The Modeling of Viscous Dissipation in a Saturated Porous Medium

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A critical review is made of recent studies of the modeling of viscous dissipation in a saturated porous medium, with applications to either forced convection or natural convection. Alternative forms of the viscous dissipation function are discussed. Limitations to the concept of fully developed convection are noted. Special attention is focused on the roles of viscous dissipation and work done by pressure forces (flow work) in natural convection in a two-dimensional box with either lateral or bottom heating. [DOI: 10.1115/1.2755069]

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1 Introduction

An examination of the results of the survey presented by Nield and Bejan [1] reveals that in the past few years, there has been a flurry of activity on investigations of the role of viscous dissipation in convection in a porous medium. Some 30 papers on this topic bear publication dates of 2000 or later, while prior to 2000, only a handful of such papers were published. Some of this work is routine, but much is of fundamental importance and several aspects are controversial. Of particular interest are the asymptotic boundary layers and backward boundary layers that exist with natural and mixed convection past vertical walls [2–9], and the changes in fully developed and developing forced convection in channels and ducts [10–16] that result from the viscous dissipation. In this note, various aspects of this work are discussed in turn.

2 Modeling the Viscous Dissipation Term

A prime matter for discussion is the form of the mathematical expression for representing the viscous dissipation in the thermal energy equation. This expression depends on the way in which the drag force is modeled in the momentum equation, which following Eq. (1.18) of Ref. [1] (with a typographical error corrected) is written as

$$\rho\left[\frac{1}{\phi}\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{\phi}\nabla\left(\frac{\mathbf{v}\cdot\mathbf{v}}{\phi}\right)\right]=-\nabla P+\mu_{\mathrm{eff}}\nabla^2\mathbf{v}-\frac{\mu}{K}\mathbf{v}-\frac{c_F\rho}{K^{1/2}}|\mathbf{v}|\mathbf{v} \tag{1}$$

Here, $\mathbf{v}$ is the Darcy velocity, P is the fluid pressure, ρ and μ are the fluid density and viscosity, μ_{eff} is the effective viscosity, and ϕ, K, and c_F are the porosity, permeability, and a Forchheimer coefficient of the porous medium. (Some authors prefer to use $C=c_F/K^{1/2}$. However, c_F is preferred here because it is dimensionless and depends only weakly on the geometry of the porous medium and on the Reynolds number of the flow.) The last three terms of Eq. (1) are the Brinkman, Darcy, and Forchheimer drag terms.

The standard thermal energy equation (compare Eq. (2.3) of Ref. [1]) with local thermal equilibrium assumed and with no energy source term is

$$(\rho c)_m\frac{\partial T}{\partial t}+(\rho c)_f\mathbf{v}\cdot\nabla T=\nabla\cdot(k_m\nabla T) \tag{2}$$

Here, T is the temperature, $(\rho c)_m$ is the heat capacity (per unit volume) of the porous medium, $(\rho c)_f$ is the heat capacity of the porous fluid, and k_m is the thermal conductivity of the porous medium. It appears that Ene and Sanchez-Palencia [17] were the first to introduce an extra term Φ on the right-hand side of Eq. (2) to account for the effect of viscous dissipation, and if the fluid is incompressible, the porous medium is isotropic and Darcy's law holds then

$$\Phi=(\mu/K)\mathbf{v}\cdot\mathbf{v} \tag{3}$$

To see this, note that the average of the rate of doing work by the pressure, on a unit volume of a representative elementary volume (REV), is given by the negative of $\nabla\cdot(P\mathbf{v})=\mathbf{v}\cdot\nabla P$ since $\nabla\cdot\mathbf{v}=0$. Thus, Eq. (3) tells us that in this case, Φ is the power of the drag force. In fact, an argument from first principles given by Nield [18] indicates that Φ should remain equal to the power of the total drag force in more general situations. This means that the Forchheimer drag term should contribute an amount $(c_F\rho/K^{1/2})|\mathbf{v}|\mathbf{v}\cdot\mathbf{v}$ to the viscous dissipation Φ despite the fact that the viscosity μ does not appear explicitly in this expression. The apparent anomaly was resolved by Nield [18] who pointed out that the Forchheimer drag term models essentially a form-drag effect and involves the separation of boundary layers and wake formation behind solid obstacles on the pore scale. The pore scale convective inertial effects contributing to the form drag lead to a substantial modification of the velocity field and, in particular, to an enlargement of the macroscopic region in which pore scale velocity gradients are large. This leads to an increase in the total viscous dissipation (summed over the whole region occupied by fluid) and hence, because of the fundamental equality of viscous dissipation (within a volume) and the power of the drag force (on that volume) to the increase in the drag. Viscosity acts throughout the fluid, and not just at the solid boundaries. The Forchheimer form-drag term arises from the action of viscosity, mediated by the inertial effects affecting the distribution of pressure that also contributes to stress at the solid boundaries. Nield [18] also pointed out that once a division is made into a term linear in the velocity, and one quadratic in the velocity, it is then inevitable, on dimensional grounds, that the Darcy term will appear with the viscosity as a coefficient, whereas the Forchheimer term will not explicitly involve the viscosity. Pertinent to this view of form-drag effects is the work of Narasimhan and Lage (summarized in Ref. [19]) on temperature-dependent viscosity. This work clearly shows changes of dynamic viscosity affecting the total drag, even though the form drag does not carry any explicit relation to the viscosity. As far as the present author is aware, there is now a consensus on how the Darcy and Forchheimer contributions should be modeled. However, how the Brinkman contribution should be modeled is a matter of current controversy. Nield [18] argued that the Brinkman contribution should be the power of the Brinkman drag term, so that the combined Darcy and Brinkman contributions are given by

$$\Phi=(\mu/K)\mathbf{v}\cdot\mathbf{v}-\mu_{\mathrm{eff}}\mathbf{v}\cdot\nabla^2\mathbf{v} \tag{4}$$

In the case of unidirectional flow in a circular tube, this takes the form

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$$\Phi = \frac{\mu u^2}{K} - \mu_{\text{eff}} u \left(\frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr} \right) \tag{5}$$

On the other hand, Al-Hadhrami et al. [20] proposed a form which is compatible with an expression derived from the Navier-Stokes equation for a fluid clear of solid material, so that instead of Eq. (5) they would set

$$\Phi = \frac{\mu u^2}{K} + \mu \left(\frac{du}{dr} \right)^2 \tag{6}$$

They correctly noted that in the case where the Darcy number $\mathrm{Da} \to \infty$ (where Da is defined as K/L^2, where L is the characteristic length scale), the expression in Eq. (5) becomes negative and so is physically unrealistic. However, it is the opinion of the present author (expressed in Nield [21]) that the Brinkman equation (with a term involving a constant coefficient μ_{eff}) itself is expected to break down as $\mathrm{Da} \to \infty$. Consequently, one should regard a change in sign of the expression in Eq. (5) as just a warning that one is in uncharted waters. The present author is of the opinion that the equality of the viscous dissipation and the power of the drag, when volume averages are taken, is a fundamental principle. The principle is valid for a Darcy flow. It is also valid for a fluid clear of solid material. It is true that this is not immediately obvious. The argument involves a mathematical identity involving derivatives. When averaging is done over a representative elementary volume, information about these derivatives is lost. This is why for finite values of the Darcy number, Eq. (6) is not consistent with the Brinkman equation. Thus, the situation is that we have two expressions for Φ, one (Eq. (5)) that is based on the Brinkman drag, consistent with the power-of-drag principle but is invalid for infinite Da, and a second one (Eq. (6)) that is valid at infinite Da, consistent with the power-of-drag principle at infinite Da but is not consistent with that principle at finite Da if the Brinkman drag is employed. That inconsistency means that the Brinkman drag assumption is not uniformly valid. The present author concludes that Eq. (6) is not properly based on the Brinkman equation. Rather, it is an ad hoc formula, based on two separate versions of the momentum equation (Darcy and Navier-Stokes), with each version valid for a different Da range.

In their paper on thermally developing forced convection in a circular duct, Nield et al. [12] and Kuznetsov et al. [22] obtained values of the Nusselt number for each of the models of Nield and Al-Hadhrami et al., so in principle these results provide a basis for an experimental test between the two models. However, the differences are significant only at high values of Da and Pe (the Peclet number) and achieving these values in an experiment is likely to be difficult.

Likewise, it should be possible in principle to test whether the Brinkman equation (with a term as written in Eq. (1)) breaks down at large values of the Darcy number by performing numerical simulations of flow past a sparse array of circular cylinders, but again there are practical difficulties. Some preliminary work has been done by Gerritsen et al. [23] that suggests that the Brinkman equation is indeed not uniformly valid as the porosity tends to unity.

3 Scaling Considerations

An important practical consideration is the a priori estimation of whether or not viscous dissipation is important in a particular case. Nield [18] noted that scale analysis, involving the comparison of the magnitude of the viscous dissipation term to the thermal diffusion term, shows that viscous dissipation is negligible if $N_v \ll 1$, where

$$N_v = \frac{\mu U^2 L^2}{K c_P k_m \Delta T} = \frac{\mathrm{Br}}{\mathrm{Da}} \tag{7}$$

where the Darcy number $\mathrm{Da} = K/L^2$ and the Brinkman number Br is defined by

$$\mathrm{Br} = \frac{\mu U^2}{c_P k_m \Delta T} = \mathrm{EcPr} \tag{8}$$

and in turn the Prandtl number Pr and the Eckert number Ec are defined by

$$\mathrm{Pr} = \frac{\mu/\rho}{k_m/(\rho c_P)_f} \tag{9a}$$

$$\mathrm{Ec} = \frac{U^2}{c_P \Delta T} \tag{9b}$$

Here, c_P is the specific heat at constant pressure of the fluid, k_m is the effective thermal conductivity of the porous medium, and U, L, and ΔT are characteristic velocity, length, and temperature scales, respectively. For most situations, the Darcy number is small, so viscous dissipation is important at even modest values of the Brinkman number. These comments have been made on the assumption that the Peclet number Pe is not large, where Pe is defined by

$$\mathrm{Pe} = \frac{(\rho c_P)_f U L}{k_m} \tag{10}$$

If it is large, then the proper comparison is the one between the magnitudes of the viscous dissipation term and the convective transport term. This ratio is of order Ec/DaRe, where the Reynolds number $\mathrm{Re} = \rho U L/\mu$.

For forced convection, the choice of the characteristic velocity is obvious (some mean or maximum value of the forced velocity). However, for natural convection, more care is needed. For the general situation, scale analysis (such as that presented in Sec. 4.3 of Ref. [24]) leads to the estimate

$$U \sim [k_m/(\rho c_P)_f L] \mathrm{Ra}_D^{1/2} \tag{11}$$

and the condition that viscous dissipation becomes negligible becomes $\mathrm{Ge} \ll 1$, where Ge is the Gebhart number defined by

$$\mathrm{Ge} = \frac{g\beta L}{c_P} \tag{12}$$

and Ra_D is the Rayleigh-Darcy number defined by

$$\mathrm{Ra}_D = \frac{\rho g \beta K L \Delta T}{\mu [k_m/(\rho c_P)_f]} \tag{13}$$

The topic of scaling is discussed further in Sec. 6.

4 Forced Convection Problem

When the effect of viscous dissipation is introduced into the classical forced convection problem, some interesting complications arise. Some of these have been discussed by Nield [25]. It is customary to define a Brinkman number Br (like the Nusselt number) in terms of the difference between a bulk mean temperature T_m and a wall temperature T_w. In the case of uniform flux boundaries (the H boundary condition), Br can be readily treated as a constant for the fully developed problem. However, for the uniform-temperature boundaries (the T boundary condition), T_w is a constant but T_m depends on x, and so Br depends on x. One response is to freeze T_m at its value for some representative value of x when determining the value of Br. An alternative response is to declare that in this situation, fully developed flow cannot be treated in isolation but only as a limiting case of the solution of a thermally developing problem with a specified inlet temperature that can be used to define Br.

There are other complications. In the presence of volumetric heating due to the viscous dissipation, and with the wall heat flux depending on the axial coordinate for the case of the T boundary condition, one has to explicitly satisfy the first law of thermodynamics instead of assuming that integration of the local thermal energy equation leads to satisfaction of this law. A further peculiarity with the T boundary condition is that the Nusselt number as usually defined takes one value when Br=0 and another value when Br$\neq$0, no matter how small. It appears that the existence of this jump means that the concept of fully developed convection is of limited utility in the situation where viscous dissipation is important.

The situation is further complicated when the viscosity of the liquid varies with temperature. For the T boundary condition with Br=0, the situation is relatively simple, because then the temperature is tending to the wall value throughout the medium as one moves axially downstream, and consequently the Nusselt number tends to that for the constant viscosity case. In other situations, the temperature continually changes as the axial coordinate increases and there is no truly fully developed value of the Nusselt number in a general situation. The effect of viscous dissipation then leads to a new situation that is worthy of further investigation.

The effect of flow work (work done by pressure forces) as well as viscous dissipation was examined by Nield [21]. The magnitudes of these effects are governed by independent parameters, whose ratio involves a parameter N_d defined by

$$N_d = \frac{GK}{\mu U} \tag{14}$$

where G is the magnitude of the applied pressure gradient.

5 Natural Convection Problem

The problem of convection in a two-dimensional laterally heated square cavity (with thermally insulated top and bottom) is of great interest to the computational fluid dynamics community (e.g., see de Vahl Davis [26]) as well as being applicable to a wide variety of practical problems. The case of a porous medium has been discussed by Costa [27]. The corresponding problem with a clear fluid was treated by Costa [28] and Pons and le Quéré [29,30].

Costa [27] concluded that

The unique energy formulation compatible with the first law of thermodynamics informs us that if the viscous dissipation term is taken into account, also the work of pressure forces term needs to be taken into account. In integral terms, the work of pressure forces must equal the energy dissipated by viscous effects, and the net energy generation must be zero. If only the (positive) viscous dissipation term is considered in the energy conservation equation, the domain behaves like a heat multiplier, with a heat output greater than the heat input. …[T]he main ideas and conclusions apply equally for any general natural or mixed convection heat transfer problem.

It is now argued that the statement in the last sentence is incorrect. Rather, Costa's conclusion is specific to natural convection in a laterally heated box and does not apply generally. The argument involves a close examination of the Boussinesq approximation, and to this we now turn.

6 Boussinesq Approximation

A systematic discussion of the validity of the Boussinesq approximation for liquids and gases has been made by Gray and Giorgini [31]. They expressed their result using a velocity scale $(\beta g L \Delta T)^{1/2}$ in the present notation. Using this velocity scale, and the corresponding time scale, the thermal energy equation (Eq. (32) of Ref. [31]) (that represents the thermal energy equation in what they call the extended Boussinesq approximation and what Pons and le Quéré [30] call the thermodynamic Boussinesq equation) can be written as

$$\frac{DT}{Dt} = \frac{1}{(\mathrm{PrRa})^{1/2}}\nabla^2 T - \mathrm{Ge}\frac{T_0}{\Delta T}w + \mathrm{Ge}\left(\frac{\mathrm{Pr}}{\mathrm{Ra}}\right)^{1/2}\Phi \tag{15}$$

Here, the variables are all dimensionless: they are the temperature T, time t, spatial coordinates x, y, and z, vertical velocity component w, and viscous dissipation function Φ. The new dimensionless parameters are the fluid Rayleigh number Ra and the temperature ratio $T_0/\Delta T$, where Ra is defined as

$$\mathrm{Ra} = \frac{\beta g L^3 \Delta T}{\kappa \nu} \tag{16}$$

where β is the isobaric coefficient of volumetric thermal expansion, L is a characteristic length scale that we can take as the height of the enclosure, ΔT is a temperature difference that we can take as $T_h - T_c$, where T_c and T_h are, respectively, the cold and hot wall temperatures, and T_0 is a representative absolute temperature that we can take as the arithmetic mean of T_c and T_h, while κ is the thermal diffusivity and ν is the kinematic viscosity of the fluid. The coefficient of w in Eq. (15), namely, $\mathrm{Ge}T_0/\Delta T$, is Pons and le Quéré's [30] parameter ϕ.

The last term in Eq. (15) represents the effect of viscous dissipation and the second to last term is the largest contribution from the work done by pressure forces (flow work). It is clear that both of these terms will be negligible if Ge is sufficiently small. It is also clear that the ratio of flow work term to the viscous dissipation term is of order $(T_0/\Delta T)(\mathrm{Ra}/\mathrm{Pr})^{1/2}$ and usually this will be large, even for modest values of Ra/Pr, since $T_0/\Delta T$ will commonly be quite large. Thus, in the general situation, one cannot ignore the flow work when the viscous dissipation is important. However, when $(\mathrm{Ra}/\mathrm{Pr})^{1/2}$ is small in comparison with $\Delta T/T_0$, it is the viscous dissipation term that is more important. It is emphasized that the above conclusion applies only when Eq. (15) is applicable. For the case of a liquid, this equation will usually be applicable, but in the case of a gas, another term due to the work of pressure forces, one involving the product of Ge and the parameter $\beta\Delta T$, becomes significant. For air, the Prandtl number equals 0.71 and so the effective Prandtl number Pr is of order unity or smaller, and in a typical CFD calculation, Ra could be 10^6 (or larger) and $T_0/\Delta T$ could be 10 (or larger). In these circumstances, the viscous dissipation will be negligible compared with the flow work.

We interpolate the remark that if all terms are retained in an equation, then altering the velocity scale does not matter. However, when it comes to deciding which terms are negligible (and hence can be ignored), then it does matter. In the thermal energy equation for a natural convection problem, the viscous dissipation term is quadratic in the velocity, while the major flow work term (involving the product of the pressure with a velocity component) is cubic in the velocity (since the pressure itself is quadratic in the velocity). Hence, the velocity scale is critical in deciding which of viscous dissipation or flow work is playing the major role in given circumstances.

For the case of the laterally heated box, the appropriate velocity scale is that based on the thermal conductivity, namely, κ/L, where κ is the effective thermal diffusivity. This is the scaling assumption made by Costa [27]. Now, in place of Eq. (15), we have

$$\frac{DT}{Dt} = \frac{1}{(\mathrm{PrRa})^{1/2}}(\nabla^2 T - \phi w + \mathrm{Eck}\Phi) \tag{17}$$

where the Eckert number Eck is defined as

$$\mathrm{Eck} = \frac{(\kappa/L)^2}{c_p \Delta T} \tag{18}$$

One can get rid of the parameter Pr in Eq. (17) by incorporating it into the time scale (and the corresponding velocity scale), so that then, one has

$$\frac{DT}{Dt} = \frac{1}{\mathrm{Ra}^{1/2}}(\nabla^2 T - \phi w + \mathrm{Eck}\Phi) \tag{19}$$

There is a fundamental difference between the bottom heating and lateral heating cases, as represented by Eqs. (15) and (19), respectively. For the former case, the characteristic velocity, and hence the ratio of the pressure work and viscous dissipation terms, depends on the intensity of the convective flow as measured by the parameter Ra. In the latter case, this ratio does not depend on the intensity of the convective flow. This conclusion is consistent with Costa's conclusion that in the latter case, the ratio for the integrated terms is, in fact, unity.

It is possible to make a further scaling move and incorporate a factor Ra into the velocity scale, which was done by Pons and le Quéré, ending up with

$$\frac{DT}{Dt} = \frac{1}{\mathrm{Ra}^{1/2}}(\nabla^2 T - \phi \mathrm{Ra}^{1/2} w + \mathrm{Ge}\Phi) \tag{20}$$

which can be compared with Eq. (3) in Ref. [30]. It is the opinion of the present author that this move is ill advised because it complicates the discussion unnecessarily, and the more direct approach of Costa [27] is preferable.

A related study is that by Breugem and Rees [32]. They have explicitly derived the volume-averaged Boussinesq equations for flow in porous media with viscous dissipation for the Brinkman model. They introduced a velocity scale on the assumption that the momentum equation is dominated by the balance between the buoyancy force and the Darcy drag force. This yields the velocity estimate

$$U \sim [k_m/(\rho c_P)_f L]\mathrm{Ra}_D \tag{21}$$

They noted that this scale is only appropriate when $\mathrm{Ra}_D\mathrm{Da}/\mathrm{Pr} \ll 1$ and $F \ll 1$, where F is a Forchheimer parameter. It appears to the present author that this means that this scale is applicable to weak natural convection but is generally not appropriate for natural convection problems where the Rayleigh number is large, when one would expect that the expression given in Eq. (11) would be the appropriate velocity scale.

7 Lateral Heating and Bottom Heating

In this section, the difference between natural convection in a box resulting from uniform lateral heating (the de Vahl Davis problem) and that from uniform heating from below (the Rayleigh-Bénard problem) is discussed. In the first case, the basic temperature gradient is horizontal. In the second case, it is vertical, and it is easy to explain what drives the convection. The rate of release of kinetic energy due to the buoyancy force (per unit volume) is proportional to the product of the vertical velocity component w, and the temperature excess θ and $w\theta$ is positive throughout the fluid region. Indeed, the analysis of Chandrasekhar [33] shows that the onset of convection occurs at the minimum temperature gradient at which a balance can be maintained between the kinetic energy dissipated by viscosity and the internal energy released by the buoyancy force. Thus, one can say that convection resulting from a basic vertical temperature gradient is caused by buoyancy.

However, in the case of a basic horizontal temperature gradient, the case is different. Now, when a fluid particle is displaced vertically, no kinetic energy is released since there is no change in basic temperature in that direction. It is only when the fluid particle is displaced at a nonzero angle to both the vertical and the horizontal that kinetic energy is released. This situation occurs in the corners of the box. If one considers a box with two horizontal walls and two vertical walls, with the left-hand one heated and the right-hand one cooled, then the release of kinetic energy is positive in the top left and bottom right corners but negative in the top right and bottom left corners, and by symmetry, the net effect is zero, at least to first order in the small perturbation variables. Thus, it is a glib statement to say that the natural convection is caused by buoyancy in this case. It is obvious that the situation is more subtle.

The approach of Bejan [24] is to consider the box as a closed system and to argue that the only energy exchanges between the system and its surroundings are the heat fluxes through the hot and cold walls (Q_h, positive, and Q_c, negative) and that the first law of thermodynamics requires that $Q_h+Q_c=0$. It is assumed that gravity plays a passive role and that any additional heating due to viscous dissipation is balanced by work done by pressure forces. As Bejan [24] points out, the cellular flow is the succession of four processes (heating, expansion, cooling, and compression) so that the convection loop is equivalent to the cycle executed by the working fluid in a heat engine. The buoyancy effect due to gravity releases energy that would increase the kinetic energy of the fluid were it not that in a steady state the braking effect of viscosity immediately converts this energy to heat. The net effect of the heat engine is to produce degraded energy in the form of heat. In other words, one has a heat source within the box that has to be taken into account in the energy balance. In other books, Bejan [34,35] discusses a "temperature gap system," in which a would-be Carnot engine has no shaft to the outside but instead dissipates its power, dumping it to one or both of the hot and cold walls. He illustrated this by figures based on the assumption that the system is closed (with respect to mass flow) and isolated and therefore the incoming and outgoing heat fluxes have to be equal.

Thus, the conclusion of Costa [27] is consistent with the explanation of Bejan. Costa's work shows that the following items form a consistent set: (1) zero net release of energy as a result of buoyancy, (2) computations based on equations derived using a diffusive velocity scale, and (3) balancing heat fluxes at the hot and cold boundaries.

However, it should now be clear why his conclusion about the balance between pressure work and viscous dissipation is limited to the case of a laterally heated box. In the case of a bottom heated box, an additional agency, namely, the net kinetic energy released due to the buoyancy force, comes into play. As a result, the velocity scale is changed from a thermal diffusion (conduction) value to one dependent on the Rayleigh number. No longer is there a requirement that the pressure work and viscous dissipation be in balance. Rather, the relative magnitude of these effects depends on the intensity of the convection as measured by the value of the Rayleigh number. For a bottom heated box, the enclosure is closed as far as mass flow is concerned, but energy is exchanged with the outside when uniform temperatures at the bottom and top are maintained at fixed values and the heat fluxes at the bottom and top adjust accordingly. The occurrence of viscous dissipation modifies the energy balance but not in a radical way. The work done by pressure forces also modifies the energy balance but in a way that is not completely tied with the viscous dissipation. For forced convection, the characteristic velocity is again different, and so the roles of pressure work and viscous dissipation are different in this situation, as was noted in Sec. 4 above.

It is worth adding that the result of Chandrasekhar [33] mentioned above can be extended from the Boussinesq case to a non-Boussinesq case. In the case of a compressible fluid, it is known [36] that the main effect of compressibility can be taken into account simply by modifying the definition of the Rayleigh number, replacing the basic temperature gradient by the difference between that basic gradient $\Delta T/L$ and the adiabatic gradient $\beta T_0 g/c_P$ $(=\phi \Delta T/L)$. This implies that at the onset of convection, the viscous dissipation is balanced by the difference between the energy released by buoyancy and the work done by the pressure.

8 Conclusion

In this note, the discussion has ranged over a number of topics, some of which are currently controversial. It is hoped that the discussion will spur further investigations. In particular, the author would like to see an experimental test of the alternative expressions proposed for the viscous dissipation performed. He would also like to see a CFD investigation of natural convection in a square box with both lateral heating and bottom heating, with the computations being done employing the appropriate velocity scales and over the full range of pertinent parameters.

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My thinking has been greatly stimulated by discussions with my University of Auckland colleague Professor Gordon Mallinson. It was the paper by Mallinson et al. [37] and an associated conference poster presented by Mallinson that stimulated the recent flurry of work on natural convection in a laterally heated square box. I am also grateful for the comments of several other colleagues with whom I have discussed this matter.

Nomenclature

Br = Brinkman number, defined by Eq. (8)
c = specific heat
c_F = Forchheimer coefficient
Da = Darcy number, K/L^2
Ec = Eckert number, defined by Eq. (9*b*)
Eck = Eckert number, defined by Eq. (18)
Ge = Gebhart number, defined by Eq. (12)
K = permeability
k = thermal conductivity
L = length scale
N_d = parameter defined by Eq. (14)
N_v = parameter defined by Eq. (7)
P = pressure
Pe = Peclet number, defined by Eq. (10)
Pr = Prandtl number, defined by Eq. (9*a*)
Ra = Rayleigh number, defined by Eq. (16)
Ra_D = Rayleigh-Darcy number, defined by Eq. (13)
Re = Reynolds number, $\rho UL/\mu$
Q = heat flux
T = temperature
t = time
U = velocity scale
$\mathbf{v}$ = Darcy velocity
w = vertical component of the Darcy velocity
x,y,z = spatial coordinates

Greek Symbols

β = volumetric thermal expansion coefficient
μ = fluid viscosity
μ_{eff} = effective viscosity
ρ = fluid density
θ = perturbation temperature
Φ = viscous dissipation function
ϕ = $\mathrm{Ge}T_0/\Delta T$

Subscripts

c = cold
f = fluid
h = hot
m = bulk, porous medium
w = wall
0 = reference

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