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Effects of Viscous Dissipation and Flow Work on Forced Convection in a Channel Filled by a Saturated Porous Medium

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Abstract. Fully developed forced convection in a parallel plate channel filled by a saturated porous medium, with walls held either at uniform temperature or at uniform heat flux, with the effects of viscous dissipation and flow work included, is treated analytically. The Brinkman model is employed. The analysis leads to expressions for the Nusselt number, as a function of the Darcy number and Brinkman number.

Key words: viscous dissipation, flow work, forced convection, channel.

Nomenclature

Bn	Brinkman number defined by Equation (47).
Br	Darcy–Brinkman number defined by Equation (21).
c_P	specific heat at constant pressure.
Da	Darcy number, K/H^2.
G	applied pressure gradient, $-\mathrm{d}p^*/\mathrm{d}x^*$.
H	half channel width.
k	fluid thermal conductivity.
K	permeability.
M	μ_{eff}/μ.
N	$GK/\mu U^*$.
Nu	Nusselt number defined by Equation (12).
Pe	Péclet number defined by Equation (3).
q''	wall heat flux.
S	$(M\,Da)^{-1/2}$.
T^*	temperature.
T_{m}^*	bulk mean temperature.
T_{w}^*	wall temperature.
u	$\mu u^*/GH^2$.
u^*	filtration velocity.
$\hat{u}$	u^*/U^*.
U^*	mean velocity.

x^* longitudinal coordinate.
y^* transverse coordinate.
y_1, y_2 dimensionless functions defined in Equation (27a) and (27b).

Greek Symbols
η y^*/H.
θ $(T^* - T_w^*)/(T_m^* - T_w^*)$.
μ fluid viscosity.
μ_{eff} effective viscosity in the Brinkman term.
ρ fluid density.
ϕ dimensionless heat source expression defined by Equation (19).
Φ viscous dissipation function.

1. Introduction

Because of the use of hyperporous media in the cooling of electronic equipment, there has recently been renewed interest in the problem of forced convection in a porous medium channel. In their recent survey of the literature, Nield and Bejan (1999) refer to over 30 papers on this topic, but none of them deals with the effect of viscous dissipation. This gap in the literature has been partly filled by Nield *et al.* (2003, in press). These papers were concerned with thermally developing flow, and the downstream thermal boundary condition adopted therein was an approximate one that is inappropriate for fully developed flow. The Brinkman model has been used to model the flow through the porous medium. For this flow model, the modelling of the viscous dissipation term in the thermal energy equation is at present controversial, and accordingly we treat in turn alternative models. (The validity of the Brinkman model itself is independent of whether or not viscous dissipation is significant.) We have in mind that our results might serve as a basis for an experimental test of the competing viscous dissipation models. When the fluid concerned is a gas, the effect of flow work is generally comparable with the effect of viscous dissipation. When the fluid is a liquid, the effect of flow work is negligible. In this paper we deal in turn with the two types of fluid.

Previous work on the effects of viscous dissipation and flow work in ducts, in the case of fluids clear of solid material, has been surveyed by Shah and London (1978).

2. Analysis

2.1. BASIC EQUATIONS

For the steady-state hydrodynamically-developed situation we have unidirectional flow in the x^*-direction between impermeable boundaries at $y^* = -H$ and $y^* = H$, as illustrated in Figure 1. The Brinkman momentum equation is

$$\mu_{\text{eff}} \frac{\mathrm{d}^2 u^*}{\mathrm{d}y^{*2}} - \frac{\mu}{K} u^* + G = 0, \tag{1}$$

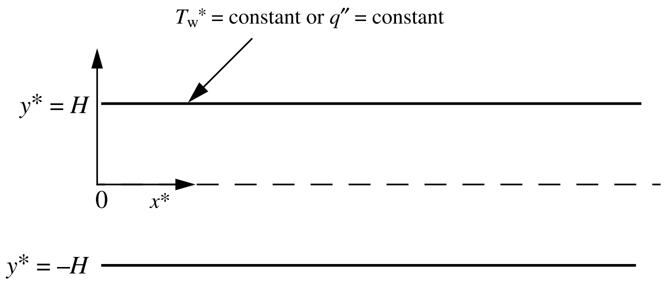

Figure 1. Definition sketch.

where μ_{eff} is an effective viscosity, μ is the fluid viscosity, K is the permeability, and G is the applied pressure gradient.

We define dimensionless variables

$$\xi = \frac{x^*}{Pe\,H}, \qquad \eta = \frac{y^*}{H}, \qquad u = \frac{\mu u^*}{GH^2}. \tag{2}$$

Here the Péclet number Pe is defined by

$$Pe = \frac{\rho c_{\mathrm{P}} H U^*}{k}. \tag{3}$$

The dimensionless form of Equation (1) is

$$M\frac{\mathrm{d}^2 u}{\mathrm{d}\eta^2} - \frac{u}{Da} + 1 = 0. \tag{4}$$

We have defined the viscosity ratio M and the Darcy number Da by

$$M = \frac{\mu_{\text{eff}}}{\mu}, \qquad Da = \frac{K}{H^2}. \tag{5}$$

The solution of this equation subject to the boundary condition $u = 0$ at $\eta = 1$, and the symmetry condition $\mathrm{d}u/\mathrm{d}\eta = 0$ at $\eta = 0$ is

$$u = Da\left(1 - \frac{\cosh S\eta}{\cosh S}\right), \tag{6}$$

where

$$S = \left(\frac{1}{M\,Da}\right)^{1/2}. \tag{7}$$

The mean velocity U^* and the bulk mean temperature T^*_{m} are defined by

$$U^* = \frac{1}{H}\int_0^H u^*\,\mathrm{d}y^*, \tag{8a}$$

$$T^*_{\mathrm{m}} = \frac{1}{HU^*}\int_0^H u^* T^*\,\mathrm{d}y^*. \tag{8b}$$

Further dimensionless variables are defined by

$$\hat{u} = \frac{u^*}{U^*}, \tag{9a}$$

$$\theta = \frac{T^* - T^*_{\mathrm{w}}}{T^*_{\mathrm{m}} - T^*_{\mathrm{w}}}. \tag{9b}$$

This implies that

$$U^* = \frac{GK}{\mu}\left(1 - \frac{\sinh S}{S\cosh S}\right), \tag{10}$$

$$\hat{u} = \frac{S\cosh S - S\cosh S\eta}{S\cosh S - \sinh S}. \tag{11}$$

Equation (10) may be used to relate G to U^* if that is required for practical purposes. The Nusselt number Nu is defined as

$$Nu = \frac{2Hq''}{k(T^*_{\mathrm{w}} - T^*_{\mathrm{m}})}. \tag{12}$$

(The reader should note that we have followed Nield and Bejan (1999) and defined Nu in terms of the channel width rather than the hydraulic diameter. The Nusselt number defined in terms of the hydraulic diameter is twice Nu.)

It is assumed that there is local thermal equilibrium. A criterion (that is met in most circumstances) for the validity of this assumption for steady forced convection was given by Nield (1998). It is also assumed that the Péclet number is sufficiently large so that the axial conduction may be neglected.

The steady-state thermal energy equation is then

$$\rho c_{\mathrm{p}} u^* \frac{\partial T^*}{\partial x^*} = k\frac{\partial^2 T^*}{\partial {y^*}^2} + \Phi, \tag{13}$$

where Φ is the contribution due to viscous dissipation. The modelling of this viscous term is controversial. The simplest expression, which is appropriate to the Darcy equation, in the present case is

$$\Phi = \frac{\mu {u^*}^2}{K}. \tag{14a}$$

Nield (2000, 2002) argued that the viscous dissipation function should remain equal to the power of the drag force when the Brinkman equation is considered, and in the present case this implies that

$$\Phi = \frac{\mu {u^*}^2}{K} - \mu_{\mathrm{eff}} u^* \frac{\mathrm{d}^2 u^*}{\mathrm{d}{y^*}^2}. \tag{14b}$$

On the other hand, Al-Hadhrami *et al.* (2002, 2003) proposed a form which is compatible with an expression derived from the Navier–Stokes equation for a fluid clear of solid material, in the case of large Darcy number, and in this case we have

$$\Phi = \frac{\mu {u^*}^2}{K} + \mu\left(\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right)^2. \tag{14c}$$

In each case the added Brinkman term is O(Da) in comparison with the Darcy term. Consequently, in the case of small Da the three models are effectively equivalent to each other. Nield (in press) has commented on the alternative models.

For the case where the fluid is a gas, flow work contributes a term $\beta\, T^* u^*\, \mathrm{d}p^*/\mathrm{d}x^*$ to the right-hand side of Equation (13) that is generally comparable with the viscous dissipation. Here p^* denotes the pressure and β is the isobaric coefficient of volumetric thermal expansion coefficient, and in the case of a perfect gas $\beta = 1/T^*$. (See, for example, Shah and London, 1978, p. 13.) In subsequent work in this paper, it is assumed that if the fluid is a gas then the gas is a perfect one, so one can replace $\beta\, T^*$ by unity. In the case where the fluid is a liquid, the effect of flow work is negligible. The effect of flow work is equivalent to the introduction of an additional heat source term, and so for mathematical convenience this effect can be incorporated with the source term due to viscous dissipation.

We can combine the three Equations (14a), (14b) and (14c), together with the flow work term into a single equation:

$$\Phi = \frac{\mu {u^*}^2}{K} - c_1 \mu_{\mathrm{eff}} u^* \frac{\mathrm{d}^2 u^*}{\mathrm{d}y^2} + c_2 \mu \left(\frac{\mathrm{d}u^*}{\mathrm{d}y}\right)^2 + c_3 u^* \frac{\mathrm{d}p^*}{\mathrm{d}x^*}, \tag{15}$$

where (Model 1) for the Darcy model $c_1 = 0$ and $c_2 = 0$; (Model 2, Nield) for the 'power of drag force' model $c_1 = 1$ and $c_2 = 0$; (Model 3, Al-Hadrami *et al.*) for the 'clear fluid compatible' model $c_1 = 0$ and $c_2 = 1$; in each case, (A) $c_3 = 0$ for a liquid and (B) $c_3 = 1$ for a gas. (The negative of the pressure gradient $\mathrm{d}p^*/\mathrm{d}x^*$ that appears in Equation (15) is identical with G.)

Equation (13) must be compatible with the First Law of Thermodynamics, which when applied to a thin cross-sectional slice of the channel leads to the requirement that

$$\frac{\mathrm{d}T^*_{\mathrm{m}}}{\mathrm{d}x^*} = \frac{q'' + \int_0^H \Phi\, \mathrm{d}y^*}{\rho c_{\mathrm{P}} U^* H}. \tag{16}$$

Case 1. Constant-temperature Boundary Condition

In this case one has $T^*_{\mathrm{w}} = \text{constant}$, and so

$$\frac{\partial T^*}{\partial x^*} = (T^*_{\mathrm{m}} - T^*_{\mathrm{w}})\frac{\partial \theta}{\partial x^*} + \theta \frac{\mathrm{d}T^*_{\mathrm{m}}}{\mathrm{d}x^*}. \tag{17}$$

This follows from differentiation of a rearrangement of Equation (9b).

Using Equation (17), one can write Equation (13) in the dimensionless form

$$\frac{\partial^2\theta}{\partial\eta^2} - \hat{u}\frac{\partial\theta}{\partial\xi} + \left(\frac{1}{2}Nu - Br\langle\phi\rangle\right)\hat{u}\theta = -Br\,\phi \tag{18}$$

where

$$\phi = \hat{u}^2 - c_1 M\, Da\, \hat{u}\frac{\mathrm{d}^2\hat{u}}{\mathrm{d}\eta^2} + c_2\, Da\left(\frac{\mathrm{d}\hat{u}}{\mathrm{d}\eta}\right)^2 - c_3 N\hat{u} \tag{19}$$

and as shorthand notation we have introduced $\langle f(\eta)\rangle \equiv \int_0^1 f(\eta)\,\mathrm{d}\eta$.

The parameter N is defined by

$$N = \frac{GK}{\mu U^*}, \tag{20}$$

and N takes the value 1 for the special case of Darcy flow, and for a general flow differs from 1 by an amount of order Da.

Here the Darcy–Brinkman number Br is defined as

$$Br = \frac{\mu U^{*2} H^2}{k(T_\mathrm{m}^* - T_\mathrm{w}^*)K}. \tag{21}$$

The choice of temperature difference in the denominator has been made in light of the fact that an inlet temperature is not available to replace T_m^* in the definition, and the fact that viscous dissipation is equivalent to a positive heat source, so Br will be positive for the fully developed situation.

For the case of fully developed (both thermally and hydrodynamically) convection, Equation (18) reduces to

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} + \left(\frac{1}{2}Nu - Br\langle\phi\rangle\right)\hat{u}\theta = -Br\,\phi. \tag{22}$$

Substituting from Equation (11) one obtains

$$\begin{aligned}\phi = {} & \frac{S^2}{(S\cosh S - \sinh S)^2}\Big[\cosh^2 S + (c_1 - 2)\cosh S\cosh S\eta + \\ & + (1 - c_1)\cosh^2 S\eta + \frac{c_2}{M}\sinh^2 S\eta\Big] - c_3 N\frac{S\cosh S - S\cosh S\eta}{S\cosh S - \sinh S},\end{aligned} \tag{23}$$

$$\begin{aligned}\langle\phi\rangle = {} & \frac{S^2}{(S\cosh S - \sinh S)^2}\Big[\cosh^2 S + \frac{1}{2} - \frac{c_1}{2} - \frac{c_2}{2M} - \\ & - \left(\frac{3}{4} - \frac{c_1}{4} - \frac{c_2}{4M}\right)\frac{\sinh 2S}{S}\Big] - c_3 N.\end{aligned} \tag{24}$$

Equation (22) has to be solved subject to the boundary conditions

$$\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = 0 \quad \text{at } \eta = 0, \qquad \theta = 0 \quad \text{at } \eta = 1. \tag{25}$$

The first of these equations is a symmetry condition. The second is a compatibility condition following from the definition Equation (9b) and the fact that $T^* = T_w^*$ at $y^* = H$ independently of whether or not T_w^* is a constant.

Once the solution for θ has been found, the value of Nu is determined by the compatibility condition

$$\langle \hat{u}\theta \rangle = 1. \tag{26}$$

For computation, one can let

$$y_1 = \theta, \tag{27a}$$

$$y_2 = \frac{\mathrm{d}\theta}{\mathrm{d}\eta}, \tag{27b}$$

so that

$$y_1' = y_2, \tag{28a}$$

$$y_2' = -\left(\frac{1}{2}Nu - Br\,\langle\phi\rangle\right)\hat{u}\,y_1 - Br\,\phi, \tag{28b}$$

and integrate forward (stepping y_1 and y_2 alternately) using the initial values

$$y_1(0) = a, \tag{29a}$$

$$y_2(0) = 0, \tag{29b}$$

and vary the constant a together with Nu to satisfy simultaneously

$$y_1(1) = 0, \tag{30a}$$

$$\int_0^1 \hat{u}\,y_1\,\mathrm{d}\eta = 1. \tag{30b}$$

Case 2. Uniform-flux Boundary Conditions

Now the expression in Equation (16) is independent of x^*, $\partial T^*/\partial x^* = \mathrm{d}T_m^*/\mathrm{d}x^*$ and instead of Equation (22) one obtains

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} = -Br\,\phi + \left(Br\,\langle\phi\rangle - \frac{Nu}{2}\right)\hat{u}. \tag{31}$$

This has to be solved subject to the conditions given by Equation (25). For this case an analytical solution is possible. Because flow work contributes a term to ϕ that is proportional to $\hat{u}$, and because $\langle\hat{u}\rangle = 1$, the contributions of flow work to Equation (31) cancel. This means that flow work has no effect on the distribution of the dimensionless temperature θ or the Nusselt number in the case of uniform-flux boundary conditions. Accordingly, in the remaining analysis we take $c_3 = 0$ at the outset. We then have

$$\phi = \lambda^2(A + B\cosh S\eta + C\cosh 2S\eta) \tag{32}$$

where, for algebraic convenience, we have introduced

$$\lambda = \frac{S}{S\cosh S - \sinh S}, \tag{33}$$

$$A = \cosh^2 S + \frac{1 - c_1 - c_2/M}{2}, \tag{34}$$

$$B = (c_1 - 2)\cosh S, \tag{35}$$

$$C = \frac{1 - c_1 + c_2/M}{2}. \tag{36}$$

Then

$$\langle\phi\rangle = \lambda^2\left(A + \frac{B\sinh S}{S} + C\frac{\sinh 2S}{2S}\right) \tag{37}$$

$$\hat{u} = \lambda(\cosh S - \cosh S\eta). \tag{38}$$

Now the solution of Equation (31) subject to Equation (25) is

$$\theta = \int_\eta^1 \int_0^{\eta''} \frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2}(\eta')\,\mathrm{d}\eta'\,\mathrm{d}\eta'' \tag{39}$$

which gives

$$\begin{aligned}\theta = {} & \left(Br\,\langle\phi\rangle - \frac{Nu}{2}\right)\lambda\left\{\frac{\cosh S}{2}(\eta^2 - 1) - \frac{(\cosh S\eta - \cosh S)}{S^2}\right\} - \\ & - Br\,\lambda^2\left\{A\frac{\eta^2 - 1}{2} + B\frac{(\cosh S\eta - \cosh S)}{S^2} + \right. \\ & \left. + C\frac{(\cosh 2S\eta - \cosh 2S)}{4S^2}\right\}.\end{aligned} \tag{40}$$

Then the compatibility condition gives

$$\begin{aligned}1 = \langle\theta\hat{u}\rangle = {} & \left(Br\,\langle\phi\rangle - \frac{Nu}{2}\right)\lambda\left\{\frac{\cosh S}{2}f_1(S) - \frac{1}{S^2}f_2(S)\right\} - \\ & - Br\,\lambda^2\left\{\frac{A}{2}f_1(S) + \frac{B}{S^2}f_2(S) + \frac{C}{4S^2}f_3(S)\right\},\end{aligned} \tag{41}$$

where

$$f_1(S) \equiv \langle(\eta^2 - 1)\hat{u}\rangle = \lambda\left\{\left(\frac{1}{3} + \frac{2}{S^2}\right)\cosh S - \left(\frac{1}{S} + \frac{2}{S^3}\right)\sinh S\right\} - 1 \tag{42a}$$

$$f_2(S) \equiv \langle(\cosh S\eta - \cosh S)\hat{u}\rangle = \lambda\left\{-\frac{1}{2} + \frac{\sinh 2S}{4S}\right\} - \cosh S \tag{42b}$$

$$f_3(S) \equiv \langle(\cosh 2S\eta - \cosh 2S)\hat{u}\rangle = \lambda\left\{-\frac{\sinh S}{4S} + \frac{\sinh 3S}{12S}\right\} - \cosh 2S \tag{42c}$$

This can now be solved for Nu in terms of Br, to give

$$Nu = 2\left[Br\,\langle\phi\rangle - \frac{1 + Br\,\lambda^2\{(A/2)f_1(S) + (B/S^2)f_2(S) + (C/4S^2)f_3(S)\}}{\lambda\{((\cosh S)/2)f_1(S) - (1/S^2)f_2(S)\}}\right]. \tag{43}$$

For the special case $Br = 0$, this gives

$$Nu = \frac{12S(S - \tanh S)^2}{2S^3 + 3S\tanh^2 S + 15(\tanh S - S)}, \tag{44}$$

in agreement with the expression given in Equation (4.120) of Nield and Bejan (1999).

For the case of very small Darcy number (slug flow) one finds that

$$Nu = 6, \tag{45}$$

independent of the value of Br and independent of the viscous dissipation model.

For the case of very large Darcy number and $M = 1$, the result for Model 3 is

$$Nu = \frac{70}{17} + \frac{54}{17}Bn. \tag{46}$$

The expression (46) is that relevant to a clear fluid. Here Bn is Brinkman number appropriate to a fluid clear of solid material, related to the porous-medium Darcy–Brinkman number Br by

$$Bn = Da\,Br = \frac{\mu {U^*}^2}{k(T^*_\mathrm{m} - T^*_\mathrm{w})}. \tag{47}$$

For the general case, one can calculate Nu from Equations (43), (42) and (32)–(36).

3. Results and Discussion

We have the capability to calculate temperature profiles. For the case of uniform-temperature boundary condition, our shooting procedure yields θ as the eigenfunction corresponding to the eigenvalue Nu, while for the uniform flux case we can calculate θ from Equation (40) once Nu is found from Equation (43). However, we have an extensive parameter space to explore and so for brevity we have chosen to present in this paper just the values of Nu.

Figures 2–4 apply to the uniform-temperature boundary condition and a liquid saturating the porous medium (negligible flow work effect). The three viscous dissipation models are considered in turn. A very small Darcy number corresponds to the Darcy flow limit, and a very large Darcy number to the clear fluid limit. As one would expect, for the case of small Da there is relatively little difference between the results of the three models (though comparison between the figures is made

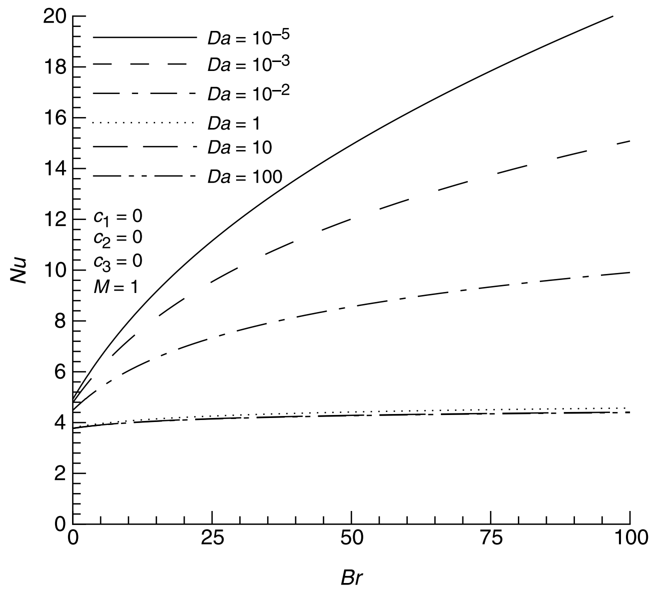

Figure 2. Uniform-temperature boundary condition. Model 1A (viscous dissipation modelled by a Darcy term, liquid). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

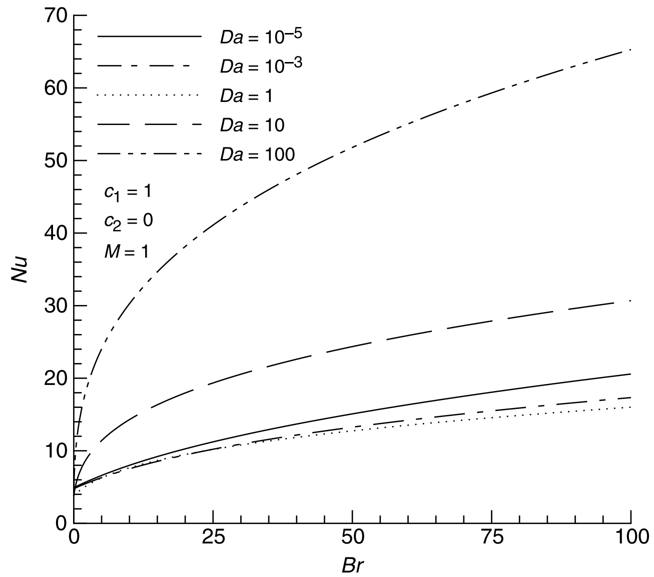

Figure 3. Uniform-temperature boundary condition. Model 2A (viscous dissipation modelled by a power-of-drag term, liquid). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

somewhat difficult by the necessarily different vertical scales). At larger values of *Da*, the difference between the various models is more dramatic. For Model 1, the Nusselt number decreases as the Darcy number is increased. For Models 2 and 3,

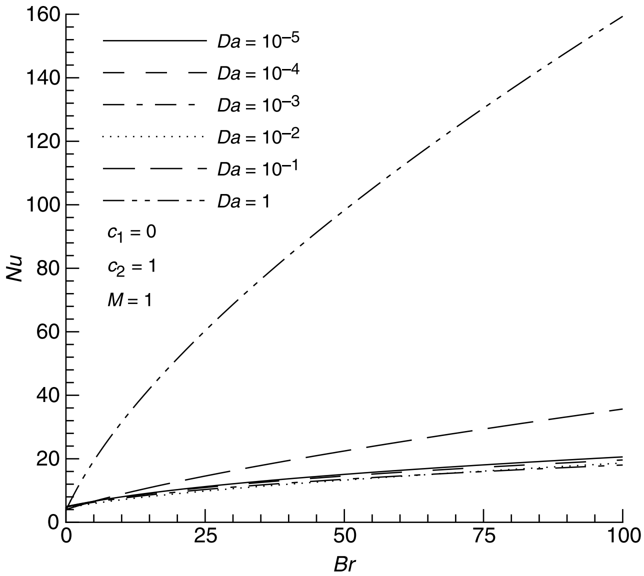

Figure 4. Uniform-temperature boundary condition. Model 3A (viscous dissipation model by a clear-fluid-compatible term, liquid). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

Nu initially decreases as *Da* increases for fixed *Br*, passes through a minimum, and then increases again. For the case of Model 3 the ultimate increase is very large. In all cases *Nu* increases with *Br* for fixed *Da*. It is only Model 3 that claims to be relevant at very large values of *Da*, and in Figure 5 we have a confirmation that this model does lead to the limit as expected when *Da* tends to infinity.

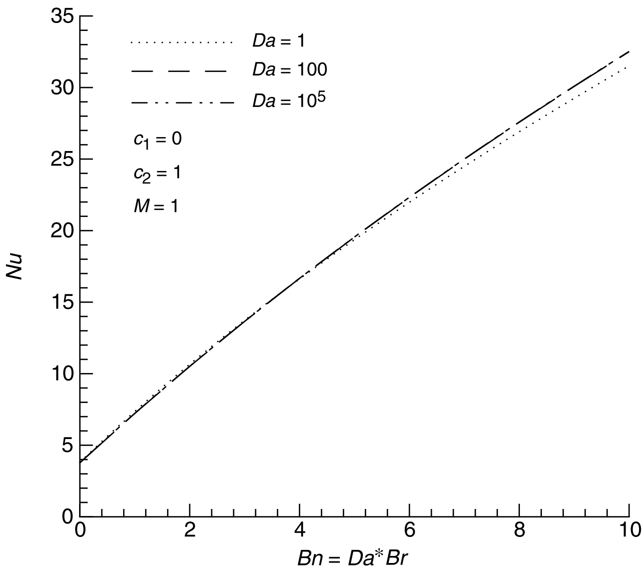

Figure 5. As for Figure 4, but with Nusselt number versus the clear-fluid Brinkman number.

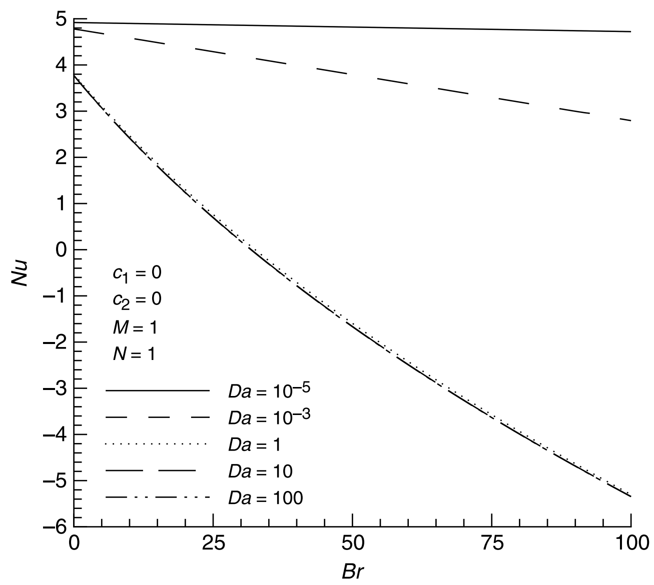

Figure 6. Uniform-temperature boundary condition. Model 1B (viscous dissipation modelled by a Darcy term, perfect gas). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

Figures 6–8 correspond to Figures 2–4, but now the effect of flow work, in the case of a perfect gas, has been added. The contrast between Figures 6 and 2 is dramatic. For small *Da*, the effect of flow work more or less cancels out the effect

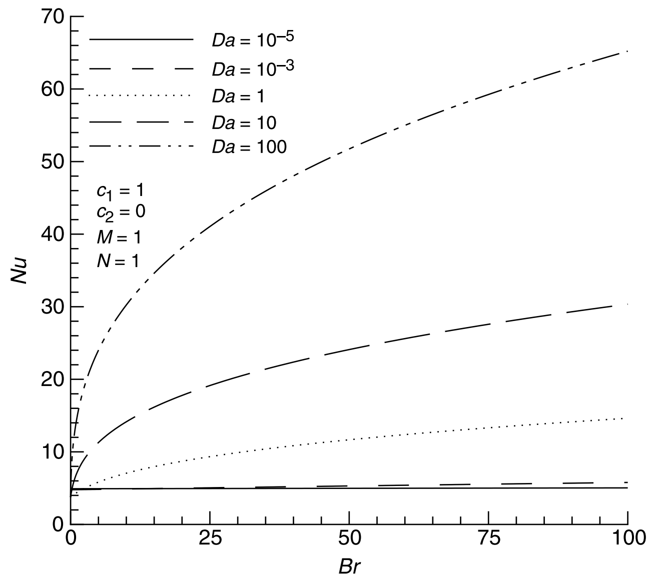

Figure 7. Uniform-temperature boundary condition. Model 2B (viscous dissipation modelled by a power-of-drag term, perfect gas). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

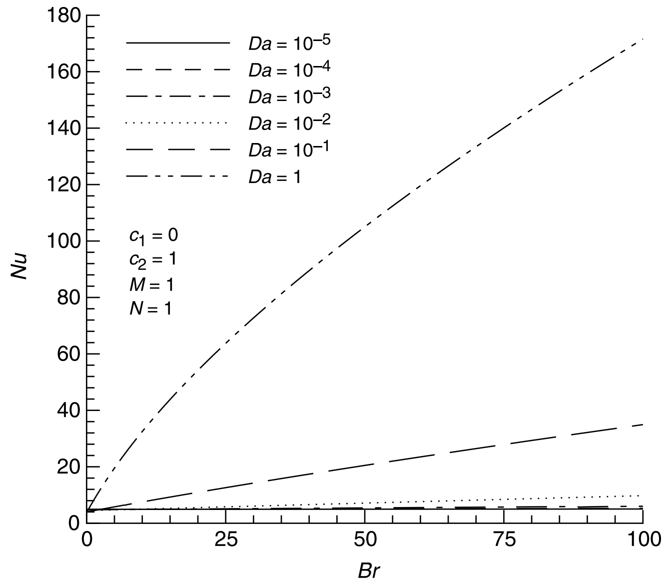

Figure 8. Uniform-temperature boundary condition. Model 2C (viscous dissipation model by a clear-fluid-compatible term, perfect gas). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

of viscous dissipation in the case of Model 1. For other values of *Da*, the effect of flow work is to produce a net reduction in the values of *Nu*. The contrast between Figures 7 and 3, or between Figures 8 and 4, is less dramatic. For Models 2 and 3,

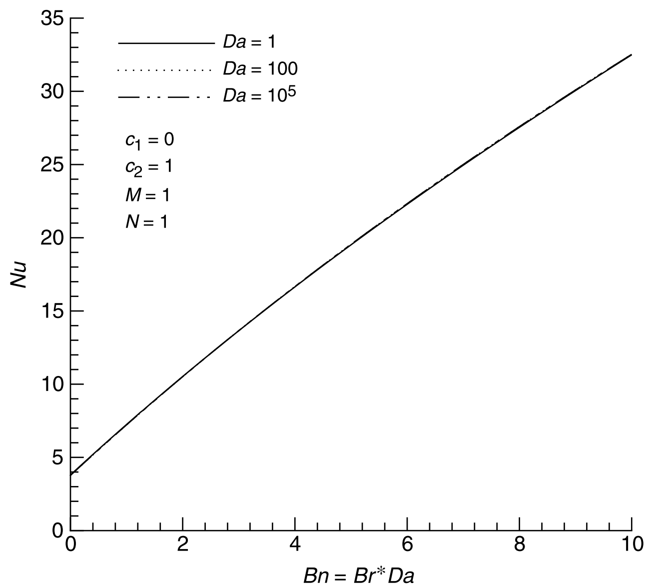

Figure 9. As for Figure 8, but with Nusselt number versus the clear-fluid Brinkman number.

flow work does not have much effect in the case of large *Da*. For small *Da* the flow work again more or less cancels out the effect of viscous dissipation (Figure 9).

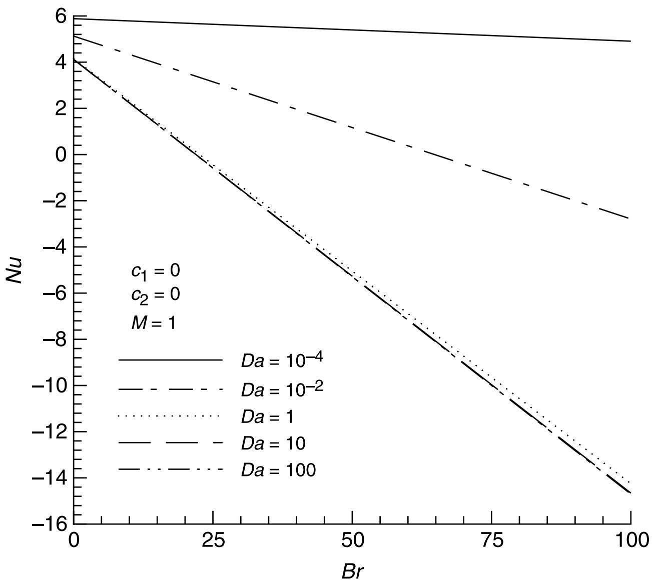

Figure 10. Uniform-flux boundary condition. Model 1 (viscous dissipation modelled by a Darcy term, liquid or gas). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

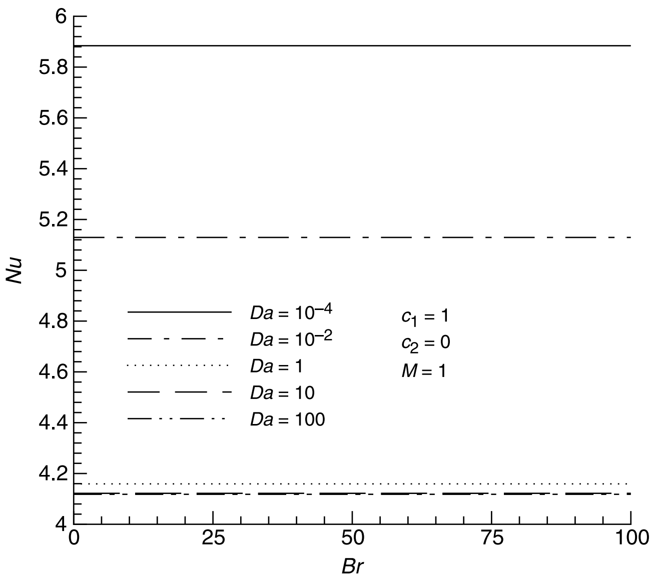

Figure 11. Uniform-flux boundary condition. Model 2 (viscous dissipation modelled by a power-of-drag term, liquid or gas). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

Figures 10–13 for the uniform-flux boundary condition correspond to Figures 2–5 for the uniform-temperature boundary condition. As we have already discovered from our analysis, flow work has no effect for the uniform-flux case. As

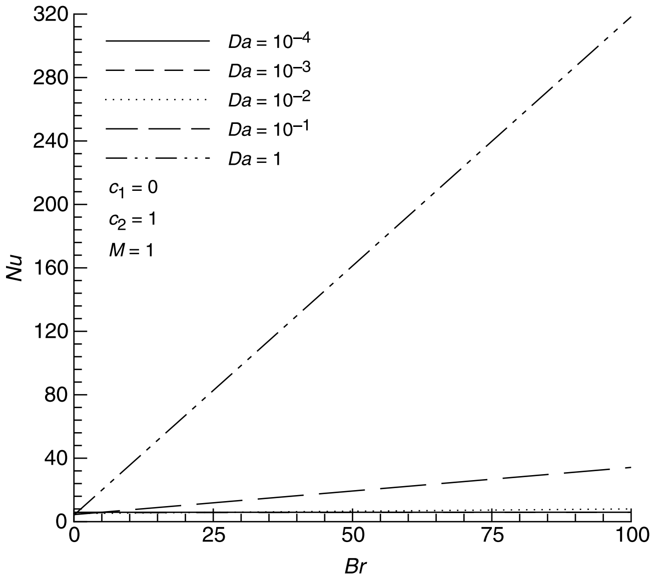

Figure 12. Uniform-flux boundary condition. Model 3 (viscous dissipation modelled by a clear-fluid-compatible term, liquid or gas). Plots of Nusselt number versus Darcy–Brinkman number for various values of Darcy number.

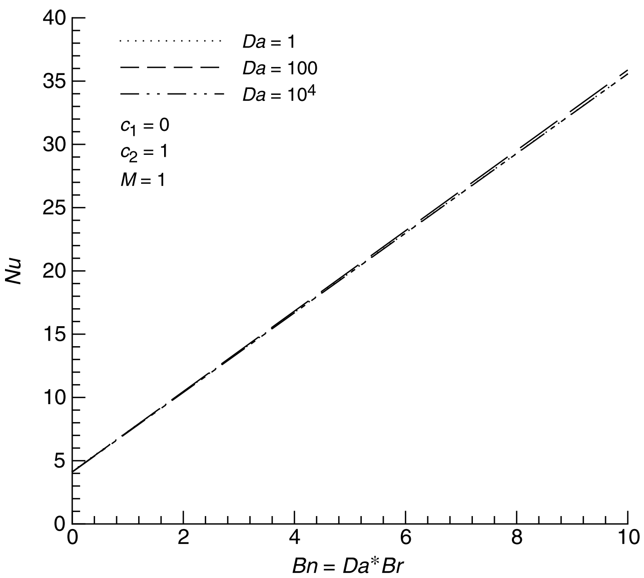

Figure 13. As for Figure 12, but with Nusselt number versus the clear-fluid Brinkman number.

expected from the fact that Equation (43) indicates that *Nu* is a linear function of *Br*, the plots in Figures 10–13 are all straight lines. It is very clear that when *Da* is small there is little difference between the results of the three models. Now, however, for Model 1 *Nu* decreases as *Br* increases (and also as *Da* increases), for Model 2 *Nu* is independent of *Br* (something that can be demonstrated directly from the analysis) but decreases as *Da* increases, while for Model 3 the situation is more complicated. That *Nu* is independent of *Br* for Model 2 is evident when one recognizes that the flow work term is equal to the power (per unit volume of the porous medium) of the pressure gradient, which is equal to the power of the drag force (in this case of steady unidirectional flow) which in turn is equal to the viscous dissipation term for this case. We have already observed that flow work has no effect on *Nu*, and it follows that for this case viscous dissipation will also have no effect on *Nu*. In the case of Model 3, the effect of *Br* for the uniform-flux boundary condition is qualitatively the same as that for the uniform-temperature boundary condition.

As far as the authors are aware, there is no work in the published literature with which the present results can usefully be compared. For example, the work by Ou and Cheng (1973) is limited by the fact that their solution violates the First Law of Thermodynamics far downstream.

4. Conclusions

The effect of viscous dissipation, either without or with the additional effect of flow work, on fully developed forced convection in a parallel-plate channel filled with a porous medium, has been studied analytically, both for uniform-temperature and uniform-flux boundary conditions. The effect of various models for the effect of viscous dissipation has been investigated. For small Darcy number, the difference between these models is negligible, but for larger Darcy number the difference is substantial, and the present results offer a basis for an experimental investigation to adjudicate between these models.

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