ASSIGNMENT-OPTIMIZATION

Contents

1 Problem

Let $f(x)$ is a cubic polynomial which has a local maxima at x=-1.If $f(2) = 18$, f(1)= -1 and $f'(x) = -1$ has local minimum at x=0,then
(a) The distance between (-1,2) and (a,f(a)) where x=a is a point of local minima is $2\sqrt{5}$
(b) $f(x)$ is increasing for $x \in [1, 2\sqrt{5}]$
(c) $f(x)$ has local minima at x=1
(d) The value of $f(0)$ is 5

2 Solution

1.Let the cubic polynomial equation be

$$f(x) = ax^3 + bx^2 + cx + d \tag{1}$$

2.Given that $f(2) = 18$

$$8a + 4b + 2c + d = 18 \tag{2}$$

3.Given that $f(1) = -1$

$$a + b + c + d = -1 \tag{3}$$
$$\frac{df(x)}{dx} = 3ax^2 + 2bx + c \tag{4}$$
$$f'(-1) = 0 \tag{5}$$
$$3a - 2b + c = 0 \tag{6}$$
$$\frac{df'(x)}{dx} = 6ax + 2b \tag{7}$$
$$f''(0) = 0 \tag{8}$$
$$b = 0 \tag{9}$$

4.Solving Eqs(2)(3)and (6) by substituting b=0 we get

$$8a + 2c + d = 18 \tag{10}$$
$$a + c + d = -1 \tag{11}$$
$$3a + c = 0 \tag{12}$$

5.Solving Eqs(10)(11)(12)

$$\begin{pmatrix} 8 & 2 & 1 & 18 \\ 1 & 1 & 1 & -1 \\ 3 & 1 & 0 & 0 \end{pmatrix}$$

$$R_1 = \frac{R_1}{8}$$

$$\begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{8} & \frac{9}{4} \\ 1 & 1 & 1 & -1 \\ 3 & 1 & 0 & 0 \end{pmatrix}$$

$$R_2 = R_2 - R_1$$

$$\begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{8} & \frac{9}{4} \\ 0 & \frac{3}{4} & \frac{7}{8} & -\frac{13}{4} \\ 3 & 1 & 0 & 0 \end{pmatrix}$$

$$R_3 = R_3 - 3R_1$$

$$\begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{8} & \frac{9}{4} \\ 0 & \frac{3}{4} & \frac{7}{8} & -\frac{13}{4} \\ 0 & \frac{1}{4} & -\frac{3}{8} & -\frac{27}{4} \end{pmatrix}$$

$$R_2 = \frac{4}{3} R_2$$

$$\begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{8} & \frac{9}{4} \\ 0 & 1 & \frac{7}{6} & -\frac{13}{3} \\ 0 & \frac{1}{4} & -\frac{3}{8} & -\frac{27}{4} \end{pmatrix}$$

$$R_1 = R_1 - \frac{R_2}{4}$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{6} & \frac{10}{3} \\ 0 & 1 & \frac{7}{6} & -\frac{13}{3} \\ 0 & \frac{1}{4} & -\frac{3}{8} & -\frac{27}{4} \end{pmatrix}$$

$$R_3 = R_3 - \frac{R_2}{4}$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{6} & \frac{10}{3} \\ 0 & 1 & \frac{7}{6} & -\frac{13}{3} \\ 0 & 0 & -\frac{2}{3} & -\frac{17}{3} \end{pmatrix}$$

$$R_3 = -\frac{3}{2} R_3$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{6} & \frac{10}{3} \\ 0 & 1 & \frac{7}{6} & -\frac{13}{3} \\ 0 & 0 & 1 & \frac{17}{2} \end{pmatrix}$$

$$R_1 = R_1 + \frac{R_3}{6}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{19}{4} \\ 0 & 1 & \frac{7}{6} & -\frac{13}{3} \\ 0 & 0 & 1 & \frac{17}{2} \end{pmatrix}$$

$$R_2 = R_2 - \frac{7}{6} R_3 \begin{pmatrix} 1 & 0 & 0 & \frac{19}{4} \\ 0 & 1 & 0 & -\frac{57}{4} \\ 0 & 0 & 1 & \frac{17}{2} \end{pmatrix}$$

5. From which we can get a = $\frac{19}{4}$, b=0 , c = $\frac{57}{4}$,

d = $\frac{17}{2}$
eq(1) can be modified as

$$19x^3 - 57x + 34 = 0 \tag{13}$$

For Maxima :
Using gradient ascent method,

$$x_n = x_{n-1} + \mu \frac{df(x)}{dx} \tag{14}$$

$$\frac{df(x)}{dx} = 3ax^2 + 2bx + c \tag{15}$$

substituting eq 15 in 14
$x_n = x_{n-1} + \mu(3ax^2 + 2bx + c)$
taking
$x_0 = -1, \mu = 0.001, precision = 0.00000001$ values obtained using python are :

$$\boxed{\text{Maxima} = 72.0} \tag{16}$$

$$\boxed{\text{Maxima Point} = -1.0} \tag{17}$$

For Minima :
Using gradient decent method,

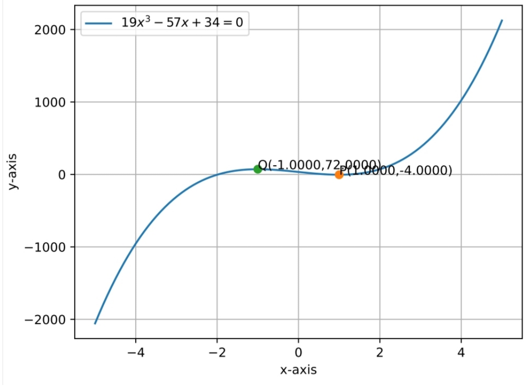
$$x_n = x_{n-1} - \mu \frac{df(x)}{dx} \tag{18}$$

$$\tag{19}$$

substituting eq 15 in 14
$x_n = x_{n-1} - \mu(3ax^2 + 2bx + c)$
taking
$x_0 = -1, \mu = 0.001, precision = 0.00000001$ values obtained using python are :

$$\boxed{\text{Maxima} = -4.0} \tag{20}$$

$$\boxed{\text{Maxima Point} = 1.0} \tag{21}$$



The python code provided in the below source code link.

https://github.com/sivagayathri
/FWC/blob/main/opt/opt-2.py