CS 70 Discussion 0B

August 30, 2024

Propositional Logic

We define A and B as **propositions** (true or false statements). We have the following notation and translations in propositional logic:

Propositional Logic	English
$\neg A$	A is not true
$A \wedge B$	A and B are true
$A \vee B$	A or B is true
$A \implies B$	If A is true, then B is true
$A \equiv B$	A is true **if and only if (iff)** B is true

Set Notation

A set is a collection of unique, unordered elements. Here is some notation for sets and their translations (P is a **propositional function**, which is just a function that outputs "true" or "false" based on an input x):

Set Notation	English
$S = \{a_1, a_2, \ldots, a_n\}$	S is a set with n elements: $a_1, a_2, \ldots, a_n$
$x \in S$	S contains element x
$x \notin S$	S does not contain element x
$\{x \| P(x)\}$	Set with all values x where $P(x)$ is true
$A \cap B$	Set with all x where $x \in A \wedge x \in B$
$A \cup B$	Set with all x where $x \in A \vee x \in B$
$A \backslash B$	Set with all x where $x \in A \wedge x \notin B$

Special Sets

Here are some useful sets to know:

Set	English
$\mathbb{R}$	Set of all real numbers
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{Z}$	Set of all integers
$\mathbb{N}$	Set of all natural numbers (in this class, $0 \in \mathbb{N}$)

Miscellaneous

Here are some notations to know:

Propositional Logic	English
$(\forall x \in S)(P(x))$	For all elements $x \in S$, $P(x)$ is true
$(\exists x \in S)(P(x))$	There is some $x \in S$ where $P(x)$ is true
$a \mid b$	b is divisible by a

The following are always true for any propositions A, B, and C:

$$(A \implies B) \equiv (\neg A \vee B)$$
$$[A \wedge (B \vee C)] \equiv [(A \wedge B) \vee (A \wedge C)]$$
$$[A \vee (B \wedge C)] \equiv [(A \vee B) \wedge (A \vee C)]$$

DeMorgan's Law

There is a specific way in which you can propagate negations ($\neg$) through conjunctions ($\wedge$) and disjunctions ($\vee$). The following facts always hold true (for any set S and propositions A and B):

$$\neg(A \wedge B) \equiv \neg A \vee \neg B$$
$$\neg(A \vee B) \equiv \neg A \wedge \neg B$$
$$\neg(\forall x \in S)(P(x)) \equiv (\exists x \in S)(\neg P(x))$$
$$\neg(\exists x \in S)(P(x)) \equiv (\forall x \in S)(\neg P(x))$$