1. Eigenvalues for the domain regular7

Regular polygon with n sides.

Adaptive finite element method of degree 2 with 111643 triangles.

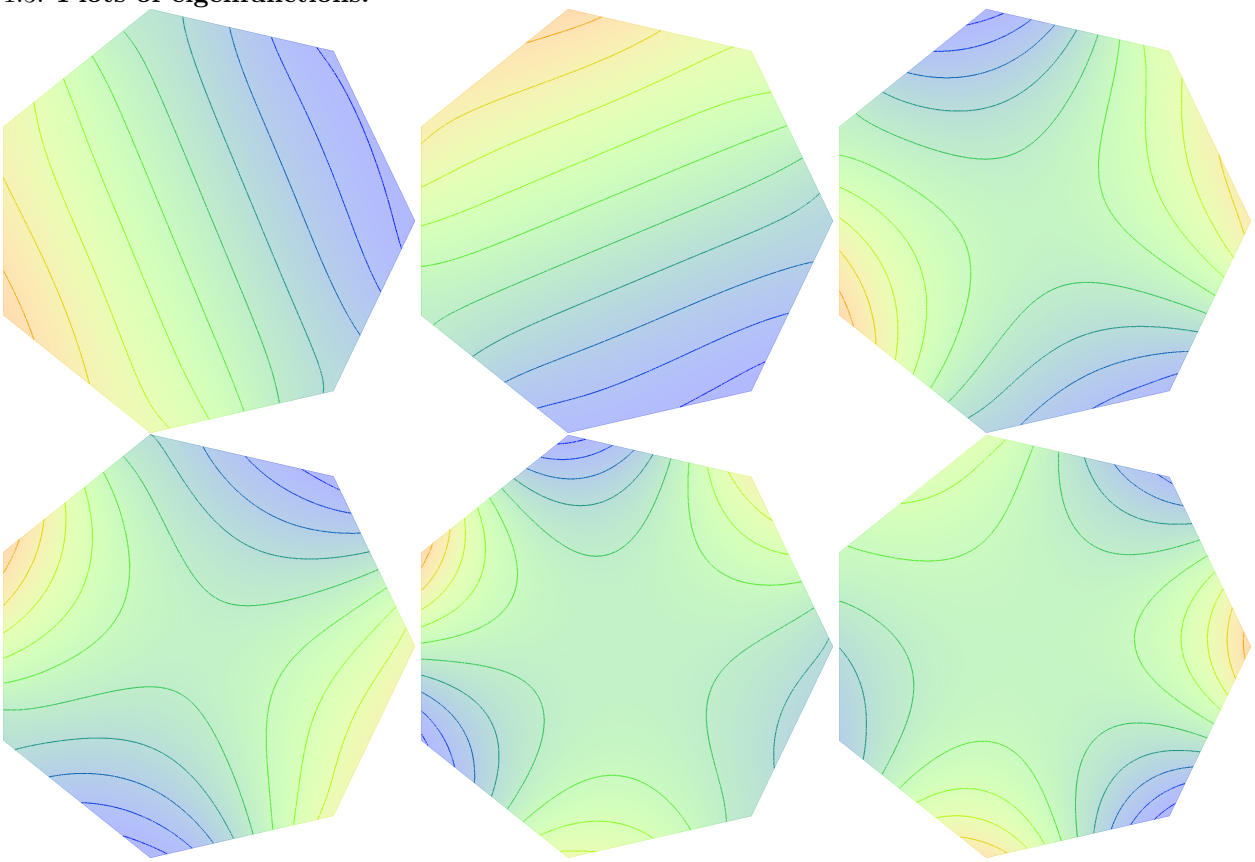
1.1. **Geometric quantities (perimeter L and constants g_i).** All quantities evaluated using boundary integral for a very fine mesh. For g_i we use geometric representations from Lemma 6.2.

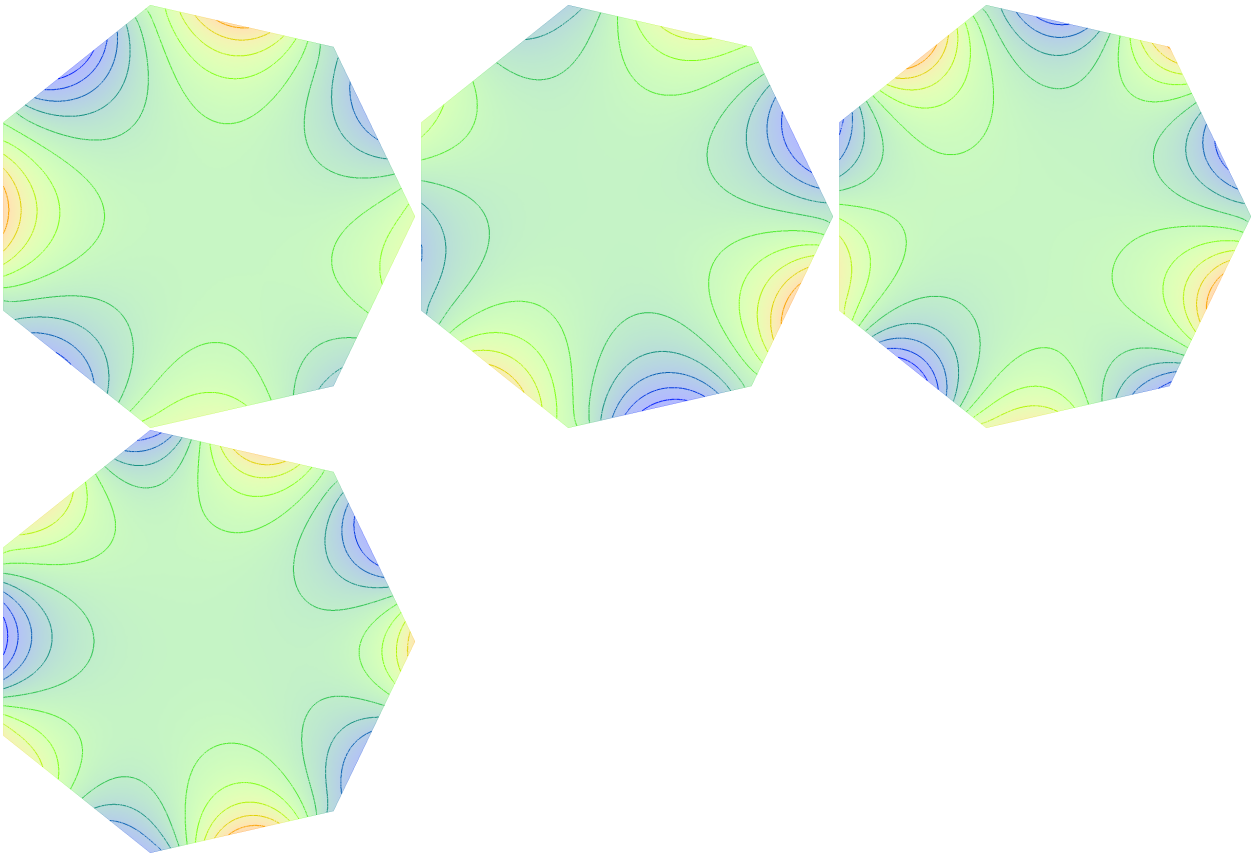
$$L = 6.07437234765 \quad g_0 = 1.07302973471 \quad g_1 = 1.003984018 \quad g = 1.0379328998$$

1.2. **Eigenvalues.** Upper bounds are obtained by plugging numerical eigenfunctions into the Rayleigh quotient. Numbers in parentheses are numerical eigenvalues of the discrete problem. Finally ρ_i are the rescaled sums $(\sigma_1 + \cdots + \sigma_n)L$ on the domain divided by the same quantity on the disk.

$\sigma_1 \le 1.0208704035$	(1.0208704034)	$\rho_1 = 0.986943189861$
$\sigma_2 \le 1.0208704035$	(1.0208704034)	$\rho_2 = 0.986943189861$
$\sigma_3 \le 1.99686022381$	(1.99686022373)	$\rho_3 = 0.976095930066$
$\sigma_4 \le 1.9968602238$	(1.99686022373)	$\rho_4 = 0.972480176799$
$\sigma_5 \le 2.81568032866$	(2.81568032858)	$\rho_5 = 0.950776239368$
$\sigma_6 \le 2.81568032867$	(2.8156803286)	$\rho_6 = 0.939924270654$
$\sigma_7 \le 4.48784690137$	(4.48784690135)	$\rho_7 = 0.976111923406$
$\sigma_8 \le 4.48784690141$	(4.48784690137)	$\rho_8 = 0.99782451506$
$\sigma_9 \le 5.24534197919$	(5.24534197914)	$\rho_9 = 1.00110042424$
$\sigma_{10} \le 5.24534197973$	(5.24534197974)	$\rho_{10} = 1.00328436372$

1.3. **Plots of eigenfunctions.**





1.4. **Sizes of mesh triangles and list of other parameters.**

Initial polygon: n sides. Adaptive: try to find 4 eigenfunctions, use at most None. Refine to at least 400000 triangles.

Sizes of mesh triangles after adaptive refinement (blue - small):