Heat Transfer in Porous Media

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1. Introduction

Heat transfer phenomena play a vital role in many problems which deals with transport of flow through a porous medium. One of the main applications of study the heat transport equations exist in the manufacturing process of polymer composites [1] such as liquid composite molding. In such technologies, the composites are created by impregnation of a preform with resin injected into the mold's inlet. Some thermoset resins may undergo the cross-linking polymerization, called curing reaction, during and after the mold-filling stage. Thus, the heat transfer and exothermal polymerization reaction of resin may not be neglected in the mold-filling modeling of LCM. This shows the importance of heat transfer equations in the non-isothermal flow in porous media.

Generally, the energy balance equations can be derived using two different approaches: (1) two-phase or thermal non-equilibrium model [2-6] and (2) local thermal equilibrium model [7-18]. There are two different energy balance equations for two phases (such as resin and fiber in liquid composite molding process) separately in the two-phase model, and the heat transfer between these two equations occur via the heat transfer coefficient. In the thermal equilibrium model, we assume that the phases (such as resin and fiber) reach local thermodynamic equilibrium. Therefore, only one energy equation is needed as the thermal governing equation, [3,5]. Firstly, we consider the heat transfer governing equation for the simple situation of isotropic porous media. Assume that radioactive effects, viscous dissipation, and the work done by pressure are negligible. We do further simplification by assuming the thermal local equilibrium that $T_s = T_f = T$ where T_s and T_f are the solid and fluid phase temperature, respectively. A further assumption is that there is a parallel conduction heat transfer taking place in solid and fluid phases.

Taking the average over an REV of the porous medium, we have the following for solid and fluid phases,

$$(1-\varphi)(\rho c)_s \frac{\partial <T_s>^s}{\partial t} = (1-\varphi)\nabla.(k_s \nabla <T_s>^s) + (1-\varphi)q_s''' \tag{1}$$

$$\varphi(\rho c_P)_f \frac{\partial <T_f>^f}{\partial t} + (\rho c_P)_f v.\nabla <T_f>^f = \varphi\nabla.(k_f \nabla <T_f>^f) + \varphi q_f''' \tag{2}$$

where c is the specific heat of the solid and c_p is the specific heat at constant pressure of the fluid, k is the thermal conductivity coefficient and q''' is the heat production per unit volume. By assuming the thermal local equilibrium, setting $T_s = T_f = T$, one can add Eqs. (1) and (2) to have:

$$(\rho c)_m \frac{\partial <T>}{\partial t} + (\rho c)_f v.\nabla <T> = \nabla.(k_m \nabla <T>) + q'''_m \tag{3}$$

where $(\rho c)_m$, k_m and q'''_m are the overall heat capacity, overall thermal conductivity, and overall heat conduction per unit volume of the porous medium, respectively. They are defined as follows:

$$(\rho c)_m = (1-\varphi)(\rho c)_s + \varphi(\rho c_p)_f \tag{4}$$

$$k_m = (1-\varphi)k_s + \varphi k_f \tag{5}$$

$$q'''_m = (1-\varphi)q'''_s + \varphi q'''_f \tag{6}$$

2. Governing equations

2.1 Macroscopic level

Pillai and Munagavalasa [19] have used volume averaging method with the local thermal equilibrium assumption to derive a set of energy and species equations for dual-scale porous medium. The schematic view of such volume is presented in the figure 1. Unlike the single scale porous media, there is an unsaturated region behind the moving flow-front in the dual-scale porous media. The reason for such partially saturated flow-front can be mentioned as the flow resistance difference between the gap and the tows where the flow goes faster in the gaps rather than the wicking inside the tows. Pillai and Munagavalasa [19] have applied the volume averaging method to the dual-scale porous media. Using woven fiber mat in the LCM, they considered the fiber tows and surrounding gaps as the two phases.
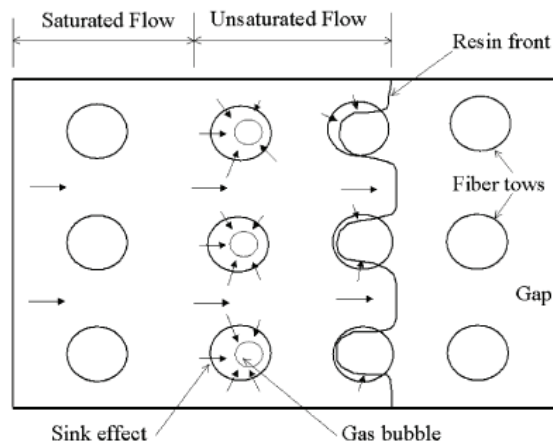

Fig. 1. Schematic view of dual-scale porous-medium [19]

The pointwise microscopic energy balance and species equations for resin inside the gap studied at first, and then the volume average of these equation is taken. Finally, they came up with the macroscopic energy balance and species equations.

The macroscopic energy balance equation in dual-scale porous medium is given by

$$\rho_g C_{P,g}\left[\varepsilon_g \frac{\partial}{\partial t}\langle T_g\rangle^g + \langle v_g\rangle.\nabla\langle T_g\rangle^g\right] = \nabla.K_{th}.\nabla\langle T_g\rangle^g + \varepsilon_g\rho_g H_R f_c + Q_{conv} - Q_{cond} \tag{7}$$

where the ρ_g and $C_{P,g}$ are the resin density and specific heat respectively. T_g is the temperature of resin in the gap region, ε_g is gap fraction, H_R is the heat reaction and f_c is the reaction rate. The $\varepsilon_g\rho_g H_R f_c$ term represents the heat source due to exothermic curing reaction. The term K_{th} is the thermal conductivity tensor for dual-scale porous medium defined as

$$K_{th} = k_g\varepsilon_g\delta + \frac{k_g}{V}\int_{A_{gt}} n_{gt}bdA - \frac{\rho_g C_{P,g}\varepsilon_g}{V_g}\int \hat{v}_g b dV \tag{8}$$

where k_g, δ and $\hat{v}_g$ are thermal conductivity of the resin, a unit tensor and the fluctuations in the gap velocity with respect to the gap averaged velocity respectively. The vector b relates temperature deviations in the gap region to the gradient of gap-averaged temperature in a closure. Considering the temperature closure formulation as $\hat{T}_g = b.\nabla < T_g >^g$, the local temperature deviation is related to the gradient of the gap-averaged temperature through the vector b, [19].

Q_{conv} in the Eq. (8) is the heat source term due to release of resin heat prior to the absorption of surrounding tows given by

$$Q_{conv} = \rho_g C_{P,g} S_g[\langle T_g\rangle^g - \langle T_g\rangle^{gt}] \tag{9}$$

where Sg , the sink term and areal average of temperature on the tow-gap interface are expressed as following respectively

$$S_g = \frac{1}{\varepsilon_g V}\int v_g.n_{gt}dA \tag{10}$$

and

$$\langle T_g\rangle^{gt} = \frac{1}{A_{gt}}\int T_g dA \tag{11}$$

and Q_{cond} is the heat sink term caused by conductive heat loss to the tows given by

$$Q_{cond} = \frac{1}{V}\int k_g(-\nabla T_g).n_{gt}dA \tag{12}$$

Using the analogy between heat and mass transfer to derive the gap-averaged cure governing equation following the Tucker and Dessenberger [6] approach, one can derive the following equation

$$\varepsilon_g \frac{\partial}{\partial t}\left\langle c_g \right\rangle^g + \left\langle v_g \right\rangle . \nabla \left\langle c_g \right\rangle^g = \nabla . D . \nabla \left\langle c_g \right\rangle^g + \varepsilon_g f_c + M_{conv} - M_{diff} \tag{13}$$

where c_g is the degree of cure in a resin which value of 0 and 1 correspond to the uncured and fully cured resin situation, D is diffusivity tensor for the gap flows and is given by

$$D = D_1 \varepsilon_g \delta + \frac{D_1}{\mathbf{V}} \int n_{gt} b \, d\mathbf{A} - \frac{\varepsilon_g}{V_g} \int \hat{v}_g b \, dV \tag{14}$$

where D_1 is the molecular diffusivity of resin. In the Eq. (13), M_{conv} is the convective source due to release of resin cure when absorbing into tows as a results of sink effect, given by

$$\mathrm{M}_{conv} = \mathrm{S}_g [\left\langle c_g \right\rangle^g - \left\langle c_g \right\rangle^{gt}] \tag{15}$$

where $\left\langle c_g \right\rangle^{gt}$ is the areal average of temperature on the tow-gap interface, expressed as

$$\left\langle c_g \right\rangle^{gt} = \frac{1}{A_{gt}} \int\limits_{A_{gt}} c_g dA \tag{16}$$

and M_{diff} is the cure sink term as a result of the diffusion of cured resin into the tows, given by

$$M_{diff} = \frac{1}{V} \int D_1 (-\nabla c_g) . n_{gt} dA \tag{17}$$

It should be noted that the only way to compute the Q_{conv}, Q_{cond}, M_{conv} and M_{diff} is solving for flow and transport inside the tows.

2.2 Microscopic level

Phelan et al. [20] showed that the conventional volume averaging method can be directly used to derive the transport equation for thermo-chemical phenomena inside the tows for single-scale porous media. The final derivation for microscopic energy equation is

$$[\varepsilon_t \left(\rho C_p \right)_l + \left(1 - \varepsilon_t \right) \left(\rho C_P \right)_f] \frac{\partial T_t}{\partial t} + \left(\rho C_P \right)_l v_t . \nabla T_t = \nabla . K_{th,t} . \nabla T_t + \varepsilon_t \rho_l H_R f_c \tag{18}$$

where the subscript t refer to tows. The microscopic species equation is given by

$$\varepsilon_t \frac{\partial c_t}{\partial t} + v_t . \nabla c_t = \nabla . \varepsilon_t D_t . \nabla c_t + \varepsilon_t f_c \tag{19}$$

The complete set of microscopic and macroscopic energy and species equations as well as the flow equation should be solved to model the unsaturated flow in a dual-scale porous medium.

3. Dispersion term

In some cases, a further complication arises in the thermal governing equation due to thermal dispersion [21]. The thermal dispersion happens due to hydrodynamic mixing of fluid at the pore scale. The mixings are mainly due to molecular diffusion of heat as well as

the mixing caused by the nature of the porous medium. The mixings are mainly due to molecular diffusion of heat as well as the mixing caused by the nature of the porous medium. Greenkorn [22] mentioned the following nine mechanisms for most of the mixing;

1. Molecular diffusion: in the case of sufficiently long time scales
2. Mixing due to obstructions: The flow channels in porous medium are tortuous means that fluid elements starting a given distance from each other and proceeding at the same velocity will not remain the same distance apart, Fig. 2.
3. Existence of autocorrelation in flow paths: Knowing all pores in the porous medium are not accessible to the fluid after it has entered a particular fluid path.
4. Recirculation due to local regions of reduced pressure: The conversion of pressure energy into kinetic energy gives a local region of low pressure.
5. Macroscopic or megascopic dispersion: Due to nonidealities which change gross streamlines.
6. Hydrodynamic dispersion: Macroscopic dispersion is produced in capillary even in the absence of molecular diffusion because of the velocity profile produced by the adhering of the fluid wall.
7. Eddies: Turbulent flow in the individual flow channels cause the mixing as a result of eddy migration.
8. Dead-end pores: Dean-end pore volumes cause mixing in unsteady flow. The main reason is as solute rich front passes the pore, diffusion into the pore occurs due to molecular diffusion. After the front passes, the solute will diffuse back out and thus, dispersing.
9. Adsorption: It is an unsteady-state phenomenon where a concentration front will deposit or remove material and therefore tends to flatten concentration profiles.
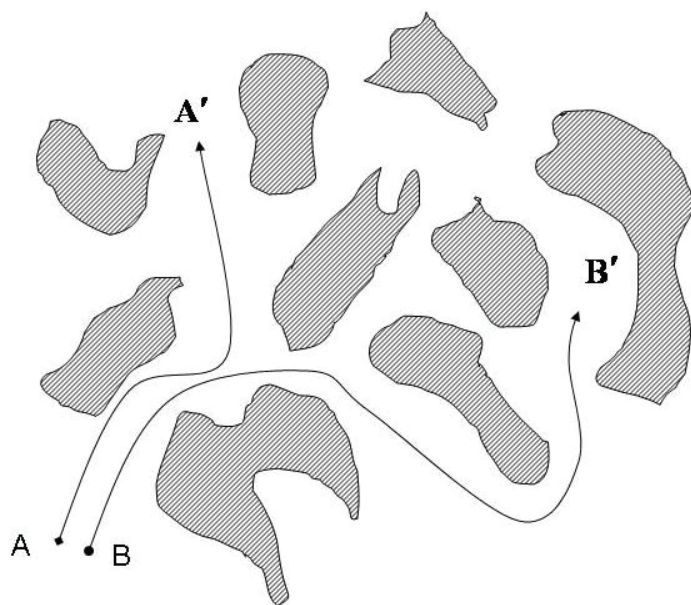

Fig. 2. Mixing as a result of obstruction

Rubin [23] generalized the thermal governing equation

$$\left(\rho c\right)_m \frac{\partial T}{\partial t} + \left(\rho c\right)_f v.\nabla T = \nabla.(k_m \nabla T) + q_m''' \tag{20}$$

where K is a second-order tensor called dispersion tensor.

Two dispersion phenomena have been extensively studied in the transport phenomena in porous media are the mass and thermal dispersions. The former involves the mass of a solute transported in a porous medium, while the latter involves the thermal energy transported in the porous medium. Due to the similarity of mass and thermal dispersions, they can be described using the dimensionless transport equations as

$$\frac{\partial \langle \Omega \rangle}{\partial \theta} + \langle U_i \rangle \frac{\partial \langle \Omega \rangle}{\partial X_i} = \frac{1}{Pe} \frac{\partial}{\partial X_i} \left(D_{ij} \frac{\partial \langle \Omega \rangle}{\partial X_j} \right) \tag{21}$$

where $\langle \Omega \rangle$ is either averaged concentration for mass dispersion or averaged dimensionless temperature for thermal dispersion, θ is dimensionless time, $\langle U_i \rangle$ is averaged velocity vector, Pe is Peclet number, D_{ij} is dispersion tensor of 2nd order. It should be noted that $Pe = \frac{uL}{\mathcal{D}}$ in mass dispersion and $Pe = \frac{uL}{\alpha}$ in thermal dispersion where u and L are characteristic velocity and length, respectively. $\mathcal{D}$ and α are molecular mass and thermal diffusivities, respectively.

3.1 Dispersion in porous media

Most studies on dispersion tensor so far have been focusing on the isotropic porous media. Nikolaveskii [24] obtained the form of dispersion tensor for isotropic porous media by analogy to the statistical theory of turbulence. Bear [25] obtained a similar result for the form of the dispersion tensor on the basis of geometrical arguments about the motion of marked particles through a porous medium. Bear studied the relationship between the dispersive property of the porous media as defined by a constant of dispersion, the displacement due to a uniform field of flow, and the resulting distribution. He used a point injection subjected to a sequence of movements. The volume averaged concentration of the injected tracer, C_0, around a point which is displaced a distance $L = ut$ in the direction of the uniform, isotropic, two dimensional field of flow from its original position is considered in his research.

$$C(x,y;x_0,y_0) = \frac{C_0}{2\pi {\sigma_x}^2 {\sigma_y}^2} .\exp \left\{ -\frac{m^2}{2{\sigma_x}^2} - \frac{n^2}{2{\sigma_y}^2} \right\} \tag{22}$$

where L is the distance of mean displacement, u is the uniform velocity of flow, t is the time of flow, σ_x and σ_y are standard deviations of the distribution in the x and y directions, respectively and, finally m and n are the coordinates of the point (x,y) in the coordinate system centered at $\left(\xi, \eta\right)$ given by $m = x - (x_0 + L)$ and $n = y - y_0$, figure 3. This figure shows a point injection as a result of subsequence movement where initially circle tracer gets an elliptic shape at $L = ut$.
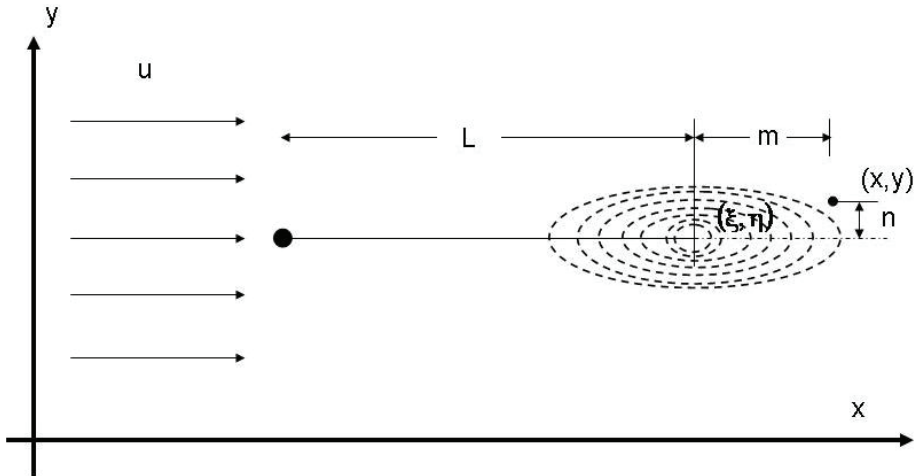
Fig. 3. Dispersion of a point injection displaced a distance L

Standard deviations are defined by $\sigma_x = \left(2D_I L\right)^{0.5}$ and $\sigma_y = \left(2D_{II} L\right)^{0.5}$ where D_I and D_{II} are the longitudinal and transverse constants of dispersion in porous media, respectively. One should note that the D_I and D_{II} used in the Bear work depend only upon properties of the porous medium such as porosity, grain size, uniformity, and shape of grains. From Eq. (22), it follows that, after a uniform flow period, lines of the similar concentration resulting from the circular point injection of the tracer take the ellipse shape centered at the displaces mean point and oriented with their major axes in the direction of the flow.

$$\frac{x^2}{\sigma_x{}^2} + \frac{y^2}{\sigma_y{}^2} = 1 \tag{23}$$

Bear conjectured that the property which is defined by the constant of dispersion, D_{ijkl}, depends only upon the characteristics of porous medium and the geometry of its pore-channel system. In a general case, this is a fourth rank tensor which contains 81 components. These characteristics are expressed by the longitudinal and lateral constants of dispersion of the porous media. Scheidegger [26] used the dispersion tensor D_{ij} in the following form

$$D_{ij} = a_{ijkm} \frac{v_k v_m}{|v|} \tag{24}$$

where v is the average velocity vector, v_k is the k_{th} component of velocity vector, a_{ijkm} is a fourth rank tensor called geometrical dispersivity tensor of the porous medium. Bear demonstrated how the dispersion tensor relates to the two constants for an isotropic medium: $a_{||}$ = longitudinal dispersion[1], and $a_{\perp}$ = transversal dispersion[2]. Scheidegger [26] has shown that there are two symmetry properties for dispersivity tensor

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[1] The longitudinal direction is along the mean flow velocity in porous media, whereas the transverse direction is perpendicular to the mean flow velocity.

$$a_{ijkm} = a_{jikm} \quad \text{and} \quad a_{ijkm} = a_{ijmk} \tag{25}$$

Therefore, only 36 of 81 components of fourth rank tensor a_{ijkl} is independent. For an isotropic porous medium, the dispersivity tensor must be isotropic. An isotropic fourth rank tensor can be expressed as

$$a_{ijkm} = \alpha\delta_{ij}\delta_{km} + \beta\delta_{ik}\delta_{jm} + \gamma\delta_{im}\delta_{jk} \tag{26}$$

where α, β, γ are constants and δ_{ij} is Kronecker symbol. Because of symmetry properties expressed by Eq.(23), we get

$$\beta = \gamma \tag{27}$$

So the dispersivity tensor can be written as

$$a_{ijkm} = \alpha\delta_{ij}\delta_{km} + \beta\left(\delta_{ik}\delta_{jm} + \delta_{im}\delta_{jk}\right) \tag{28}$$

On substituting Eq. (26) into Eq. (22), we can obtain the dispersion tensor as

$$D_{ij} = \alpha|v|\delta_{ij} + \frac{2\beta}{|v|}v_iv_j \tag{29}$$

If we define $a_\perp = \alpha|v|$, $a_{||} - a_\perp = 2\beta|v|$ and $n_i = v_i/|v|$ (n_i is the mean flow direction), then dispersion tensor D_{ij} can be written as

$$D_{ij} = a_\perp\delta_{ij} + \left(a_{||} - a_\perp\right)n_in_j \tag{30}$$

From Eq. (28), it is quite clear that the three principle directions of dispersion tensor D are orthogonal to each other (due to the symmetry of D_{ij}), and one principle direction is along the mean flow direction (**n**) and the other two are perpendicular to the mean flow direction. Therefore, for isotropic medium, the dispersion tensor can be expressed by longitudinal and transverse dispersion coefficients. If we consider the mean flow is along x-axis, D_{ij} can be written as

$$D = \begin{bmatrix} a_{||} & 0 & 0 \\ 0 & a_\perp & 0 \\ 0 & 0 & a_\perp \end{bmatrix} \tag{31}$$

Therefore, transport equation can be written as

$$\frac{\partial\langle\Omega\rangle}{\partial\theta} + \langle U_1\rangle\frac{\partial\langle\Omega\rangle}{\partial X_1} = \frac{1}{Pe}\left(a_{||}\frac{\partial^2\langle\Omega\rangle}{\partial X_1^2} + a_\perp\frac{\partial^2\langle\Omega\rangle}{\partial X_2^2} + a_\perp\frac{\partial^2\langle\Omega\rangle}{\partial X_3^2}\right) \tag{32}$$

It has been shown that one of the principle axes of the dispersion tensor in isotropic porous medium is along the mean flow direction. Unlike the isotropic media, there are nine independent components in the dispersion tensor for the case of anisotropic porous media. Bear [25] noted that the dispersion problem in a nonisotropic material still remains unsolved. He suggested to distinguishing between various kinds of anisotropies and doing

statistical analysis with different frequency functions for the spatial distribution of channels in each case. Unless some specific types of porous media, like axisymmetric or transversely isotropic, it is not possible to simplify the form of dispersion tensor. In 1965, Poreh [27] used the theory of invariants to give a dispersion tensor for axisymmetric porous media. The average properties of axisymmetric porous medium which affecting the macroscopic dispersion pattern are invariants to rotaion about given line. He establish the general form of D_{ij} with two arbitrary vectors **R** and **S** as following

$$D_{ij}R_iS_j = B_1\delta_{ij}R_iS_j + B_2v_iv_jR_iS_j + B_3\lambda_i\lambda_jR_iS_j + B_4v_iR_i\lambda_jS_j + B_5\lambda_iR_iv_jS_j \qquad (33)$$

where **λ** is the axis of symmetry, B_1, B_2, B_3, B_4, and B_5 are arbitrary functions of v^2 and $v_k\lambda_k$. For arbitrary **R** and **S** and symmetric D_{ij}, one can have

$$D_{ij} = B_1\delta_{ij} + B_2v_iv_j + B_3\lambda_i\lambda_j + B_4\left(v_i\lambda_j + \lambda_iv_j\right) \qquad (34)$$

The dimensionless form of Eq. (32) is obtained as

$$\frac{D_{ij}}{D_0} = G_1\delta_{ij} + G_2\left(\frac{l^2}{D_0^2}\right)v_iv_j + G_3\lambda_i\lambda_j + G_4\left(\frac{l}{D_0}\right)\left(v_i\lambda_j + \lambda_iv_j\right) \qquad (35)$$

where G_1, G_2, G_3 and G_4 are dimensionless functions of $(vl/D_0)^2$, $(vl/v)^2$, and D_0 is molecular diffusivity coefficient, l is length characterizing the size of the pores, v is kinematic viscosity. Finally, the dispersion tensor for axisymmetric porous media is

$$\frac{D_{ij}}{D_0} = \left(\beta_1 + \beta_2\frac{v^2l^2}{D_0^2}\right)\delta_{ij} + \beta_3\left(\frac{l^2}{D_0^2}\right)v_iv_j + \left(\beta_4 + \beta_5\frac{v^2l^2}{D_0^2}\right)\lambda_i\lambda_j + \beta_6\left(\frac{vl^2}{D_0^2}\right)\left(v_i\lambda_j + \lambda_iv_j\right) \qquad (36)$$

where β_1 and β_4 are dimensionless numbers, β_2, β_3 and β_5 are even functions of $\cos\omega$, and β_6 is an odd function of $\cos\omega$.
By assuming no motion within an axisymmetric medium, D_{ij} is simplify to

$$\frac{D_{ij}}{D_0} = \beta_1\delta_{ij} + \beta_4\lambda_i\lambda_j \qquad (37)$$

Eq.(37) indicating that one of the principal axes of D_{ij} is, in this case, co-directional with **λ**. He followed these arguments that for sufficiently large Reynolds number, the dispersivity tensor for axisymmetric porous medium can be expressed as

$$\frac{D_{ij}}{lv} = \varepsilon_1\delta_{ij} + \frac{\varepsilon_2v_iv_j}{v^2} + \varepsilon_3\lambda_i\lambda_j + \frac{\varepsilon_4(v_i\lambda_j + v_j\lambda_i)}{v} \qquad (38)$$

where ε_1, ε_2, ε_3, and ε_4 are parameters determined by the dimensionless geometry of the medium and depends slightly on the Reynolds number and value of $\cos\omega$. On should note that the above derivation, primarily based on symmetry considerations, can not reveal the scalar nature of general dispersivity tensor. Bear [25] also noted that his analysis was based on an unproved assumption that D_{ij} may be expanded in a power series. In 1967, Whitaker [28] applied pointwise volume-averaging method for transport equation in anisotropic porous media and obtained following dispersion tensor D_{ij}

$$D_{ij} = D_0 \left(\delta_{ij} + RB_{ij} \right) + C_{ikj} v_k + E_{ikmj} v_k v_m \tag{39}$$

where second order tensor B_{ij} is a function of tortuosity vector

$$\tau_j = \int_S \Omega n_j ds \tag{40}$$

The third-order tensor C_{ikj} is given by

$$C_{ikj} = \frac{\partial^2 \langle \tilde{\Omega} \tilde{v}_i \rangle}{\partial \langle v_k \rangle \partial \left(\dfrac{\partial \langle \Omega \rangle}{\partial x_j} \right)} \tag{41}$$

And the fourth-order tensor E_{ikmj} is introduced as

$$E_{ikmj} = \frac{\partial^3 \langle \tilde{\Omega} \tilde{v}_i \rangle}{\partial \langle v_k \rangle \partial \langle v_m \rangle \partial \left(\dfrac{\partial \langle \Omega \rangle}{\partial x_j} \right)} \tag{42}$$

where $\tilde{\Omega}$ is deviation of concentration or temperature from the average and $\tilde{v}_i$ is velocity deviation given respectively as

$$\tilde{\Omega} = \Omega - \langle \Omega \rangle^f \text{ and } \tilde{v}_i = v_i - \langle v_i \rangle^f \tag{43}$$

One should keep in mind that from Eqs. (41) and (42), we know that C_{ikj} and E_{ikmj} are completely symmetrical.

On comparing Eq. (38) with Eq. (24), one can note that there are both third- and fourth-order symmetric tensors associated with velocity in the Whitaker's derivation, while Nikolaveskii [24], Bear [25], and Scheidegger's [26] derivations only contain fourth-order symmetric tensor. Patel and Greenkorn [29] suggested that Whitaker's expression for dispersion tensor is the correct one for anisotropic media. There are two distinct components of dispersion tensor for isotropic medium while Whitaker's expression of dispersion tensor resulted in only one component for isotropic medium.

The diffusion term becomes less important at higher velocities which gives

$$D_{ij} \approx C_{ikj} v_k + E_{ikmj} v_k v_m \tag{44}$$

For isotropic media, the tensors C_{ikj} and E_{ikmj} must be isotropic. Hence, $C_{ikj} = 0$, and E_{ikmj} is a linear combination of the Kronecker deltas as expressed in Eq. (26). Since E_{ikmj} is completely symmetric, Eq. (42), the tensor E_{ikmj} can be shown as

$$E_{ikmj} = \alpha \left(\delta_{ik} \delta_{mj} + \delta_{im} \delta_{kj} + \delta_{ij} \delta_{km} \right) \tag{45}$$

Therefore Eq. (39) reduces to

$$D_{ij} = \alpha \left(2 v_i v_j + \delta_{ij} |v|^2 \right) \tag{46}$$

Assuming 1-D flow in Cartesian coordinate frame where

$$v_1 = u \quad v_2 = v_3 = 0 \tag{47}$$

then, Eq. (46) can be written as

$$D = \alpha u^2 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{48}$$

Equation (48) shows that the longitudinal coefficient of dispersion tensor in this case, is three times the transverse coefficient. This equation clearly indicates the huge difference between the isotropic and anisotropic porous media. Greenkorn [29] showed experimentally that the ratio $D_{||}/D_\perp$ varies approximately from lower value of 3 to the higher value of 60. He showed that this ration is a function of the flow velocity.

Experimental results by Patel and Greenkorn [29] show that the ratio $D_{||}/D_\perp$ varies from a lower value of about 3 to a high value of about 60. This ratio of longitudinal to transverse dispersion coefficients is shown to be actually a function of the velocity of flow. Although the Whitaker's method is at variance with Greenkorn experimental results, it still do give the correct lower limit result for a homogeneous, uniform, isotropic medium.

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