CS 70 Discussion 10A

November 6, 2024

Joint Distributions

Problem: What if we want to calculate the probabilities of two different random variables taking on particular values simultaneously?
Solution: We define the probability that $X = x$ and $Y = y$ as:

$$\mathbb{P}[X = x, Y = y] = \mathbb{P}[X = x \cap Y = y]$$

X and Y are independent iff for all $x, y \in \mathbb{R}$:

$$\mathbb{P}[X = x, Y = y] = \mathbb{P}[X = x]\mathbb{P}[Y = y]$$

Conditional Distributions

Problem: How do we get the probability of a random variable taking a specific value, conditioned on the value of another random variable?
Solution: We say that the probability that $X = x$ conditioned on $Y = y$ is:

$$\mathbb{P}[X = x | Y = y] = \frac{\mathbb{P}[X = x, Y = y]}{\mathbb{P}[Y = y]}$$

You can also condition on events. For example, for event A:

$$\mathbb{P}[X = x | A] = \frac{\mathbb{P}[X = x \cap A]}{\mathbb{P}[A]}$$

Expectation

Problem: How do we get the "average" value of a random variable?

Solution: We define the **expectation** of a random variable X as the "average" value that the random variable takes. For a discrete random variable X:

$$\mathbb{E}[X] = \sum_x x\mathbb{P}[X = x]$$

For a non-negative discrete random variable X (**tail-sum**):

$$\mathbb{E}[X] = \sum_{x=1}^{\infty} \mathbb{P}[X \geq x]$$

You can also take the expectation of functions of random variables (**Law of the Unconscious Statistician (LOTUS)**):

$$\mathbb{E}[f(X)] = \sum_y y\mathbb{P}[f(X) = y] = \sum_x f(x)\mathbb{P}[X = x]$$

Linearity of Expecations

For a set of random variables $X_1, X_2, \ldots, X_n$, and constants $c_1, c_2, \ldots, c_n$, we have:

$$\mathbb{E}\left[\sum_{i=1}^{n} c_i X_i\right] = \sum_{i=1}^{n} c_i \mathbb{E}[X_i]$$

Key Importance: It doesn't matter if each of the X_is are independent or not! We can always use the above formula.