CS 70 Discussion 8A

October 23, 2024

Sample Space

Background: Every discrete probability problem can be represented as an "experiment". You have a bunch of possible outcomes of the experiment, and each of these outcomes has a particular probability of happening.

Notation:

- Ω is the sample space of our experiment, and it contains all the possible outcomes of our experiment.
- Ex: for flipping two coins, you can have $\Omega_1 = \{\mathsf{TT}, \mathsf{TH}, \mathsf{HT}, \mathsf{HH}\}$ or $\Omega_2 = \{0, 1, 2\}$ be the sample space.
- There are often multiple ways to define the sample space! Pick which definition is better based on the granularity you need for the problem.

Events

Sample Point: Some possible outcome ω from our experiment (i.e. $\omega \in \Omega$). An example for the coin-flipping scenario is: $\omega = \text{TH}$.
Event: Some subset E of our sample space (i.e. $E \subseteq \Omega$). An example for the coin-flipping scenario is: $E =$ event that we get an even number of heads $= \{\text{TT}, \text{HH}\}$

Probability Space

Probability Measure: We define some function/measure $\mathbb{P}$ that maps sample points and events to real numbers. $\mathbb{P}$ satisfies all of the following:

- Non-Negativitiy: $(\forall \omega \in \Omega)(\mathbb{P}[\omega] \geq 0)$
- Unit-Measure: $\sum_{\omega \in \Omega} \mathbb{P}[\omega] = 1$

$\mathbb{P}[\omega]$ tells us the probability of a particular sample point being the result of our experiment, and $\mathbb{P}[E]$ tells us the probability of a particular event being observed (i.e. one of the sample points $\omega \in E$ are observed: $\mathbb{P}[E] = \sum_{\omega \in E} \mathbb{P}[\omega]$).

Probability Space (Cont.)

Here are some useful formulas/facts:

1. For a **uniform probability space** (each experiment outcome is equally-likely):

$$\mathbb{P}[E] = \frac{|E|}{|\Omega|}$$

2. Principle of Inclusion-Exclusion:

$$\mathbb{P}\left[\bigcup_{i=1}^{n} E_i\right] = \sum_{s=1}^{n} (-1)^{s-1} \sum_{\substack{S \subseteq \{1,2,\ldots,n\} \\ |S|=s}} \mathbb{P}\left[\bigcap_{i \in S} E_i\right]$$