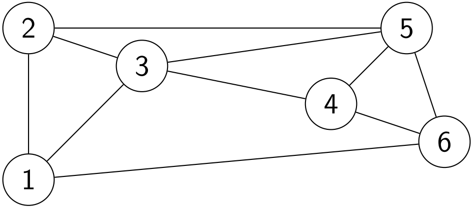
CS 70 Discussion 3A

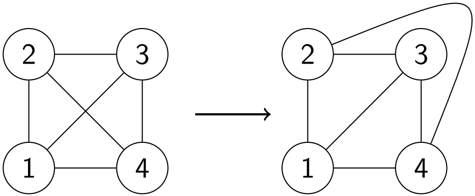
September 18, 2024

Planar Graphs

Graphs that can be drawn on a 2D plane without crossing edges:



Sometimes planarity can be non-obvious (both of the following graphs are equivalent, and are both planar):



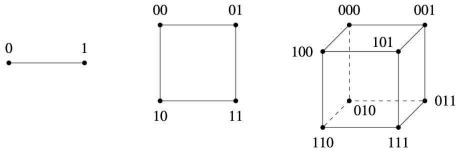
Planar Graphs (Cont.)

Here is some planar graph vocabulary:

- **Faces**: Separated regions in our 2D embedding (ex: 6 faces in first example on previous slide)
- **Euler's Formula**:
  Planar and connected $\implies |V| + |F| = |E| + 2$
- **Euler's Edge Formula**:
  Planar and $|V| \geq 3 \implies |E| \leq 3|V| - 6$
- **Four-Color Theorem**:
  Planar $\implies$ 4-colorable (vertex coloring)
- **Kuratowski's Theorem**: Planar $\iff$ No $K_{3,3}$ or K_5 exists
  - A $K_{m,n}$ is a **complete, bipartite graph** with m vertices in one group and n vertices in the other group, and a K_n is a **complete graph** with n vertices.

Hypercubes

These are n-dimensional cubes. Examples of $n = 1$, $n = 2$, and $n = 3$, respectively:



Here is a useful non-geometric interpretation:

- View each node as a bitstring of n bits (there are 2^n nodes in an n-dimensional hypercube)
- An edge between vertex u and vertex v exists iff u and v's bitstring labels differ by exactly one bit
- To construct $n+1$-dimensional hypercube with n-dimensional hypercubes A and B: prepend 1s to all vertices in A, prepend 0s to all vertices in B, and add edges connecting previously identical pairs of vertices in A and B

Hamiltonian Walks/Tours

Hamiltonian Walk: A walk that visits every vertex in the graph exactly once

Hamiltonian Tour: A tour that visits every vertex in the graph exactly once (except starting vertex: visited once at start and once at end)

Note: There is no efficient algorithm to find a Hamiltonian walk/tour in a graph