CS 70 Discussion 1B

September 6, 2024

Induction Proof

Goal: Prove that for propositional function P, $(\forall n \in \mathbb{N})(P(n))$
Approach: Do the following:

1. Base Case: Prove $P(0)$ is true
2. Inductive Hypothesis: Just state: "Assume that for some $n \in \mathbb{N}$, $P(n)$ is true"
3. Inductive Step: Prove that $P(n) \implies P(n+1)$ for any $n \in \mathbb{N}$

Strong Induction

Goal: Prove that for propositional function P, $(\forall n \in \mathbb{N})(P(n))$
Approach: Do the following:

1. Base Case(s): Prove enough base cases (ex: if proving $P(n)$ requires $P(n-2)$, you need base cases for $n = 0$ and $n = 1$). We will say that $n = 0, n = 1, \ldots, n = k - 1$ are base cases.
2. Inductive Hypothesis: Just state: "Assume that for some $n \in \mathbb{N}$ where $n \geq k$, $P(m)$ is true for all $m < n$"
3. Inductive Step: Prove that
   $[(\forall m \in \{0, 1, ..., n-1\})(P(m))] \implies P(n)$
   i.e. Instead of just using $P(n-1)$ to prove $P(n)$, we use as many of the previous n "steps" to prove $P(n)$

Some Tips

- Try to use simple induction whenever possible (less to write)
- **Strengthening Induction Hypothesis**: Oddly, it can be easier to prove a statement under even stricter conditions (ex. it can be easier to prove $f(n) \leq 2$ instead of $f(n) \leq 4$ using induction)
- A very important sign to use induction is if you are trying to prove a formula/statement for *only natural numbers*.