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Effect of car acceleration on traffic flow in 1D stochastic CA model

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Abstract

The one-dimensional (1D) asymmetric simple-exclusion model with parallel update (the deterministic cellular automaton (CA) 184) is extended to take into account acceleration of cars. Velocity and acceleration are assigned to each particle (car). The acceleration of a car is determined by the dynamical equation of motion. The stochastic cellular automaton model mimics the traffic flow on a highway. In congested traffic flow, the properties of traffic jams are studied by computer simulation. The dependence of the propagating velocity of traffic jams upon car acceleration is shown. Also, a simple mean-field theory is given for the congested traffic flow. It is found that the propagating velocity of a traffic jam obtained from simulation agrees with the analytical result for not small values of car acceleration.

1. Introduction

Recently, traffic problems have attracted considerable attention. A variety of approaches have been applied to describe the collective properties of traffic flows [1–5]. Cellular automaton (CA) models are being applied successfully to simulations of traffic. The one-dimensional (1D) asymmetric simple-exclusion model has been applied to model traffic flow on a one-dimensional road [6,7]. The 1D asymmetric simple-exclusion model with parallel update is consistent with the deterministic CA model 184 [8]. When the number of cars is large, traffic flows can be modelled phenomenologically in terms of a 1D compressible gas. Such a hydrodynamic approach predicts the appearance of traffic solitons, shock waves and traffic jams [9]. However, the hydrodynamic approach does not naturally describe the behavior of traffic flows in the low-density limit where there are large heterogeneities in traffic density [10]. For this situation, a microscopic model will provide a more appropriate description. Also, to mimic traffic flow in a city, two-dimensional CA models have been proposed [8,11–16].

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Very recently, Nagel and Schreckenberg [1] introduced a stochastic cellular automaton model to simulate freeway traffic. They showed that the start-stop waves (traffic jams) appear in the congested traffic region as are observed in real freeway traffic. In their model, each car has an integer velocity v with values between zero and v_{max}. Each car is accelerated, slowed down, randomized, or advanced by the following four consecutive steps. However, car acceleration is not assigned explicitly to each car. The dependence of the propagating velocity of start-stop waves upon v_{max} and the randomization parameter is unclear.

Bando et al. [4] proposed a dynamical model whose equation of motion is expressed as a second order differential equation. The acceleration in the dynamical model was expressed as

$$\frac{d^2 x_n(t)}{dt^2} = a\left(V(\Delta x_n(t)) - \frac{dx_n(t)}{dt}\right), \tag{1}$$

where $d^2x_n(t)/dt^2$ is the acceleration of the nth car, $\Delta x_n(t)$ is the headway of the nth car, $dx_n(t)/dt$ is the velocity of the nth car and $V(\Delta x_n(t))$ is the legal velocity. They found that a traffic jam spontaneously appears in the congested traffic region. They showed that the traffic jam is structurally stable and a limit cycle appears in the phase space of velocity and headway. In the dynamical model, space and time are continuous variables.

Yukawa and Kikuchi [5] proposed a coupled-map lattice model to describe the traffic flow on a highway. The velocity was determined by the headway. The fluctuation of the car velocity was taken into account by introducing the return map. The above authors studied the traffic patterns under various conditions of traffic flow. However, the effect of acceleration on traffic flow is not taken into account in the coupled-map lattice model.

In the CA models proposed until now, the acceleration of each car has not been taken into account. The effect of car acceleration on traffic flow is unclear. There is an open question as to how the structure and propagating velocity of traffic jams depend on the car acceleration.

In this paper, we propose the CA model to take into account the acceleration of cars. We extend the 1D asymmetric simple-exclusion model with parallel update to include velocity and acceleration of cars. We assign the velocity and acceleration to each particle (car) in the 1D asymmetric simple-exclusion model. The acceleration of a car is determined by the dynamical equation of motion. The velocity is given by its integrated acceleration. Each car adapts its velocity to the circumstances of traffic flow when a car interacts with other cars. We study the propagation of the traffic jams (or start-stop waves) in the congested traffic region by computer simulation. We show the dependence of the propagating velocity of traffic jams upon the car acceleration. We find that the propagating velocity of traffic jams increases with the strength of car acceleration. We investigate the behavior of cars in the phase space of velocity and headway. Also, we give a simple mean-field theory for the traffic jams. We derive the analytical result for the propagating velocity of traffic jams. We compare the analytical result with the simulation result.

The organization of the paper is as follows. In Section 2, we present the CA model to take into account car acceleration. We show the simulation result for the CA model. In Section 3, we give a simple mean-field theory for the traffic jams. We compare the analytical result with the simulation result. Finally, Section 4 contains a brief summary.

2. Model and simulation

We present the CA model with velocity and acceleration for freeway traffic. We extend the 1D asymmetric simple-exclusion model with parallel update to take into account car acceleration. The CA model is defined on a one-dimensional lattice of L sites with periodic boundary condition. Each site is occupied by a single car or it is empty. For an arbitrary configuration, one update of the system consists of the following rules which are performed in parallel for all cars. Car n moves ahead by one step with probability $p_n(t)$, or otherwise, car n does not move with probability $1 - p_n(t)$. In the dilute limit of car density, car n moves ahead with the mean velocity $p_n(t)$ at coarse-grained time scales since car n is not blocked by other cars. The probability $p_n(t)$ corresponds to the velocity $v_n(t)$ of car n. When the probability $p_n = 1$ for all cars, the model reproduces the 1D asymmetric simple-exclusion model. The velocity $v_n(t)$ is given by integrating acceleration. The acceleration of individual cars is determined by the dynamical equation of motion. The acceleration is assigned to each car. The acceleration of nth car is given by

$$\frac{\mathrm{d}^2 x_n(t)}{\mathrm{d}t^2} = \begin{cases} a, & \Delta x_n(t) \geqslant x_c \\ -a, & \Delta x_n(t) < x_c \end{cases}, \tag{2}$$

where $a > 0$, $\Delta x_n(t) (= x_{n-1}(t) - x_n(t))$ is the headway and x_c is the safety distance. When the headway $\Delta x_n(t)$ is larger than the safety distance x_c, the nth car is accelerated by $a(>0)$. If the headway is less than the safety distance, the nth car is decelerated by a. The velocity $v_n(t)$ of the nth car is given by integrating acceleration $\mathrm{d}^2 x_n(t)/\mathrm{d}t^2$ of car n. If the velocity $v_n(t)$ is larger than $v_{\max}$ ($\leqslant 1$), the velocity $v_n(t)$ becomes $v_{\max}$. If the velocity $v_n(t)$ is less than zero, the velocity $v_n(t)$ becomes zero. The velocity $v_n(t)$ ranges from zero to $v_{\max}$,

$$0 \leqslant v_n(t) \leqslant v_{\max} . \tag{3}$$

Thus, the acceleration $\mathrm{d}^2 x_n(t)/\mathrm{d}t^2$ and the velocity $v_n(t)$ are assigned to the nth car. The car (or particle) moves ahead by the transition probability $p_n(t) (= v_n(t))$. The transition probability $p_n(t)$ of the nth car changes from time to time. If car n is blocked ahead by another car, car n does not move and its velocity becomes zero.

In this model, the traffic problem on a highway is reduced to its simplest form while the essential features are maintained. The feature includes the flow in one direction of cars which cannot overlap. Furthermore, this model possesses the property that the car moves with its velocity which is determined by integrating acceleration. Our

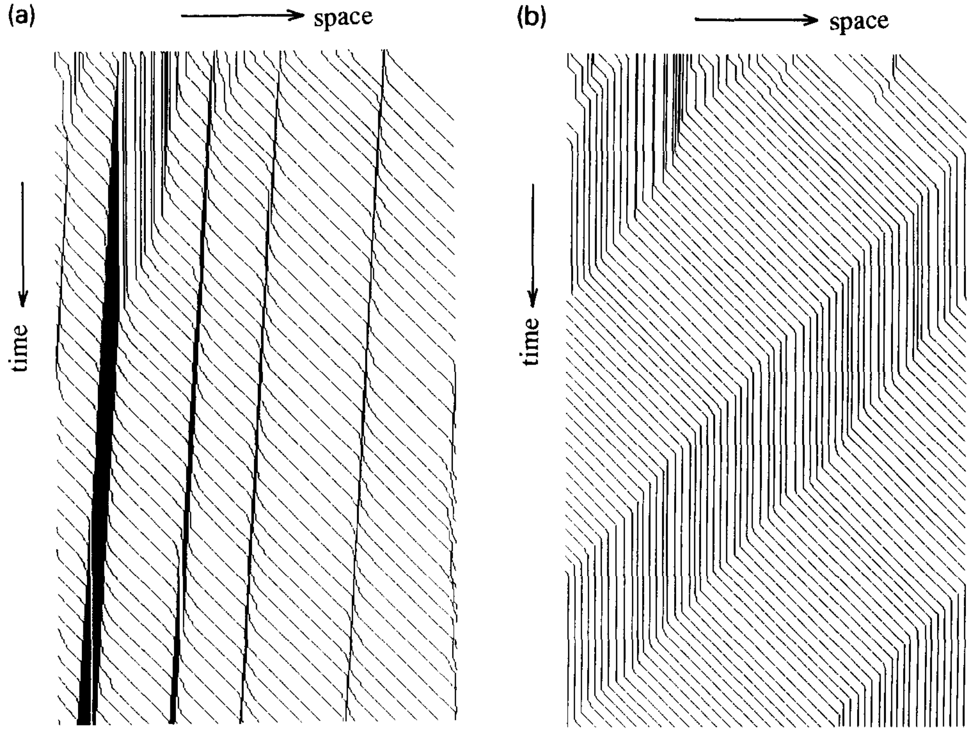

Fig. 1. Typical configurations of cars up to 500 time steps for initial car density $p_0 = 0.15$, initial car velocity $v_0 = 0.3$, safety distance $x_c = 7$, maximal velocity $v_{max} = 1$ and minimal velocity $v_{min} = 0$ where the system size L is 300. The horizontal and vertical directions indicate respectively space and time. A car is indicated by a dot. The trajectory of a car is represented by a curve. (a) The traffic pattern for acceleration $a = 0.05$. The four start–stop waves are formed and propagate backward. (b) The traffic pattern for acceleration $a = 0.2$. A single start–stop wave is formed and propagates backward.

model is simpler than the dynamical model by Bando et al. [4]. Due to this simplicity, it is possible to apply to our model an analytical method. The analytical approach will be given by Section 3.

We perform the computer simulation for our model. Initially, cars are randomly distributed on the sites of a one-dimensional lattice with initial density p_0. Furthermore, the initial velocity v_0 is assigned to each car. The acceleration of each car is determined by Eq. (2). The velocity $v_n(t)$ of the nth car is given by integrating the acceleration. If the velocity $v_n(t)$ is larger than the maximal velocity v_{max}, its velocity is limited to the maximal velocity. If the velocity $v_n(t)$ is less than zero, its velocity is limited to zero.

Fig. 1 shows the typical traffic patterns. Pattern (a) indicates the car trajectories for $p_0 = 0.15$, $v_0 = 0.3$, $x_c = 7$, $v_{max} = 1$ and $a = 0.05$ up to 500 time steps where the system size is $L = 300$. Pattern (b) indicates the car trajectories for $a = 0.2$ where the parameters p_0, v_0, x_c and v_{max} are the same as in pattern (a). The horizontal direction indicates the direction in which cars move ahead. The vertical direction indicates time.

A car is indicated by a dot. The trajectory of a car is represented by a curve. In the pattern (a), four start–stop waves are formed and propagate backwards. Within the start–stop waves, the cars are stopped with no spacing (the headway is one) and the car velocity $v(t)$ becomes zero. In the region outside of the start–stop waves, the cars move with the maximal velocity $v_{max}(=1)$ and the headway becomes larger than the safety distance x_c. In pattern (b), a single start–stop wave is formed and propagates backward. Within the start–stop waves, the cars are stopped with a finite spacing less than the safety distance and the car velocity becomes zero. The width of the start–stop wave is larger than that in pattern (a). The propagating velocity of the start–stop wave in pattern (b) is larger than that in pattern (a). In the region outside of the start–stop wave, the cars move with the maximal velocity, the headway becomes larger than the safety distance and the headway is less than that in pattern (a). With increasing magnitude of acceleration, the car density within the start–stop wave becomes low. The car density within the maximal velocity region becomes high with increasing magnitude of acceleration.

Fig. 2 shows the plot of the propagating velocity v_p of the sart–stop wave against the magnitude a of acceleration for $p_0 = 0.15$, $x_c = 7$ and $v_{max} = 1$. The data are plotted by full circles. The propagating velocity v_p increases linearly with magnitude of acceleration when the acceleration a is larger than 0.07. When the acceleration a is less than 0.07, the propagating velocity v_p decreases slowly with decreasing strength of acceleration. The solid line indicates the analytical result obtained from the mean-field theory in Section 3. The analytical result agrees well with the simulation result for not small values of acceleration.

Fig. 3 shows the plot of the mean traffic current Q against the car density p for safety distance $x_c = 13$, acceleration $a = 0.05$ and maximal velocity $v_{max} = 1$. The data are

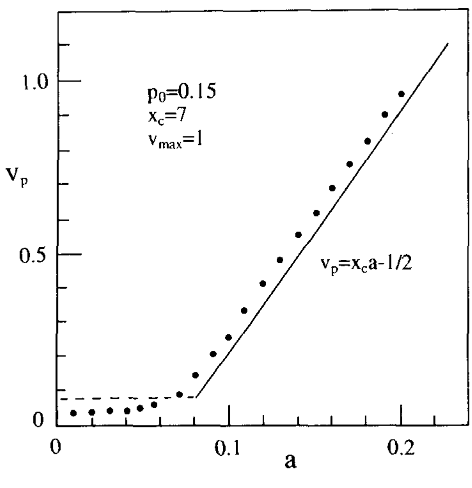

Fig. 2. Plot of the propagating velocity v_p of the start stop wave against acceleration a for $p_0 = 0.15$, $x_c = 7$, $v_{max} = 1$ and $v_{min} = 0$. The data are plotted by full circles. The solid line represents the analytical result obtained from mean-field theory.

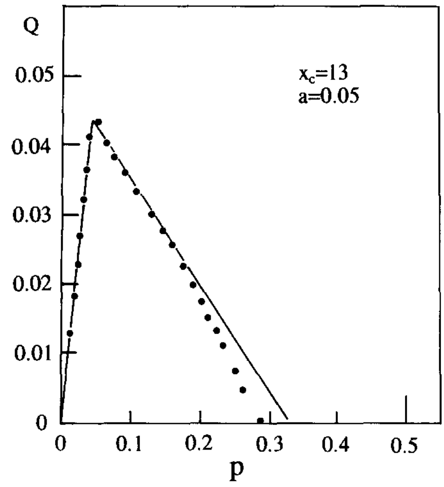

Fig. 3. Plot of the mean traffic current Q against car density p for safety distance $x_c = 13$, acceleration $a = 0.05$ and maximal velocity $v_{max} = 1$. The data are plotted by full circles. The solid curve indicates the analytical result obtained from the mean-field theory.

plotted by full circles. The traffic current Q is defined as

$$Q \equiv \frac{1}{L} \sum_{i=1}^{N} v_i , \tag{4}$$

where N is the total number of cars and L is the system size. The mean traffic current is obtained from averaging over 1000 time steps after a sufficiently large time. At lower density than the maximal current, cars are moving with the maximal velocity where the headway of each car is larger than the safety distance x_c. At higher density than the maximal current, cars are in congested traffic, and traffic jams appear as start–stop waves. Furthermore, at high density, all cars are stopped by traffic jams and the traffic current becomes zero at car density less than one. The solid curve indicates the analytical result obtained from the mean-field theory in Section 3. The analytical result agrees well with the simulation result.

In Fig. 4, we show the plot of the velocity $v(t)$ against the headway $\Delta x(t)$ to describe the behavior of a car in a congested traffic flow. The trajectory indicates the behavior of a car in the phase space of velocity and headway for safety distance $x_c = 15$, acceleration $a = 0.05$, maximal velocity $v_{max} = 1$ up to 2000 time steps, where the system size is $L = 600$ and the car density is $p_0 = 0.07$. The trajectory shows a periodic behavior. The left point on the lowest line represents the property within the traffic jam (or start–stop wave) where a car is within the state of minimal velocity (zero) and minimal headway. The right point on the highest line represents the property within the freely moving region where a car is moving with the maximal velocity (one) and the maximal headway larger than the safety distance. The right hand side of the trajectory represents the behavior of going out of the traffic jam. The left hand side of

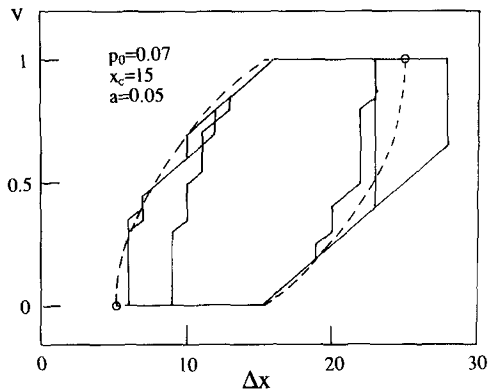

Fig. 4. Plot of the velocity $v(t)$ against the headway $\Delta x(t)$ for safety distance $x_c = 15$, acceleration $a = 0.05$ and maximal velocity $v_{max} = 1$ where the car density is $p_0 = 0.07$. The trajectory (solid curves) represents the behavior of a car in the phase space of velocity and headway. The trajectory indicated by the broken line shows the analytical result obtained from mean-field theory.

the trajectory represents the behavior of going into the traffic jam. The trajectory representing by the broken line shows the analytical result obtained from the mean-field theory in Section 3.

3. Mean-field theory

We present a simple mean-field theory for a traffic jam in congested traffic. In the congested traffic flow, a phase separation appears where there are two phases: of the traffic jam region, and the freely moving region. In the jam phase, the cars are stopped and their velocity is zero. The headway Δx within the jam phase is less than the safety distance x_c. In the freely moving phase, the cars are moving with the maximal velocity v_{max}. The headway Δx within the freely moving phase is larger than the safety distance x_c. We assume that the headway within the jam region has a constant value of x_{min} and the car density within the jam phase is homogeneous. Similarly, we assume that the headway within the freely moving region has a constant value of x_{max} and the car density within the freely moving phase is homogeneous.

We consider the process in which a car goes out of the traffic jam. Here we set the maximal velocity $v_{max} = 1$. The process is schematically shown in Fig. 5a. Car 2 is accelerated when its headway is larger than the safety distance x_c. The point a′ indicates the position at which the headway of car 2 becomes to be equal to the safety distance x_c. In time, car 2 reaches the maximal velocity $v_{max} = 1$. The point b′ indicates the position at which the velocity of car 2 reaches to the maximal velocity. The time t necessary to accelerate from zero velocity to the maximal velocity is given by

$$v = \int_0^t a\,\mathrm{d}t = at = 1, \qquad t = 1/a\,. \tag{5}$$

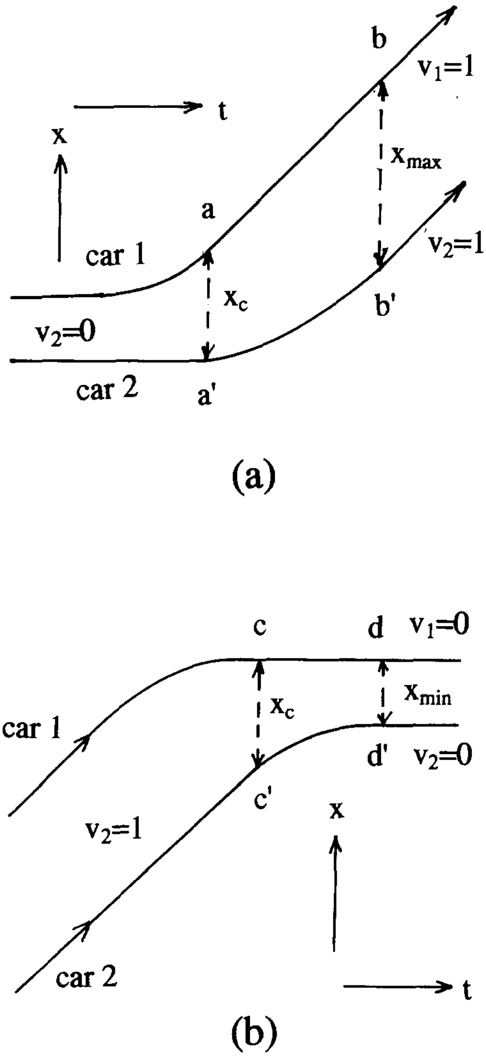

Fig. 5. The behavior of two cars in the mean-field theory. (a) The trajectories of the two cars, where car 2 goes out of the traffic jam. (b) The trajectories of the two cars where car 2 goes into the traffic jam.

The distance x covered by car 2 until it is accelerated to the maximal velocity is given by

$$x = \int_0^t v \, \mathrm{d}t = at^2/2, \quad x = 1/2a. \tag{6}$$

The headway $x_{\max}$ between points b and b′ is given by

$$x_{\max} = x_c + 1/2a. \tag{7}$$

Eq. (7) gives the maximal headway within the freely moving phase.

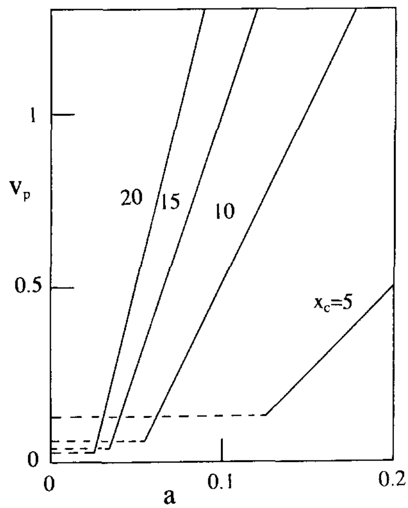

Fig. 6. Plot of the propagating velocity v_p against acceleration a for safety distances $x_c = 5, 10, 15$ and 20 which is obtained from the mean-field theory.

Similarly, we calculate the minimal headway x_{min} within the jam phase. We consider the process in which a car goes into the traffic jam. The process is shown in Fig. 5b. Car 2 is decelerated when its headway is less than the safety distance x_c. Point c′ indicates the position at which the headway of car 2 becomes to be equal to the safety distance x_c. In time, car 2 reaches the minimal velocity $v_{min} = 0$. Point d′ indicates the position at which the velocity of car 2 reaches zero. The time t necessary to decelerate from velocity one to velocity zero is given by

$$t = 1/a . \tag{8}$$

The distance x covered by car 2 until it is decelerated to zero velocity is given by

$$x = 1/2a . \tag{9}$$

The headway x_{min} between the points d and d′ is given by

$$x_{min} = x_c - 1/2a . \tag{10}$$

Eq. (10) gives the minimal headway within the jam phase. The minimal headway must be larger than one since car 2 cannot overtake car 1. Therefore, the above analysis is satisfied subject to the condition

$$a \geqslant 1/2(x_c - 1) . \tag{11}$$

The propagating velocity v_p of the traffic jam (or start–stop wave) is determined from the slope from the point c to the point d′. The propagating velocity is given by

$$v_p = x_c a - 1/2 . \tag{12}$$

Fig. 6 shows the plot of the propagating velocity v_p against the acceleration a for the safety distances $x_c = 5, 10, 15, 20$. The solid lines represent the result of Eq. (12). The broken lines represent $a = 1/2(x_c - 1)$. Above the broken line, condition (11) is satisfied and Eq. (12) holds. The solid line in Fig. 2 shows Eq. (12). The analytical result of Eq. (12) is well consistent with the simulation result.

We derive the fundamental diagram (a diagram of traffic current versus car density). In the congested traffic flow, there are two phases with different densities. In the freely moving phase, the density ρ_f is given by the inverse of the maximal headway (7)

$$\rho_f = 1/(x_c + 1/2a). \tag{13}$$

In the jam phase, the density ρ_j is given by the inverse of the minimal headway (10)

$$\rho_j = 1/(x_c - 1/2a). \tag{14}$$

The total density p is given by

$$x\rho_f + (1 - x)\rho_j = p, \tag{15}$$

where x is the fraction of the freely moving phase. The traffic current Q is given by

$$Q = x\rho_f v_{\max} = \rho_f(\rho_j - p)/(\rho_j - \rho_f), \tag{16}$$

where $v_{\max} = 1$. The car density at the maximal current is determined by the density ρ_f since the car velocity is maximal when the headway is larger than $x_{\max}$. For lower densities than ρ_f, the traffic current Q is given by

$$Q = pv_{\max} = p, \quad p \leqslant \rho_f. \tag{17}$$

For $p > \rho_f$, the traffic flow becomes congested and the traffic jam appears. Eq. (16) holds for $p > \rho_f$. Fig. 7 shows the fundamental diagram for $x_c = 11$, 13 and 15 where the acceleration is $a = 0.05$. The solid line in Fig. 3 shows Eqs. (16) and (17). The analytical result is well consistent with the simulation result.

We derive the analytical expression of the trajectory in the phase space of velocity and headway representing the behavior of a car in a congested traffic flow. In the

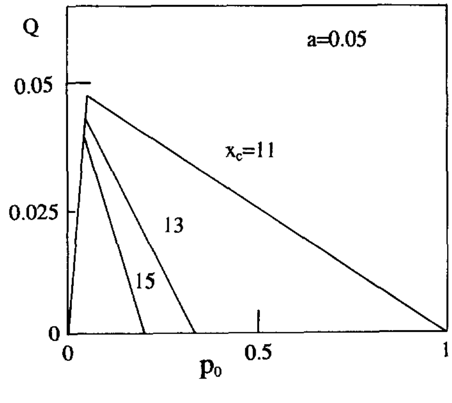

Fig. 7. The fundamental diagram (the traffic current Q versus the car density p) obtained from the mean-field theory for safety distances $x_c = 11$, 13 and 15 where the acceleration is $a = 0.05$.

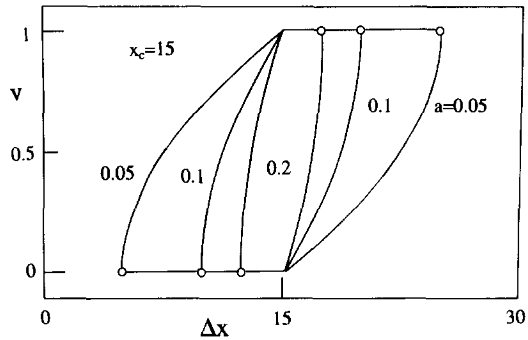

Fig. 8. The trajectories of a car in the phase space of headway Δx and velocity v obtained from the mean-field theory for acceleration $a = 0.05, 0.1$ and 0.2 where the safety distance $x_c = 15$.

process where a car goes out the traffic jam (see Fig. 5a), the headway of car 2 is described by

$$\Delta x = x_c + t - at^2/2 = x_c + v/a - v^2/2a\,. \tag{18}$$

Similarly, when a car goes into the traffic jam (see Fig. 5b), the headway of car 2 is given by

$$\Delta x = x_c - (1 - v)/a + (1 - v)^2/2a\,. \tag{19}$$

Fig. 8 shows the trajectories of a car in the phase space of headway and velocity for $a = 0.05, 0.1$ and 0.2 where $x_c = 15$. The lowest-left point represents the state (velocity $v = 0$ and headway $\Delta x = x_c - 1/2a$) in the traffic jam. The top-right point represents the state (velocity $v = 1$ and the headway $\Delta x = x_c + 1/2a$) in the freely moving phase. With increasing acceleration, the trajectory shrinks. This means that the minimal headway increases with acceleration and the maximal headway decreases with increasing acceleration. The trajectory representing by the broken line in Fig. 4 shows the analytical result (18) and (19). The analytical result is nearly consistent with the simulation result.

4. Summary

We presented an extended cellular automaton model to simulate traffic flow. In this model, the motion of individual car was determined by the dynamical equation of motion. We investigated the properties of traffic jams in congested traffic flow by computer simulation. Also, we presented a simple mean-field theory for the congested traffic flow. We compared the analytical result with the simulation result. We found that the mean-field result agrees with the simulation result.

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