1. Eigenvalues for the domain disk

Radius function:

$$R(\theta) = 1$$

Adaptive finite element method of degree 2 with 109340 triangles.

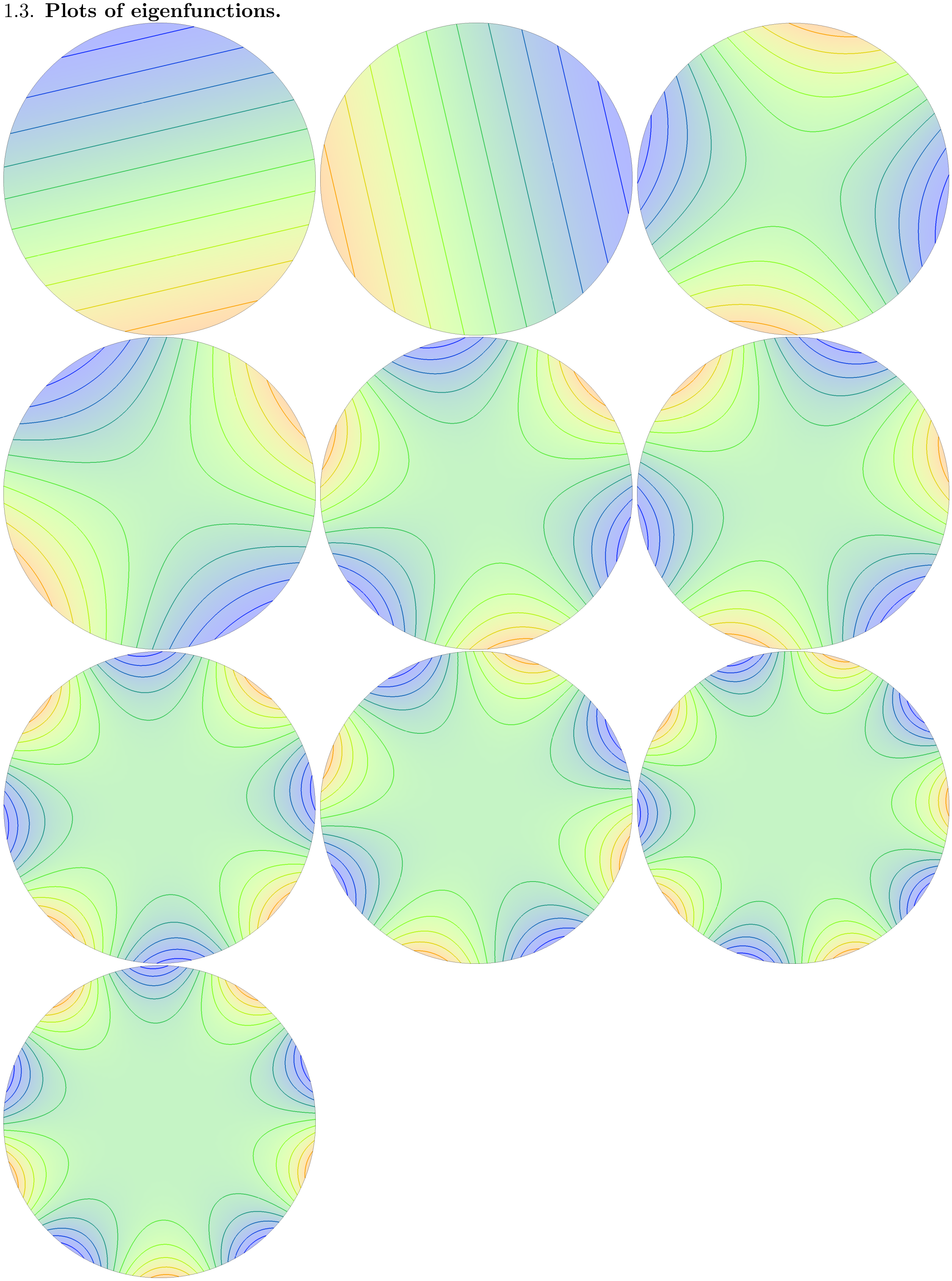
1.1. Geometric quantities (perimeter L and constants g_i). All quantities evaluated using boundary integral for a very fine mesh. For g_i we use geometric representations from Lemma 6.2.

$$L = 6.28318520641 \quad g_0 = 1.00000003208 \quad g_1 = 1.0 \quad g = 1.00000001604$$

1.2. Eigenvalues. Upper bounds are obtained by plugging numerical eigenfunctions into the Rayleigh quotient. Numbers in parentheses are numerical eigenvalues of the discrete problem. Finally ρ_i are the rescaled sums $(\sigma_1 + \cdots + \sigma_n)L$ on the domain divided by the same quantity on the disk.

$\sigma_1 \leq 1.00000001562$	(1.00000001553)	$\rho_1 = 0.999999999586$
$\sigma_2 \leq 1.00000001638$	(1.00000001633)	$\rho_2 = 0.999999999964$
$\sigma_3 \leq 2.00000003076$	(2.00000003066)	$\rho_3 = 0.999999999652$
$\sigma_4 \leq 2.00000003338$	(2.00000003327)	$\rho_4 = 0.999999999985$
$\sigma_5 \leq 3.00000006857$	(3.00000006849)	$\rho_5 = 1.00000000226$
$\sigma_6 \leq 3.00000007367$	(3.00000007362)	$\rho_6 = 1.00000000383$
$\sigma_7 \leq 4.00000023091$	(4.00000023087)	$\rho_7 = 1.00000001329$
$\sigma_8 \leq 4.00000024074$	(4.00000024066)	$\rho_8 = 1.00000001946$
$\sigma_9 \leq 5.00000073535$	(5.00000073539)	$\rho_9 = 1.00000004178$
$\sigma_{10} \leq 5.00000075564$	(5.00000075557)	$\rho_{10} = 1.00000005733$

1.3. Plots of eigenfunctions.



1.4. **Sizes of mesh triangles and list of other parameters.**

Initial polygon: 20 sides. Adaptive: try to find 4 eigenfunctions, use at most None. Refine to at least 400000 triangles.

Sizes of mesh triangles after adaptive refinement (blue - small):