A Rigorous Formation Damage Skin Factor and Reservoir Inflow Model for a Horizontal Well

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Summary

In this paper, we present a new analytical model for formation damage skin factor and the resulting reservoir inflow, including the effect of reservoir anisotropy and damage heterogeneity. The shape of the damaged region perpendicular to the well is based on the pressure equation for an anisotropic medium and, thus, is circular near the well and elliptical far from the well. The new model can be used for various distributions of damage along the well, depending on the time of exposure during drilling and completion. The inflow equation for a damaged, parallel-piped-shape reservoir illustrates the importance of the ratio of the reservoir thickness to the drainage length perpendicular to the well on the influence of formation damage for horizontal well productivity. Our model gives a simple, analytical expression for determining this effect.

Introduction

Horizontal well completion technology has become an important part of oil and gas recovery. Horizontal wells have proven to be excellent producers for thin reservoirs or for thicker reservoirs with good vertical permeability. A horizontal well creates a drainage pattern that is quite different from that for a vertical well. The flow geometry in a horizontal well is more likely to be radial near the well and linear far from the well while, in general, the vertical well has radial flow geometry. Another major difference between horizontal and vertical wells is the strong influence of horizontal to vertical permeability anisotropy on horizontal well productivity. Because of these factors, near-wellbore formation damage has a different effect on a horizontal well than on a vertical one and must be described with a different skin-factor model.

Another critical difference between vertical and horizontal wells is that the damage distribution around a horizontal well is likely to be highly nonuniform. Reservoir anisotropy may lead to an elliptically shaped damage zone perpendicular to the well, depending on the ratio of the vertical to horizontal permeability. Because of the large formation length contacted by a horizontal well, formation damage is not likely to be uniformly distributed. Therefore, the damage zone around a horizontal wellbore cannot be assumed to be simply a cylindrical region of reduced permeability, as is the usual assumption for vertical wells.

The objective of this paper is to provide a basis for estimating the overall damage-skin effect for horizontal wells and for determining the horizontal well productivity, including the productivity loss caused by formation damage near the wellbore in an anisotropic reservoir. Our model accounts for the effects of permeability anisotropy and can be used for various damage distributions along the well.

Formation-Damage-Skin Model for a Horizontal Well

There are two key parts to our new model of the formation damage skin factor. The first is a model of the local skin factor, $s(x)$, describing the effect of damage in the y-z plane perpendicular to the wellbore **(Fig. 1).** The second element of the new model is the manner of accounting for any arbitrary distribution of damage along the horizontal well, as illustrated in **Fig. 2.**

Local Skin-Factor Model, $s(x)$. To derive an analytical model of the local skin factor at Position x along a horizontal well, we must make some assumptions about the distribution of damage in the y-z plane. We assume that the cross section of damage perpendicular to the well (Fig. 1) mimics the isobars given by Peaceman's solution[1] for flow through an anisotropic permeability field to a cylindrical wellbore. This solution shows that isobars are a series of concentric ellipses with aspect ratios (ratio of the major to minor axis lengths) being 1 at the wellbore and increasing as the distance from the wellbore increases. Because formation damage is often directly related to flux or velocity, we assume that the damage is distributed similar to the pressure field (i.e., the outer boundary of the damaged zone will lie on an isobar).

With this assumption about the distribution of the damage in the y-z plane, Hawkins' formula[2] can be transformed for anisotropic space, and an analytical expression for local skin can be derived, as shown in Appendix A.

$$s(x)=\left[\frac{k}{k_d(x)}-1\right]\ln\left\{\frac{1}{I_{\text{ani}}+1}\left[\frac{r_{dh}(x)}{r_w}+\sqrt{\left(\frac{r_{dh}(x)}{r_w}\right)^2+I_{\text{ani}}^2-1}\right]\right\}, \quad (1)$$

$$\text{where } I_{\text{ani}}=\sqrt{k_H/k_V}. \quad (2)$$

Here, r_{dH} = the half-length of the horizontal axis of the damage ellipse, r_w = wellbore radius, k_d = the permeability in the damaged zone, and k = the undamaged permeability. Eq. 1 can be called the general form of the Hawkins formula for an anisotropic permeability field. For an isotropic medium ($I_{\text{ani}}=1$), Eq. 1 reduces to the usual Hawkins formula.

Overall Horizontal Well Damage-Skin-Factor Model. Next, we derive a model for the overall damage skin factor for a horizontal well for any arbitrary distribution of local damage, as shown in Fig. 2.

For an isotropic permeability field, we assume that radial flow is dominant in the near-wellbore region. This gives the following relationship between the overall skin, s_{eq}, and local skin, $s(x)$.

$$s_{\text{eq}}=\frac{L}{\int_0^L\{\ln[h/(2r_w)]+s(x)\}^{-1}\,\mathrm{d}x}-\ln\left(\frac{h}{2r_w}\right). \quad (3)$$

Applying an appropriate coordinate transformation into the equivalent isotropic space gives the following overall skin equation for an anisotropic medium (see Appendix A).

$$s_{\text{eq}}=\frac{L}{\int_0^L\left\{\ln\left[\frac{I_{\text{ani}}h}{r_w(I_{\text{ani}}+1)}\right]+s(x)\right\}^{-1}\mathrm{d}x}-\ln\left[\frac{I_{\text{ani}}h}{r_w(I_{\text{ani}}+1)}\right]. \quad (4)$$

Eqs. 1 and 4 are the formation-damage-skin model for a horizontal well that takes into account the reservoir anisotropy and

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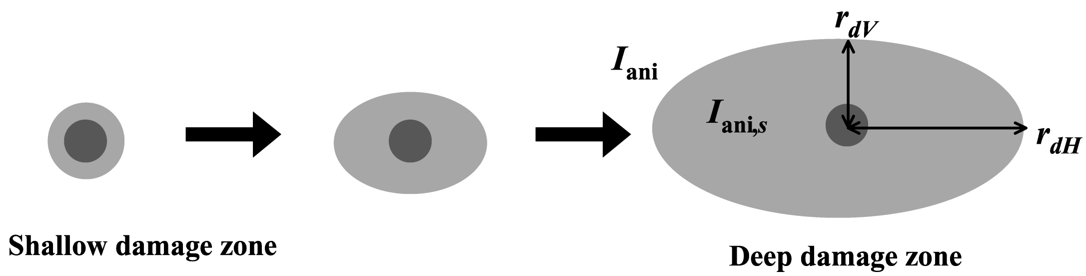

Fig. 1—Assumed damage region in the *y-z* plane.

damage heterogeneity. Compared with Hawkins' formula for a vertical well, these expressions contain additional parameters, such as reservoir thickness, length of a horizontal well, and index of anisotropy. The effects of the reservoir thickness and the length of a horizontal well on the skin calculation will be almost negligible. However, the anisotropy index will be one of the key parameters influencing the damage skin factor for a horizontal well.

Examples

Skin Calculation for a Truncated Elliptical Cone Model. Frick and Economides[3] developed equations for the horizontal well damage-skin effect for a particular damage distribution around a horizontal well, as depicted in **Fig. 3.** For damage in the *y-z* plane (i.e., at any cross section of the wellbore), they assumed that the damaged region is elliptical in shape with the ratio of the maximum to the minimum axis of the ellipse equal to I_{ani}. In contrast, in our model we assume that the aspect ratio of the elliptical damaged region increases in the same manner as isobars as the damaged region gets larger.

For damage distribution along the well, Frick and Economides assumed that the depth of damage would be greatest at the heel of the well, decreasing linearly to a minimum value at the toe of the well. This assumption is based on damage being caused by the time of exposure to drilling fluid. From these geometric assumptions about the distribution of damage, they derived the following horizontal well skin equation.

$$s_{\text{eq}} = \left(\frac{k}{k_d} - 1\right)\ln\left[\frac{1}{I_{\text{ani}}+1}\sqrt{\frac{4}{3}\left(\frac{r_{dH,\max}^2}{r_w^2} + \frac{r_{dH,\max}}{r_w} + 1\right)}\right] \quad \ldots (5)$$

This equation was obtained by applying Hawkins' formula to a cylindrical damage region, with the volume of the cylinder calculated as the volume of the damage cone. In terms of the damage distribution along the well (the *x*-direction), this is equivalent to calculating the overall skin factor as the linear average of the local skin, $s(x)$. In the derivation of our overall skin-factor model (Eq. 4), we find that a geometric average is correct.

The following examples compare these two skin-factor models for the conditions given in **Table 1.** We consider penetrations of damage (maximum axis of the ellipse near the vertical section of the well) up to 5 ft and permeability impairment ratios (k/k_d) of 5, 10, and 20. To solve the integral in Eq. 4, the damaged region was divided into small segments, with a constant damage depth in each segment. For an isotropic reservoir, cross sections of the damage cone normal to the well are circular. The damage shapes in the *y-z* plane presumed by both models are exactly the same for the isotropic case. However, our skin model and the existing one give different results for all permeability impairment ratios except at $r_{dH} = r_w$ **(Fig. 4).** This difference occurs because the Frick and Economides model uses a linear average of the local skin factor to find the overall skin, while our model uses a geometric average derived by equating the total flow rate with the sum of the local flow rates. For an isotropic reservoir, Frick and Economides' model estimates a larger skin effect for all permeability impairment ratios than our model. The higher the permeability impairment ratio, the larger the differences between the models.

For an anisotropic reservoir (see **Fig. 5**), the difference between the two models becomes more significant. This difference is primarily caused by the different assumptions made about the shape of damage in the *y-z* plane. For a damage extent of less than approximately 1 ft in the horizontal direction, the Frick and Economides model calculates a negative skin factor, even though there is damage around the well. This anomalous result is because the equivalent radius of the elliptical damage region calculated in their model is actually smaller than the wellbore radius for shallow damage. Because the elliptical damage regions in our model approach a circular shape as the damage zone becomes small, this problem does not occur. For deep damage, our model predicts a lower skin factor than the Frick and Economides model, as was the case for an isotropic reservoir.

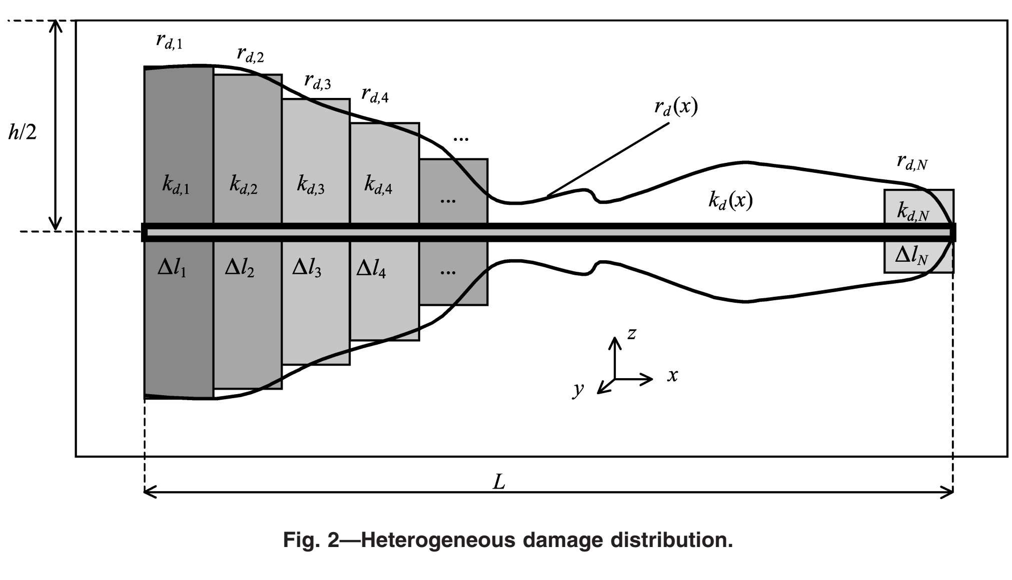

Fig. 2—Heterogeneous damage distribution.

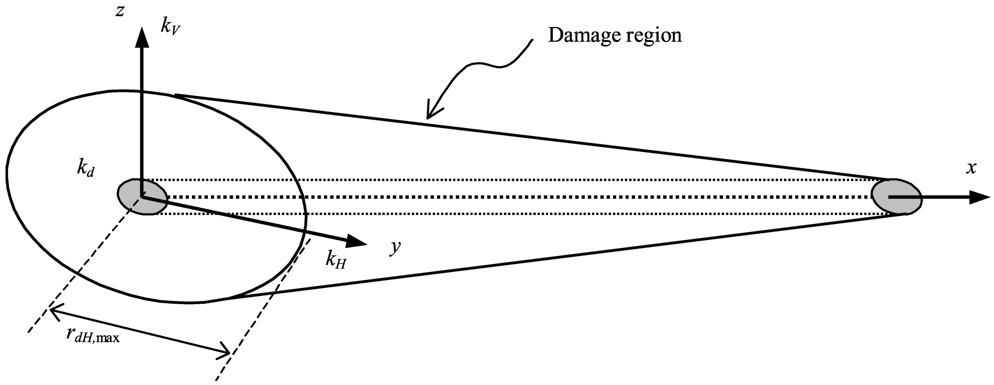

Fig. 3—Truncated, elliptical cone model.

Effect of Damage on Horizontal Well Productivity. The ratio of the damaged-well productivity index to the ideal, undamaged productivity index quantifies the effect of formation damage on total well productivity. An analytical equation for inflow to a horizontal well from a rectangular reservoir, derived in Appendix B, shows that the index of anisotropy and the ratio of the reservoir thickness, h, to the drainage length perpendicular to the well, y_b, are the key parameters dictating the importance of damage on horizontal well productivity.

This equation is derived by assuming steady-state flow of an incompressible fluid for a fully penetrating, horizontal well. The equation also can be rewritten as:

$$\frac{J_d}{J_o}=\frac{\beta_r+\beta_l-1.224}{\beta_r+\beta_l-1.224+s}, \quad (6)$$

where $\beta_r=\ln\left[\frac{hI_{\text{ani}}}{r_w(I_{\text{ani}}+1)}\right]$, $\quad (7)$

and $\beta_l=\frac{\pi y_b}{hI_{\text{ani}}}$. $\quad (8)$

β_r and β_l = the geometric factors resulting from radial and linear flow, respectively. As the reservoir thickness increases, β_r becomes dominant. At $h=2y_b$, the flow geometry is almost radial (linear flow term vanishes). However, on a practical reservoir scale, the reservoir thickness is almost always much smaller than the drainage length perpendicular to the well. In addition, because β_r is a logarithmic function, it does not change much compared with β_l. Therefore, β_l will be the dominant factor for the impact of skin on horizontal well production. **Fig. 6** shows the productivity index ratio for isotropic reservoirs with different skin factors. The drainage length perpendicular to the well is assumed to be 2,000 ft, and the wellbore radius is 0.3 ft. Obviously, the impact of formation damage becomes more significant as h/y_b increases. We can conclude that formation damage may not be significant for thin reservoirs, but it may be more important for relatively thick reservoirs. This difference comes from the flow geometry of a horizontal well, which consists of a combination of radial and linear flows.

Fig. 7 shows the productivity index ratio for anisotropic reservoirs with a constant skin ($s=10$). The index of anisotropy decreases the equivalent reservoir thickness and increases the equivalent drainage length. Therefore, reservoir anisotropy magnifies the influence of damage for horizontal wells.

TABLE 1—DATA FOR THE EXAMPLES

Example Data	
Index of anisotropy, I_{ani}	1 and 3
Wellbore radius, r_w	0.328 ft
Reservoir height, h	53 ft
Horizontal well length, L	2,000 ft

We have considered the case for which the horizontal well is assumed to penetrate the entire reservoir. In the case of a partially penetrating well, we need to consider the effect of the well length relative to the drainage radius for the influence of the damage skin on the productivity index. According to Renard and Dupuy,[4] decreasing the horizontal well length only slightly increases the influence of skin on the total production, which will be negligible compared with the effect of the reservoir anisotropy and the ratio of h to y_b for long horizontal wells.

Conclusions

A new damage-skin model has been derived that includes damage heterogeneity along the well and reservoir anisotropy. The model is applicable for various distributions of damage. We derived a skin equation for an anisotropic medium analogous to the well-known Hawkins formula for a vertical well. Combining both equations gives a general model of the damage-skin factor for a horizontal well.

For a cone-shaped damage region, the commonly used damage-skin-factor model underpredicts skin for shallow damage and overpredicts it for deeper damage. Moreover, it may yield negative skin for shallow damage in anisotropic reservoirs.

We also developed an analytical inflow equation to quantify the impact of formation damage on horizontal well production that strongly depends on reservoir geometry, particularly reservoir thickness and drainage length perpendicular to the well. Horizontal well flow geometry can be described by radial flow near the wellbore and linear flow far from the well. The existence of the linear flow region reduces the effect of formation damage on overall well performance. A thin reservoir has more contribution from the linear pressure-drop region than a thick reservoir. Therefore, formation damage in a thin reservoir has less of an effect on well productivity than in a thick reservoir. Reservoir anisotropy increases the influence of formation damage on the productivity of a horizontal well because it lengthens the equivalent reservoir height and shortens the equivalent drainage length. This means that the radial flow region increases and the linear flow region decreases.

Nomenclature

b = constant of conformal mapping, dimensionless
B = formation volume factor, res bbl/STB (m^3/Sm^3)
h = reservoir height, ft (m)
I_{ani} = anisotropy index, dimensionless
J = productivity index, STB/D/psi (m^3/sec/Pa)
k = permeability, md (m^2)
L = horizontal well length, ft (m)
N = total number of segments
p = pressure, psi (Pa)
q = flow rate, bbl/D (m^3/sec)
q^* = flow rate per unit length, bbl/D-ft (m^3/sec-m)
r = radius, ft (m)
r_e = drainage radius, ft (m)

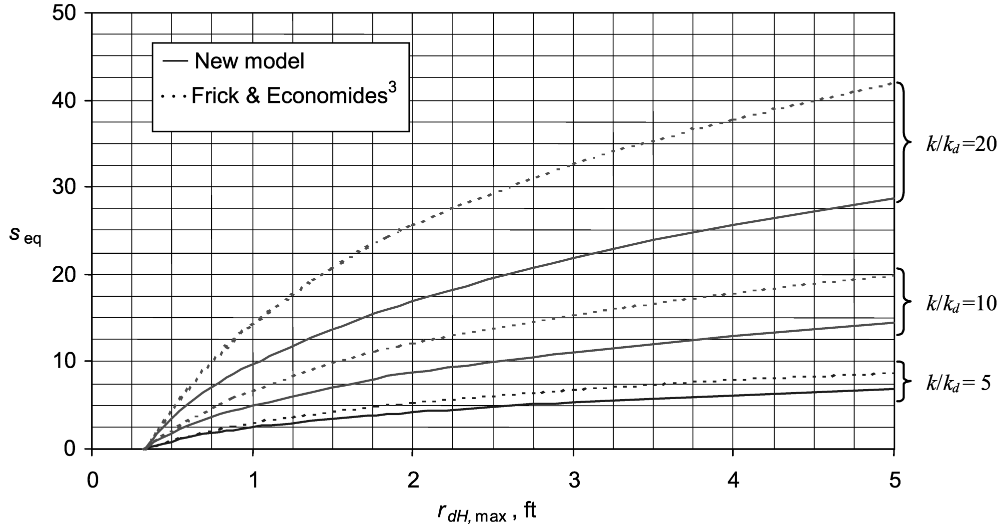

Fig. 4—Comparison of skin calculation for I_{ani}=1 (isotropic case).

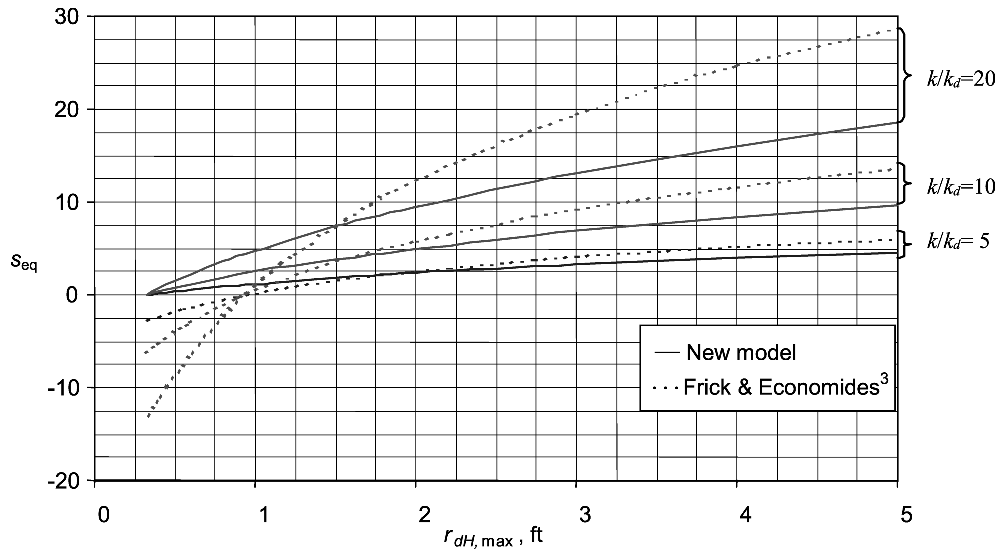

Fig. 5—Comparison of skin calculation for I_{ani}=3.

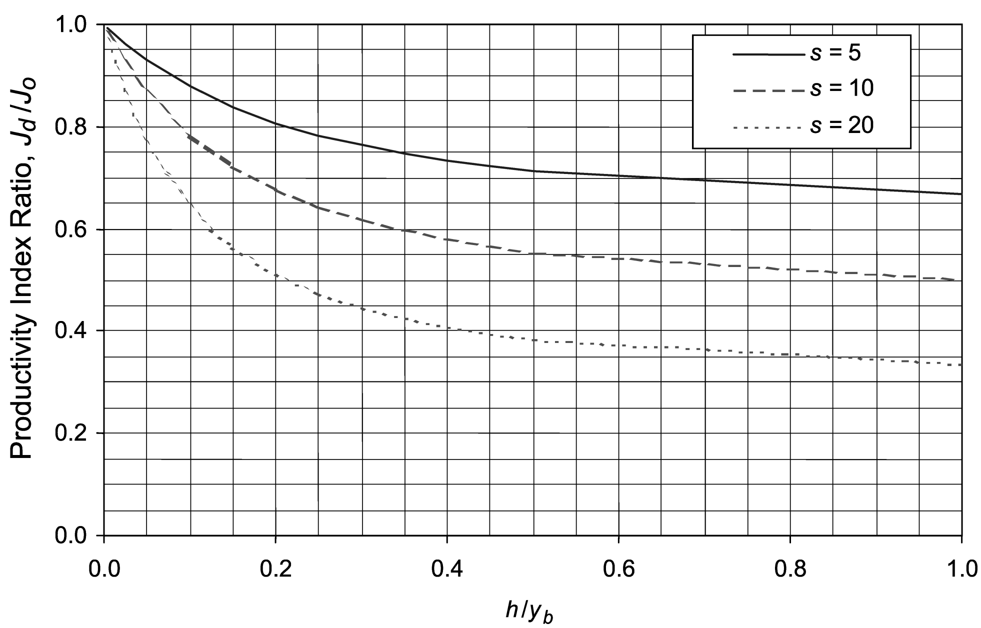

Fig. 6—Effect of formation damage on horizontal well production in an isotropic reservoir.

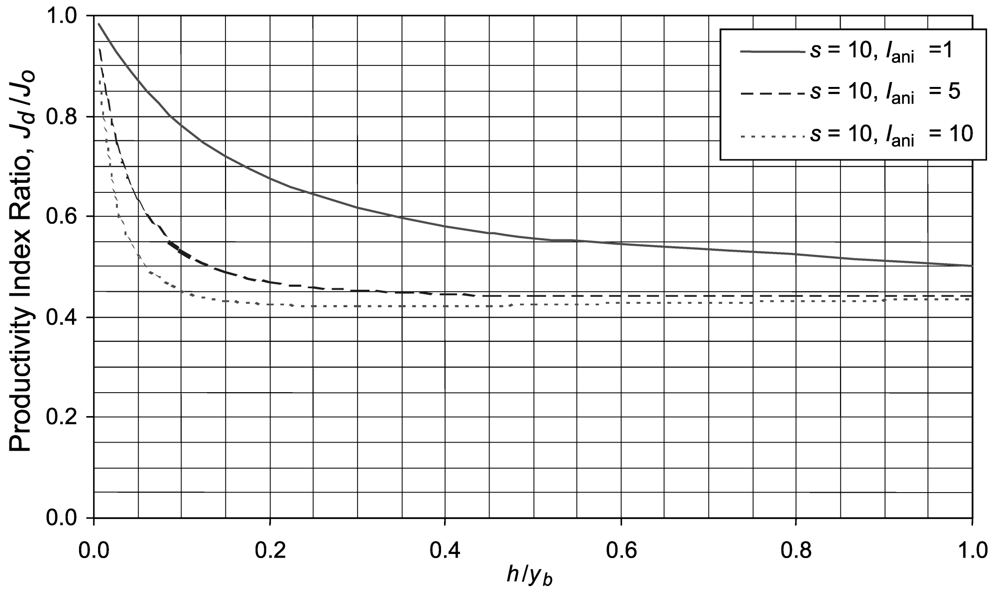

Fig. 7—Effect of formation damage on horizontal well production in an anisotropic reservoir.

r_w = wellbore radius, ft (m)
s = skin factor, dimensionless
u = transformed coordinate in z direction, ft (m)
v = transformed coordinate in y direction, ft (m)
x = x-coordinate (original coordinate), ft (m)
y = y-coordinate (original coordinate), ft (m)
z = z-coordinate (original coordinate), ft (m)
β = geometric quantity
Δl = length of well segment, ft (m)
Δp = pressure drop, psi (Pa)
μ = fluid viscosity, cp (m·Pa·s)
ρ = variable of conformal mapping

Subscripts

b = boundary
d = damage zone
eq = equivalent
H = horizontal
i = index of segment
l = linear flow
o = original (or undamaged)
r = radial flow
t = transition
V = vertical

Acknowledgments

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Appendix A—Formation-Damage-Skin Model

Skin Calculation for an Anisotropic Reservoir. In discussing flow through an anisotropic medium, it is assumed that the principal axes of the permeability tensor are parallel to the coordinate axes. To derive a skin equation, we assume that the cross section of damage perpendicular to the well **(Fig. 8)** mimics the isobars given by Peaceman's solution[1]; that is,

$$p = p_{wf} + \frac{q\mu}{2\pi(k_H k_V)^{0.5} L}\ln\left(\frac{r_{\mathrm{eq}}}{r_{w,\mathrm{eq}}}\right), \qquad \text{(A-1)}$$

where r_{eq} = the mean of the maximum and minimum axes of an isobar in the equivalent isotropic space. The coordinate transformation into the equivalent isotropic space is given by:

$$v = y/\sqrt{I_{\mathrm{ani}}}, \qquad \text{(A-2)}$$

$$u = z\sqrt{I_{\mathrm{ani}}}, \qquad \text{(A-3)}$$

where $I_{\mathrm{ani}} = \sqrt{k_H/k_V}$. (A-4)

Because we have an elliptic boundary condition at the wellbore in the transformed coordinate, isobars are not radial; rather, the isobars are a family of concentric ellipses. According to Peaceman's work, the major and minor axes of isobars in the equivalent isotropic space are given by

$$u_0 = b\cos h\rho, \qquad \text{(A-5)}$$

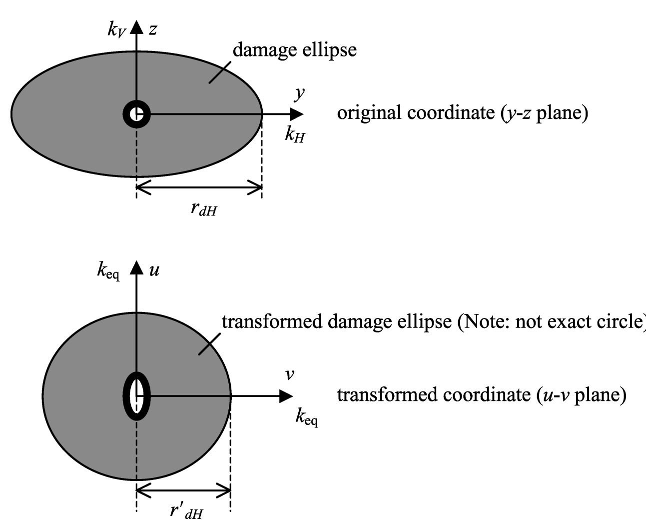

Fig. 8—Coordinate transformation into the equivalent isotropic space.

and $v_0 = b \sin h\rho$, ... (A-6)

in which ρ = a variable of conformal mapping and b = a constant of conformal mapping, which is

$$b = \frac{r_w\sqrt{k_H - k_V}}{(k_H k_V)^{0.25}}, \text{ for } k_H > k_V. \quad \text{(A-7)}$$

For a damage ellipse,

$$\rho_d = \sin h^{-1}\left(\frac{v_d}{b}\right) = \sin h^{-1}\left(\frac{r_{dH}/\sqrt{I_{\text{ani}}}}{b}\right). \quad \text{(A-8)}$$

The mean axis of a damage ellipse can be estimated by:

$$\begin{aligned} r_{d,\text{eq}} &= \frac{b}{2}(\cos h\rho_d + \sin h\rho_d) \\ &= \frac{r_w}{2}\sqrt{I_{\text{ani}} - \frac{1}{I_{\text{ani}}}}\exp\left[\sin h^{-1} = \left(\frac{r_{dH}}{r_w\sqrt{I_{\text{ani}}^2 - 1}}\right)\right] \\ &= \frac{r_w}{2}\sqrt{I_{\text{ani}} - \frac{1}{I_{\text{ani}}}}\left[\frac{r_{dH}}{r_w\sqrt{I_{\text{ani}}^2 - 1}}\right. \\ &\quad \left. + \sqrt{\left(\frac{r_{dH}}{r_w\sqrt{I_{\text{ani}}^2 - 1}}\right)^2 - 1}\,\right] \quad \text{(A–9)} \end{aligned}$$

Hawkins' formula for an anisotropic reservoir can be transformed to

$$s = \left(\frac{k_{\text{eq}}}{k_{d,\text{eq}}} - 1\right)\ln\left(\frac{r_{d,\text{eq}}}{r_{w,\text{eq}}}\right), \quad \text{(A-10)}$$

where $\dfrac{k_{\text{eq}}}{k_{d,\text{eq}}} = \dfrac{\sqrt{k_V k_H}}{\sqrt{k_{dV} k_{dH}}} = \dfrac{k_H}{k_{dH}} = \dfrac{k}{k_d}$, ... (A-11)

and $r_{w,\text{eq}} = r_w[(I_{\text{ani}} + 1)/(2\sqrt{I_{\text{ani}}})]$. .... (A-12)

To derive Eq. A-11 it is assumed that $I_{\text{ani},d} = I_{\text{ani}}$. (This may not always be true in field practices, but it is an essential assumption to achieve the rigorous skin representation.) Thus, the following skin equation is derived by substituting into Eq. A-10 for $r_{d,\text{eq}}$ from Eq. A-9 and for $r_{w,\text{eq}}$ from Eq. A-12.

$$s = \left(\frac{k}{k_d} - 1\right)\ln\left[\frac{1}{I_{\text{ani}} + 1}\left(\frac{r_{dH}}{r_w} + \sqrt{\frac{r_{dH}^2}{r_w^2} + I_{\text{ani}}^2 - 1}\right)\right]. \quad \text{(A-13)}$$

The previous equation can be used to calculate the skin factor for an anisotropic reservoir. If the permeability is isotropic, it reduces to the conventional Hawkins formula.

Heterogeneous Skin Calculation. If the distribution of formation damage is heterogeneous, the distribution of the flow rate per unit length along the well is not uniform but a function of the position of x along a well.

$$q^*(x) = \frac{2\pi k \Delta p/(\mu B)}{\ln[h/(2r_w)] + s(x)}. \quad \text{(A-14)}$$

Frick and Economides[3] postulated that the damage distribution along a horizontal well was not uniform because of the exposure time during drilling and completion. We also assume that the damage permeability and damage radius are functions of x to account for local variation in k, local fractures, changes in drilling fluid properties, etc.

$$k_d = k_d(x), \quad \text{(A-15)}$$

$$r_d = r_d(x). \quad \text{(A-16)}$$

The local skin factor, $s(x)$, is also given by a function of x.

$$s(x) = \left[\frac{k}{k_d(x)} - 1\right]\ln\left[\frac{r_d(x)}{r_w}\right]. \quad \text{(A-17)}$$

Integrating Eq. A-14 over $[0,L]$ gives the total flow rate to the well.

$$q = \int_0^L q^*(x)\mathrm{d}x = \frac{2\pi k \Delta p}{\mu B}\int_0^L \frac{\mathrm{d}x}{\ln[h/(2r_w)] + s(x)}. \quad \text{(A-18)}$$

The overall skin effect can be derived by comparing Eq. A-18 with the following equation.

$$q = \frac{2\pi k L \Delta p/(\mu B)}{\ln[h/(2r_w)] + s_{\text{eq}}}. \quad \text{(A-19)}$$

Hence,

$$s_{\text{eq}} = \frac{L}{\displaystyle\int_0^L \frac{\mathrm{d}x}{\ln[h/(2r_w)] + s(x)}} - \ln\left(\frac{h}{2r_w}\right) \quad \text{(A-20)}$$

for an isotropic reservoir.

If k_d and r_d = constants along a well, we can solve the integral analytically to yield:

$$s_{\text{eq}} = \left(\frac{k}{k_d} - 1\right)\ln\left(\frac{r_d}{r_w}\right). \quad \text{(A-21)}$$

This is the well-known Hawkins' formula. Because the integral is usually difficult to solve analytically, it needs to be divided into a number of segments, as shown in Fig. 2. Rewriting the equation in the form of a Riemann sum over $[0,L]$ gives

$$s_{\text{eq}} = \frac{L}{\displaystyle\sum_{i=1}^{N}\left\{\frac{\Delta l_i}{\ln[h/(2r_w)] + s_i}\right\}} - \ln\left(\frac{h}{2r_w}\right), \quad \text{(A-22)}$$

in which $s_i = \left(\dfrac{k}{k_{d,i}} - 1\right)\ln\left(\dfrac{r_{d,i}}{r_w}\right)$, ... (A-23)

and N = the total number of the segments. To ensure the accuracy of the discretization in Eq. A-22, N must be chosen sufficiently high, depending on $r_d(x)$. The choice of $N = 100$ is enough to achieve an error of less than 1% for the truncated elliptical cone damage model (a linear case).

For an anisotropic reservoir, Eq. A-20 becomes

$$s_{\text{eq}} = \frac{L}{\displaystyle\int_0^L \left\{\ln\left[\frac{I_{\text{ani}}h}{r_w(I_{\text{ani}} + 1)}\right] + s(x)\right\}^{-1}\mathrm{d}x} - \ln\left[\frac{I_{\text{ani}}h}{r_w(I_{\text{ani}} + 1)}\right], \quad \text{(A-24)}$$

in which

$$s(x) = \left[\frac{k}{k_d(x)} - 1\right]\ln\left[\frac{1}{I_{\text{ani}} + 1}\left(\frac{r_{dH}(x)}{r_w} + \sqrt{\left[\frac{r_{dH}(x)}{r_w}\right]^2 + I_{\text{ani}}^2 - 1}\right)\right]. \quad \text{(A-25)}$$

Eqs. A-24 and A-25 are the formation-damage-skin equations for a horizontal well, which take into account the reservoir anisotropy and damage heterogeneity.

Appendix B—Horizontal Inflow Model for a Parallel-Piped Reservoir

Assuming the length of the horizontal well can be approximated to fully penetrate in a parallel-piped reservoir with no flow boundaries at the top and bottom of the reservoir (see **Fig. 9**), the total pressure drop can be simply calculated as follows:

$$\Delta p = \Delta p_r + \Delta p_l, \quad \text{(B-1)}$$

in which Δp_r and Δp_l = the pressure drops for the radial and linear flow regions. From Darcy's law, the pressure drop caused by the radial flow is given by:

$$\Delta p_r = \frac{q\mu}{2\pi k L}\ln\left(\frac{r_t}{r_w}\right). \quad \text{(B-2)}$$

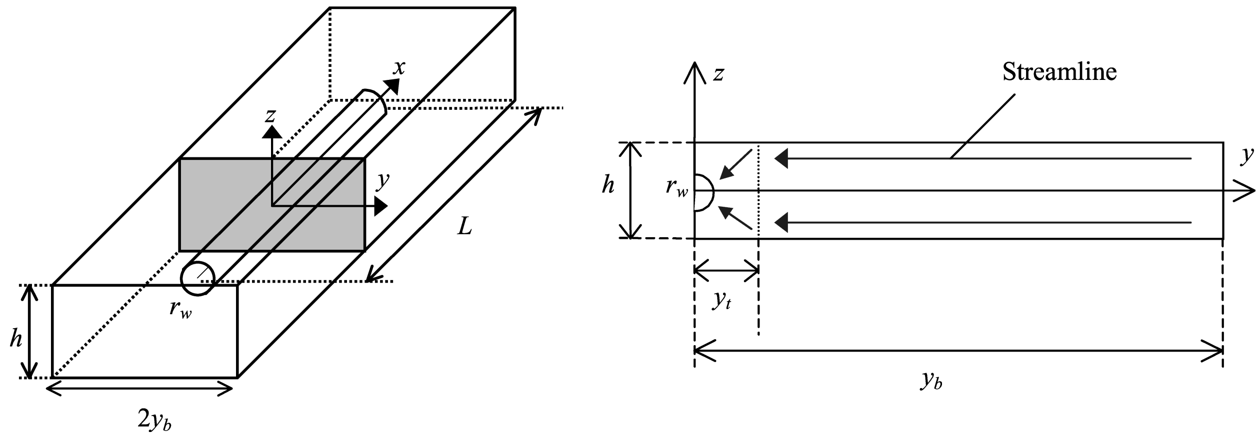

Fig. 9—Horizontal well flow geometry in a rectangular reservoir.

Similarly, the pressure drop caused by linear flow is given by:

$$\Delta p_l = \frac{(q/2)\mu(y_b - y_t)}{khL}. \quad \text{(B-3)}$$

Based on finite-element-method (FEM) simulations, we may choose r_t and y_t in the following manner.

$$y_t = h/2, \quad \text{(B-4)}$$

and $r_t = y_t\sqrt{2} = (h\sqrt{2})/2.$ (B-5)

Substituting the previous equations into Eqs. B-2 and B-3 gives

$$\Delta p_r = \frac{q\mu}{2\pi kL}\ln\left(\frac{h\sqrt{2}}{2r_w}\right), \quad \text{(B-6)}$$

and $\Delta p_l = \dfrac{q\mu(y_b - h/2)}{2khL}.$ (B-7)

Hence, the total pressure drop for the rectangular reservoir is

$$\Delta p = \frac{q\mu}{2\pi kL}\left[\ln\left(\frac{h\sqrt{2}}{2r_w}\right) + \pi(y_b/h - 1/2)\right]. \quad \text{(B-8)}$$

The existence of formation damage around a well results in an additional pressure drop.

$$\Delta p_d = \frac{q\mu}{2\pi kL}s. \quad \text{(B-9)}$$

The total pressure drop is given by:

$$\Delta p = \frac{q\mu}{2\pi kL}\left[\ln\left(\frac{h\sqrt{2}}{2r_w}\right) + \pi(y_b/h - 1/2) + s\right]$$
$$= \frac{q\mu}{2\pi kL}[\ln(h/r_w) + \pi y_b/h - 1.917 + s]. \quad \text{(B-10)}$$

The productivity index ratio of the damaged reservoir to the undamaged reservoir is given by:

$$\frac{J_d}{J_o} = \frac{\ln(h/r_w) + \pi y_b/h - 1.917}{\ln(h/r_w) + \pi y_b/h - 1.917 + s}. \quad \text{(B-11)}$$

For an anisotropic formation, Eq. B-11 is

$$\frac{J_d}{J_o} = \frac{\ln\left[\dfrac{hI_{\text{ani}}}{r_w(I_{\text{ani}}+1)}\right] + \dfrac{\pi y_b}{hI_{\text{ani}}} - 1.224}{\ln\left[\dfrac{hI_{\text{ani}}}{r_w(I_{\text{ani}}+1)}\right] + \dfrac{\pi y_b}{hI_{\text{ani}}} - 1.224 + s}. \quad \text{(B-12)}$$

This equation gives the ratio of the productivity index of a damaged horizontal well to that of an undamaged well.

SI Metric Conversion Factors

cp × 1.0*	E−03	=	Pa·s
ft × 3.048*	E−01	=	m
psi × 6.894 757	E+00	=	kPa
STB × 1.589 873	E−01	=	m^3

*Conversion factor is exact.

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Discussion of "A Rigorous Formation Damage Skin Factor and Reservoir Inflow Model for a Horizontal Well"

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Furui *et al.*[1] present two associated models in the paper. The first model estimates the productivity of a horizontal well and is referred to as a productivity model. The second is for assessing the effect of nonuniform skin distribution around the well and is referred to as a heterogeneous-skin model. The equations for both the productivity and the heterogeneous-skin models are concise, simple, and easy to use. However, both models may be less than rigorous, and both are critiqued here.

Productivity Model. The model assumes that there are two constant-pressure lateral boundaries parallel to the well axis and two other impermeable lateral boundaries normal to the well axis. The mathematical treatment also requires that the well be centralized with respect to all reservoir boundaries. Additionally, in the model, the horizontal well penetrates the formation fully, extending from one impermeable lateral boundary to another. These assumptions on formation boundaries, well location, and well length are quite restrictive in terms of to how many real reservoir/well pairs the model could be applied. The assumptions mentioned also make the proposed productivity model less rigorous and deficient compared to the models in the literature.[2–5]

The productivity model proposed by Furui *et al.*[1] was compared against the models presented by Goode and Kuchuk,[2] Babu and Odeh,[3] and Yildiz.[4] For the sake of simplicity, an isotropic formation was considered. The data set used in the comparison, almost identical to that in Fig. 6 of Ref. 1, was chosen such that the geometric limitations of the authors' model were satisfied. Hence, a fully penetrating horizontal well was selected. The data set included $B_o = 1.5$ res bbl/STB, $\mu = 1$ cp, $k = 100$ md, $L_h = 2{,}000$ ft, $r_w = 0.3$ ft, $x_e = 2{,}000$ ft, $y_e = 2{,}000$ ft, 50 ft $\leq h \leq$ 500 ft, and and $y_{e1/2} = y_w = 1{,}000$ ft. As in Ref. 1, constant and uniform skin values of $s = 5$, 10, and 20 were assigned along the wellbore.

The results are shown in **Fig. 1.** The results from the models presented in Refs. 2 through 4 agree very well. However, the productivity-index values computed with the Furui *et al.*[1] model deviate significantly from the rest.

Remember that Goode and Kuchuk[2] verified their model against the results from a commercial simulator. Gilman and Jargon[6] compared both the Goode-Kuchuk[2] and Babu-Odeh[3] models against the numerical simulator and showed that both models were accurate. Additionally, in Refs. 7 through 10, the Babu-Odeh model was tested against the numerical and analytical models and verified. The selectively completed horizontal well model presented in Ref. 4 also was compared against the models in Refs. 11 and 12 and was proven to be accurate.

Heterogeneous Skin Model. The authors claimed that a geometric average of the segmentwise, variable, nonuniform skin distribution is the equivalent skin factor for the well. However, the mathematical treatment presented does not properly formulate the interaction between the local skin factors and its effect on flux distribution along the wellbore. To assess the effect of variable skin distribution on well performance, a multisegment well model is needed, as described in Refs. 4, 5, and 7. A multisegment model can solve for the pressure losses and flux distribution simultaneously.

The heterogeneous skin model proposed by the authors was compared against the results from the multisegment model described in Ref. 4 and the partially penetrating horizontal well models described in Refs. 2 and 3. The data set was the same as that used in Fig. 1, with $h = 100$ ft.

For the models presented in Refs. 2 through 4, the equivalent skin factor is calculated as follows. In the first step, using the referenced models, total pressure drop in the damaged well, Δp_{ws}, is computed for a given flow rate and skin distribution. In the second step, pressure drop in the undamaged well, Δp_w, is calculated for the same flow rate. The difference between the pressure drops in the damaged and undamaged wells gives the additional pressure drop due to formation damage. The equivalent additional pressure drop caused by damage can be related to the equivalent skin factor as

$$\Delta p_{\text{seq}} = \Delta p_{ws} - \Delta p_w = 141.2 \frac{q_w \mu B_o}{k L_h} s_{\text{eq}}. \quad (1)$$

By rearranging Eq. 1,

$$s_{\text{eq}} = \frac{k L_h (\Delta p_{ws} - \Delta p_w)}{141.2 q_w \mu B_o}. \quad (2)$$

Seven different skin distributions were considered. The skin distributions are given in **Fig. 2** and **Table 1.** The local skin values are related to the radius and permeability of the damaged zone through Hawkin's formula. In Case 1, the well is undamaged. In Cases 2 through 6, the well has undamaged and damaged sections with $s_j = 1{,}000$. A formation damage skin of $s_j = 1{,}000$ practically nullifies the contribution from the damaged segments. The skin distributions in Cases 2 through 6 make the well behave as if it is partially or selectively completed. It should be pointed out that the models in Refs. 2 and 3 assume a partially penetrating well and do not consider variable skin or multisegments. However, the models in Refs. 2 and 3 could be extended to simulate the flow into horizontal wells in Cases 2 through 5 by treating the segments with very high skin factors as uncompleted and taking advantage of the symmetrical skin distribution. The models in Refs. 2 and 3 are not applicable to the skin distributions in Cases 6 and 7.

The computed equivalent skin factors are given in **Table 2.** The results from Refs. 2 through 4 agree very well. The heterogeneous skin model proposed by Furui *et al.*[1] yields significantly different results. The heterogeneous skin model predicts the same well performance and equivalent skin factor for Cases 2 through 6. However, as also shown in Refs. 4, 11, and 12, different distributions of open segments along the wellbore make a difference in well performance and result in a different equivalent skin factor for each segment distribution, even for a fixed penetration ratio.

For Case 7, the multisegment model predicts that the first, second, third, and fourth quarters of the well produce 32.8, 25.8, 23.7, and 17.7%, respectively, of the total fluid. However, the heterogeneous skin model proposed by Furui *et al.*[1] does not account for each quarter of the well producing at different flow rates.

When there is nonuniform damage around a horizontal well, even a fully penetrating horizontal well would be subject to a 3D flow field around it. The heterogeneous skin model formulated by Furui *et al.*[1] is based on 1D geometry only. Hence, it does not have the capability to account for the changes in the flux distribution

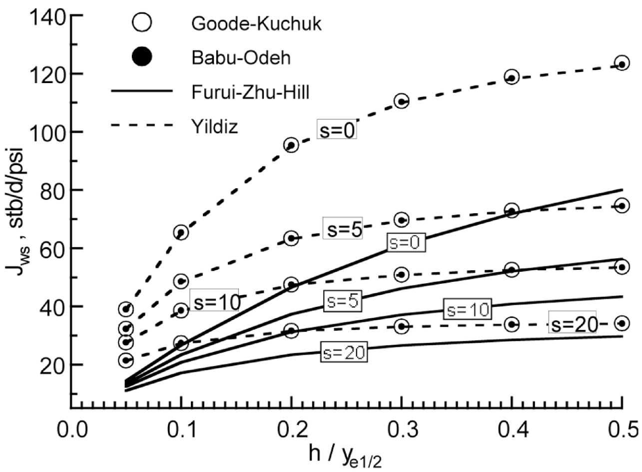

Fig. 1—Comparison of horizontal well-productivity models.

and 3D nature of the flow around the wellbore. The heterogeneous skin model described in Ref. 1 is incorrect.

Nomenclature

B_o = formation volume factor, res bbl/STB
h = formation thickness, ft
k = permeability, md
L_h = horizontal well length, ft
L_{hj} = length of segment j on the well, ft
n_s = number of segments along the wellbore, ft
q_w = flow rate, STB/D
s_{eq} = effective equivalent skin factor for the well
s_j = skin factor around segment j along the well
x_e = reservoir length in x-direction, ft
y_e = reservoir width in y-direction, ft
$y_{e1/2}$ = reservoir half-width in y-direction, ft
y_w = distance from the reservoir boundary to well location in y-direction, ft
Δp_{seq} = additional pressure drop owing to damage, psi
Δp_w = formation pressure drop for an undamaged horizontal well, psi
Δp_{ws} = total pressure drop for a damaged horizontal well, psi
μ = viscosity, cp

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TABLE 1—SKIN DISTRIBUTIONS
1: Undamaged
2: n_s=2, s_1=0, s_2=1,000, L_{h1} = L_{h2}=1,000 ft
3: n_s=3, s_1 = s_3=0, s_2=1,000, L_{h2}=1,000 ft, L_{h1} = L_{h3}=500 ft
4: n_s=3, s_1 = s_3=1,000, s_2=0, L_{h2}=1,000 ft, L_{h1} = L_{h3}=500 ft
5: n_s=9, s_1 = s_3 = s_5 = s_7 = s_9=0, s_2 = s_4 = s_6 = s_8 = 1,000
L_{h1} = L_{h9}=125 ft, L_{h2}=L_{h3} = L_{h4}=L_{h5} = L_{h6} = L_{h7}=L_{h8}=250 ft
6: n_s=9, s_1 = s_3 = s_5 = s_7 = s_9=0, s_2 = s_4 = s_6 = s_8 = 1,000
L_{h1}=L_{h3}=L_{h5}=L_{h7} = L_{h9}=200 ft, L_{h2}=L_{h4}=L_{h6}=L_{h8}=250 ft
7: n_s=4, s_1=0, s_2=5, s_3=10, s_4=20, L_{h1} = L_{h2} = L_{h3}=L_{h4} =500 ft

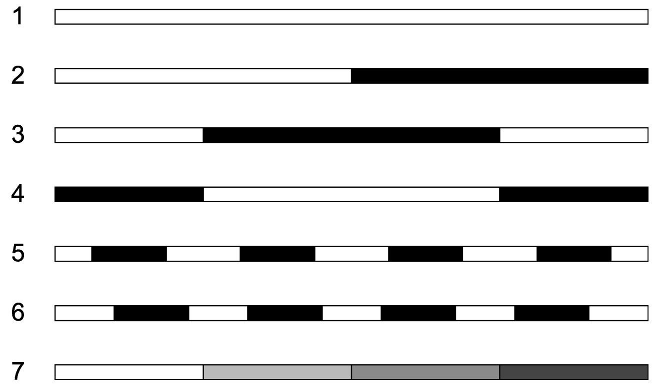

Fig. 2—Skin distributions considered in the comparison.

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SI Metric Conversion Factors

bbl × 1.589 873	E–01	= m^3
cp × 1.0*	E–03	= Pa·s
ft × 3.048*	E–01	= m
psi × 6.894 757	E+03	= Pa

*Conversion factor is exact.

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TABLE 2—COMPARISON FOR EQUIVALENT SKIN FACTOR

	s_{eq}			
Case	Goode-Kuchuk[2]	Babu-Odeh[3]	Yildiz[4]	Furui *et al.*[1]
1	0.0	0.0	0.0	0.0
2	22.4	23.0	22.1	5.0
3	12.5	13.5	12.7	5.0
4	12.5	13.5	12.7	5.0
5	5.8	6.4	5.9	5.0
6			6.6	5.0
7			7.2	14.9

Authors' Reply to "Discussion of 'A Rigorous Formation Damage Skin Factor and Reservoir Inflow Model for a Horizontal Well'"

K. Furui, SPE, **D. Zhu,** SPE, and **A.D. Hill,** SPE, U. of Texas at Austin

We appreciate Professor Yildiz taking the time to study our paper and provide the following responses to his critiques.

Productivity Model. It is always enlightening to see another proof that apples are, in fact, not equal to oranges. In Fig. 1 of the Discussion, a comparison of the horizontal well inflow model that we presented (referred to in the discussion as the Productivity Model) with the previous models of Babu and Odeh, Goode and Kuchuk, and Yildiz (Refs. 2 through 4) showed that our model does not predict the same productivity as the others. This is reassuring, because our model is a steady-state one, while Refs. 2 through 4 present pseudosteady-state models. The difference among these models shows in the boundary conditions at the reservoir boundary in the horizontal direction, perpendicular to the wellbore. In our steady-state model, constant pressure is assumed at this boundary, while the pseudosteady-state models (see Refs. 2 through 4) assume no flow boundaries. We would certainly anticipate different reservoir performances under these greatly differing conditions, and that is what we observed.

More appropriate ways to validate the productivity model we presented include comparing the model with numerical simulation results or to the analytical solutions for steady-state flow to a fully penetrating horizontal well. Such a comparison is shown in **Fig. 1,** in which we compare our result with the analytical model presented by Butler[1] and with our finite-element simulator.[2] We conclude from this comparison that our productivity model is a reasonable approximation for steady-state flow to a fully penetrating horizontal well, and is, in fact, a surprisingly good model, considering its simplicity. By way of reference, we also plot the pseudosteady-state result in Fig. 1 to show again that constant pressure and a no-flow boundary are very different assumptions about reservoir flow behavior. We will leave it to the reader to ponder the interesting fact that the productivity index of a horizontal well in a steady-state flow system is significantly lower than that in a pseudosteady-state system and that this difference decreases as the thickness of the reservoir increases relative to the reservoir extent perpendicular to the well.

A final point about this productivity model is as follows. We presented this model in a paper primarily about formation damage effects in horizontal wells because this simple model separates the effects of linear flow to the horizontal wellbore from the effects of the near-wellbore radial flow region. This division makes it easy to discern the relative effects of these regions on overall well performance and, hence, draw conclusions about the importance of near-well effects, such as formation damage. It is again reassuring to see that this simple, illustrative model predicts steady-state flow to a fully penetrating horizontal well so accurately. We were not attempting to develop yet another variation on the excellent work of Babu and Odeh for the case of a horizontal well producing at pseudosteady state in a closed reservoir.

Heterogeneous Skin Model. Now that we have proved that apples are not oranges, we can move on to another maxim—use the right tool for the job. Our paper presents a model of the formation-damage skin factor, not a model of an overall-equivalent skin factor or of partial-completion skin factor. All but one of the examples discussed by Yildiz are partial-completion cases in which sections of the wellbore are either completely open or completely closed (Yildiz assumes a skin factor of 1,000 to approximate completely closed sections). For cases such as these, a partial completion skin model, several of which have been presented in the past (Muskat,[3] Odeh,[4] Cinco-Ley *et al.*,[5] Papatzacos,[6] Goode and Wikinson,[7] etc.), is appropriate to apply. Our heterogeneous skin model accounts for nonuniform skin distribution where the variation of damage (skin) is not this abrupt. It can be applied where the flowlines in the near-well vicinity are radial or close to radial. When there is a sudden change in the local skin factor along the well, significant flow in the axial direction will result, and this effect is more appropriately modeled with a partial-completion skin model than a formation-damage model.

Case 7 of the Discussion is the only case for which our model should be applied, and even in this case it could be used more efficiently than simply calculating one overall skin, as we will show. First, we must correct the skin calculation result presented in the Discussion for this case. Using Eq. A-22 of our paper:

$$s_{eq} = \frac{L}{\sum_{i=1}^{4} \frac{\Delta l_i}{\ln(h/2r_w) + s_i}} - \ln\left(\frac{h}{2r_w}\right)$$
$$= \frac{4}{\frac{1}{5.12} + \frac{1}{5.12+5} + \frac{1}{5.12+10} + \frac{1}{5.12+20}} - 5.12 = 4.88. \quad (1)$$

According to Table 2, Yildiz's model gives $s = 7.2$ for Case 7 [Our finite element simulation (FEM) result gives $s = 7.1$], compared with 4.9 when calculating one overall skin factor with our damage model. The difference in these results is caused by the abrupt changes in assumed skin, which lead to axial flow effects, as

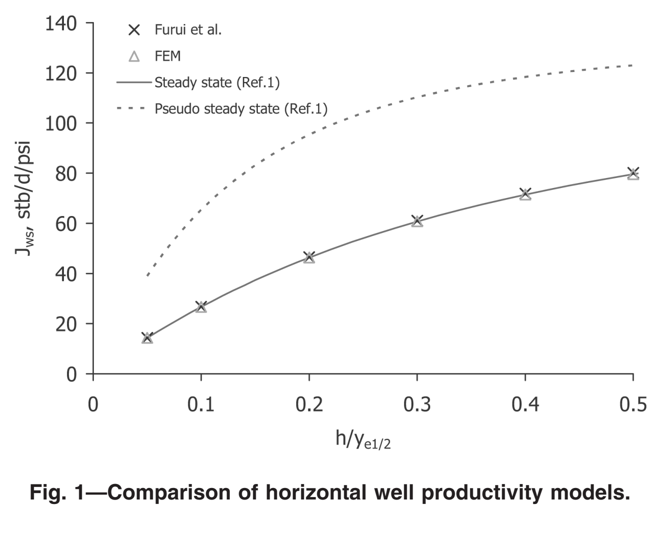

Fig. 1—Comparison of horizontal well productivity models.

discussed in the previous paragraph. It is also incorrectly stated in the Discussion that our heterogeneous skin model does not account for each quarter of the well producing at different flow rates; our model is derived based on the flux from the formation along the well being variable in response to variable damage. This is clearly shown in the Appendix of our paper. Applying Eq. A-14 for the local flow rate per unit length and using the overall skin of 4.88 obtained with our model, the four segments of the well in Case 7 produce 49, 25, 17, and 10% of the total production.

A situation like that assumed in Case 7, with abrupt transitions in the local damage conditions, are likely rare in actual wells. However, for such a case, our heterogeneous skin factor model can be applied more correctly by using it to calculate the damage skin factor for each discrete segment of the well. The skin factors for each segment can then be used in any appropriate model of horizontal well performance that can treat the well as a series of segments.

In summary, to correctly apply our damage skin model to partially completed wells or wells with abrupt changes in the completion or damage conditions, the following procedures should be used.

1. Discretize the horizontal well into open and closed segments.
2. Calculate the skin factor for each open section with our formation-damage skin model.

3a. Estimate the overall skin factor, considering the partial-completion skin factor, <u>or</u>

3b. Use a numerical simulator and assign the variable skin factors to the open-well gridblocks (segments).

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