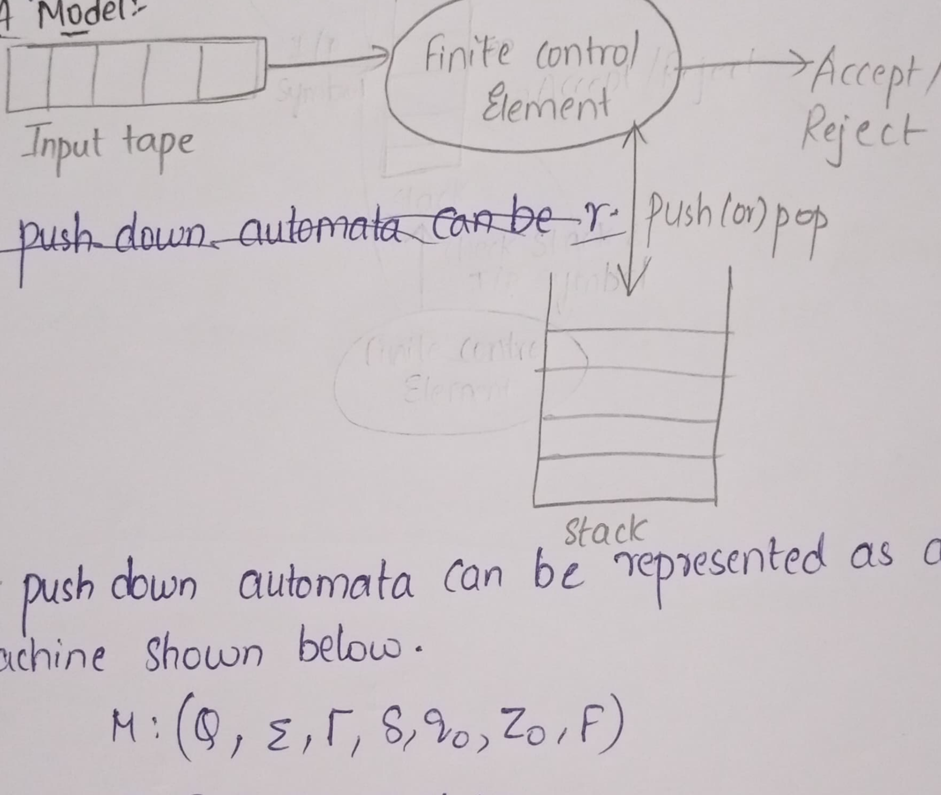
13/5/22

PUSH DOWN AUTOMATA

* Push down automata is the way to construct a language from the Context free Grammar. There are 3 basic components of push down automata.
  1) Input tape
  2) Finite Control element
  3) Stack

push down automata can be represented with these three components as shown below:-

PDA Model:-



~~A push down automata can be r.~~

A push down automata can be represented as a 7 tuple machine shown below.

$$M : (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

where Q : Finite set of states

Σ : finite set of Symbols

Γ : Stack Symbols

δ : Transition function

$$(Q \times \{\Sigma \cup \epsilon\} \times \Gamma) \rightarrow (Q, \Gamma^*)$$

q_0 : Initial State where $q_0 \in Q$.

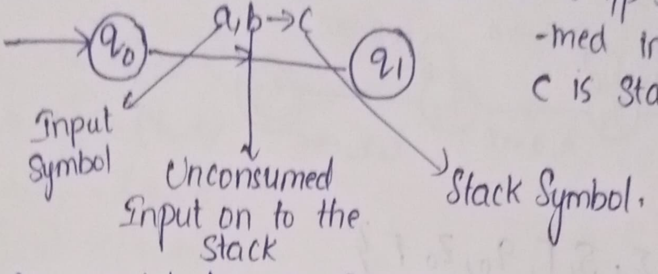
Z_0 : Initial Stack Symbol

F : Final State where $F \in Q$

Terminologies in PDA:-

1) push down Automata can be termed as finite State machine + Stack.

2) The graphical representation of transition in PDA is given as q_0 tends to q_1 on $a,b \to c$ where a is i/p Symbol, b is unconsumed input on to the stack c is Stack Symbol.



Here a, b & c could be ϵ Sometimes.

3) An instantaneous description is applied in PDA represented as (q,a,x) where $q \in Q$, $a \in$ Input Symbol & x is a Stack operation Symbol.

4) Turnstile notation.

If an instantaneous description is providing a transition or multiple transitions then it is represented using a turnstile notation ($\vdash$).

$$(q,a,x) \vdash (q,a)$$

(or)

$$(q,a,x) \vdash^{*} (q,a^{*})$$

5) The acceptance of a String in PDA is possible in 2 ways

i) If the transition reaches the final State.

ii) At the end of the transitions for an input tape the Stack is empty.

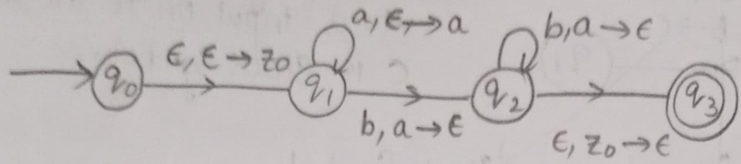
Ex:- Design a PDA for the language $L = \{a^n b^n / n \geq 0\}$.

Sol:- Note:- As per PDA design algorithm, in the transition representation like $a,b \to c$ in which 'b' is a push symbol & c is a pop Symbol. This is incomplete as per Churchy's finding. Alen turing has given a Solution for the same transition $a,b \to c$ in Such a way that b is a pop

Symbol & c is a push symbol. So the PDA design always follow the turing model for context free Grammars.

Sol: let $\omega = aabb$.

I/P Pop Push
$a, b \rightarrow c$



$M: \{Q, \Sigma, \delta, \Gamma, q_0, Z_0, F\}$

$Q: \{q_0, q_1, q_2, q_3\}$

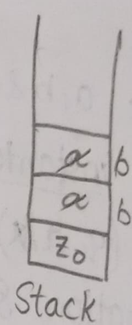
$\Sigma: \{a, b, \epsilon\}$

$\Gamma: \{a, b, Z_0\}$

$q_0: \{q_0\}$

$Z_0: \{Z_0\}$

$F: \{q_3\}$

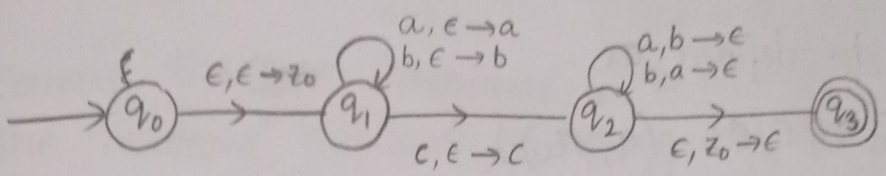


$\delta: (Q \times (\Sigma \cup \epsilon) \times \Gamma) \longrightarrow (Q, \Gamma^*)$

δ
$\delta(q_0, \epsilon, \epsilon) = (q_1, Z_0)$
$\delta(q_1, a, Z_0) = (q_1, a)$
$\delta(q_1, b, a) = (q_2, \epsilon)$
$\delta(q_2, b, a) = (q_2, \epsilon)$
$\delta(q_2, \epsilon, Z_0) = (q_3, \epsilon)$

Eg: Design a PDA for the language $L = \{wcw^R\} \mid w \in (a+b)^*\}$

Sol: let $w = abcba$



$\delta(q_0, a, Z_0) = (q_0, a)$

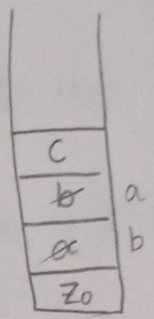
$\delta(q_0, b, Z_0) = (q_0, b)$

$\delta(q_0, a, a) = (q_1, a)$

$\delta(q_0, b, b) = (q_1, b)$

$\delta(q_0, b, a) = (q_1, b)$

$\delta(q_0, a, b) = (q_1, a)$



Q) Check acceptance of string for PDA of $L = \{a^n b^n / n \geq 0\}$

Sol: Given $L = \{a^n b^n / n \geq 0\}$

PDA can be defined as

$$\delta(q_0, \epsilon, \epsilon) = (q_1, z_0)$$
$$\delta(q_1, a, z_0) = (q_1, a)$$
$$\delta(q_1, a, a) = (q_1, a)$$
$$\delta(q_1, b, a) = (q_2, \epsilon)$$
$$\delta(q_2, b, a) = (q_2, \epsilon)$$
$$\delta(q_2, \epsilon, z_0) = (q_3, \epsilon)$$

Let us consider $w = aaabbb$ for $n = 3$.

Acceptance of string is checked by Instantaneous Description

$$(q_0, aaabbb, z_0) \vdash (q_1, aabbb, az_0)$$
$$\vdash (q_1, abbb, aaz_0)$$
$$\vdash (q_1, bbb, aaaz_0)$$
$$\vdash (q_2, bb, aaz_0)$$
$$\vdash (q_2, b, az_0)$$
$$\vdash (q_2, \epsilon, z_0)$$
$$\vdash (q_3, \epsilon, \epsilon)$$

$\boxed{\text{ie } q_3 \in F}$

Eg: Check the acceptance of $w = aaaabbbb$.

Sol: $$(q_0, aaaabbbb, z_0) \vdash (q_1, aaabbbb, az_0)$$
$$\vdash (q_1, aabbbb, aaz_0)$$
$$\vdash (q_1, abbbb, aaaz_0)$$
$$\vdash (q_1, bbbb, aaaaz_0)$$
$$\vdash (q_2, bbb, aaaz_0)$$
$$\vdash (q_2, bb, aaz_0)$$
$$\vdash (q_2, b, az_0)$$
$$\vdash (q_2, \epsilon, z_0) \vdash (q_3, \epsilon, \epsilon)$$ $\boxed{\text{ie } q_3 \in F}$

CFG to PDA Conversion :- For a given CFG 'G' the PDA can be constructed by the following Steps :-

(GNF ⇒ NT → T (NT)*) T → terminal, NT → nonterminal

1) Convert the given grammar 'G' to GNF (Griebach Normal form)
2) The resultant PDA will be having only one State $\{q\}$.
3) The initial Symbol of the Context-free Grammar is the initial Symbol of PDA.
4) For a transition with non-terminal Symbols the PDA rule is $\delta(q, \epsilon, A) = (q, a)$ for $A \to a$
5) For each terminal Symbol add the following rule to PDA $\delta(q, a, a) = (q, \epsilon)$ where $a \in T$

Ex :- Construct the PDA for the CFG with production rules

$S \to aAA$

$A \to aS / bS / a$

Sol: Given a G' $< V, T, P, S >$

For the given G the PDA is constructed as follows :-

1) Given grammar 'G' is in GNF.

for $S \to aAA$ (apply rule 4),

$$\delta(q, \epsilon, S) = (q, aAA)$$

for $A \to aS/bS/a$ (apply rule 4)

$$\delta(q, \epsilon, A) = (q, aS) / (q, bS) (q, a).$$

for the terminals $T = \{a, b\}$

apply rule 5:

for $T \in a$

$$\delta(q, a, a) = (q, \epsilon)$$

for $T \in b$

$$\delta(q, b, b) = (q, \epsilon)$$

The PDA for given CFG is

$$< Q, \Sigma, \delta, \Gamma, q_0, Z_0, F >$$

$Q = \{q\}$, $\Sigma = \{a, b\}$, δ,

$q_0 = \{q\}$, $Z_0 = \{\}$; $\Gamma = \{\}$, $F = \{q\}$.

Eg: $A \rightarrow aXY / 0$ where $V = \{A, X, Y\}$.

$X \rightarrow aA / bY / x$ $T = \{a, 0, x, y, b\}$

$Y \rightarrow y$

Sol:- Given a G <V,T,P,S>.

For the given G the PDA is constructed as follows:-

1) Given grammar G' is in GNF.

for $A \rightarrow aXY/0$:- (apply rule 4)

$$\delta(q, \epsilon, A) = (q, aXY) / (q, 0)$$

~~for A → 0 (apply rule 5)~~

$$\cancel{\delta(q, 0, 0) = (q, \epsilon)}$$

for $X \rightarrow aA / bY / x$ (apply rule 4)

$$\delta(q, \epsilon, X) = (q, aA) / (q, bY) / (q, x)$$

for $Y \rightarrow y$: (apply rule 4)

$$\delta(q, \epsilon, Y) = (q, y)$$

for the terminals $T = \{a, 0, x, y, b\}$

for $T \in a$

$$\delta(q, a, a) = (q, \epsilon)$$

for $T \in b$

$$\delta(q, b, b) = (q, \epsilon)$$

for $T \in 0$

$$\delta(q, 0, 0) = (q, \epsilon)$$

for $T \in x$

$$\delta(q, x, x) = (q, \epsilon)$$

for $T \in y$

$$\delta(q, y, y) = (q, \epsilon)$$

The PDA for given CFG is $<Q, \Sigma, \delta, \Gamma, q_0, Z_0, F>$

$Q - \{q\}$, $\Sigma - \{a, 0, b, x, y\}$, δ:

$q_0 - \{q\}$, $Z_0 - \{\}$, $\Gamma - \{\}$, $F - \{q\}$

Eg: PDA for given CFG

$S \rightarrow 0BB$

$B \rightarrow aS / bS / b$

Check the acceptance of String $w = 0a0bbb$.

Sol:

$\delta(q, \epsilon, S) = (q, 0BB)$

$\delta(q, \epsilon, B) = (q, aS) / (q, bS) / (q, b)$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 0, \ ) = (q, 0)$

$\delta(q, a, \ ) = (q, a)$

$\delta(q, b, \ ) = (q, b)$

Check:

$(q, 0a0bbb, \ ) \vdash^* (q, a0bbb, 0)$

$\vdash (q, 0bbb, a0)$

$(q, bbb, c$