Multi-Mode Project Scheduling with Limited Resource and Budget Constraints

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***Abstract*—In this paper, we consider multi-mode based approach to modeling project scheduling problem where resources are limited and project scheduling cost is fixed. These multi-modes are depending on project duration time, time lags to other activities and resource requirements. A multi-mode resource constraint project scheduling problem can be formally defined by a number of activities-nodes with several modes that should be scheduled to minimize the project duration, subject to generalized precedence relations with a limited resource. The most common exact approaches for solving this problems are based on branch-and-bound algorithms, integer linear programming and Boolean satisfiability (SAT). We first consider, Integer Linear Programming for modeling and solving the project scheduling with limited resources, budget constraints and precedence relation. We proposed a mathematical model for finding its optimal solution. We jointly consider to make sure the project duration and project cost is minimized, and incorporate multiple types of computational resources.**

I. Introduction

A multi-mode resource-constrained project scheduling problem is an operational research problem that which can be defined as an optimal allocation of resources, operating in different modes, to a certain project activities having predefined precedence relationship. Each activity can be performed with several execution modes, each having different resource requirements- renewable and nonrenewable resources and a related estimated duration. This problem is further constraint by limited resources and budget.Many r

In multi-mode resource constrained project scheduling problems (MRCPSP), there are some activities which maintain a precedence relation and each activity is executed in one of several modes, $m \in M_i$, representing a combination of limited resources and durations. Assume that, building a software in a software project farm where are some activities and fixed manpower like that senior programmer and junior programmer to complete a project fully. Assume that, the activities are: System requirement specification, Analysis, Design, Coding, Error checking, Testing, Implementation etc. Here, we can see that, second activity is preceded by customer requirement, after customer requirements analysis project manager informs programmers to make a design for the project, and completing the design, code implementation activity can be ready to start and error checking activity can be performed parallel with code implementation. Here, is a problem can be come out if the programmers or resources are not available to perform this activity then the activity will be waited for its required resources, and when the resources are available then it assigns to start. And another criteria is to be mentioned that, each activity requires a fixed budget. Assume that, the activities maintain the showing Figure-1 precedence relation and a demo example is shown in Table-I where the required resources and fixed cost is given for each and every activity with several modes.

Fig. 1: Activity precedence relation

Here, we can see that every task has a fixed budget constraints. Before project scheduling a fixed budget cost(threshold value) will be mentioned previous so that, sum of scheduling task's cost won't be cross the threshold value. So, a project manager of a software farm assigns a budget constraint for a project. Now, the goal is minimize the project scheduling time under considering activity precedence relation, limited resources and a fixed budget.

And there are many other real life problems like, Airplane flight scheduling, A cutting-stock problem, Class routine operation, Batch scheduling in process industries etc can be known as resource constraint project scheduling problem.

The research problem is to find out the optimal scheduling of the precedence activity in such a way that the project duration is minimize and it can be completed within the estimated budget. A subsidiary part of the problem is to select the mode of the individual activities in which they will be executed.

It is a challenging problem, and several techniques have been proposed to solve this problem. Authors in [1] investigated that, when flexible constraints and uncertain processing times are to be jointly considered, the use of possibility decision theory leads to the computation of robust schedules. Authors in [3] proposed that, In RCPSP and MRCPSP, when once an activity starts, it will be executed until its completion. Authors in [4] investigated that, project scheduling problems are distinctly complex and would benefit from evolutionary techniques for finding optimal solutions or near optimal solutions.

Authors in [5] considered project duration as their primary objective with renewable resources but there are limitations, they did not work with cost or any kind of non- renewable resources. Their solution methodology based on Branch and

TABLE I: Operating modes, duration, resources type, and cost of different activities

	Mode 1 (M1)				Mode 2 (M2)				Mode 3 (M3)			
Activity	Duration	R1	R2	Cost	Duration	R1	R2	Cost	Duration	R1	R2	Cost
A1	4	2	3	30	3	2	4	35	2	3	5	40
A2	6	3	5	28	4	3	5	30	3	4	5	35
A3	4	2	4	25	3	3	5	30	2	3	6	36
A4	4	4	3	21	3	4	5	27	3	4	5	27
A5	5	2	6	19	4	3	6	25	4	4	6	36
A6	3	2	3	18	3	2	3	18	2	3	3	24
A7	6	2	3	31	5	2	4	35	4	3	4	40

Bound Algorithm. But, when more activities preferred to execute in the schedule on that time this algorithm can not give efficient solutions.

In this paper we will focus on finding the optimal solution with minimum makespan at lowest possible cost. In addition, it is attempting to determine a contingency plan when a project faces a customers request for a new delivery time and/or a change in budget constraints. The key contributions of this work are summarized as follows:

- We formulate the project scheduling problem as a integer linear programming(ILP) problem defined as a set of precedence activities within Activity-on Node(AON) graph representation, which is proved to be an NP-hard problem. And, we Proposed a mathematical model named MRCP(Multi-mode resource constraints project) scheduling system and measured its performance.
- We have also implemented MRCPSP with using a heuristic algorithm (Brute Force) for comparing with our proposed model for numerical evaluation.
- The results of performance evaluation studies, experimented on our MRCP scheduling system, comparing with two heuristic algorithms named Branch and Bound, Brute Force and we also analysis the performance evaluation. Our proposed MRCP scheduling system shows better performance than two algorithms.

The rest of this paper is organized as follows. The related works and motivation points are described in Section 2. We define the problem definition, formulate the problem and the details of our proposed MRCP scheduling system is presented in Section 3. The simulation results are presented in Section 4 and finally, we conclude the paper in Section 5 along with future research directions.

II. Related Work

Project scheduling is a tool that, manages the works and resources associated with delivering a project on time. Project scheduling is important to organize, keep track of the finished and in-progress tasks and manage the quality of work which is delivered. However, many problems arise during project scheduling. Minimizing project duration is the primary objective. Project cost is also a critical matter, but there will always be a trade off between project time and cost so scheduling activities can be challenging due to precedence activities, resources, and execution modes. Schedule reduction is heavily dependent on the availability of resources [1].

Multi-mode multi resource constrained project scheduling problems (MMRCPSP) are more common in the real world. Each activity can be executed in one of a set of modes. Once the activity starts the selected mode cannot be changed. The objective is to find a minimal makespan schedule that, meets the constraints imposed by the precedence relations and by the limited resources available [2]. RCPSP and MRCPSP assume that, once an activity starts, it will be executed until its completion [3] .

Evolutionary algorithms have been developed based on a form of meta-heuristic techniques especially by genetic algorithms. Evolutionary algorithms have shown to be well suited for complex problems. Project scheduling problems are distinctly complex and would benefit from evolutionary techniques for finding optimal solutions or near optimal solutions [4] . The multi-mode resource constrained project scheduling problem has been scheduled to minimize the project duration subject to generalized precedence relation. Because this problem is NP-hard even for simple graphs, that is, it is strongly believed that they cannot be solved to optimality within polynomially bounded computation time. A common practice is to employ heuristic algorithms without performance guarantees [7], [8] which may unknowingly suffer from poor performance as compared to the optimal solution. Only a very limited amount of existing work followed a rigorous theoretical framework from approximation algorithms [9] and competitive analysis [10]. A related problem that has emerged recently is Online Placement of Multi Component Applications in Edge Computing Environments [6]. Motivated by an algorithms for solving the online application placement with provable competitive ratios and also provides an optimal solution to the placement of a linear application graph. Our important aspect to note is that, most existing work, including [5], Fuzzy Multi-mode resource constrained project scheduling problem is one of operational research problems which can be defined as a set of precedence activities and each activity can be performed with several execution modes and each mode of an activity requires a different renewable resources. They considered Branch-and-Bound procedure to solve [5] fuzzy multi-mode scheduling problem can be splitted into : a) Branching strategy steps; b) temporal analysis and possible pruning rules. They only considered working with renewable resources.

In this work, we proposed a mathematical (MRCP) model which works with both renewable and non-renewable resources. Our proposed MRCP model can gives the

efficient results and works with hundreds activities. On the other hand, Branch and Bound can not give the efficient results more than 10 activities.

III. Proposed MRCP System

A. Problem Definition

The classical RCPSP assumes that each activity can only be executed in a single way which is determined by a fixed duration and fixed resource requests. But there are many practical situations in which the duration of an activity can be decreased at the expense of providing additional resources. In such a situation, an activity may be executed in one of several modes. When there are several different modes that may be selected for an activity, the problem is known as a RCPSP with multiple modes. In this problem, activities of the project may be executed in more than one operating modes and renewable resource constraints exist. Each activity operation mode has a different duration and requires different amount of renewable and non-renewable resources. Each mode corresponds to a different time resource trade-off option for the activity under consideration. A feasible schedule specifies the implementation mode, as well as the start and finish times for each activity.

Formally, the MRCP scheduling problem can be described as follows: A project consists of n activities represented using an activity-on-node (AON) representation, where nodes represent the activities and arcs denote the precedence relationships and its represented by index $i = 1, \cdots, n$. Each activity can be executed in one of m modes, where m goes from 1 to M_i , where M_i is the number of possible modes for activity i. We assume that, all project activities are ready at the beginning of the project and no preemption is permitted during their executions. Once a mode has been selected for an activity, it must be finished without switching the mode. The duration of activity i executed in mode m is d_{im}. The non-preemptive assumption implies that once an activity i has been assigned mode m, it must be executed for d_{im} consecutive time units without interruption. Activity i cannot be started before completion of all of its predecessors. We assume that, there are R_{kt} amount of renewable resources of type k available in time period t. Index k goes from $1, \cdots, K$, where K is the total number of resource types. Activity i executed in mode m requires r_{imk} resource units per period for resource type k. We assume c_{im} the cost of activity i being performed in mode m. And the c_{th} is total project budget constraint previously given so that after scheduling the cost of the project can not crossed the limit of budget constraint. The activity durations, resource availabilities and resource requirements and cost of the per activity, budget constraint are non-negative integers. Further more, assume that E_i is the earliest start time and L_i the latest start time of activity i; E_i is calculated using a forward pass of the PERT chart using the critical path method assigning the fastest mode to each activity. L_i for the rest of the activities are calculated using a backward pass of the PERT chart assuming the slowest resource mode is assigned to activities. Also let P_i be the set of all preceding activities of activity i.

TABLE II: Notation and parameter

Notation	Parameter
A	A set of project activities
n	Number of activities
M_i	Number of modes that task i can be performed in
m	each mode(from 1 to M_i)
K	Number of resource type
r_{ikm}	Amount of resource type k required activity i being performed in mode m
R_k	The available quantity of resource type k
d_{im}	The activity duration required to perform activity i in mode m
c_{im}	The cost of activity i being performed in mode m
S_i	Starting time of task i
F_i	Finishing time of task i
R	A set of renewable resources
P_i	Precedence Constraints in set A
E_i	Earliest start time of activity i
L_i	Latest start time of activity i
c_{th}	Threshold value for budget constraint

B. MRCP Objective Function

Let, x_{imt} is a binary variable that indicates which method m is planned for activity i
Now, x_{imt} stands for 1, if activity i starts with m -execution mode otherwise, x_{imt} stands for 0 for all other mode.
Assume that, S_i and F_i stand for the starting time and finishing time of task i
Now, the finish time of any activity i can be written as, $F_i = S_i + d_{im}$

So, F_n stands for total project duration of any project scheduling problem .

Now, the problem can be formulated as follows:

$$Minimize \quad max\ F_n \tag{1}$$

Subject to,

$$\sum_{m=1}^{M_i} \sum_{t=E_i}^{L_i} x_{imt} = 1, \quad (\forall i) \in A \tag{2}$$

$$\sum_{m=1}^{M_j} \sum_{t=E_j}^{L_j} t \times x_{jmt} \leq \sum_{m=1}^{M_i} \sum_{t=E_i}^{L_i} (t - d_{im}) \times x_{imt}, j \in P_i \tag{3}$$

(a) Activities Vs Execution Time

(b) Resources vs Execution Time

Fig. 2: Impacts of increasing number of activities, resources on that performances of studied systems.

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i}\sum_{p=t}^{t+d_{im}-1} r_{ikm} \times x_{jmt} \leq R_{kt}, \quad (\forall k) \in K \tag{4}$$

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i}\sum_{t=E_i}^{L_i} c_{im}x_{imt} \leq c_{th} \tag{5}$$

In this mathematical model, the objective function (1) is to minimize the total project makespan. The constraints given in Constraints (2 to 5) are used to formulate the general feasibility of the problem. The constraint (2) ensures that each activity i is performed in one of its modes and is finished within its time window $[E_i \cdots L_i]$. The constraint (3) ensures that precedence of relations are maintained, i.e., no activity can be started until all its predecessors activities are completed their task. Here P_i is the set of all immediate predecessor activities of activity i and d_{im} is the duration of activity i operating in mode m. Renewable resource restrictions are enforced by constraint (4). The constraint (5) ensures that the total budget of the scheduled task should be less than or equal to the threshold value which can be given by the project manager.

In solving this problem heuristically, two decisions are involved (1) which activity to be assigned if two or more activities are precedence and resource feasible and (2) which of the several feasible modes of execution to select for the assigned activity. Two priority rules are therefore needed to solve the problem - one for activity selection and one for execution-mode selection. Finally although other constrained could be considered the objective function considered in this work is to minimize the project completion time.

The execution time(s) required for solving the above optimization problem for various kinds of activities, modes, resources are given in the Figure-2

The values in the y axis depict that the problem is solvable in polynomial time(second) what follows next, We present the numerical evaluation of the proposed project scheduling problem and compare the results with The-state-of-the-art works

IV. Numerical Evaluation

In this section, we compare our proposed MRCP model against two heuristic algorithms. The first approach is Brute Force algorithm to greedily minimizes the project makespan considering budget constraints. The second approach is the Branch and Bound algorithm proposed in [11]. Both the Brute Force and Branch and Bound algorithms required an optimization problem to be solved as resource constraints project scheduling problem for the multi-mode approach . This optimization problem can be expressed as a integer linear program (ILP). ILPs are generally not solvable in polynomial time. It means that, we can not provide a time window which can gives an approximate solution. We solve the ILP scheduling problem directly using CPLEX. This gives an exact solution for our proposed MRCP scheduling system, rather than the Brute Force and Branch and Bound algorithms may perform better than they would in reality, and we are conservative in showing the effectiveness of the proposed mathematical model.

A. Environmental Setup

We evaluate the proposed multi-mode project scheduling system in **CPLEX**, a constrained programming solver. We used CP optimization tool named **IBM ILOG CPLEX Optimization Studio** and **12.6.1** version. Unlike many meta-heuristic and heuristic methods in the literature, the CPLEX model is fast and provides near-optimum solutions to projects with hundreds (1,000 to 2000) of activities.

In our experiments, the number of activity nodes for each Scheduling is randomly chosen from the interval [3, 7], and the number of modes chosen from the interval [3, 7]. We use a sequential experiments to understand the relationship among this three (MRCP, Brute Force, Branch and Bound). The cost threshold value of an every experiments are linearly increases with the size of the activities and modes. In Figure-3, the number of modes, number of resource types, the limit of the resources are same for all (3a, 3b, 3c) and a fixed budget constraints is assumed for all. We just sequentially change the number of activity to see the differences. Now, in Figure-4, the number of activities, number of resources type, the limit of the resources are same for all (4a, 4b, 4c) and a fixed budget constraint is assumed for all. We just sequentially change the number of modes to see the result comparing with these three. The optimization is calculated following equations [1-5]. Each simulation was run for 0.1s and the graph data points are plotted for the average of the results from 20 simulation runs.

B. Performance Metrics

- Project completion time: The project completion of time measured by- after maintaining mode assignment of activities, precedence relationship and also check the resources. If resources are available then it will assigns to start otherwise, it will be waited until resources will be available. When all the activities of a project fulfill all of this criteria and found a solution time is called the project completion time.
- Cost of the project: Cost of the project is measured the total cost of the project after scheduling the project and it will be less than or equal to cost threshold value.
- Cost threshold value: The cost threshold value is defined by the fixed budget constraints of a scheduling problem which is provided by the project manager.
- Percentage of resources saving: The percentage of resources saving are measured as the ratio of total number

(a) Activity Vs Project Completion Time (b) Activity vs Cost of the Project (c) Activity vs Saving Resources

Fig. 3: Impacts of increasing number of activities on that performances of studied systems.

(a) Modes Vs Project completion time (b) Modes Vs Cost of the Project (c) Modes Vs saving resources

Fig. 4: Impacts of increasing number of operational modes on that performances of studied systems

of saving resources after scheduling and the total number of resources.

C. Results and Discussion

1) Impacts of increasing number of activities: We first vary the increasing of the number of activities to study the performance of our proposed MRCP scheduling system and two different heuristic algorithm(Brute force and BB). The graphs in Fig.3a depict that the project completion time of a project is efficient for MRCP scheduling system than others. However, the completion time is same for both (MRCP and Brute Force) at a certain point when the number of activity is lower. The total cost of a project for MRCP model, Brute Force and BB are relatively close, as shown in Fig.3b. The Branch and bound gives lowest cost but highest project completion time. But our target is to be scheduled a problem with lowest project duration simultaneously considering with lowest possible cost. A point to be noticed that, when the requirement of resources of any activity is something like that, when a single activity can run only a particular time means no two activities can run in parallel in a same time considering with available resources, our MRCP model and Brute Force gives same results (shown in Figure-3). There is no noticeable relation in Fig. 3c, which represents the percentage of resources saving by varying the increasing of number of activities, because we just considering our focus is to be minimized our total project duration and total project cost not to be focused on lowest amount of resources are taken first or resources saving are very much important or anything else.

2) Impacts of increasing number of operational modes: In this section, we have evaluated the numerical evaluation by varying the increasing of number of modes. Here, we can see something different than Figure-3. The graphs of Fig. 4a depicts that our MRCP scheduling system outperforms than the Brute Force and Branch and Bound since the former project completion times are significantly decrease for the number of increasing modes. The total cost of the project are increased for simultaneously increasing different modes, as shown in Fig. 4b. Therefore, proposed MRCP performs efficient with focusing on both(minimum project duration and cost). Again Branch and Bound gives lowest cost but highest project completion time. in Fig. 4c we can see that, the percentage of resources saving is same for all experimental data for both of our proposed MRCP and brute force but BB is different from them. So, we have found that the proposed MRCP scheduling system performs well than the other state-of-the art works for its better decision making capability.

3) Impacts of cost threshold value (c_{th}): We have kept the cost threshold value which is greater than the optimal cost of the project. Let, take an example from Figure-3 where we can see that, in activity number 3, in which project duration is 10 and the total cost of the project is 280. It means that, if we consider cost threshold value is greater than 280, but it can not minimize total project duration. Because of, a strong precedence relationship of activities must be maintained.

Again, if we have kept the cost threshold value is so much lowest that, it's difficult for the project to schedule the problem with focusing on the functionality of cost threshold value after maintaining the activity precedence. In that situation, our proposed MRCP model provides no solution.

V. Conclusion

In this paper, we present a research to solving multi-mode resources constraint scheduling problem. To the best of our knowledge, our proposed MRCP model gives the best solution and better performance than all of existing solution. In our MRCP scheduling system, each activity selects an individual mode from severals modes, maintaining a strong precedence relation of activities, works with renewable resources and scheduling cost must be less or equal than the cost (Cost threshold value) which is provided by the project manager.

Although our proposed MRCP model achieves better lowest project completion time, lowest cost of the project for any multi-mode resources constraints project scheduling problem. The focus of future work in this area should then be on introducing efficient methods to handle practical size problems, time-cost trade-off problem that work best with the proposed MRCP model.

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