that a term apparently independent of the viscosity represents the viscous dissipation, must be resolved by other means. The paradox is related to the famous D'Alembert paradox. For D'Alembert, who did not know about boundary layers, the problem was how to explain the appearance of finite drag in circumstances where viscous effects appeared to be negligible and the predicted drag, computed from the pressure distribution, was zero. Now the problem is to explain an increase in drag, and hence an increase in viscous dissipation, produced by inertial effects which are apparently independent of the viscosity. When discussing inertial effects it is worth noting that some authors use a momentum equation that includes a convective inertial term as well as the Forchheimer drag term, and this has been questioned (Nield, 1991, 1994). It is clear that the convective inertial term is irrelevant here in the sense that it does not contribute to the viscous dissipation (though, of course, it does contribute to the kinetic energy balance), and that is another good reason why it may be better to drop this term from the equation in some circumstances. Incidentally, this argument provides a good reason why one should not use the terminology 'inertial effect' without further qualification when referring to the Forchheimer drag term.

The explanation of the apparent paradox lies in the recognition that, as pointed out by Joseph *et al*. (1982), the Forchheimer drag term models essentially a form drag effect, and involves the separation of boundary layers and wake formation behind solid obstacles. (In fact, the basic idea goes back at least as far as Bakhmeteff and Feodoroff (1937). At the time when the present author co-authored the paper Joseph *et al*. (1982), he did not realize the full significance of the observation.) The pore scale convective inertial effects contributing to the form drag lead to a substantial modification of the velocity field, and in particular, to an enlargement of the macroscopic region in which pore scale velocity gradients are large. This leads to an increase in the total viscous dissipation (summed over the whole region occupied by fluid), and hence, because of the fundamental equality of viscous dissipation (within a volume) and the power of the drag force (on that volume), to the increase in the drag. Lage (1998) has emphasized the distinction between porous media of bluff-body type and those of conduit type, so it is worthwhile pointing out that the above argument applies to both types of porous media. In the case of bluff bodies the separation of boundary layers and wake formation occurs behind the bodies, while in the case of conduits of converging–diverging type the phenomenon occurs downstream of the shoulders.

3. Further Comments on Forchheimer Drag

It seems that previous confusion about the nature of form drag in a porous medium was primarily due to not appreciating the implications of the fact that viscosity acts throughout the fluid, and not just at the solid boundaries. When it is explicitly pointed out, it is obvious that viscous dissipation is a process that occurs throughout the space occupied by fluid, whereas the Darcy drag term is something that arises from the viscous contribution to stress at the solid boundaries. Although the Forchheimer form drag term does not explicitly involve the viscosity as a factor in the expression, it does arise from the action of viscosity, mediated by the inertial effects affecting the distribution of pressure that also contributes to stress at the solid boundaries. As D'Alembert found, without viscosity there is no drag force, the reason being that there is then no boundary layer and hence no boundary layer separation. In dealing with the effect of viscosity it is essential to get things right at the microscopic (pore) scale before moving on to consider the macroscopic (representative elementary volume) scale. In the present context this is achieved essentially by relating a volume integral over a representative elementary volume to a surface integral (see below).

Another point worthy of note is that the dichotomy into viscous (Darcy) drag and inertial form (Forchheimer) drag is not arbitrary, as might be thought from the way in which some early workers added terms to an empirical relationship between pressure drop and velocity. Rather, the two terms arise naturally once a division is made into a term linear in the velocity and one quadratic in the velocity. It is then inevitable on dimensional grounds that the Darcy term will appear with the viscosity as a coefficient, whereas the Forchheimer term will not explicitly involve the viscosity. Thus, it is not an accident that the viscosity cancels in the inertial term in the transformation of Equation (3) to Equation (4). The cancellation of viscosity is inevitably involved in the overall process of the production of increased drag by wake formation. Another point is that the Forchheimer drag term and the Darcy term are intrinsically linked. While it may make mathematical sense to treat the asymptotic case where the flow is so rapid that the Forchheimer term dominates the Darcy term, as several authors have done, the results may not have much physical significance, at least in the absence of careful interpretation.

The Forchheimer formulation is applicable to what Skjetne and colleagues (Skjetne and Auriault, 1999) have called the strong inertial flow regime. This is a regime in which the pore Reynolds number Re_p (based on the particle or pore diameter) is of order unity or greater. For the standard Forchheimer model there is no cubic term present but, as Lage *et al*. (1997) have pointed out, when the complications resulting from transition to turbulence are taken into account the coefficient of the quadratic term varies slowly with velocity, and so the drag is effectively cubic within a restricted Reynolds number range. This behavior is to be distinguished from the cubic drag phenomenon recently discussed by, for example, Mei and Auriault (1991), Wodie and Levy (1991), Rasoloarijaona and Auriault (1994), Firdaouss *et al*. (1997) and Skjetne and Auriault (1999), a phenomenon that is applicable to the weak inertial regime, one for which $\mathrm{Re_p}$ is small compared with unity. For this regime the quadratic term is absent. When an expansion in powers of $\mathrm{Re_p}$ is made, it turns out that the coefficient of the quadratic term is zero. It appears that the expansion methodology employed by these authors is inadequate for the treatment of boundary layer separation, because it does not lead to the prediction of the quadratic drag that is associated with that separation.

For flow past a single solid obstacle in an unbounded region, the size of the region in which velocity gradients are large can grow without limit, but in the case of a porous medium a limit is set by the pore size. For example, in the case of a porous medium with spatially periodic structure, the limiting region is a period cell. This suggests that at very large velocities the rate of increase of drag with increase of velocity may fall below that predicted by the Forchheimer expression with an unchanged constant. The situation is complicated by transition to unsteady and chaotic flow regimes, but it is interesting that Kaviany (1995, p. 49) refers to experimental results reported in Macdonald *et al*. (1979) and Dybbs and Edwards (1984) indicating an asymptotic behavior in which the normalized pressure drop does not change with the Reynolds number.

In their study of natural convection, Fand *et al*. (1994) argued that at high flow rates the boundary layers are sufficiently thin as to render viscous dissipation negligible compared to conduction at the heated surface. We can now see that their argument is invalid, and that in their problem viscous dissipation should be important for large values of the Rayleigh number as well as for small values.

I am grateful to the referee for the following general comment, which provides an alternative way of looking at the situation. Consider a laminar flow and assume that it can be described by the Navier–Stokes equation and an adherence condition on the pore surface. Then the seepage law appears as a nonlinear relation between Darcy velocity $\mathbf{v}$, macroscopic pressure gradient ∇p, viscosity μ and density ρ, and this can be put in the form

$$\nabla p = \mathbf{G}(\mathbf{v}, \mu, \rho). \tag{10}$$

In the case of the Navier–Stokes equation, the viscous term and the inertial term are well identified as separate terms, but this is not so in the case of the macroscopic seepage law. It is possible to demonstrate that the macroscopic dissipation at the Darcy scale is $\mathbf{v} \cdot \nabla p$ and that the drag is ∇p, no matter what is the form of the seepage law (10). Thus, in general, viscous and inertial effects are intimately mixed in the expression of the dissipation and of the drag, as well as in (10). When $\mathbf{v}$ becomes small, relation (10) reduces to the classical Darcy law. By making use of this, relation (10) can be rewritten in the form

$$\nabla p = -\left(\frac{\mu}{K}\right)\mathbf{v} + \mathrm{H}(\mathbf{v}, \mu, \rho). \tag{11}$$

However, as a general rule the nonlinear function **H** depends on both μ and ρ. This can be checked in the particular case of small Reynolds numbers (Mei and Auriault, 1991).

4. Extension to Other Models

The recognition that the viscous dissipation is equal to the power of the total drag force leads to a major simplification in the modelling of viscous dissipation. Rather than having to perform the difficult task of evaluating the r.e.v. average of the intrinsic viscous dissipation, which involves an expression which is nonlinear in the velocity derivatives, one can simply evaluate the power of the drag force. (Looking back to the derivation of Equations (1.66)–(1.68) of Kacaç and Yener (1987) we can see that the simplification results from the relating of a volume integral to a surface integral for a small control volume, and it is the surface integral that involves the power of the drag force.) For example, we can now see how to model viscous dissipation in a porous medium when the Brinkman–Forchheimer model is used. (As far as the author is aware, this has not been hitherto discussed explicitly in the literature, except by Nield and Bejan (1999).) If the local drag is correctly modeled by $(\mu/K)\mathbf{v} - \mu_{\text{eff}}\nabla^2\mathbf{v}$, then the viscous dissipation is equal to the power of the drag, and so is equal to $(\mu/K)\mathbf{v}\cdot\mathbf{v} - \mu_{\text{eff}}\mathbf{v}\cdot\nabla^2\mathbf{v}$.

5. Estimation of the Importance of Viscous Dissipation

Finally, we note that the viscous dissipation term in the thermal energy equation is of the order of magnitude $\mu U^2/Kc_{\text{p}}$. On the other hand, the thermal conduction term that appears in the same equation is of the order of magnitude $k\Delta T/L^2$ where U, L and ΔT are the characteristic velocity, length, and temperature difference scales, respectively. Hence the viscous dissipation is negligible, if

$$N \ll 1, \tag{12}$$

where

$$N = \frac{\mu U^2 L^2}{Kc_{\text{p}}k\Delta T} = \frac{\text{Ec Pc}}{\text{Da}} = \frac{\text{Br}}{\text{Da}}, \tag{13}$$

where the Eckert number, Prandtl number, Darcy number and Brinkman number are respectively defined by

$$\text{Ec} = \frac{U^2}{c_{\text{p}}\Delta T}, \quad \text{Pr} = \frac{\mu/\rho}{k/\rho c_{\text{p}}}, \quad \text{Da} = \frac{K}{L^2}, \quad \text{Br} = \frac{\mu U^2}{k\Delta T}. \tag{14}$$

For most situations the Darcy number is small, so viscous dissipation is important at even modest values of the Eckert number. The circumstances in which viscous dissipation is important are those involving flows of relatively large velocity. The author believes that the results in this paper are likely to be applicable in the context of particle bed nuclear reactors.

In the case of forced convection, a suitable choice for the characteristic velocity U is obvious. In the case of natural convection, scale analysis arguments such as those employed in Section 7.1.1 of Nield and Bejan (1999) lead to the estimate $U \sim (\kappa_e/L)\mathrm{Ra}^{1/2}$, where κ_e is the effective thermal diffusivity of the porous medium, and Ra is the Rayleigh–Darcy number defined by

$$\mathrm{Ra} = \frac{g\beta K L \Delta T}{\kappa_e \nu}, \tag{15}$$

(where β is the coefficient of thermal expansion) so in this case

$$\mathrm{Ec} \sim \mathrm{Pr}^{-1}\mathrm{Da\ Ge}, \tag{16}$$

where Ge is the Gebhart number defined by

$$\mathrm{Ge} = \frac{g\beta L}{c_p}, \tag{17}$$

and the condition (12) becomes

$$\mathrm{Ge} \ll 1, \tag{18}$$

in agreement with the statement by Nield and Bejan (1999, p. 25). The analysis of Nakayama and Pop (1989) and Murthy and Singh (1997) confirms that a Gebhart number (denoted in those papers by ϵ) is the pertinent group in natural convection problems in the Darcy flow case. Murthy and Singh (1997) found that the Nusselt number is decreased due to viscous dissipation by a fraction approximately equal to Ge, for small values (0.01 and 0.1) of Ge.

The above comments on forced convection are made on the assumption that the Péclet number $\mathrm{Pe} = UL/\kappa_e$ is not large. If it is large, then the proper comparison is one between magnitudes of the viscous dissipation term and the convective transport term in the thermal energy equation. This ratio is of order EcPr/DaPe = Ec/DaRe, where the Reynolds number $\mathrm{Re} = UL/\nu$.

6. Summary

It has been demonstrated that for the Forchheimer model of a porous medium, the viscous dissipation is represented by a term that is apparently independent of the viscosity, and this paradox is resolved. Finally, scale analysis is employed to estimate the importance of viscous dissipation in various circumstances.

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