

Calculating Viscosities of Reservoir Fluids From Their Compositions

JOHN LOHRENZ
BRUCE G. BRAY
MEMBERS AIME
CHARLES R. CLARK

CONTINENTAL OIL CO.
PONCA CITY, OKLA.
U. OF KANSAS
LAWRENCE, KANS.

ABSTRACT

Procedures to calculate the viscosities of in situ reservoir gases and liquids from their composition have been developed and evaluated. Given a composition expressed in methane through heptanes-plus, hydrogen sulfide, nitrogen and carbon dioxide together with the molecular weight and specific gravity of the heptanes-plus fraction, the procedures are capable of calculating the viscosity of the gas or liquid at the desired temperature and pressure.

The procedure for reservoir liquids was developed using the residual viscosity concept and the theory of corresponding states, and was evaluated by comparing experimental and calculated results for 260 different reservoir oils ranging from black to highly volatile. The average absolute deviation was 16 per cent. This is the first known procedure for calculating the viscosity of reservoir liquids from their compositions as normally available, i.e., including the heptanes-plus fraction.

The procedure for reservoir gases uses a sequence of previously published correlations. Evaluation of the procedure was accomplished by comparison of 300 calculated and experimental viscosities for high-pressure gas mixtures in the literature. The average absolute deviation was 4 per cent.

The calculations are useful for (1) determining viscosities in compositional material balance computations and (2) predicting the viscosity decrease which occurs when gases, LPG, or carbon dioxide dissolve in reservoir oils.

INTRODUCTION

Methods to predict viscosities of reservoir fluids from the normally available field-measured variables have been presented. Beal,¹ Standing,² and Chew and Connally³ correlated oil viscosities with temperature, pressure, oil gravity and gas-oil ratio. Carr, Kobayashi, and Burrows⁴ and Katz *et al.*⁵ have presented correlations for reservoir gas viscosities as a function of temperature, pressure and gas gravity.

Like all intensive physical properties, viscosity is completely described by the following function:

$$\mu = F(p, T, x_1, x_2, \dots, x_{n-1}) \quad (1)$$

where $\sum_{j=1}^n x_j = 1$.

Original manuscript received in Society of Petroleum Engineers office July 10, 1964. Revised manuscript received Aug. 28, 1964. Paper presented at SPE Annual Fall Meeting, held in Houston, Tex., Oct. 11-14, 1964.

¹References given at end of paper.

Eq. 1 simply states that viscosity is a function of pressure, temperature and composition. These previous correlations¹⁻⁵ may be viewed as modifications of Eq. 1, wherein one assumes more simple functions may be used. The assumptions are practical, because the composition is frequently not known. Further, the assumptions are sufficiently valid so that these correlations are frequently used for reservoir engineering computations.

In compositional material balance⁶⁻⁹ computations, the compositions of the reservoir gases and oils are known. The calculation of the viscosities of these fluids using this composition information is required for a true and complete compositional material balance. For reservoir gases, Carr, Kobayashi and Burrows⁴ have presented a suitable compositional correlation. For reservoir oils, no correlation is available, and data from reservoir fluid analyses have been used⁷⁻⁹ for compositional material balance calculations.* From a theoretical point of view, this is entirely invalid. The reservoir fluid analysis, whether flash, differential, or other process, does not duplicate the compositions which occur during the actual reservoir depletion process, therefore the viscosities measured during reservoir fluid analysis are not those which occur in the reservoir. From a practical point of view, the "error" of using viscosities from reservoir fluid analysis is of varying and unknown significance. One can say qualitatively that the error is greatest where compositional effects are greatest, i.e., for volatile oil and gas condensate reservoirs and pressure maintenance operations. The first requirement to obtain a quantitative estimate of the significance of the error is to develop a reliable compositional correlation for the viscosities of reservoir oils. No such correlation has been available.

Consistent with this requirement, the objective of this study was to develop a procedure to predict the viscosity of reservoir fluids from their compositions. Normally, the compositions of reservoir fluids are available expressed as mole fractions of hydrogen sulfide, nitrogen, carbon dioxide and the hydrocarbons methane through the heptane-plus fraction, with the average molecular weight and specific gravity of the latter. The final correlation was to use the composition in this form. While the more challenging objective of the study was the development of a correlation for the viscosities of reservoir oils, the viscosities of reservoir gases were also studied.

The end result of the study was a procedure to calculate the viscosities of reservoir gases and liquids suitable

*Brinkman and Weinaug⁶ used the correlation of Beal¹ to calculate the viscosities.

for insertion in compositional material balance digital computer programs. The intended use of these predictions determined the desired accuracy. In compositional material balance calculations, the oil-gas viscosity ratio is always used as a multiplier with the relative permeability ratio. Since the relative permeability ratio is subject to large uncertainties, the accuracy requirement of the viscosity predictions is not severe. Average deviations of ± 25 per cent should be acceptable. Of course, better accuracy is desirable, since the viscosity prediction procedures have other uses apart from compositional material balances.

The objectives of this study were achieved with the development of the procedures of this paper. For reservoir oils, a new correlation was formed by adapting a correlation for pure components. Several correlations already in the literature were sequenced to obtain a convenient viscosity calculation procedure for reservoir gases.

CALCULATION PROCEDURES

The composition must be given in mole fractions of hydrogen sulfide, nitrogen, carbon dioxide and the hydrocarbons methane through the heptane-plus fraction. The butane and pentane fractions are divided in *n*- and iso-components. The hexane fraction includes all C_6 isomers. The average molecular weight and specific gravity of the heptane-plus fraction must also be given. From this composition, the procedures can be used to calculate the viscosity as a function of temperature and pressure for the gas or liquid.

Fig. 1 is a computational flow sheet of the procedures. The following description of the calculation procedures follows the sequence shown on Fig. 1.

BREAKUP OF THE HEPTANE-PLUS FRACTION

The heptane-plus fraction is divided into a mixture of normal paraffin hydrocarbons from C_7 through C_{40} . Mathematically, the mole fraction of each hydrocarbon C_i is determined as follows:

$$x_{C_i} = x_{C_6} e^{A(1-i)^2 + B(i-6)} \quad (2)$$

where i ranges from 7 to 40. The constants A and B are determined such that

$$x_{C_{7+}} = \sum_{i=7}^{40} x_{C_i} \quad (3A)$$

and

$$x_{C_{7+}} M_{C_{7+}} = \sum_{i=7}^{40} (x_{C_i} M_{C_i}) \quad (3B)$$

CALCULATION OF THE LOW-PRESSURE, PURE-COMPONENT GAS VISCOSITIES AT THE TEMPERATURE OF INTEREST

The low-pressure, pure-component gas viscosities at the temperature of interest must be calculated for all com-

ponents in the mixture. The Stiel and Thodos³⁰ correlation is used as follows:

$$\mu_j^* \zeta_j = 34(10^{-5}) T_{rj}^{0.94} \quad (T_{rj} < 1.5), \quad (4A)$$

$$\mu_j^* \zeta_j = 17.78(10^{-5}) (4.58 T_{rj} - 1.67)^{5/8} \quad (T_{rj} > 1.5), \quad (4B)$$

where

$$T_{rj} = \frac{T}{T_{c,j}}, \quad (5)$$

and

$$\zeta_j = \frac{T_{c,j}^{1/6}}{M_j^{1/2} p_{c,j}^{2/3}} \quad (6)$$

Scientific units, °K and atm, must be used in Eq. 6 for the pure component viscosity parameters, ζ_j . The result of the calculation is a table of values of μ_j^* for all pure components.

CALCULATION OF THE LOW-PRESSURE MIXTURE GAS VISCOSITY AT THE TEMPERATURE OF INTEREST

The Herning and Zipperer¹¹ equation is used.

$$\mu^* = \frac{\sum_{j=1}^n (x_j \mu_j^* \sqrt{M_j})}{\sum_{j=1}^n (x_j \sqrt{M_j})} \quad (7)$$

Calculations to this point are the same for both gas and liquid viscosity calculations, however, separate procedures are used beyond this point.

CALCULATION OF THE REDUCED TEMPERATURE AND PRESSURE OF THE MIXTURE (FOR GASES ONLY)

Molal average pseudocritical values are used as follows:

$$T_r = \frac{T}{\sum_{j=1}^n (x_j T_{c,j})} \quad (8)$$

$$p_r = \frac{p}{\sum_{j=1}^n (x_j p_{c,j})} \quad (9)$$

CALCULATION OF THE VISCOSITY OF THE GAS MIXTURE AT THE TEMPERATURE AND PRESSURE OF INTEREST

The chart of Baron, Roof and Wells¹² giving

$$\frac{\mu}{\mu^*} = F(T_r, p_r), \quad (10)$$

is "read" using two-dimensional, four-point Lagrangian interpolation of a 15×15 array, then the desired viscosity μ is calculated. This completes the calculation procedure for gases.

CALCULATION OF THE DENSITY OF THE LIQUID MIXTURE AT THE TEMPERATURE AND PRESSURE OF INTEREST

The Alani and Kennedy¹³ method is used.

CALCULATION OF THE REDUCED DENSITY OF THE LIQUID MIXTURE

The equation

$$\rho_r = \frac{\rho}{\rho'_c}, \quad (11)$$

defines the reduced density, ρ_r . The pseudocritical density is calculated as follows:

$$\rho'_c = \frac{1}{V'_c} = \frac{1}{\sum_{j=1}^n (x_j V_{c,j}) + x_{C_{7+}} V_{c,C_{7+}}} \quad (12)$$

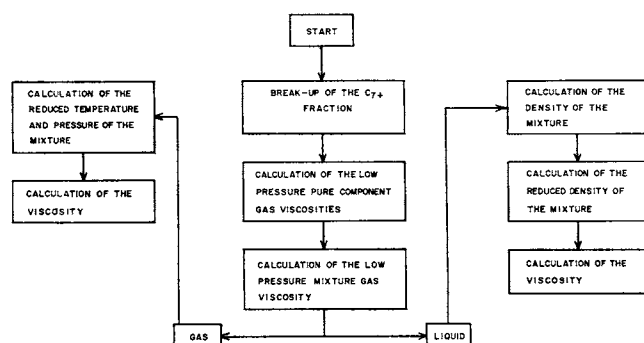


FIG. 1—COMPUTATION FLOW SHEET OF THE CALCULATION PROCEDURES.

$$\text{where } V_{C_{7+}} = 21.573 + 0.015122 M_{C_{7+}} - 27.656 G_{C_{7+}} + 0.070615 M_{C_{7+}} G_{C_{7+}} \quad (13)$$

CALCULATION OF THE VISCOSITY OF THE LIQUID MIXTURES

The relation

$$[(\mu - \mu^*)\zeta + 10^{-1}]^{1/4} = 0.1023 + 0.023364\rho_r + 0.058533\rho_r^2 - 0.40758\rho_r^3 + 0.0093324\rho_r^4 \quad (14)$$

which Jossi, Stiel and Thodos¹⁴ developed for pure components is used and solved for the viscosity, μ . For mixtures, μ^* and ρ_r are defined by Eqs. 7 and 11, respectively. The mixture viscosity parameter is given by

$$\zeta = \frac{\left[\sum_{j=1}^n (x_j T_{cj}) \right]^{1/6}}{\left[\sum_{j=1}^n (x_j M_j) \right]^{1/2} \left[\sum_{j=1}^n (x_j p_{cj}) \right]^{1/6}} \quad (15)$$

Scientific units of measurement must be used in Eq. 15.

DEVELOPMENT OF THE CALCULATION PROCEDURES

The computation flow sheet of Fig. 1 was the initial framework about which the calculation procedures were developed. The transition from the framework to the detailed procedures of the previous section required numerous selections of methods and techniques. Some selections were based on extensive evaluations. Other selections were made for convenience and some were quite arbitrary. This section summarizes the background or reasons for the selections made.

BREAKUP OF HEPTANES-PLUS FRACTION

Use of Eq. 2 assumes a distribution of individual C_{7+} components which passes through the hexane mole fraction x_{C_6} and tails off to extremely small amounts of heavy hydrocarbons well beyond that equivalent to the average molecular weight of the C_{7+} fraction. The assumption that the individual C_{7+} components are normal paraffins is convenient for later critical property estimations.

Admittedly, the breakup procedure does not recognize the differences in character, e.g., paraffinic or aromatic, of C_{7+} fractions. It does, at least, agree with the fact that even C_{7+} fractions of low average molecular weights contain small amounts of exceedingly heavy hydrocarbons. The breakup procedure was developed specifically for phase equilibria calculations¹⁵ in compositional material balances. Here, neglecting the small amounts of exceedingly heavy hydrocarbons causes serious discrepancies.⁸ For viscosity prediction, it is included to be consistent with the intended use in compositional material balance computations.

CRITICAL PROPERTIES OF PURE COMPONENTS

For normal paraffins, the critical constants are calculated by the method of Thodos.¹⁶ The calculated values agree closely with available experimental values and were used throughout for consistency. Critical constants reported in the literature^{17,18} were used for isobutane, isopentane, hydrogen sulfide, nitrogen and carbon dioxide.

HYDROGEN SULFIDE

All of the components of interest are generally classed as non-polar except hydrogen sulfide, which is moderately polar. The same calculation procedures are used for hydrogen sulfide, however, as for all of the other components. Some restriction must be expected, therefore, on the use

of the procedures for fluids with high hydrogen sulfide content.

CALCULATION OF THE VISCOSITY OF A LOW-PRESSURE GAS MIXTURE

In 1954, Carr, Kobayashi and Burrows¹ recommended the Herning and Zipperer¹¹ relation (Eq. 7). A comparative evaluation was made of Eq. 7 with another relation given by Herning and Zipperer:¹¹

$$\mu^* = \frac{\sum_{j=1}^n (x_j \mu_j^* \sqrt{M_j T_{cj}})}{\sum_{j=1}^n (x_j \sqrt{M_j T_{cj}})} \quad (16)$$

and one by Wilke:¹⁹

$$\mu^* = \frac{\sum_{j=1}^n \frac{\mu_j^*}{1 + \frac{1}{x_j} \sum_{k=1, k \neq j}^n (x_k \phi_{jk})}}{\sum_{j=1}^n \frac{\mu_j^*}{1 + \frac{1}{x_j} \sum_{k=1, k \neq j}^n (x_k \phi_{jk})}} \quad (17)$$

$$\text{where } \phi_{jk} = \frac{1 + \left\{ \left[\frac{\mu_j^*}{\mu_k^*} \right]^{1/2} \left[\frac{M_k}{M_j} \right]^{1/4} \right\}^2}{\sqrt{2} \left[1 + \frac{M_j}{M_k} \right]^{1/2}} \quad (18)$$

Several simple relations, such as the molal average viscosity and fluidity, were also evaluated.

Evaluations were made by comparing calculated and experimental results reported in the literature for methane-ethane,²⁰ methane-propane,^{20,21} ethane-propane,²⁰ methane-carbon dioxide,²² propane-carbon dioxide,²⁰ and carbon dioxide-nitrogen^{23,24} mixtures. Pure component viscosities were calculated from the Stiel and Thodos correlation.¹⁰

Comparisons of results showed that the simple relations and Eq. 16 were consistently inferior. The proper choice between Eqs. 7 and 17 was not so clear. Table 1 was prepared to summarize the situation and shows why Eq. 7 was selected.

Before the development of Table 1, a favored predisposition toward Eq. 17 seemed plausible. Eq. 7 is wholly empirical, while Eq. 17 follows from consideration of molecular mechanics. Table 1 shows, however, that Eq. 7 is as good or better than Eq. 17. From a theoretical viewpoint, the meaning of the comparative evaluation is diluted by the fact that calculated pure component viscosities¹⁰ were used. From the practical viewpoint, this is not a consideration. A final practical consideration in selecting Eq. 7 was that Eq. 17 required about 10 times more computation time.

THE VISCOSITY RATIO CHART

Baron, Roof and Wells¹² cite the several viscosity ratio charts which have been published. The charts are similar, but moderately small differences exist. The Baron, Roof and Wells chart was tentatively selected and used because it covered the necessary ranges of reduced temperature and pressure, could be read with sufficient resolution, and described a complete rectangular array of

*Stiel and Thodos¹⁰ report an average absolute deviation of slightly less than 2 per cent for their correlation.

TABLE 1—SUMMARY OF COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS OF THE VISCOSITY OF GAS MIXTURES AT LOW PRESSURES

	No. of Points	Eq. 7	Eq. 17
HYDROCARBONS ONLY	49		
Average Absolute Deviation, %		1.17	1.44
Bias, %		+0.82	+0.37
MIXTURES CONTAINING A NONHYDROCARBON	26		
Average Absolute Deviation, %		9.88	10.07
Bias, %		-4.09	-5.58
TOTAL	75		
Average Absolute Deviation, %		4.19	4.43
Bias, %		-0.88	-1.69

viscosity ratios. Subsequent successful use showed that only a very minor improvement from any other chart could be expected.

LIQUID DENSITY CALCULATION

The method of Alani and Kennedy¹³ is used without modification except for the nonhydrocarbon components. Some newly determined constants are used for hydrogen sulfide, nitrogen, and carbon dioxide. The constants and the source of the pure component liquid density data from which they were derived are given in Table 2. Use of these constants should permit improved liquid density predictions for reservoir mixtures in which these nonhydrocarbons are present.

REDUCED DENSITY AND VISCOSITY PARAMETER FOR MIXTURES

Eq. 14 was developed for pure components. Its use for mixtures required the definition of suitable procedures to calculate the reduced densities and viscosity parameters.

The combination of Eqs. 11 and 12 for the reduced density and Eq. 15 for the viscosity parameter was chosen after considering various alternatives. The definitions of the pseudocritical volume $V_{c'}$ considered are given in the following general equation:

$$V_{c'}^m = \sum_{j=1}^n (x_j V_{c_j}^m) + x_{C_{7+}} V_{c_{C_{7+}}}^m \quad (19)$$

The exponents m in Eq. 19 which were considered were -3, -2, -1, 1 (Eq. 12), 2 and 3. The mixture parameter definitions considered were:

$$\zeta = \frac{\sum_{j=1}^n (x_j T_{c_j}^{1/6})}{\sum_{j=1}^n (x_j M_j^{1/2}) \sum_{j=1}^n (x_j p_{c_j}^{2/3})} \quad (20)$$

and

$$\zeta = \sum_{j=1}^n (x_j \zeta_j) \quad (21)$$

in addition to Eq. 15.

Data for methane-propane³¹ and methane-n-butane²⁸ mixtures were used to prepare plots of $(\mu - \mu^*)\zeta$ vs ρ_r using the various definitions of $V_{c'}$ and ζ . Experimental densities and low-pressure gas viscosities were used. The definitions of ζ and $V_{c'}$ used in the calculation procedures were selected because this combination gave the most uniform plot of $(\mu - \mu^*)\zeta$ with ρ_r for all mixtures of the data considered.

CRITICAL VOLUME OF HEPATANES-PLUS FRACTION

Eq. 13 was developed from experimental viscosity data of reservoir fluids at their bubble points. The same 260 reservoir fluid analyses subsequently used to evaluate the calculation procedures for liquids were used. For each analysis, the value of $V_{c_{C_{7+}}}$ in Eq. 12 which forced exact agreement with the data was calculated. Regression analysis was used to correlate these values of $V_{c_{C_{7+}}}$ with the average molecular weight and specific gravity of the C_{7+} fraction. Of the various equations tried Eq. 13 appeared most suitable. The final Eq. 13 is the result of regression analysis on 236 of the original 260 data points. Twenty-four points were beyond the 99 per cent confidence limit

and excluded from the regression analysis. The standard deviation of Eq. 13 was ± 2.2 per cent.

Considering the development of Eq. 13, it is natural to explore the agreement of Eq. 13 with the critical volumes of pure hydrocarbons which make up C_{7+} fractions. Comparison shows that the available¹⁷ critical volumes and Eq. 13 have the same trends with the molecular weight and specific gravity, but any sort of quantitative agreement cannot be claimed. Without further development and verification, Eq. 13 must be restricted to use in these calculation procedures for viscosity prediction.

EVALUATION OF THE CALCULATION PROCEDURES

Comparisons of calculated results and experimental gas and liquid viscosity data were made for final evaluation of the calculation procedures.

Table 3 summarizes the results for gases. For the 300 points considered, the average absolute deviation was ± 4.03 per cent. Not unexpectedly, there is a tendency for the deviations to increase near the critical point. Actually, evaluation with experimental viscosity data of mixtures of reservoir gases would be preferable. Such data do not appear to be available, however.

Viscosity data for reservoir liquids were available, however, and were used to evaluate this part of the calculation procedures. Two viscosity data points, one at the bubble point and one at a higher pressure, were selected from each of 260 reservoir fluid analyses. All major domestic oil-producing areas except Pennsylvania were included in this group of analyses.

The viscosity was calculated for each of these 520 points using the procedures. A second viscosity calculation was performed using experimental density data instead of the Alani and Kennedy method.¹³ The results of both viscosity calculations were compared with the experimental values. Four 2-point sets were beyond the 99 per cent confidence interval for both viscosity calculations. Not considering these eight points, the average absolute deviation for 512 points was ± 16 per cent when calculated¹³ liquid densities were used.

For 6 two-point sets, the viscosity calculations using experimental densities were in very poor agreement with experimental viscosity data. Deviations were as high as 476 per cent. For the same data, agreement was excellent when calculated¹³ liquid densities were used. This indicates inaccurate liquid density data. Not considering these data, the average absolute deviation for 500 points was ± 12.6 per cent when experimental liquid densities were used.

Table 4 gives the ranges of the important variables from the 260 analyses used for evaluation. The reservoir liquids included ranged from black to highly volatile oils, so the calculation procedures are generally applicable. Further, the evaluation shows that treating moderately polar hydrogen sulfide as a non-polar component introduces no restrictions in the concentration range studied.

The evaluations, both for gases and liquids, show the calculation procedures predict viscosities well within acceptable limits of error for this study.

TABLE 2—ALANI-KENNEDY CONSTANTS FOR NONHYDROCARBONS

	Alani-Kennedy Constant			
	k	n	m	c
Hydrogen Sulfide ²⁶	13,200	0	0.00179	0.3945
Nitrogen ²⁸	4,300	2.293	0.000449	0.3853
Carbon Dioxide ²⁷	8,166	126	0.0001818	0.3872

TABLE 3—SUMMARY OF THE EVALUATION OF THE CALCULATION PROCEDURES FOR GAS MIXTURES UNDER PRESSURE

Mixture	Temp. Range (°F)	Press. Range (psia)	No. of Points	Average Absolute Deviation (%)
Methane-Propane ²¹	77-437	400- 5,000	147	4.99
Natural Gases ²⁹	70-220	200-10,000	96	2.1
Methane-n-Butane ²⁸	70-220	540- 7,000	45	5.2
Nitrogen-Carbon Dioxide ²³	68	65- 300	12	3.28
All Mixtures			300	4.03

TABLE 4—RANGES OF VARIABLES IN THE 260 ANALYSES USED FOR EVALUATING THE CALCULATION PROCEDURES FOR RESERVOIR OILS

Methane mole fraction	0.0002 to 0.6193
C ₂ + mole fraction	0.0163 to 0.9798
C ₂ + specific gravity	0.7784 to 0.9282
C ₂ + average molecular weight	to 338
Hydrogen sulfide mole fraction	to 0.057
Nitrogen mole fraction	to 0.069
Carbon dioxide mole fraction	to 0.0396
Temperature (°F)	73 to 262
Bubble point (psia)	40 to 5555
Pressure (psia)	to 7015
Experimental density gm/cc	0.4812 to 0.903

USE OF THE PROCEDURES

These procedures were specifically devised to use as part of a compositional material balance method. Their use, of course, is not restricted to this application alone. A pertinent example of another application is the prediction of the viscosity decrease caused by the addition of solvents in reservoir oils.

Figs. 2 and 3 show the results of such an application with experimental data for comparison. The data on Figs. 2 and 3 are experimental densities and viscosities. Data at 4,000 psig and at the saturation pressure at three different temperature levels for a reservoir oil and its mixtures with various amounts of propane are presented. The dashed curves on Fig. 2 show the calculated densities which were used to calculate the dashed curve for the viscosities on Fig. 3. The average absolute deviation of these calculated and the experimental densities and viscosities is 2.1 and 9.7 per cent, respectively.

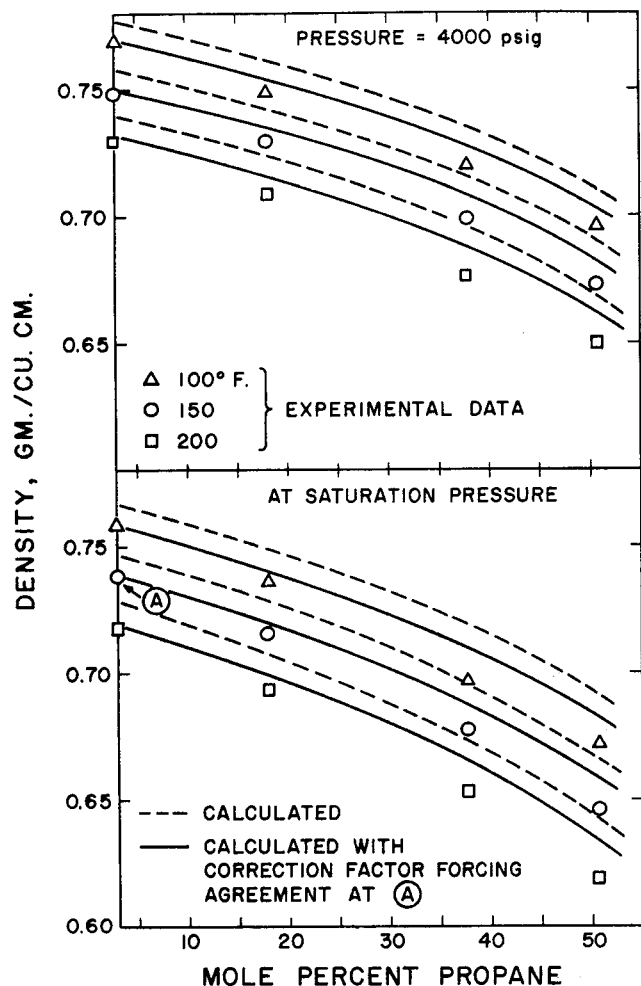


FIG. 2—CALCULATED AND EXPERIMENTAL DENSITIES OF A RESERVOIR OIL AND ITS MIXTURES WITH PROPANE.

In applications of this type, frequently the properties of the reservoir oil as it exists in the reservoirs without solvent additions will be available. One would like to have the calculations show exact agreement with the known data—to have the calculations “forced” through the reference point. The solid curves on Figs. 2 and 3 show the results of calculations forced to agree with the experimental data at one point. The data point chosen as a reference was the reservoir oil without propane additions under saturation at 150F. Forcing is done by multiplying a correction factor to the “b” constant of the Alani and Kennedy method for the density and another correction factor to the pseudocritical volume, V_c' for the viscosity. The forcing procedures reduced the average absolute deviation for the density and viscosity to 1 and 5.3 per cent, respectively, excluding the reference data point.

ALTERNATE PROCEDURES

As Fig. 1 shows, these calculation procedures involve

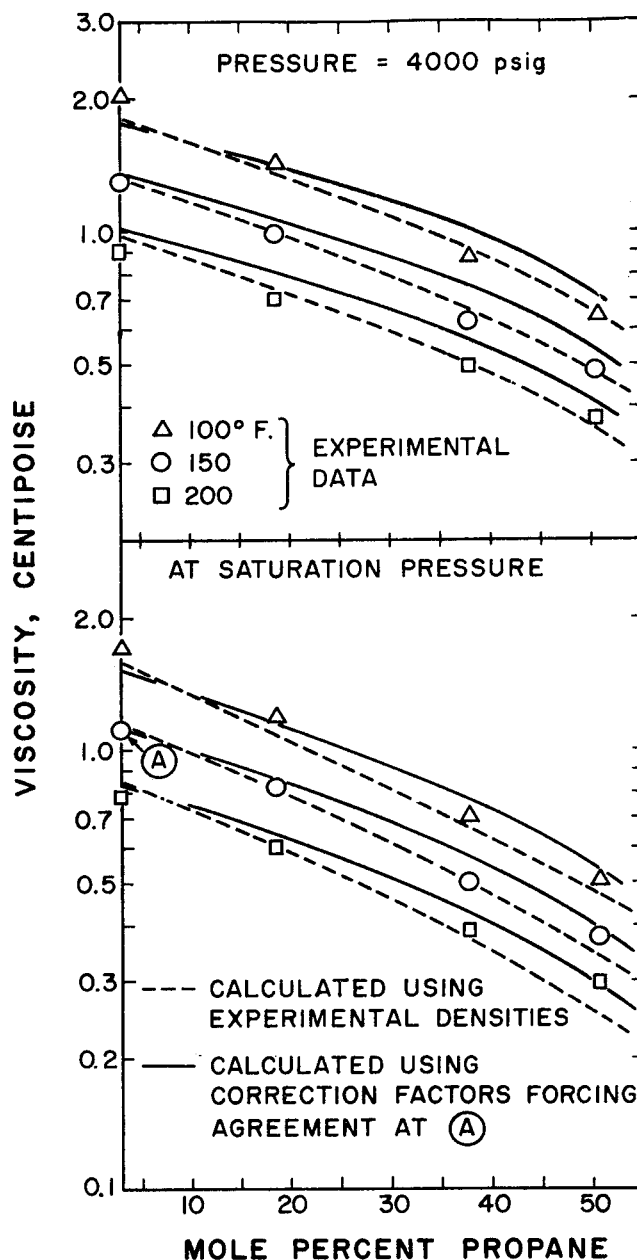


FIG. 3—CALCULATED AND EXPERIMENTAL VISCOSITIES OF A RESERVOIR OIL AND ITS MIXTURES WITH PROPANE.

steps, any one of which could be changed within the framework of the present procedure. Since these procedures were devised, some new techniques which could simplify portions of the procedures have been given.

Lee and Eakin³² and Dean and Stiel³³ have developed methods which would simplify the calculation of the low pressure viscosity of the gas mixture, μ^* . Lee and Eakin give an equation for μ^* as a function of the average molecular weight and the temperature. Dean and Stiel give equations for μ^* similar to Eq. 4A and 4B except that the viscosity parameter ζ_j and the reduced temperature T_r are defined for mixtures. Both of these methods should be somewhat more accurate and are much easier to use than the present procedure to calculate μ^* .

Other investigators^{31,33} have used different definitions of the mixture rules embodied in Eqs. 8, 9, 12 and 15; however, Dean and Stiel found no gross advantages occurred from any particular mixture rule they studied. The mixture rules used in the procedures of this paper are the simplest and are adequate.

Finally, it is readily apparent that gas viscosity predictions could be made using exactly the same procedures as for liquid viscosities. Only the lack of a gas density prediction method of proven reliability for reservoir gases prevented this.

NOMENCLATURE

- A, B = constants in Eq. 2
 F = general function
 $G_{C_{7+}}$ = specific gravity of the C_{7+} fraction
 M_j = molecular weight of component j
 $M_{C_{7+}}$ = average molecular weight of the C_{7+} fraction
 m = exponent in Eq. 19
 n = number of components in the mixture
 p = pressure
 $p_{c,j}$ = critical pressure of component j
 p_r = pseudoreduced pressure of a mixture
 T = absolute temperature
 $T_{c,j}$ = absolute critical temperature of component j
 T_r = pseudoreduced temperature of a mixture
 $T_{r,j}$ = reduced temperature of component j
 V_c' = pseudocritical volume of a mixture, cu ft/lb-mole
 $V_{c,j}$ = critical volume of component j , cu ft/lb-mole
 $V_{c,C_{7+}}$ = critical volume of the C_{7+} fraction as given by Eq. 13, cu ft/lb-mole
 $x_{C_{7+}}$ = mole fraction of the C_{7+} fraction
 x_j = mole fraction of component j
 μ = viscosity, cp
 μ^* = viscosity of a gas mixture at low pressure, cp
 μ_j^* = viscosity of component j as a gas at low pressure, cp
 ζ = viscosity parameter of a mixture, defined by Eq. 15
 ζ_j = viscosity parameter of component j , defined by Eq. 6
 ρ = density, lb-mole/cu ft
 ρ_c' = pseudocritical density of a mixture, lb-mole/cu ft
 ρ_r = reduced density
 ϕ_{jk} = function defined by Eq. 18

REFERENCES

1. Beal, Carlton: "The Viscosity of Air, Water, Natural Gas,

- Crude Oil, and Its Associated Gases at Oil Field Temperatures and Pressures", *Trans., AIME* (1946) **165**, 94.
2. Standing, M. B.: "Oil-System Correlations", *Petroleum Production Handbook*, McGraw-Hill Book Co., New York (1962).
3. Chew, Ju-Nam and Connally, C. H., Jr.: "A Viscosity Correlation for Gas-Saturated Crude Oils", *Trans., AIME* (1959) **216**, 23.
4. Carr, N. L., Kobayashi, Riki and Burrows, D. B.: "Viscosity of Hydrocarbon Gases Under Pressure", *Trans., AIME* (1954) **201**, 270.
5. Katz, D. L., et al.: *Handbook of Natural Gas Engineering*, McGraw-Hill Book Co., New York (1959) Chap. 4.
6. Brinkman, F. H. and Weinaug, C. F.: "Calculated Performance of a Dissolved-Gas Drive Reservoir by the Phase Behavior Method", *AIChE Jour.* (1957) **3**, 29.
7. Jacoby, R. H. and Berry, V. J., Jr.: "A Method for Predicting Depletion Performance of a Reservoir Producing Volatile Crude Oil", *Trans., AIME* (1957) **210**, 27.
8. Jacoby, R. H. and Berry, V. J., Jr.: "A Method for Predicting Pressure Maintenance Performance for Reservoirs Producing Volatile Crude Oil", *Trans., AIME* (1958) **213**, 59.
9. Lohrenz, John, Clark, G. C. and Francis, R. J.: "A Compositional Material Balance for Combination Drive Reservoirs with Gas and Water Injection", *Jour. Pet. Tech.* (Nov., 1963) 1233.
10. Stiel, L. I. and Thodos, George: "The Viscosity of Nonpolar Gases at Normal Pressures", *AIChE Jour.* (1961) **7**, 611.
11. Herning, F. and Zipperer, L.: "Calculation of the Viscosity of Technical Gas Mixtures from the Viscosity of Individual Gases", *Gas u. Wasserfach* (1936) **79**, No. 49, 69.
12. Baron, J. D., Roof, J. G. and Wells, F. W.: "Viscosity of Nitrogen, Methane, Ethane and Propane at Elevated Temperature and Pressure", *Jour. Chem. Eng. Data* (1959) **4**, 283.
13. Alani, G. H. and Kennedy, H. T.: "Volume of Liquid Hydrocarbons at High Temperatures and Pressures", *Trans., AIME* (1960) **219**, 288.
14. Jossi, J. A., Stiel, L. I. and Thodos, George: "The Viscosity of Pure Substances in the Dense Gaseous and Liquid Phases", *AIChE Jour.* (1962) **8**, 59.
15. Lohrenz, John and Bray, B. G.: "The Calculation of Bubble Points of Reservoir Fluids from Their Compositions", SPE 792, presented at the Rocky Mountain SPE Meeting, Casper (May 25-26, 1964).
16. Thodos, George: "Critical Constants of Saturated Aliphatic Hydrocarbons", *AIChE Jour.* (1955) **1**, 168.
17. Rossini, F. D. et al.: *Selected Values of Thermodynamic Properties of Hydrocarbons and Related Compounds*, Carnegie Press, Pittsburgh.
18. *Engineering Data Book*, Nat. Gas Supply Men's Assoc., Tulsa (1957).
19. Wilke, C. R.: "A Viscosity Equation for Gas Mixtures", *Jour. Chem. Phys.* (1950) **18**, 517.
20. Hirschfelder, J. O., Bird, R. B. and Spotz, E. L.: "The Transport Properties of Gases and Gaseous Mixtures", *Chem. Rev.* (1949) **44**, 205.
21. Bicher, L. B., Jr. and Katz, D. L.: "Viscosities of the Methane-Propane System", *Ind. Eng. Chem.* (1943) **35**, 754.
22. Jackson, W. M.: "Viscosities of the Binary Gas Mixtures. Methane-Carbon Dioxide and Ethylene-Argon", *J. Phys. Chem.* (1956) **60**, 789.
23. Kestin, J. and Leidenfrost, W.: "The Effect of Pressure on the Viscosity of N_2 - CO_2 Mixtures", *Physica* (1959) **25**, 525.
24. Kenney, M. J., Sarjant, R. J. and Thring, M. W.: "The Viscosity of Mixtures of Gases at High Temperatures", *Brit. J. Appl. Phys.* (1956) **1**, 324.
25. Reamer, H. H., Sage, B. H. and Lacey, W. N.: "Volumetric Behavior of Hydrogen Sulfide", *Ind. Eng. Chem.* (1950) **42**, 140.
26. Van Itterbeek, A. and Verbeke, O.: "Density of Liquid Nitrogen and Argon as a Function of Pressure and Temperature", *Physica* (1960) **26**, 931.
27. Newitt, D. M., Pai, M. U., Kuloor, N. R. and Huggil, J. A. W.: "Carbon Dioxide", *Therm. Func. Gases* (1962) **1**, 102.
28. Dolan, J. P., Ellington, R. T. and Lee, A. L.: "Viscosity of Methane-n-Butane Mixtures", to be published in *Jour. Chem. Eng. Data*.
29. Carr, N. L.: "Viscosities of Natural Gas Components and Mixtures", *Res. Bull., Inst. Gas. Tech.* (1953) No. 23.
30. Lee, A. L., Starling, K. E., Dolan, J. P. and Ellington, R. T.: "Viscosity Correlation for Light Hydrocarbon Systems", to be published in *AIChE Jour.*
31. Giddings, J. G.: Ph.D. Thesis, Rice U. (1963).
32. Lee, A. L. and Eakin, B. E.: "Gas-phase Viscosity of Hydrocarbon Mixtures", *Soc. Pet. Eng. Jour.* (Sept., 1964) 247.
33. Dean, D. E. and Stiel, L. I.: "The Viscosity of Nonpolar Gas Mixtures at Moderate and High Pressures", paper submitted to *AIChE Jour.* ★★★