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Zero Knowledge proof — Date: May 22, 2020

In the previous chapter, we gave an introduction on Zero-knowledge proofs and Graph isomorphism based on zero-knowledge. In this chapter, we present an implementation of Graph isomorphisms [Jan: implementation of GI *protocol*] based on zero-knowledge using Python for the original interaction [Jan: What do you mean by original interaction?] between the prover and the verifier and for the simulator S that we mentioned in chapter 2. !
!

In addition, we show different scenarios for the prover and the verifier and examine the corresponding outputs. According to our code, we suppose that the graphs we use are undirected and so the adjacency matrices are symmetric.

1 Implementation

According to the protocol that we discussed in the previous chapter, the input is two graphs and the output is a list containing: $H = \sigma(G_0)$, ch, φ and Accept or Reject.

1.1 Packages we import

[Jan: There is no need to make such a list. Instead I would suggest you discuss important tools you used in a couple of sentences. Something like "graphs were represented as adjacency matrices; graph-tools was used for handling graphs; networkx and matplotlib for drawing graph representations; we also used some combinatorial and numerical functions from numpy and sympy"] !

- Math.
- Numpy.
- Graph-tools.
- Sympy.combinatorics.
- Csv.
- Networkx.
- Matplotlib.pyplot.

1.2 Functions we use

[Jan: A similar thing here. I suggest that you put the list below in appendix as documentation of your project. In the main thesis please write a paragraph about the structure of your program, something like "we have a variety of helper functions to handle graphs and permutations, including checking equality, applying permutation to a graph, inverting a permutation; we then implement each scenario (honest prover and honest verifier for isomorphic graphs...) in a separate function".] !

[Jan: I also suggest that you describe some of the technical challenges. For example, "since the protocol involves multiple rounds of interaction, the state of prover and verifier between rounds needs to be saved. we achieve that by storing the random seed used by the prover as its additional hidden input".] !

- **generate-graph**
  "generate-graph" function enables us to transform the adjacency matrix into a graph using graph-tools package.

- **compose-permutation**
  "compose-permutation" function lets us compose two permutations (p, q) and return $p \circ q$.

- **apply-permute**
  "apply-permute" function enables us to apply a permutation P on adjacency matrix AM and return another adjacency matrix $P(AM)$.

- **inv**
  "inv" function takes a permutation P as an input and returns the inverse of P (P^{-1}).

- **are-equal**
  "are-equal" function checks if two adjacency matrices are equal or not.

- **honest-prover**
  "honest-prover" function applies the protocol honestly for prover's side: It takes two adjacency matrices to represent two corresponding graphs (G_0, G_1), the secret Π, seed s to generate random permutation, and $mess - list$ [Jan: You can use "\verb" to print code like names of variables. Google "not so short introduction to latex" and see section 2.11.5, "printing verbatim".] as an output for the whole protocol to update it during interaction with the verifier. !

- **honest-verifier** "honest-verifier" function applies the protocol honestly for the verifier's side: It takes two adjacency matrices to represent two corresponding graphs (G_0, G_1), and $mess - list$ to update it during interaction with the prover.

- **graph-isomorphism**
  "graph-isomorphism" function enables us to run the protocol between the prover and the verifier by controlling the turn of each part.

- **test-isomorphism**
  "test-isomorphism" function enables us to run "graph-isomorpism" by pass "honest-prover" and "honest-verifier" as a parameter to it.

- **cheating-prover**
  "cheating-prover" function applies the protocol for cheating prover who doesnt know the secret Π, it takes two adjacency matrices to represent two corresponding graphs (G_0, G_1),

seed s to generate random permutation, and $mess - list$ as an output for the whole protocol to update it during interaction with the verifier.

- **protocol-dishonest-prover**
  "protocol-dishonest-prover" function enables us to run "graph-isomorpism" by passing "cheating-prover" and "honest-verifier" as a parameter to it.

- **simulator**
  "simulator" function enables us to applies the protocol for simulator S that we mentioned in chapter 2.

- **get-graph-from-file**
  "get-graph-from-file" lets us take a graph from csv file.

- **get-pi-from-file**
  "get-pi-from-file" lets us take a secret Π from csv file.

- **equal**
  "equal" function is used after "get-graph-from-file" function to remove all additional data.

- **plot-graph**
  "plot-graph" function uses networkx package to plot graphs using the corresponding adjacency matrix.

1.3 Simulations and result

In this section we show different scenarios and their results. Firstly Figure 1 shows the csv file that contains our data:

Figure 1: csv file

[Jan: I could not see this file, but if it's a text file with data then I'm not sure that you should include it.] !

We have four cases, three of them using the original protocol and the last case using simulator, each one of them has two possible cases; which are **ch=0** (the fourth component is always Accept) and **ch=1**(the fourth component is different from one case to another): [Jan: Don't write "fourth component is accept", write just "the verifier accepts".] [Jan: I don't understand, why verifier always accepts if ch=0? In any case, please remind the reader what is ch.] ! !

- **Case 1: Interaction between honest prover and honest verifier**
  [Jan: I would like you to say, for every case, something like: "Inputs are two permutations of a 4-cycle, isomorphic to each other. The prover and verifer and honest, and therefore we expect the verifier to always accept. Indeed, this is the outcome we observe over several runs of the protocol. Below we present one representative run..."] !

  [Jan: I don't think it is necessary to insert print screens of code into your thesis.] !

  First we get the adjacency matrices and Π from a csv file and delete any additional data as follows:

  [Jan: You assume the figure comes here, but it does not...] !

```
In [17]: M=get_graph_from_file(0)
         AM=equal(M)
         M1=get_graph_from_file(1)
         AM1=equal(M1)
         pi=get_pi_from_file(2)
```

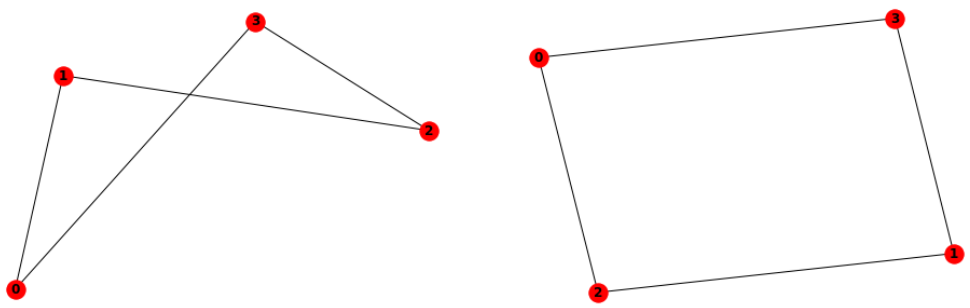
Figure 2: Read the data from csv file

```
In [105]: print('G_0 is')
          plot_graph(AM)
          print('G_1 is')
          plot_graph(AM1)
          test_isomorphism(AM,AM1,pi)
```

Figure 3: Runing the protocol

Then we plot G_0 and G_1, after that run the protocol when **ch=0** using code in Figure 3:

The first output is printing the two graphs shown by Figure 4:



(a) G_0. (b) G_1.

Figure 4: G_0 and G_1 case 1 with $ch = 0$

Let us see the situation when the verifier chooses **ch=0**:

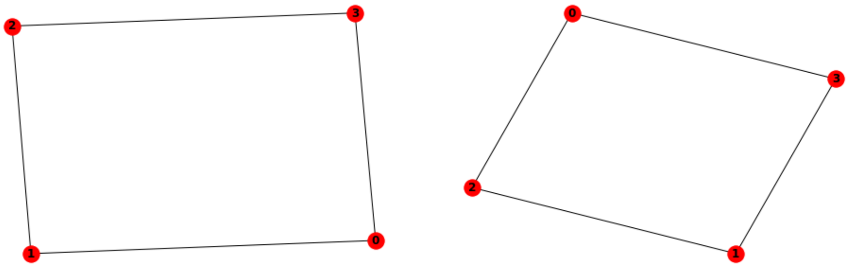
As we mentioned earlier the fourth component is Accept.
For **ch=1**, the first output is printing the two graphs shown by Figure 6:
Let us see the situation when the verifier chooses **ch=1**:

Since **ch=1** and the prover is honest, then the fourth element is Accept:

- **Case 2: Interaction between cheating prover and honest verifier**
  First, we update the second graph by reading it from csv file using code shown in Figure 8:
  when **ch=0** the first output is printing the two graphs shown by Figure 9:

```
Out[105]: [// Generated by graph-tools (version 1.0) at 2020/54/05/09/20 22:54:52
           // undirected, 4 vertices, 4 edges
           graph export_dot {
             node [color=gray90,style=filled];
             "0";
             "1";
             "2";
             "3";
             "0" -- "1";
             "0" -- "3";
             "1" -- "2";
             "2" -- "3";
           }, 0, array([3, 0, 1, 2]), 'Accept']
```

Figure 5: $mess - list$ when $ch = 0$



(a) G_0.

(b) G_1.

Figure 6: case 1, G_0 and G_1 with $ch = 1$

[Jan: I do not think it is informative for the reader to give long print screens from python. Instead, I suggest you draw the graph H and then specify permutations that map G_0 and G_1 to H.] Let us see the situation when the verifier chooses **ch=0**: !

Since **ch=0** then the fourth component is always Accept even if the prover is dishonest.

When **ch=1**, the first output is the graph of G_0 and G_1 shown by Figure 11:

The second output is $mess-list$, the last output is Reject because the prover is dishonest.

- **Case 3: Interaction between honest prover and honest verifier for big graph (v=10,e=28)**
  Now we apply ZKP for big isomorphic graphs that we can not check easily. First we read the data from the file using code shown by Figure 12:

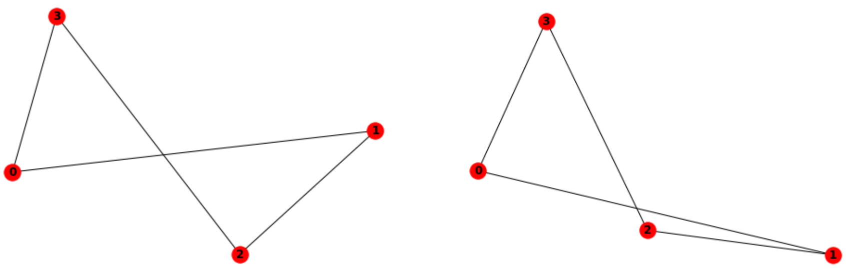
With **ch=0**, the first output is the graph of G_0 and G_1 shown by Figure 13:

```
Out[109]: [// Generated by graph-tools (version 1.0) at 2020/55/05/09/20 22:55:21
           // undirected, 4 vertices, 4 edges
           graph export_dot {
             node [color=gray90,style=filled];
             "0";
             "1";
             "2";
             "3";
             "0" -- "1";
             "0" -- "3";
             "1" -- "2";
             "2" -- "3";
           }, 1, [0, 2, 1, 3], 'Accept']
```

Figure 7: $mess - list$ when $ch = 1$

```
In [110]: M1=get_graph_from_file(3)
          AM1=equal(M1)
```

Figure 8: Update the second graph



(a) G_0.

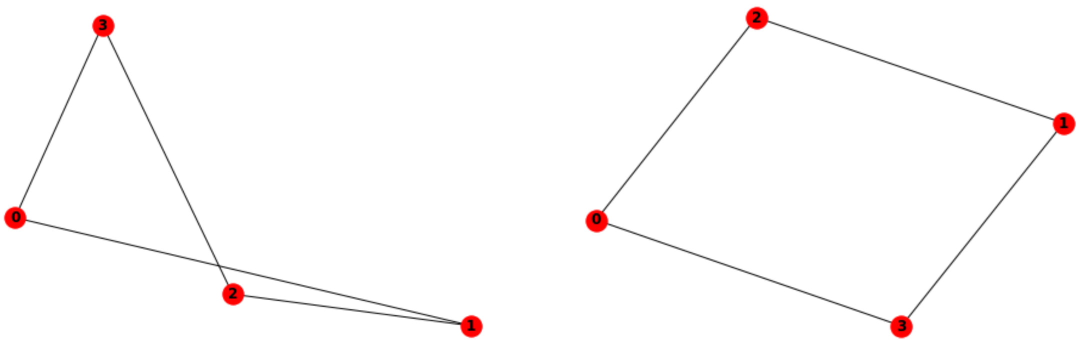
(b) G_1.

Figure 9: case 2, G_0 and G_1 with $ch = 0$

The second output is $mess-lis$, and the fourth component when $ch = 0$ is always Accept, Figure 14 shows the result:

```
Out[111]: [// Generated by graph-tools (version 1.0) at 2020/55/05/09/20 22:5
          // undirected, 4 vertices, 4 edges
          graph export_dot {
            node [color=gray90,style=filled];
            "0";
            "1";
            "2";
            "3";
            "0" -- "2";
            "0" -- "3";
            "1" -- "2";
            "1" -- "3";
          }, 0, array([3, 0, 2, 1]), 'Accept']
```

Figure 10: $mess - list$ when $ch = 0$



(a) G_0.

(b) G_1.

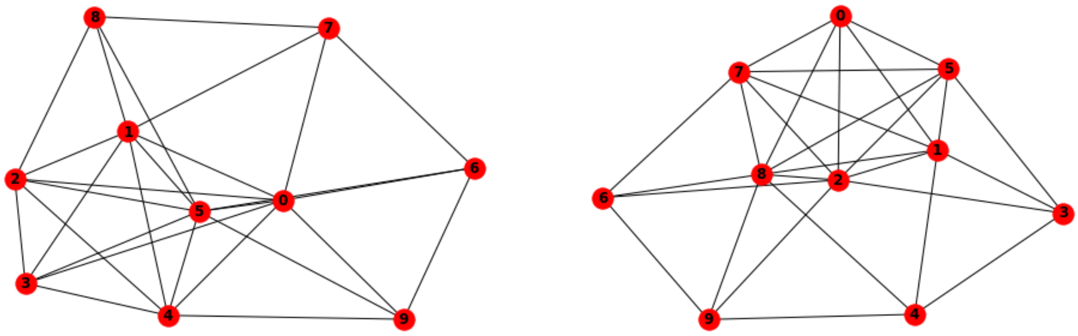
Figure 11: G_0 and G_1 case 2 with $ch = 1$

When **ch=1**, the first output is the graph of G_0 and G_1 shown by Figure 15:

Let us see the situation when the verifier chooses **ch=1**, Figure 16 shows the result:

```
In [19]: M=get_graph_from_file(4)
         AM=equal(M)
         M1=get_graph_from_file(5)
         AM1=equal(M1)
         pi=get_pi_from_file(6)
```

Figure 12: updating data and run the protocol



(a) G_0.　　　(b) G_1.

Figure 13: case 3,G_0 and G_1, with $ch = 0$

[Jan: For non-isomorphic graphs and cheating prover, I suggest you run the simulation many times, count how many times the verifier accepted and report your results.]

[Jan: I also suggest you include a case where you have large non-isomorphic graphs where it is not easily visible that they are non-isomorphic. For example, make two large isomorphic graphs as above and flip one edge randomly.]

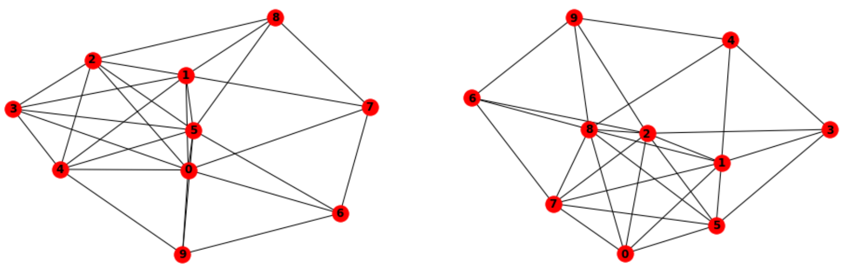
- **Case 4: Interaction between cheating prover and honest verifier using simulator**
  We use the same data in the previous case,so there is no need to update it,when $ch = 0$ the first output is G_0 and G_1 shown by Figure 17:

  The second output is $mess - list$, and the fourth component when $ch = 0$ is Accept, Figure 18 shows the result:
  When $ch = 1$ the first output is the graph of G_0 and G_1, shown by Figure 19:

  The second output is $mess - lis$, and the fourth component when $ch = 1$ is Accept since the prover is honest, Figure 20 shows the result:

```
Out[20]: [// Generated by graph-tools (version 1.0) at 2020/47/05/10/20 00:47:17
          // undirected, 10 vertices, 28 edges
          graph export_dot {
            node [color=gray90,style=filled];
            "0";
            "1";
            "2";
            "3";
            "4";
            "5";
            "6";
            "7";
            "8";
            "9";
            "0" -- "1";
            "0" -- "2";
            "0" -- "3";
            "0" -- "4";
            "0" -- "5";
            "0" -- "6";
            "0" -- "8";
            "0" -- "9";
            "1" -- "2";
            "1" -- "4";
            "1" -- "5";
            "1" -- "6";
            "1" -- "7";
            "1" -- "8";
            "1" -- "9";
            "2" -- "3";
            "2" -- "4";
            "3" -- "5";
            "3" -- "7";
            "4" -- "9";
            "5" -- "6";
            "5" -- "7";
            "5" -- "8";
            "5" -- "9";
            "6" -- "8";
            "6" -- "9";
            "7" -- "8";
            "8" -- "9";
          }, 0, array([0, 5, 8, 6, 9, 1, 2, 3, 7, 4]), 'Accept']
```

Figure 14: case 3 with $ch = 0$



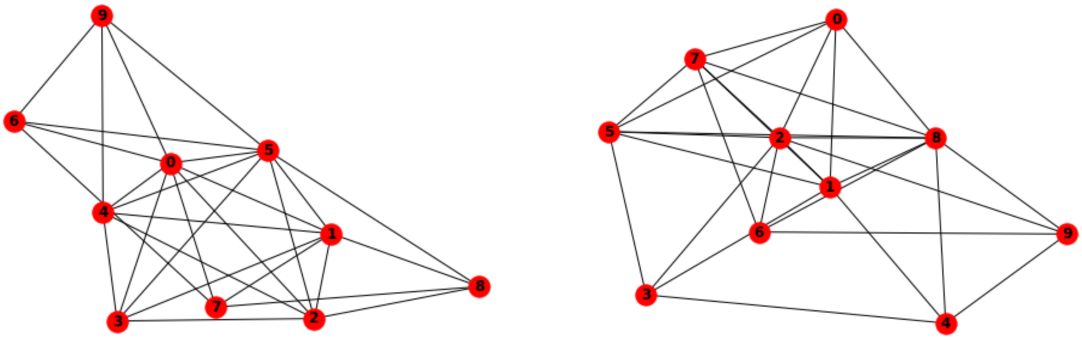
(a) G_0.

(b) G_1.

Figure 15: case 3,G_0 and G_1, with $ch = 1$

```
Out[20]: [// Generated by graph-tools (version 1.0) at 2020/47/05/10/20 00:47:17
          // undirected, 10 vertices, 28 edges
          graph export_dot {
            node [color=gray90,style=filled];
            "0";
            "1";
            "2";
            "3";
            "4";
            "5";
            "6";
            "7";
            "8";
            "9";
            "0" -- "1";
            "0" -- "2";
            "0" -- "3";
            "0" -- "4";
            "0" -- "5";
            "0" -- "6";
            "0" -- "8";
            "0" -- "9";
            "1" -- "2";
            "1" -- "4";
            "1" -- "5";
            "1" -- "6";
            "1" -- "7";
            "1" -- "8";
            "1" -- "9";
            "2" -- "3";
            "2" -- "4";
            "3" -- "5";
            "3" -- "7";
            "4" -- "9";
            "5" -- "6";
            "5" -- "7";
            "5" -- "8";
            "5" -- "9";
            "6" -- "8";
            "6" -- "9";
            "7" -- "8";
            "8" -- "9";
          }, 0, array([0, 5, 8, 6, 9, 1, 2, 3, 7, 4]), 'Accept']
```

Figure 16: G_1 case 3 with $ch = 1$

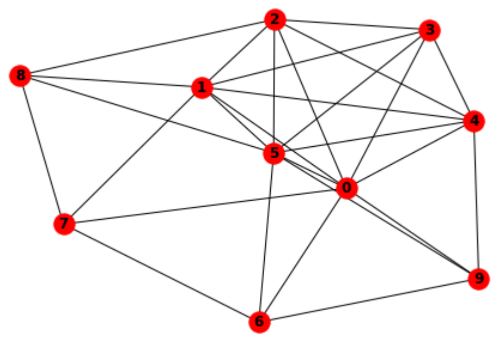


(a) G_0. (b) G_1.

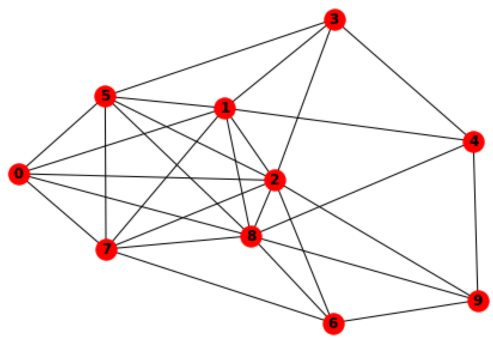
Figure 17: Case 4, G_0 and G_1 case 4 with $ch = 0$

```
Out[34]: [// Generated by graph-tools (version 1.0) at 2020/56/05/16/20 21:56:44
          // undirected, 10 vertices, 28 edges
          graph export_dot {
            node [color=gray90,style=filled];
            "0";
            "1";
            "2";
            "3";
            "4";
            "5";
            "6";
            "7";
            "8";
            "9";
            "0" -- "1";
            "0" -- "3";
            "0" -- "4";
            "0" -- "7";
            "0" -- "9";
            "1" -- "3";
            "1" -- "4";
            "1" -- "7";
            "1" -- "8";
            "1" -- "9";
            "2" -- "6";
            "2" -- "7";
            "2" -- "8";
            "2" -- "9";
            "3" -- "4";
            "3" -- "5";
            "3" -- "6";
            "3" -- "7";
            "3" -- "8";
            "3" -- "9";
            "4" -- "5";
            "4" -- "7";
            "4" -- "9";
            "5" -- "6";
            "5" -- "9";
            "6" -- "9";
            "7" -- "8";
            "7" -- "9";
          }, 0, array([9, 7, 1, 0, 4, 3, 6, 2, 8, 5]), 'Accept']
```

Figure 18: case 4, $mess - list$ when $ch = 0$



(a) G_0.



(b) G_1.

Figure 19: Case 4, G_0 and G_1 with $ch = 1$

```
Out[118]: [// Generated by graph-tools (version 1.0) at 2020/56/05/09/20 22:56:39
           // undirected, 10 vertices, 28 edges
           graph export_dot {
             node [color=gray90,style=filled];
             "0";
             "1";
             "2";
             "3";
             "4";
             "5";
             "6";
             "7";
             "8";
             "9";
             "0" -- "2";
             "0" -- "3";
             "0" -- "5";
             "0" -- "6";
             "0" -- "7";
             "0" -- "9";
             "1" -- "2";
             "1" -- "4";
             "1" -- "7";
             "1" -- "9";
             "2" -- "3";
             "2" -- "5";
             "2" -- "6";
             "2" -- "7";
             "2" -- "8";
             "2" -- "9";
             "3" -- "4";
             "3" -- "5";
             "3" -- "6";
             "3" -- "7";
             "3" -- "8";
             "3" -- "9";
             "4" -- "7";
             "4" -- "8";
             "5" -- "7";
             "5" -- "9";
             "6" -- "8";
             "7" -- "9";
           }, 1, array([5, 7, 2, 1, 4, 9, 6, 0, 3, 8]), 'Accept']
```

Figure 20: $mess - list$ case 4 with $ch = 1$