1. Eigenvalues for the domain ellipse0.75

Radius function:

$$R(\theta) = \frac{0.5}{\sqrt{-0.75\sin^2(\theta) + 1}}$$

Adaptive finite element method of degree 2 with 124749 triangles.

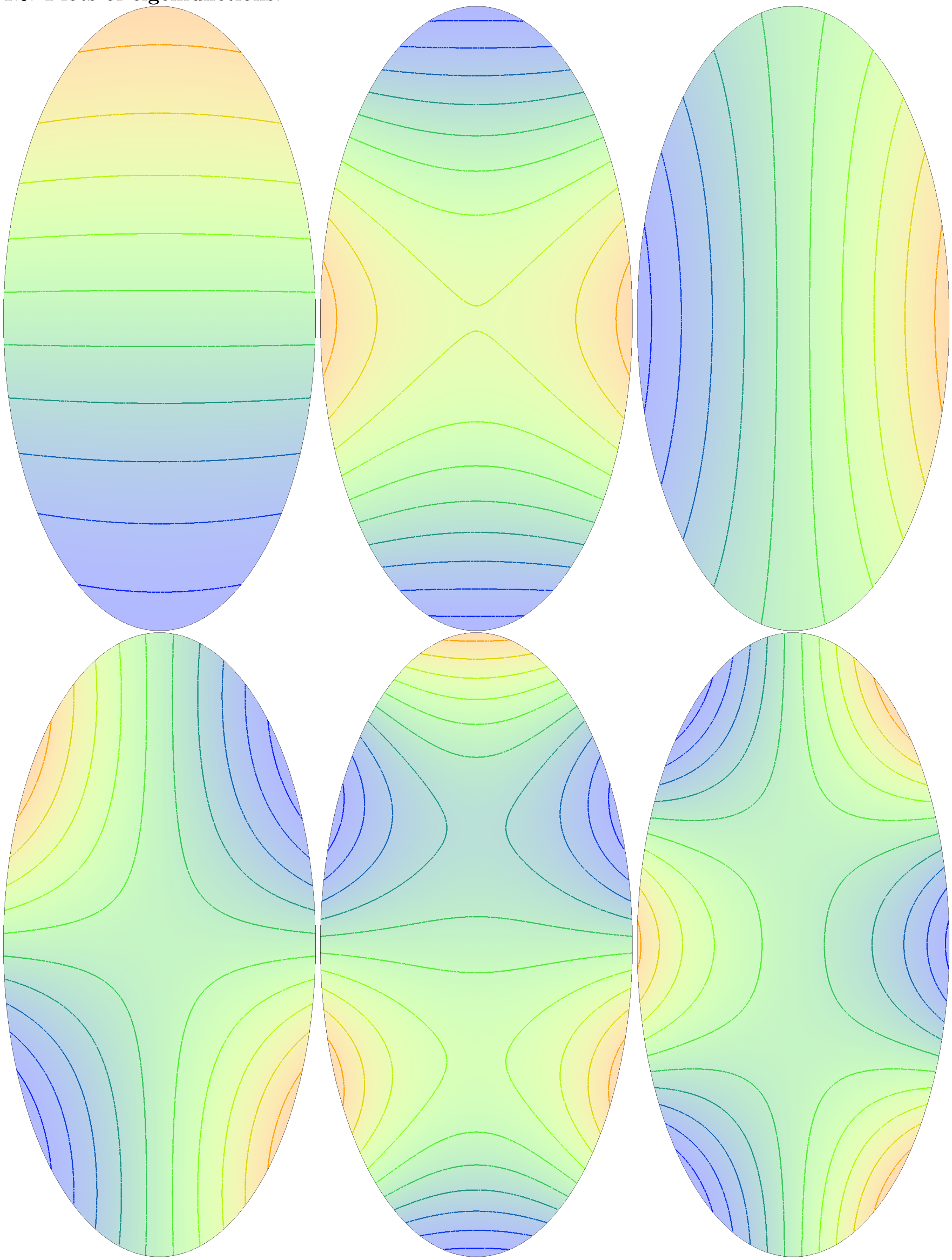
1.1. **Geometric quantities (perimeter L and constants g_i).** All quantities evaluated using boundary integral for a very fine mesh. For g_i we use geometric representations from Lemma 6.2.

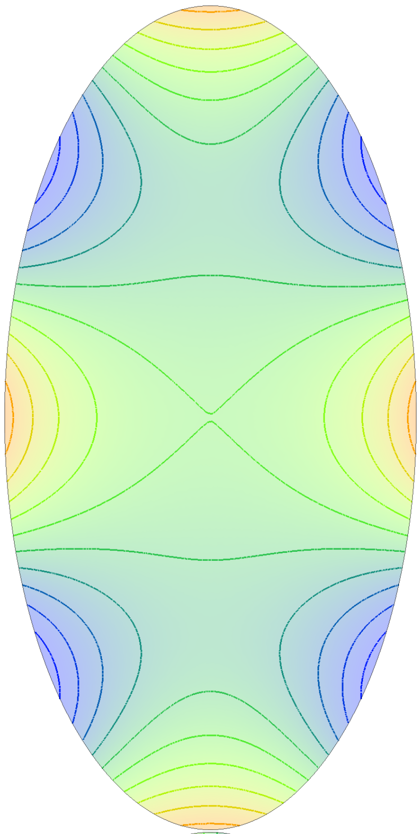
$$L = 4.84422324118 \quad g_0 = 1.25000066593 \quad g_1 = 1.07774314818 \quad g = 1.16068068517$$

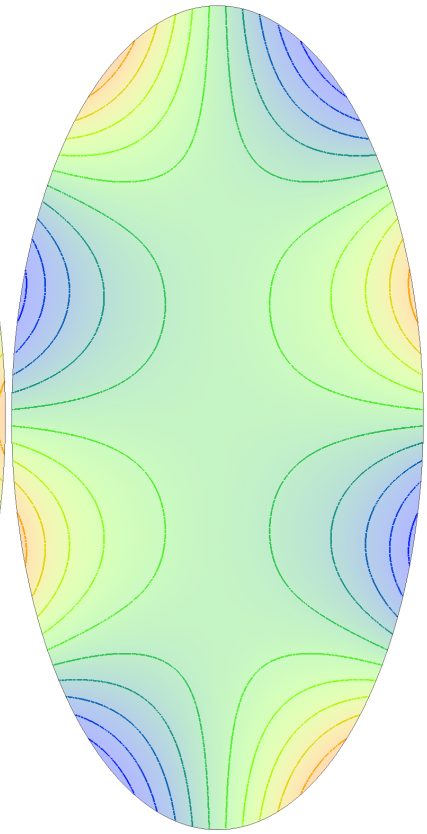
1.2. **Eigenvalues.** Upper bounds are obtained by plugging numerical eigenfunctions into the Rayleigh quotient. Numbers in parentheses are numerical eigenvalues of the discrete problem. Finally ρ_i are the rescaled sums $(\sigma_1 + \cdots + \sigma_n)L$ on the domain divided by the same quantity on the disk.

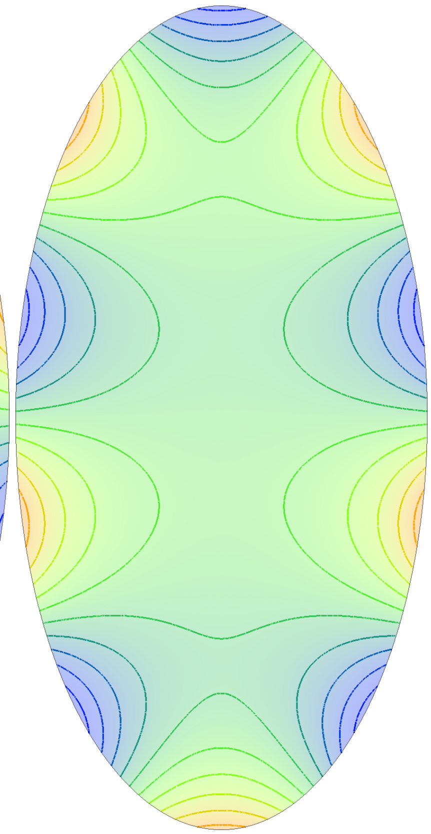
$\sigma_1 \leq 0.764598205972$	(0.764598206285)	$\rho_1 = 0.589491510827$
$\sigma_2 \leq 2.16966122971$	(2.16966123002)	$\rho_2 = 1.13113071309$
$\sigma_3 \leq 2.19166614334$	(2.19166614358)	$\rho_3 = 0.987999183873$
$\sigma_4 \leq 3.09388398255$	(3.09388398277)	$\rho_4 = 1.05622097099$
$\sigma_5 \leq 3.65528874837$	(3.65528874857)	$\rho_5 = 1.01727643636$
$\sigma_6 \leq 4.15400094939$	(4.1540009496)	$\rho_6 = 1.02984568298$
$\sigma_7 \leq 5.06743825237$	(5.06743825271)	$\rho_7 = 1.01656576567$
$\sigma_8 \leq 5.32274917448$	(5.3227491746)	$\rho_8 = 1.01843982251$
$\sigma_9 \leq 6.42297228584$	(6.42297228595)	$\rho_9 = 1.01283171784$
$\sigma_{10} \leq 6.55414660667$	(6.55414660686)	$\rho_{10} = 1.01246408307$

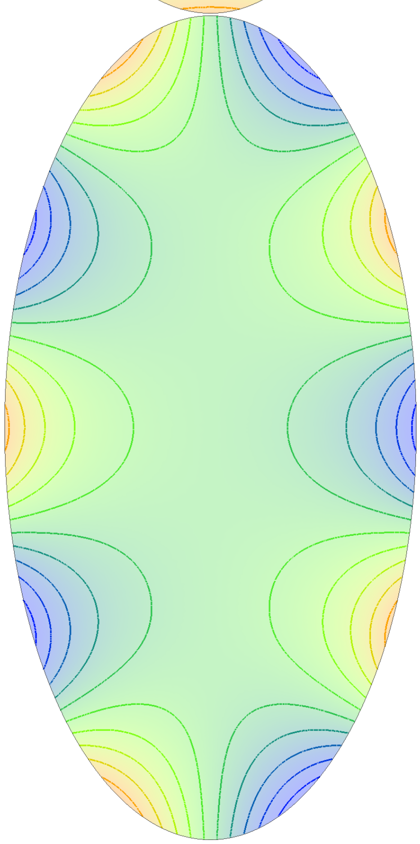
1.3. **Plots of eigenfunctions.**











1.4. **Sizes of mesh triangles and list of other parameters.**

Initial polygon: n sides. Adaptive: try to find 4 eigenfunctions, use at most None. Refine to at least 400000 triangles.

Sizes of mesh triangles after adaptive refinement (blue - small):