1. Eigenvalues for the domain hippopede0.01

Radius function:

$$R(\theta) = \sqrt{\sin^2(\theta) + 0.01\cos^2(\theta)}$$

Adaptive finite element method of degree 2 with 105320 triangles.

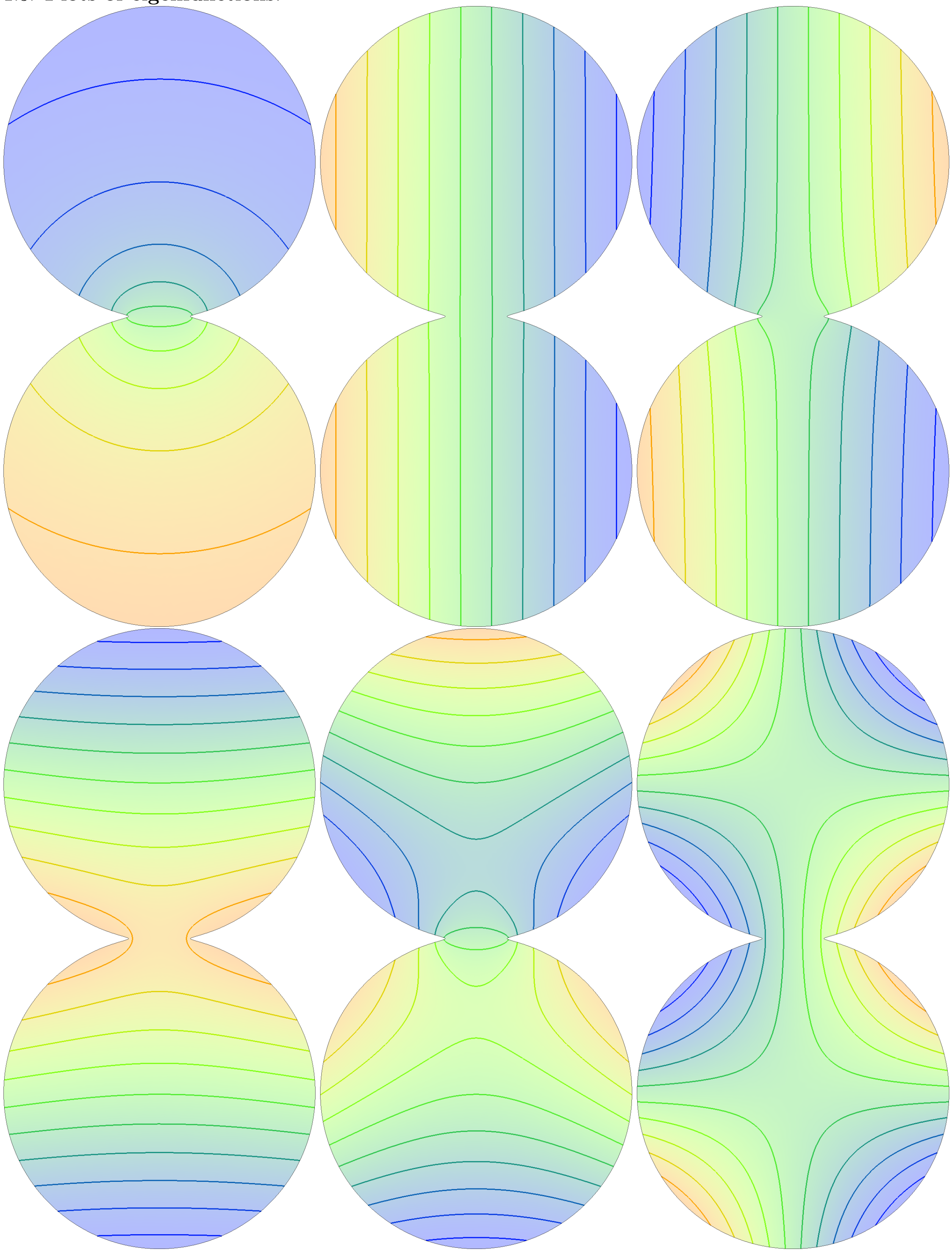
1.1. **Geometric quantities (perimeter L and constants g_i).** All quantities evaluated using boundary integral for a very fine mesh. For g_i we use geometric representations from Lemma 6.2.

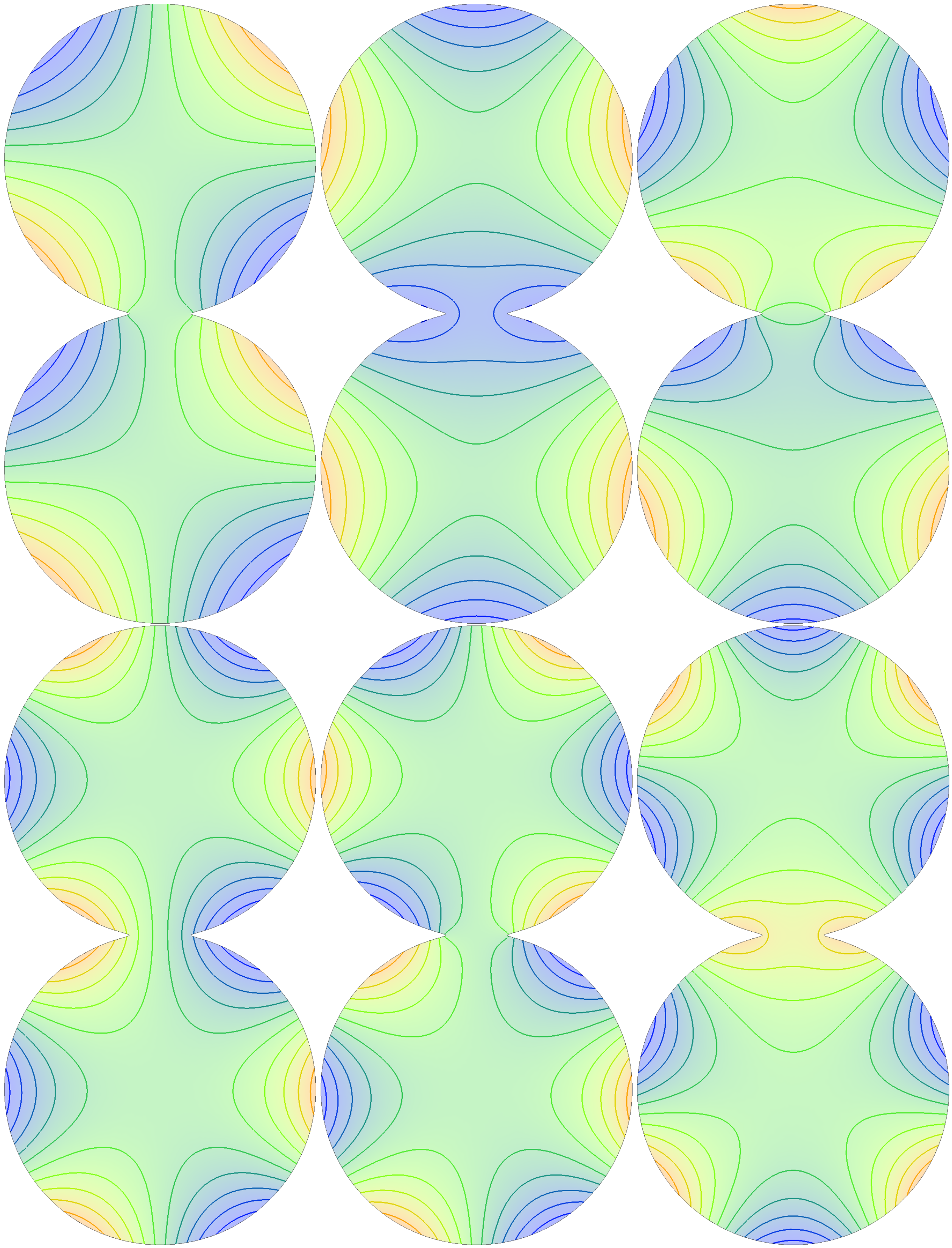
$$L = 5.92023445733 \quad g_0 = 5.05000329673 \quad g_1 = 1.02500251913 \quad g = 2.27514089691$$

1.2. **Eigenvalues.** Upper bounds are obtained by plugging numerical eigenfunctions into the Rayleigh quotient. Numbers in parentheses are numerical eigenvalues of the discrete problem. Finally ρ_i are the rescaled sums $(\sigma_1 + \cdots + \sigma_n)L$ on the domain divided by the same quantity on the disk.

$\sigma_1 \leq 0.379684031501$	(0.379684031443)	$\rho_1 = 0.35775142325$
$\sigma_2 \leq 1.99258866595$	(1.9925886659)	$\rho_2 = 1.11761868217$
$\sigma_3 \leq 2.02351672489$	(2.02351672488)	$\rho_3 = 1.035466198$
$\sigma_4 \leq 2.20444188826$	(2.20444188838)	$\rho_4 = 1.03649436077$
$\sigma_5 \leq 2.78126368521$	(2.78126368516)	$\rho_5 = 0.982174331085$
$\sigma_6 \leq 3.99886428366$	(3.99886428367)	$\rho_6 = 1.0506197651$
$\sigma_7 \leq 4.09201351612$	(4.09201351607)	$\rho_7 = 1.02894211314$
$\sigma_8 \leq 4.36830832241$	(4.36830832254)	$\rho_8 = 1.02895224798$
$\sigma_9 \leq 4.95933975901$	(4.95933975896)	$\rho_9 = 1.01007625461$
$\sigma_{10} \leq 6.02517929688$	(6.02517929683)	$\rho_{10} = 1.03096795461$
$\sigma_{11} \leq 6.18365729084$	(6.1836572916)	$\rho_{11} = 1.02098595437$
$\sigma_{12} \leq 6.51712743141$	(6.51712743189)	$\rho_{12} = 1.02133707556$

1.3. **Plots of eigenfunctions.**





1.4. **Sizes of mesh triangles and list of other parameters.**

Initial polygon: 20 sides. Adaptive: try to find 4 eigenfunctions, use at most None. Refine to at least 400000 triangles.

Sizes of mesh triangles after adaptive refinement (blue - small):