

CS 70 Discussion 11A

November 13, 2024

Conditional Expectation

Problem: How do I get the expectation of a random variable X conditioned on some other random variable Y (i.e. what is the “average” value of X given that I know $Y = k$ for constant k)?

Solution: We define the **conditional expectation** of a discrete random variable X :

$$\mathbb{E}[X|Y = k] = \sum_i i\mathbb{P}[X = i|Y = k]$$

You can also condition on any event A :

$$\mathbb{E}[X|A] = \sum_i i\mathbb{P}[X = i|A]$$

Total Expectation/Wald's Identity

Problem: How do I calculate $\mathbb{E}[X]$ given that I can only calculate $\mathbb{E}[X|Y = k]$ for some random variable Y and any constant k ?

Solution: We have the formula for **total expectation**:

$$\mathbb{E}[X] = \sum_i \mathbb{E}[X|Y = i] \mathbb{P}[Y = i]$$

If we view $\mathbb{E}[X|Y]$ as a function of Y , we get **Wald's Identity**:

$$\mathbb{E}[X] = \mathbb{E}[\mathbb{E}[X|Y]]$$

Note: Wald's Identity is significantly easier to perform calculations with!