CS 70 Discussion 14B

December 6, 2024

Markov Chains

Periodicity: The periodicity of a state x in a Markov chain is as follows:

$$\gcd\left(\left\{t \in \mathbb{N} | P_{x,x}^{(t)} > 0\right\}\right)$$

where $P_{x,x}^{(t)}$ is the probability of being at state x at time t if you started at state x at time 0.
Aperiodic: A Markov chain is aperiodic iff every state in the chain has periodicity 1 (otherwise, the chain is **periodic**).
Irreducible: A Markov chain is irreducible iff every state can reach every other state in the chain within finite time (otherwise, the chain is **reducible**).

Stationary Distribution

π (a row vector) is a **stationary distribution** iff:

$$\pi = \pi P$$

This means that if my distribution of states I can be in at time t is π, then π will be the distribution of states I can be in at time $t+1$ (i.e. my distribution of states is now constant from time t onwards). You can solve for the stationary distribution using the following system of equations:

$$(\forall x \in \{1, 2, \dots, n\}) \left(\pi(x) = \sum_{y=1}^{n} \pi(y) P_{y,x} \right)$$

$$\sum_{x=1}^{n} \pi(x) = 1$$

Fundamental Theorem of Markov Chains

This theorem says that if a Markov chain is *aperiodic* and *irreducible*, then:

$$\lim_{t \to \infty} \pi_t = \pi$$

What this says is that as we run more and more steps on our Markov chain, our distribution of states we will be in will converge to the stationary distribution.
Note: It doesn't matter what our initial distribution π_0 *is!*