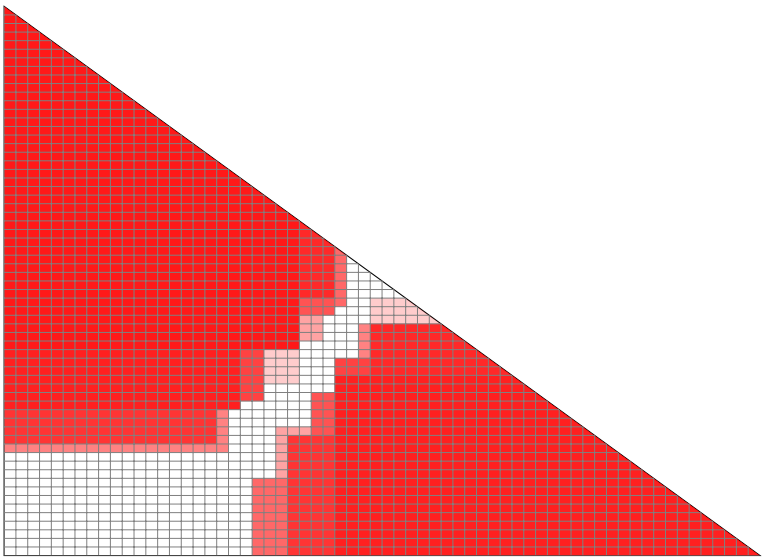


We found the excluded regions shown above.

The complement of the upper excluded region (picture below) has the smallest eigenvalue above the threshold 12.25. Yet it must contain the nodal domain which has eigenvalue below that threshold. This gives a contradiction with the domain monotonicity.

Vertices:

[(0, 12), (19, 12), (19, 17), (20, 17), (20, 18), (22, 18), (22, 20), (25, 20), (25, 25), (27, 25), (27, 28), (28, 28), (28, 29), (29, 29), (29, 35), [(64, 0), (0, 0)]]

Unprocessed eigenvalues:

[2.1633503059163846, 11.02534301446005]

Residuals:

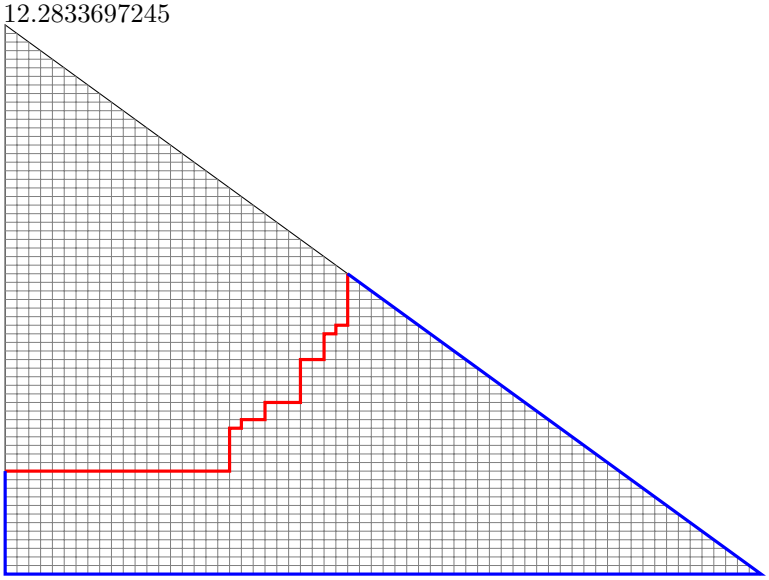
[4.598151070361678e-12, 6.9581514886418879e-12]

Rescaled eigenvalues (but not postprocessed):

[2.16335031 11.02534301]

Postprocessed eigenvalue (lower bound):

12.2833697245



1. New vertex p in upper excluded region U^1: (25, 24) (in grid coordinates).

   Vertices of the upper domain $D_U(p)$:

   [(0, 0), (25, 0), (25, 24), (40, 24), [(0, 64)]]

   Unprocessed eigenvalues for the upper domain:

   [2.1693043007060915, 10.783508211644577]

   Residuals:

   [5.195405372448488e-12, 6.945071811605383e-12]

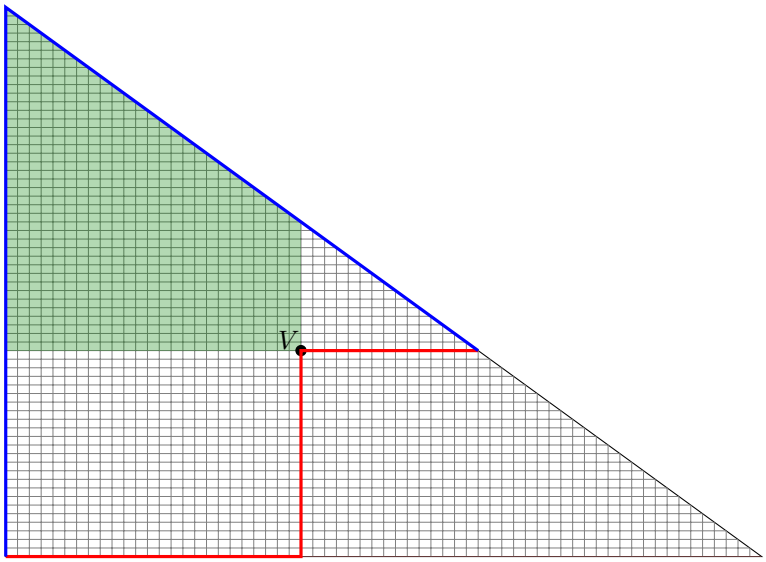
   Rescaled eigenvalues (by the bottom side length), but not postprocessed:

   [12.32808937 61.28234427]

   Postprocesses eigenvalue (lower bound):

   12.3171461311

   Eigenvalue for lower test domain D_L^{test}: 10.1477943063 (does not need to be calculated until it reaches the threshold 12.25).



2. New vertex p in lower excluded region L^1: (28, 21) (in grid coordinates).

   Vertices for the lower domain $D_L(p)$:

   [(0, 21), (28, 21), (28, 36), [(64, 0), (0, 0)]]

   Unprocessed eigenvalues for the lower domain:

   [2.1746523460623877, 8.3827823956042327]

   Residuals:

   [4.913715035163391e-12, 7.6916267788076862e-12]

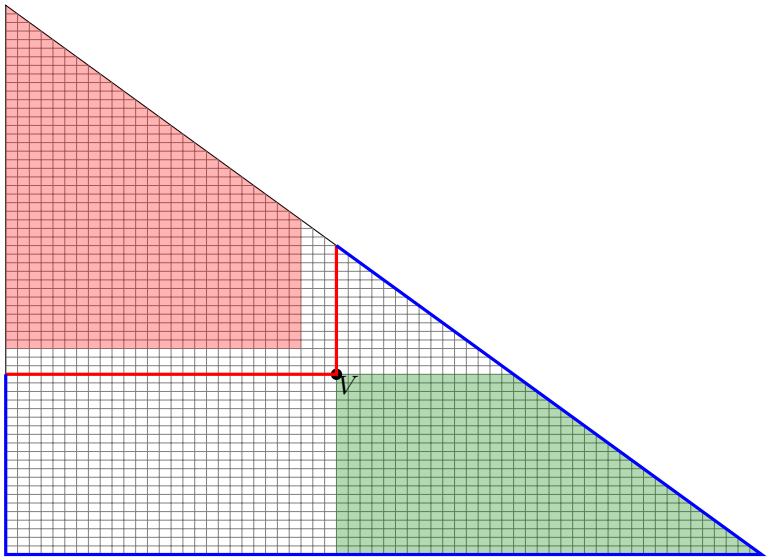
   Rescaled eigenvalues (by bottom side length), but not postprocessed:

   [12.35848215 47.63909357]

   Postprocesses eigenvalue (lower bound):

   12.347484908

   Eigenvalue for upper test domain D_U^{test}: 10.9964075789 (does not need to be calculated until it reaches the threshold 12.25).



3. New vertex p in upper excluded region U^2: (20, 17) (in grid coordinates).

   Vertices of the upper domain $D_U(p)$:

   [(0, 0), (20, 0), (20, 17), (28, 17), (28, 21), (43, 21), [(0, 64)]]

   Unprocessed eigenvalues for the upper domain:

   [2.1609297493267277, 11.105649612004703]

   Residuals:

   [4.8723116845836387e-12, 7.4090307603537676e-12]

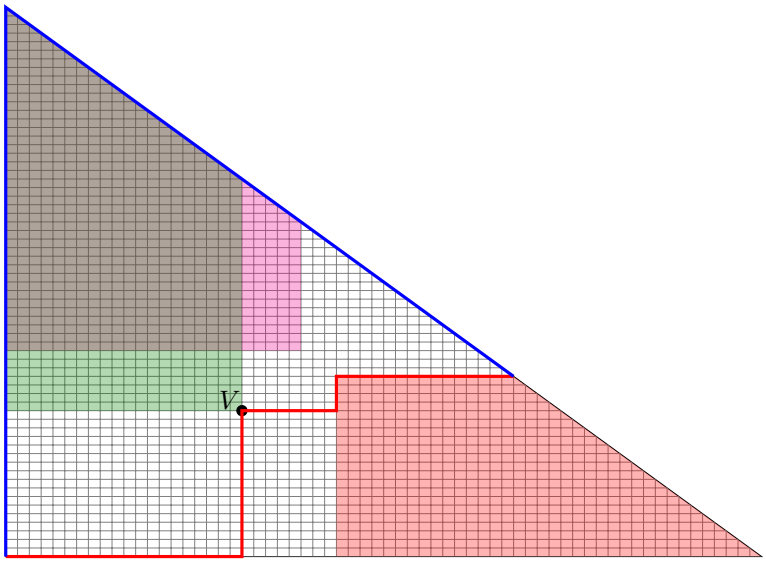
   Rescaled eigenvalues (by the bottom side length), but not postprocessed:

   [12.28049706 63.11306391]

   Postprocesses eigenvalue (lower bound):

   12.2696381062

   Eigenvalue for lower test domain D_L^{test}: 10.8384636507 (does not need to be calculated until it reaches the threshold 12.25).



4. New vertex p in lower excluded region L^2: (31, 27) (in grid coordinates).

   Vertices for the lower domain $D_L(p)$:

   [(0, 17), (20, 17), (20, 24), (25, 24), (25, 27), (31, 27), (31, 33), [(64, 0), (0, 0)]]

   Unprocessed eigenvalues for the lower domain:

   [2.1607701681727698, 9.8954267447091855]

   Residuals:

   [4.5639143759865585e-12, 7.942461693353896e-12]

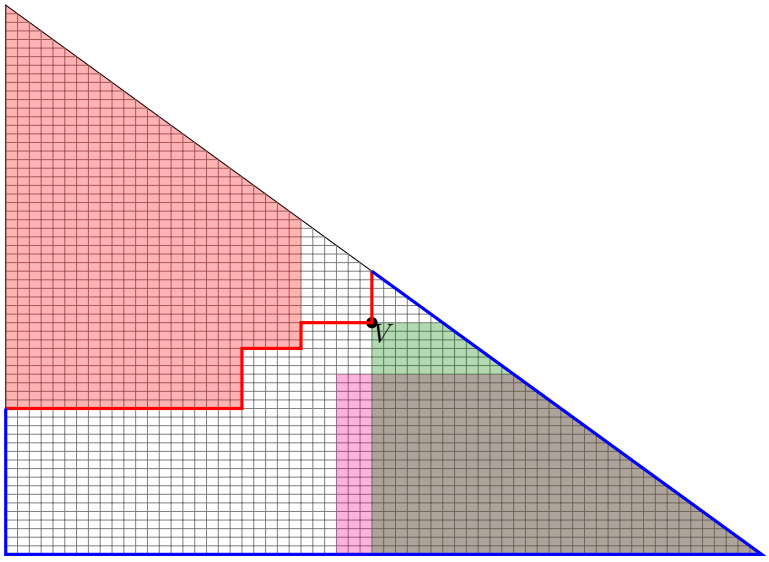
   Rescaled eigenvalues (by bottom side length), but not postprocessed:

   [12.27959016 56.23540471]

   Postprocesses eigenvalue (lower bound):

   12.2687328145

   Eigenvalue for upper test domain D_U^{test}: 11.3481893437 (does not need to be calculated until it reaches the threshold 12.25).



5. New vertex p in upper excluded region U^3: (28, 30) (in grid coordinates).

   Vertices of the upper domain $D_U(p)$:

   [(0, 0), (28, 0), (28, 30), (34, 30), [(0, 64)]]

   Unprocessed eigenvalues for the upper domain:

   [2.1592022649722211, 10.509175135495724]

   Residuals:

   [5.1257783330952429e-12, 7.9911062056934245e-12]

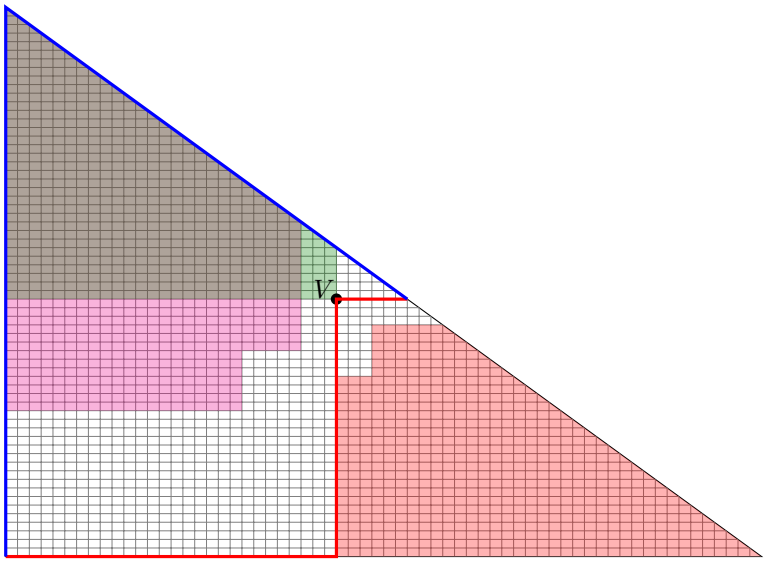
   Rescaled eigenvalues (by the bottom side length), but not postprocessed:

   [12.27067982 59.72331787]

   Postprocesses eigenvalue (lower bound):

   12.259838213

   Eigenvalue for lower test domain D_L^{test}: 11.1543395403 (does not need to be calculated until it reaches the threshold 12.25).



6. New vertex p in lower excluded region L^3: (24, 14) (in grid coordinates).

   Vertices for the lower domain $D_L(p)$:

   [(0, 14), (24, 14), (24, 24), (25, 24), (25, 30), (28, 30), (28, 36), [(64, 0), (0, 0)]]

   Unprocessed eigenvalues for the lower domain:

   [2.1837241362172217, 11.23991415603915]

   Residuals:

   [4.7787436788803418e-12, 7.0826552423146707e-12]

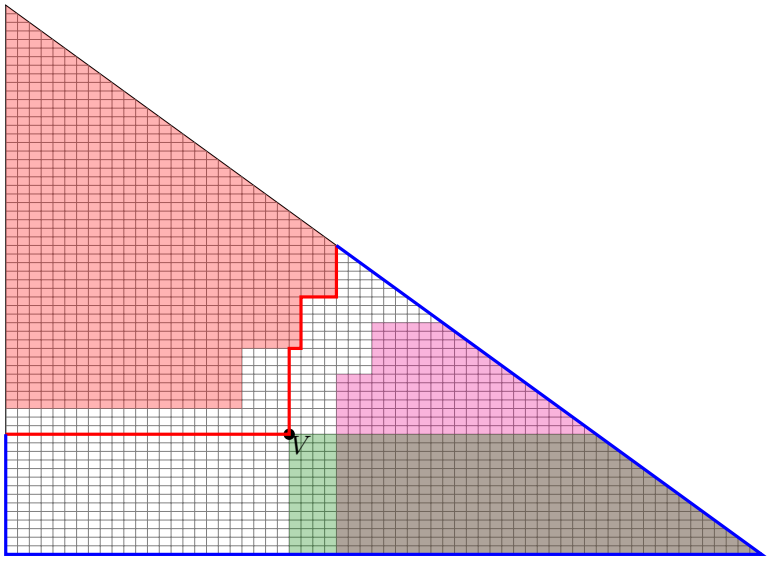
   Rescaled eigenvalues (by bottom side length), but not postprocessed:

   [12.41003685 63.87608517]

   Postprocesses eigenvalue (lower bound):

   12.3989477089

   Eigenvalue for upper test domain D_U^{test}: 11.5949767668 (does not need to be calculated until it reaches the threshold 12.25).



7. New vertex p in upper excluded region U^4: (18, 13) (in grid coordinates).

   Vertices of the upper domain $D_U(p)$:

   [(0, 0), (18, 0), (18, 13), (24, 13), (24, 14), (28, 14), (28, 21), (31, 21), (31, 27), (37, 27), [(0, 64)]]

   Unprocessed eigenvalues for the upper domain:

   [2.1616790644024646, 11.146739955047408]

   Residuals:

   [4.8383576680934513e-12, 7.4096545152980828e-12]

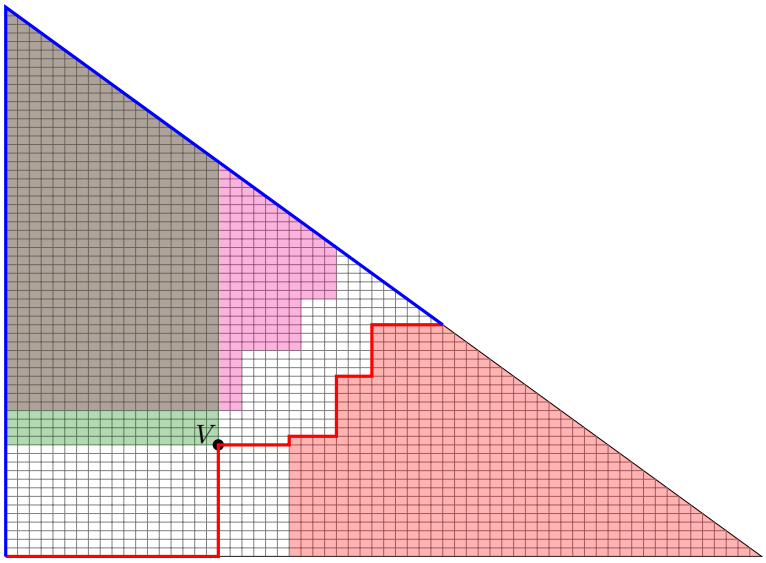
   Rescaled eigenvalues (by the bottom side length), but not postprocessed:

   [12.28475539 63.34657906]

   Postprocesses eigenvalue (lower bound):

   12.2738889118

   Eigenvalue for lower test domain D_L^{test}: 11.3641881646 (does not need to be calculated until it reaches the threshold 12.25).



8. New vertex p in lower excluded region L^4: (28, 23) (in grid coordinates).

   Vertices for the lower domain $D_L(p)$:

   [(0, 13), (18, 13), (18, 17), (20, 17), (20, 23), (28, 23), (28, 36), [(64, 0), (0, 0)]]

   Unprocessed eigenvalues for the lower domain:

   [2.1593603202112912, 10.765645575090153]

   Residuals:

   [4.6027322175280276e-12, 7.397046232424909e-12]

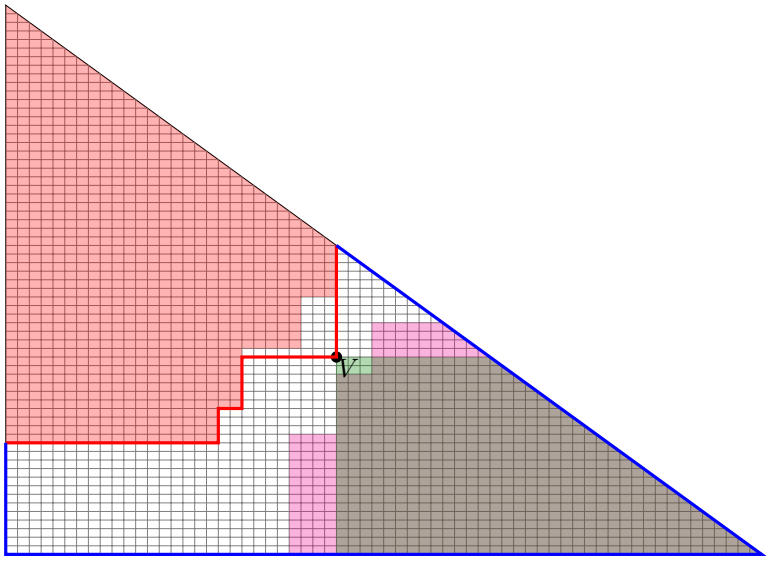
   Rescaled eigenvalues (by bottom side length), but not postprocessed:

   [12.27157804 61.18083146]

   Postprocesses eigenvalue (lower bound):

   12.2607348494

   Eigenvalue for upper test domain D_U^{test}: 11.7004951476 (does not need to be calculated until it reaches the threshold 12.25).



9. New vertex p in upper excluded region U^5: (22, 18) (in grid coordinates).

   Vertices of the upper domain $D_U(p)$:

   [(0, 0), (22, 0), (22, 18), (28, 18), (28, 23), (31, 23), (31, 27), (37, 27), [(0, 64)]]

   Unprocessed eigenvalues for the upper domain:

   [2.1696027814426753, 10.982665884435901]

   Residuals:

   [5.024149536688425e-12, 6.8260814082531892e-12]

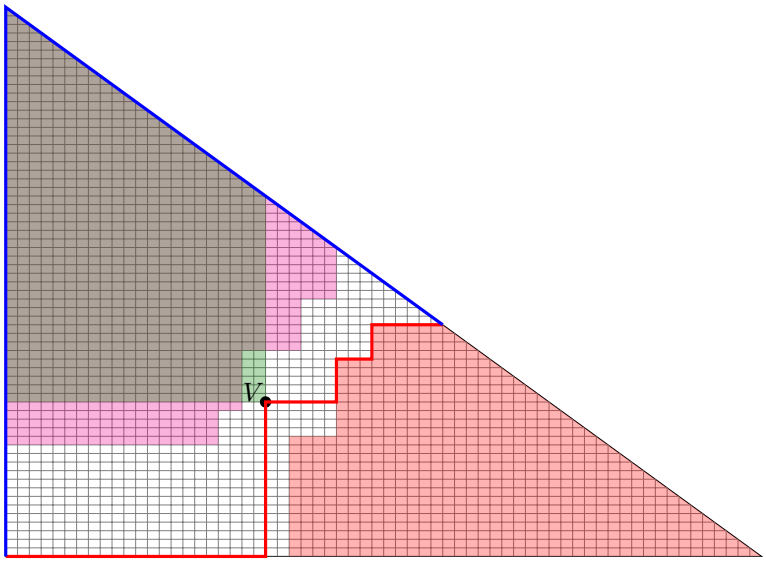
   Rescaled eigenvalues (by the bottom side length), but not postprocessed:

   [12.32978563 62.41415118]

   Postprocesses eigenvalue (lower bound):

   12.3188393777

   Eigenvalue for lower test domain D_L^{test}: 11.5163239324 (does not need to be calculated until it reaches the threshold 12.25).



10. New vertex p in lower excluded region L^5: (26, 19) (in grid coordinates).

    Vertices for the lower domain $D_L(p)$:

    [(0, 13), (18, 13), (18, 17), (20, 17), (20, 18), (22, 18), (22, 19), (26, 19), (26, 30), (28, 30), (28, 36), [(64, 0), (0, 0)]]

    Unprocessed eigenvalues for the lower domain:

    [2.1682926349150446, 10.973311705699482]

    Residuals:

    [4.4606852769163212e-12, 6.9815231469825075e-12]

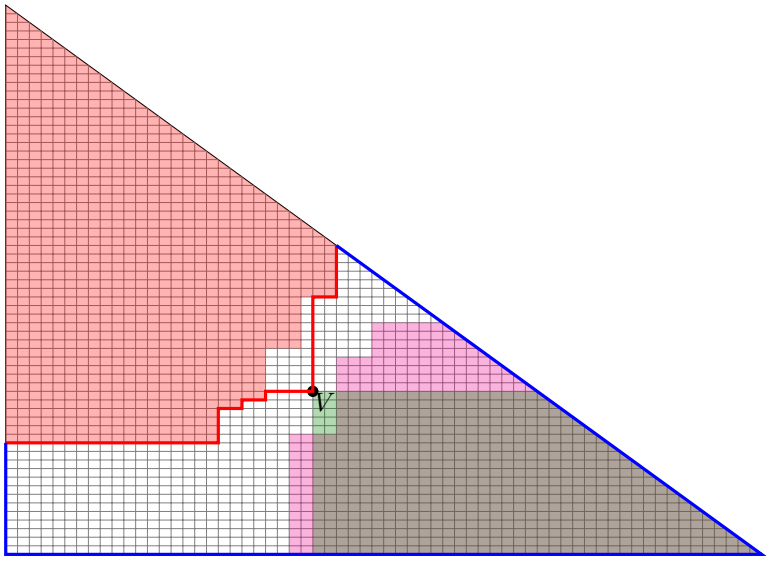
    Rescaled eigenvalues (by bottom side length), but not postprocessed:

    [12.32234011 62.36099167]

    Postprocesses eigenvalue (lower bound):

    12.3114070656

    Eigenvalue for upper test domain D_U^{test}: 11.7853482551 (does not need to be calculated until it reaches the threshold 12.25).



11. New vertex p in upper excluded region U^6: (28, 28) (in grid coordinates).

    Vertices of the upper domain $D_U(p)$:

    [(0, 0), (24, 0), (24, 14), (26, 14), (26, 19), (28, 19), (28, 28), (36, 28), [(0, 64)]]

    Unprocessed eigenvalues for the upper domain:

    [2.1591402221083222, 10.715800047494927]

    Residuals:

    [5.177504324617642e-12, 6.660217001865947e-12]

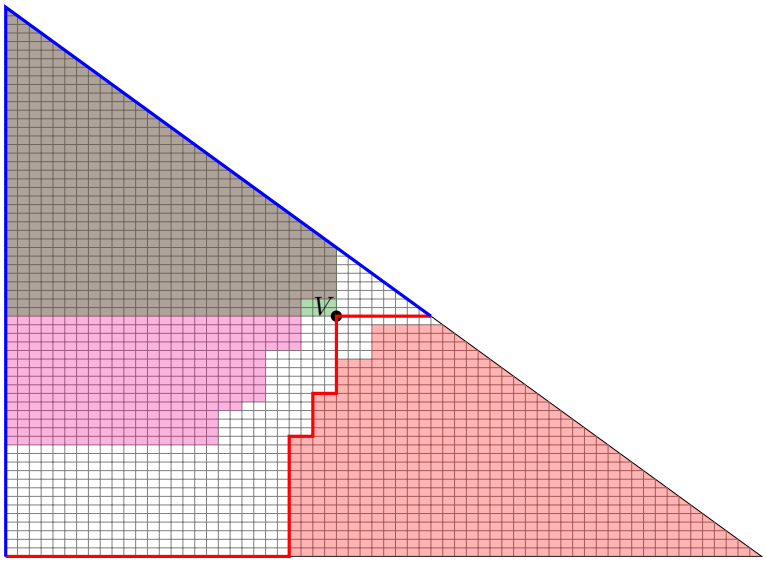
    Rescaled eigenvalues (by the bottom side length), but not postprocessed:

    [12.27032723 60.89756087]

    Postprocesses eigenvalue (lower bound):

    12.259486248

    Eigenvalue for lower test domain D_L^{test}: 11.6697779075 (does not need to be calculated until it reaches the threshold 12.25).



12. New vertex p in lower excluded region L^6: (21, 9) (in grid coordinates).

    Vertices for the lower domain $D_L(p)$:

    [(0, 9), (21, 9), (21, 18), (22, 18), (22, 24), (25, 24), (25, 28), (28, 28), (28, 36), [(64, 0), (0, 0)]]

    Unprocessed eigenvalues for the lower domain:

    [2.1648948969640949, 11.324730481768352]

    Residuals:

    [4.896413397690929e-12, 7.1156691027010294e-12]

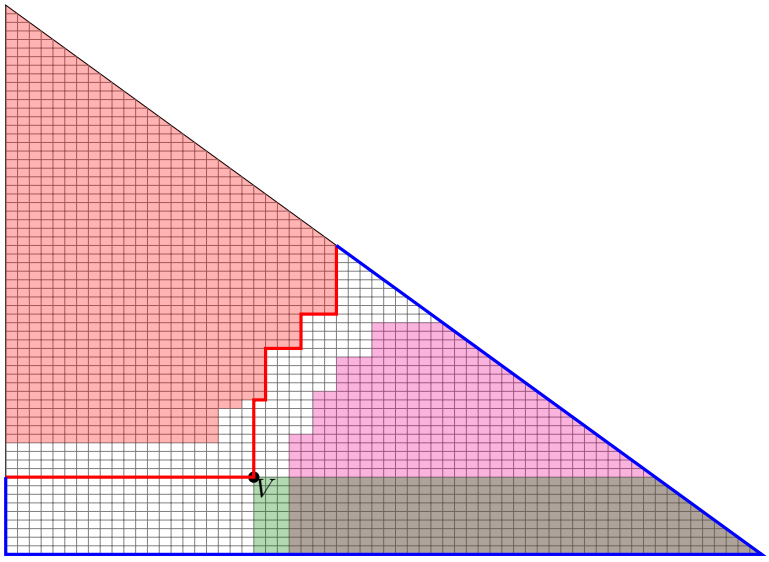
    Rescaled eigenvalues (by bottom side length), but not postprocessed:

    [12.30303087 64.35809373]

    Postprocesses eigenvalue (lower bound):

    12.2921320482

    Eigenvalue for upper test domain D_U^{test}: 11.8691352086 (does not need to be calculated until it reaches the threshold 12.25).



13. New vertex p in upper excluded region U^7: (29, 29) (in grid coordinates).

    Vertices of the upper domain $D_U(p)$:

    [(0, 0), (21, 0), (21, 9), (24, 9), (24, 14), (26, 14), (26, 19), (28, 19), (28, 23), (29, 23), (29, 29), (35, 29), [(0, 64)]]

    Unprocessed eigenvalues for the upper domain:

    [2.1588747785558096, 10.791541985379091]

    Residuals:

    [4.9585439614978746e-12, 6.8327913480466457e-12]

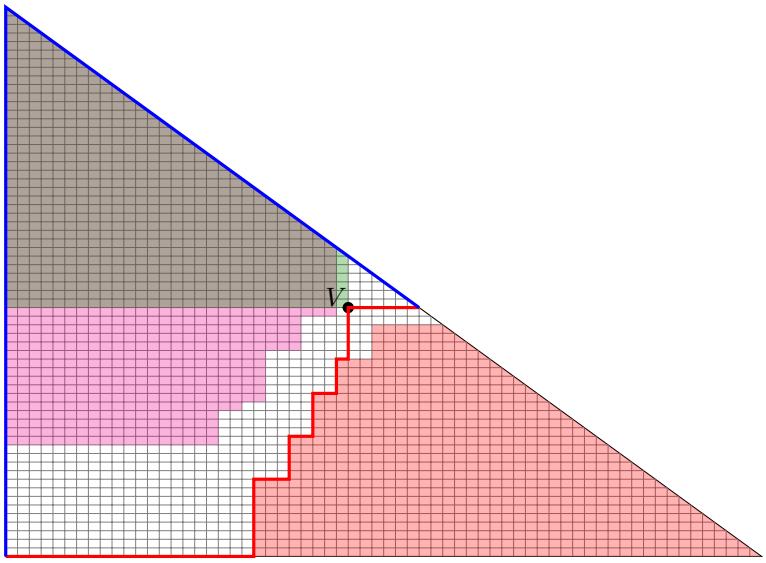
    Rescaled eigenvalues (by the bottom side length), but not postprocessed:

    [12.26881872 61.32799996]

    Postprocesses eigenvalue (lower bound):

    12.2579804048

    Eigenvalue for lower test domain D_L^{test}: 11.805661388 (does not need to be calculated until it reaches the threshold 12.25).



14. New vertex p in lower excluded region L^7: (30, 27) (in grid coordinates).

    Vertices for the lower domain $D_L(p)$:

    [(0, 13), (18, 13), (18, 17), (20, 17), (20, 18), (22, 18), (22, 24), (25, 24), (25, 27), (30, 27), (30, 34), [(64, 0), (0, 0)]]

    Unprocessed eigenvalues for the lower domain:

    [2.1640230452714846, 10.759941376290984]

    Residuals:

    [4.4907304821988718e-12, 6.8071142541062353e-12]

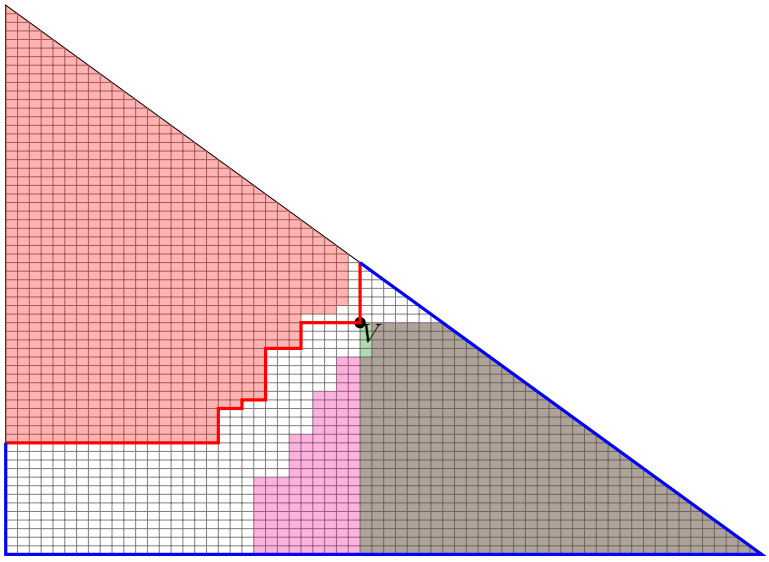
    Rescaled eigenvalues (by bottom side length), but not postprocessed:

    [12.29807616 61.14841468]

    Postprocesses eigenvalue (lower bound):

    12.2871861146

    Eigenvalue for upper test domain D_U^{test}: 11.94597256 (does not need to be calculated until it reaches the threshold 12.25).



15. New vertex p in upper excluded region U^8: (19, 12) (in grid coordinates).

    Vertices of the upper domain $D_U(p)$:

    [(0, 0), (19, 0), (19, 12), (24, 12), (24, 14), (26, 14), (26, 19), (28, 19), (28, 23), (30, 23), (30, 27), (37, 27), [(0, 64)]]

    Unprocessed eigenvalues for the upper domain:

    [2.1584009432958866, 11.023173768196036]

    Residuals:

    [5.0818345861509005e-12, 7.2845370544319738e-12]

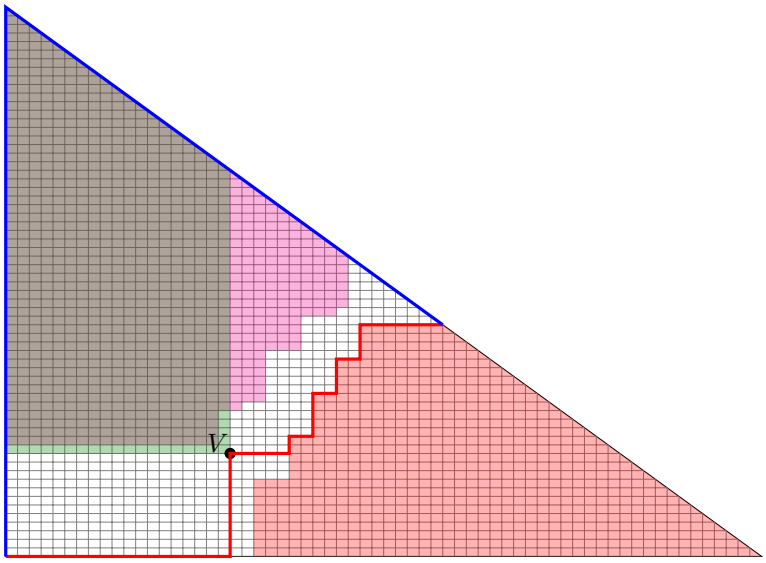
    Rescaled eigenvalues (by the bottom side length), but not postprocessed:

    [12.26612593 62.64435623]

    Postprocesses eigenvalue (lower bound):

    12.2552923686

    Eigenvalue for lower test domain D_L^{test}: 11.9274447895 (does not need to be calculated until it reaches the threshold 12.25).



16. New vertex p in lower excluded region L^8: (23, 15) (in grid coordinates).

    Vertices for the lower domain $D_L(p)$:

    [(0, 12), (19, 12), (19, 15), (23, 15), (23, 24), (25, 24), (25, 28), (28, 28), (28, 29), (29, 29), (29, 35), [(64, 0), (0, 0)]]

    Unprocessed eigenvalues for the lower domain:

    [2.1652787845040429, 11.133850073133729]

    Residuals:

    [4.4711438545278684e-12, 7.3874404717381035e-12]

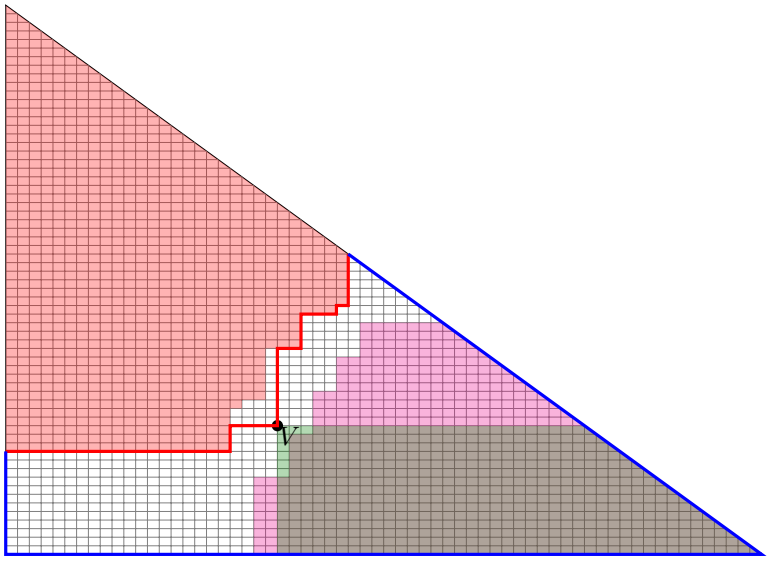
    Rescaled eigenvalues (by bottom side length), but not postprocessed:

    [12.30521249 63.27332626]

    Postprocesses eigenvalue (lower bound):

    12.2943098053

    Eigenvalue for upper test domain D_U^{test}: 12.0386343232 (does not need to be calculated until it reaches the threshold 12.25).



17. New vertex p in upper excluded region U^9: (27, 25) (in grid coordinates).

    Vertices of the upper domain $D_U(p)$:

    [(0, 0), (21, 0), (21, 9), (23, 9), (23, 15), (26, 15), (26, 19), (27, 19), (27, 25), (30, 25), (30, 27), (37, 27), [(0, 64)]]

    Unprocessed eigenvalues for the upper domain:

    [2.1624388010101683, 10.864923641996144]

    Residuals:

    [4.9533761518189615e-12, 7.0740443674823618e-12]

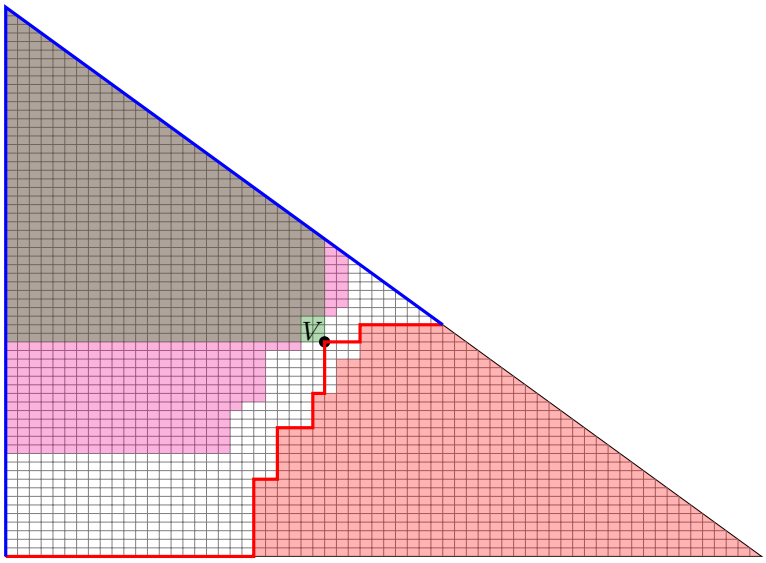
    Rescaled eigenvalues (by the bottom side length), but not postprocessed:

    [12.28907295 61.74502565]

    Postprocesses eigenvalue (lower bound):

    12.2781988354

    Eigenvalue for lower test domain D_L^{test}: 12.0541213255 (does not need to be calculated until it reaches the threshold 12.25).



18. New vertex p in lower excluded region L^9: (31, 30) (in grid coordinates).

    Vertices for the lower domain $D_L(p)$:

    [(0, 12), (19, 12), (19, 17), (20, 17), (20, 18), (22, 18), (22, 24), (25, 24), (25, 25), (27, 25), (27, 28), (28, 28), (28, 29), (29, 29), (29, 30), (31, 30), (31, 33), [(64, 0), (0, 0)]]

    Unprocessed eigenvalues for the lower domain:

    [2.1640990008677603, 10.946325258072138]

    Residuals:

    [4.6336301266502685e-12, 7.8142078360388626e-12]

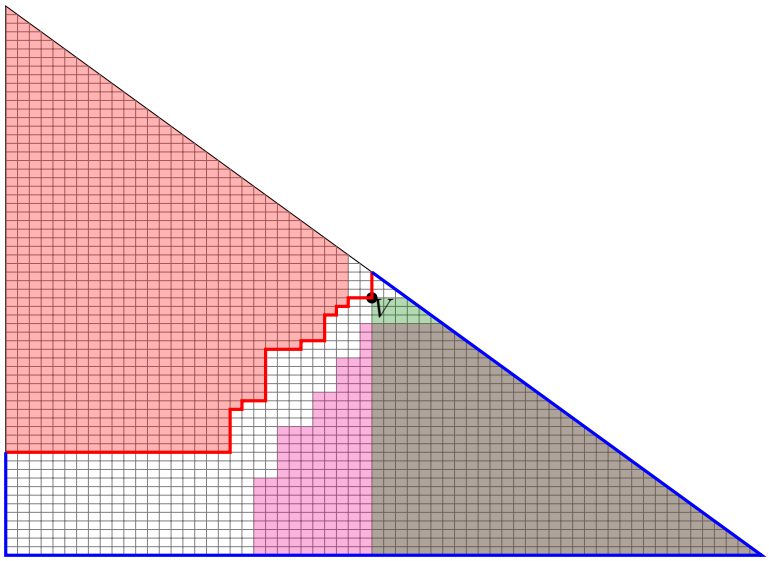
    Rescaled eigenvalues (by bottom side length), but not postprocessed:

    [12.29850781 62.20762852]

    Postprocesses eigenvalue (lower bound):

    12.2876170038

    Eigenvalue for upper test domain D_U^{test}: 12.1584700213 (does not need to be calculated until it reaches the threshold 12.25).



19. New vertex p in upper excluded region U^{10}: (25, 20) (in grid coordinates).

    Vertices of the upper domain $D_U(p)$:

    [(0, 0), (21, 0), (21, 9), (23, 9), (23, 15), (25, 15), (25, 20), (28, 20), (28, 23), (30, 23), (30, 27), (31, 27), (31, 30), (34, 30), [(0, 64)]]

    Unprocessed eigenvalues for the upper domain:

    [2.1612424885372805, 10.870469883392126]

    Residuals:

    [4.8441740893355694e-12, 6.7029987148089354e-12]

    Rescaled eigenvalues (by the bottom side length), but not postprocessed:

    [12.28227434 61.77654477]

    Postprocesses eigenvalue (lower bound):

    12.2714122515

    Eigenvalue for lower test domain D_L^{test}: 12.2833697245 (does not need to be calculated until it reaches the threshold 12.25).