CS 70 Discussion 9A

October 30, 2024

Union Bound

Problem: For a set of events $A_1, A_2, \ldots, A_n$, how do we *upper-bound* $\mathbb{P}\left[\bigcup_{i=1}^{n} A_i\right]$ without having to use the Principle of Inclusion-Exclusion?

Solution: We use the **union-bound**, which states that in all cases:

$$\mathbb{P}\left[\bigcup_{i=1}^{n} A_i\right] \leq \sum_{i=1}^{n} \mathbb{P}[A_i]$$

Symmetry

Problem: Consider a discrete sample space Ω. We have an event A for which we want to find $\mathbb{P}[A]$. How do we use the probability $\mathbb{P}[B]$ of some other event B to help us?
Solution: Find a *bijective* function $f : A \to B$ that also satisfies the property:

$$(\forall \omega \in A)(\mathbb{P}[\omega] = \mathbb{P}[f(\omega)])$$

If we manage to find such a function, we can then conclude that $\mathbb{P}[A] = \mathbb{P}[B]$.