FREQUENCY MODULATION

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CONTENTS

1. INTRODUCTION

Frequency modulation (FM) is a widely used technique in modern communication systems. It is a form of modulation that encodes information by varying the frequency of a carrier signal in proportion to the message signal
One important parameter in FM is the bandwidth, which determines the range of frequencies required to accurately represent the modulated signal. The Carson's rule provides a simple approximation for the bandwidth of an FM signal.In this the bandwidth of an FM signal can be calculated by analyzing its frequency spectrum using techniques such as Fourier analysis.

2. GENERATING FM:

A. Bandwidth calculation method

Bandwidth refers to the range of frequencies over which a signal or system operates. There are several methods to calculate the bandwidth of a signal, including the 3dB attenuation method and the spectral density method.

1) The 3dB attenuation method involves finding the frequency range where the signal's power or amplitude is reduced by half (-3dB) from its maximum value.However, it does not take into account the full frequency spectrum of the signal or system, and may not accurately reflect the actual bandwidth of the signal.

2) The spectral density method calculates the bandwidth of a signal or system by analyzing its frequency content. Specifically, it calculates the power spectral density (PSD) of the signal or system, which is a measure of how the power of the signal is distributed across different frequencies. The bandwidth is then defined as the frequency range over which 99%of the total power is contained.
   This method provides a more accurate and comprehensive measure of bandwidth as it takes into account the entire frequency spectrum of the signal or system.

B. Steps for calculating the bandwidth of the signal

1) Loading the Audio File: Load the WAV file containing the audio input signal. The audio input signal is a time-varying signal, and we need to convert it into the frequency domain to analyze its bandwidth.

2) Computing the Fourier Transform:
   To plot the spectrum of the message signal, we need to compute the Fourier Transform of the message signal, which will give us the frequency domain representation of the signal.

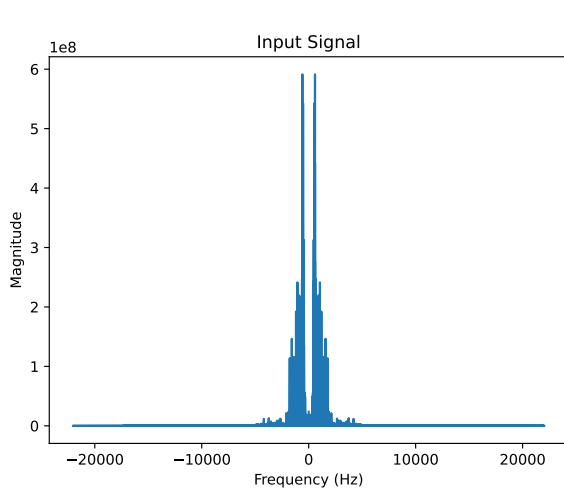$$M_k = \sum_{n=0}^{N-1} m(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \quad (2.1)$$

Fig. 1: spectrum analysis of input signal

Fig. 2: FM Transmitter Circuit

In code for computing the FFT of m(n)

$$M_k = \texttt{fft}(x(n), N) \tag{2.2}$$

The spectrum of input audio signal is plotted in Fig. 1

3) Calculating the frequency Range: Calculates the frequencies for each sample in the audio signal. The frequency range is used to plot the magnitude spectrum of the audio signal.

4) Calculating the Power Spectral Density:

$$PSD(f) = |M(f)|^2$$

Where PSD(f) is the power spectral density at frequency f, M(f) is the Fourier Transform of the input signal

5) Finding the Frequency Range with Significant Power: The frequency range with significant power in the PSD of the FM signal. This can be done by creating a mask that identifies frequencies where the PSD is greater than a certain threshold. Then the mask can be defined as:

$$mask(f) = \begin{cases} True & \text{if } fm\_psd(f) > 0.1 * \max(fm\_psd) \\ False & \text{if } fm\_psd(f) \le 0.1 * \max(fm\_psd) \end{cases}$$

python code :

```
# Find frequency range with significant power of FM
    signal
```
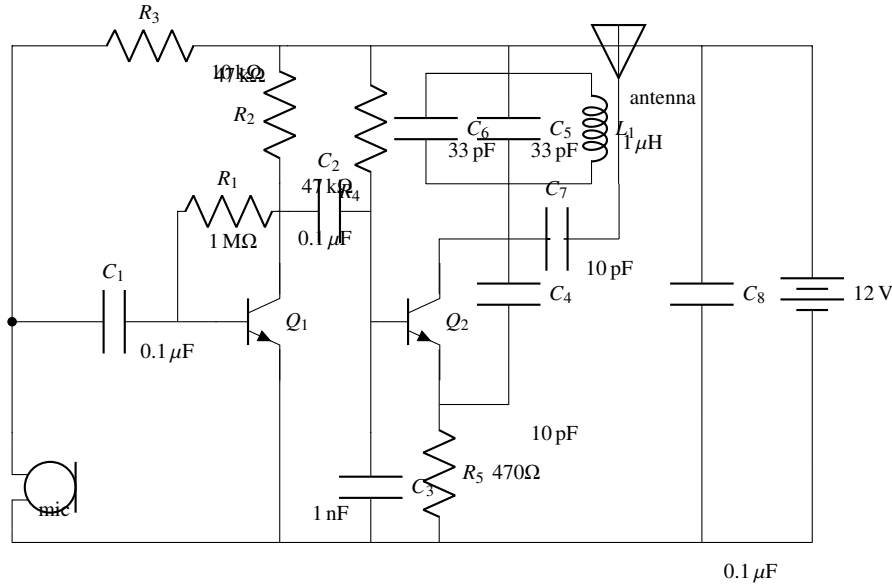
```
fm_mask = fm_psd > 0.1*np.max(fm_psd)
fm_freq_range = fm_freqs[fm_mask]
fm_bandwidth = max(fm_freq_range) − min(
    fm_freq_range)
print('FM Bandwidth:', fm_bandwidth, 'Hz')
```

Parameter	Value	Description
T	0.1	Threshold
F_s	44100 Hz	Sampling rate
t	22 μ	Sampling time

6) Calculating the Bandwidth: The bandwidth can be calculated mathematically using the following formula:

$$B = mask_{maxi} - mask_{min}$$

where $mask_{maxi}$ and $mask_{min}$ are the maximum amd minimum frequency bounds of the range respectively.
Analyzes the audio signal and plots its magnitude spectrum. The bandwidth of the audio signal is calculated using power spectral density. Obtain the bandwidth of the audio as 2 khz using below code

```
/codes/input.py
```

Component	Quantity	Value
Transistor	2	2N3904
Inductor	1	1 μH
Resistor	1	1M
	1	470Ω
	2	47kΩ
	1	10kΩ
Capacitor	1	1nF
	3	0.1μF
	2	10pF
	2	33pF
Condenser mic	1	
Antenna	1	

TABLE I: List of components

Fig. 4: spectrum analysis of fm signal

modulating signal.

We can use rectangular rule method to approximate the integral as:

$$\int_0^t m(\tau)d\tau \approx \sum_{i=0}^{N-1} m\left(i\Delta t + \frac{\Delta t}{2}\right)\Delta t \qquad (3.3)$$

where N is the number of intervals, $\Delta t = \frac{t}{N}$ is the time step, and $m\left(i\Delta t + \frac{\Delta t}{2}\right)$ is the value of the signal at the midpoint of the i-th interval.

Substituting this approximation into the original equation, we have:

$$s(t) = A_c \cos\left(2\pi F_c t + 2\pi K_f \sum_{i=0}^{N-1} m\left(i\Delta t + \frac{\Delta t}{2}\right)\Delta t\right) \quad (3.4)$$

This equation can be evaluated numerically using the discrete samples of the signal $m\left(i\Delta t + \frac{\Delta t}{2}\right)$ at each midpoint of the interval $i\Delta t + \frac{\Delta t}{2}$.

Parameter	Value	Description
T	0.1	Threshold
K_f	20 Hz/volt	Frequency sensitivity
A_c	1	amplitude of the carrier signal
F_c	100 MHz	Frequency of the carrier signal
F_s	44100 Hz	Sampling frequency
t	22 μ	Sampling time

The spectrum of FM signal is plotted in Fig. 4 using below code
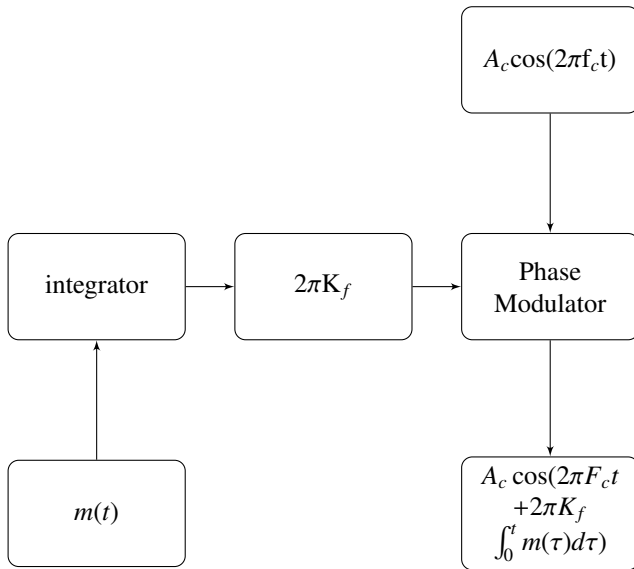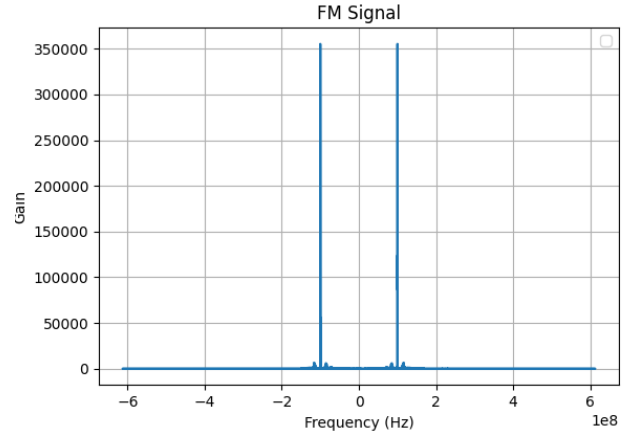
```
/codes/mod.py
```


Fig. 3: FM Transmitter Block Diagram

3. MODULATION:

1) Frequency modulation to the sound signal:

$$c(t) = A_c \cos(2\pi F_c t) \qquad (3.1)$$

Equation 3.1 shows the mathematical expression for a carrier signal with amplitude A_c, frequency F_c.

The frequency modulation is applied to the sound signal using the formula

$$s(t) = A_c \cos\left(2\pi F_c t + 2\pi K_f \int_0^t m(\tau)d\tau\right) \qquad (3.2)$$

where A_c is the amplitude of the carrier signal, F_c is the frequency of the carrier signal, $m(t)$ is the modulating signal, K_f is the frequency sensitivity constant of the

2) Calculate the bandwidth of the transmitted signal by
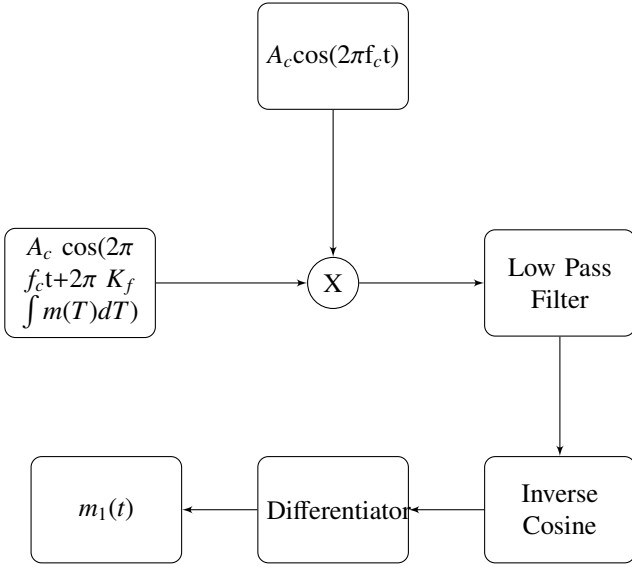
Fig. 5: FM Receiver Block Diagram

Fig. 6: local oscillator

Fig. 7: Filtered signal

computing the Fourier Transform:

$$S_k = \sum_{n=0}^{N-1} s(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \qquad (3.5)$$

In code for computing the FFT of s(n)

$$S_k = \texttt{fft}(s(n), N) \qquad (3.6)$$

Where S_k is the frequency representation of the signal, s(n) is the transmitted signal. we need to calculate its power spectral density. This can be done using the equation (3.7)

Calculating the Power Spectral Density:

$$PSD(s) = |S_k|^2 \qquad (3.7)$$

Identify the frequency range with significant power using a mask function.

4. Demodulation:

To demodulate the FM signal and recover the original message signal, we can use the following steps:

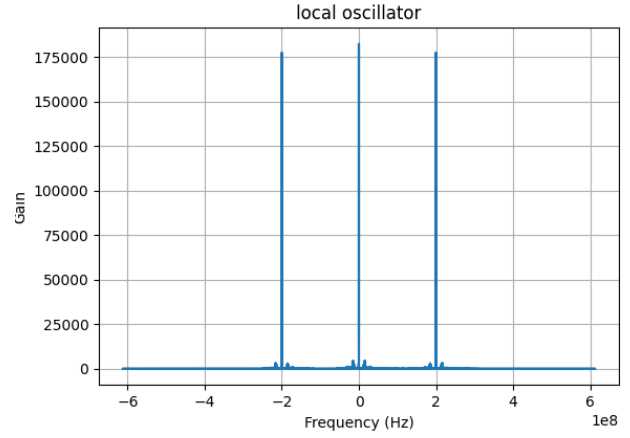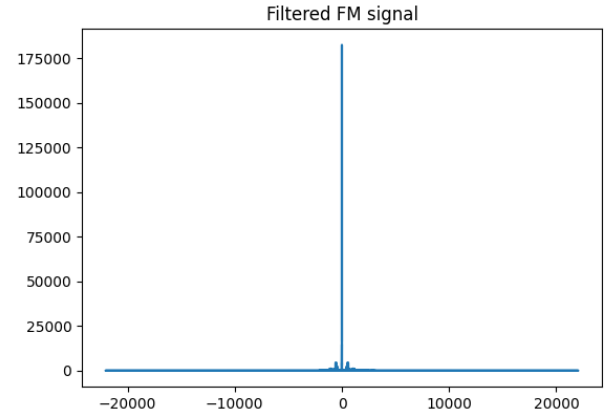1) Multiply the FM signal $s(t)$ with a local oscillator signal

$A_c \cos(2\pi F_c t)$. This gives us:

$$s(t) * c(t) = A_c \cos(2\pi F_c t) * A_c \cos(2\pi F_c t + 2\pi K_f \int_0^t m(\tau)d\tau)$$

$$(4.1)$$

$$= \frac{A_c^2}{2}[\cos(4\pi F_c t + 2\pi K_f \int_0^t m(\tau)d\tau)$$
$$+ \cos(2\pi K_f \int_0^t m(\tau)d\tau))] \qquad (4.2)$$

2) Pass the result through a low-pass filter to remove the higher frequency components and extract the lower sideband. The resulting signal can be expressed as:

$$v(t) = \frac{A_c^2}{2} \cos(2\pi K_f \int_0^t m(\tau)d\tau)$$

3) The filtered Signal is then expressed in terms of invese

Fig. 8: Recovered message signal

Upload an audio signal , converting the audio signal into a modulated signal using FM modulation, and demodulating the signal to retrieve the original audio signal using python.Fig. 8 shows the spectrum of demodulated signal. The retrieved audio signal can be saved as a file or played back in real-time using Python.
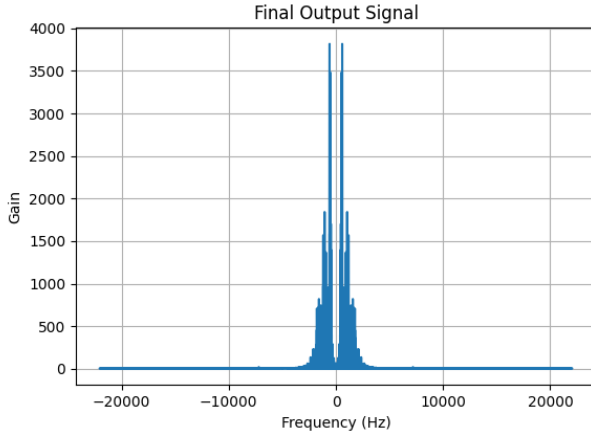
/codes/fm.py

cosin

$$V_1(t) = \cos^1(v(t))$$
$$= \frac{A_c^2}{2} \cos^1(\cos(2\pi K_f \int_0^t m(\tau)d\tau))$$
$$= A_c^2 \pi K_f \int_0^t m(\tau)d\tau$$

4) To recover the original message signal, we can differentiate the signal $V_1(t)$ with respect to time. This gives us:

$$m_1(t) = d/dx V_1(t) \tag{4.3}$$
$$= A_c^2 \pi K_f \, d/dx \int_0^t m(\tau)d\tau \tag{4.4}$$
$$= A * m(t) \tag{4.5}$$

where A=$A_c^2 \pi K_f$

In code calculates the derivative of the signal $V_1(t)$ using a finite difference method. In particular, it uses the forward difference approximation, where the derivative of a function $V_1(t)$ at point t is approximated as:

$$V'(t) \approx \frac{V(t + \Delta t) - V(t)}{\Delta t} \tag{4.6}$$

where Δt is a small increment. In the code, Δt is taken as the difference between successive time points t[i+1] - t[i]. The resulting array $m_1(t)$ contains the derivative of $V_1(t)$ at each point in t.