DISCRETE MATHEMATICS

ASSIGNMENT-1

Name :- A.p.s.s. Bhargavi
RollNo :- 20A91A05D2
Section :- II CSEC

1) Check whether the following are tautologies or contradiction or contingency

i) $\{((p \to q) \wedge (q \to r)) \to (p \to r)$

Sol:

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore ((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology.

ii) $(p \vee (q \wedge r)) \leftrightarrow ((p \vee r) \wedge (p \vee q))$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee r$	$p \vee q$	$(p \vee r) \wedge (p \vee q)$	$(p \vee (q \wedge r)) \leftrightarrow ((p \vee r) \wedge (p \vee q))$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	F	T	F	F
F	F	T	F	F	T	F	F	T
F	F	F	F	F	F	F	F	T

$\therefore (p \vee (q \wedge r)) \leftrightarrow ((p \vee r) \wedge (p \vee q))$ is a contingency.

2) Prove that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are equivalent.

Sol:- Given $\neg(p \vee (\neg p \wedge q))$

$\Rightarrow \neg p \wedge (p \vee \neg q)$

$\Rightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\Rightarrow F \vee (\neg p \wedge \neg q)$

$\Rightarrow \neg p \wedge \neg q$

$\Rightarrow$ RHS

$\therefore \neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are equivalent.

3) Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow R \vee S$.

Sol:-

$\{1\}$	1. $C \vee D$	Rule P
$\{2\}$	2. $(C \vee D) \rightarrow \neg H$	Rule P
$\{1, 2\}$	3. $\neg H$	Rule T $(P, P \rightarrow Q \Rightarrow Q)$
$\{4\}$	4. $\neg H \rightarrow (A \wedge \neg B)$	Rule P
$\{1, 2, 4\}$	5. $A \wedge \neg B$	Rule T $(P, P \rightarrow Q \Rightarrow Q)$
$\{6\}$	6. $(A \wedge \neg B) \rightarrow R \vee S$	Rule P
$\{1, 2, 4, 6\}$	7. $R \vee S$	Rule T $(P, P \rightarrow Q \Rightarrow Q)$.

4) Find the PDNF and PCNF of the following

i) $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$

ii) $P \rightarrow (P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P)$

Sol:- i) Given $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$

Truth Table

P	Q	R	$\neg P$	$\neg P \to R$	$Q \leftrightarrow P$	$(\neg P \to R) \land (Q \leftrightarrow P)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

Sum of minterms = $(P \land Q \land R) \lor (P \land Q \land \neg R) \lor (\neg P \land \neg Q \land R)$

PCNF = $\neg$(Sum of rem. minterms which are not in PDNF)

$= \neg((P \land \neg Q \land R) \lor (P \land \neg Q \land \neg R) \lor (\neg P \land Q \land R)) \lor (\neg P \land Q \land \neg R) \lor (\neg P \land \neg Q \land \neg R)$

PCNF = $(\neg P \lor Q \lor \neg R) \land (\neg P \lor Q \lor R) \land (P \lor \neg Q \lor \neg R) \land (P \lor \neg Q \lor R) \land (P \lor Q \lor R)$

ii) $P \to ((P \to Q) \land \neg(\neg Q \lor \neg P))$

$P \to ((P \to Q) \land \neg(\neg Q \lor \neg P))$

Truth Table :-

P	Q	$P \to Q$	$\neg Q$	$\neg P$	$\neg Q \lor \neg P$	$(P \to Q) \land (\neg Q \lor \neg P)$	$P \to (P \to Q) \land (\neg Q \lor \neg P)$
T	T	T	F	F	F	F	F
T	F	F	T	F	T	F	F
F	T	T	F	T	T	T	T
F	F	T	T	T	T	T	T

PDNF = Sum of minterms = $(\neg P \land Q) \lor (\neg P \land \neg Q)$

PCNF = $\neg$(Sum of rem. minterms which are not PDNF)

$= \neg((P \land Q) \lor (P \land \neg Q))$

PCNF = $(\neg P \lor \neg Q) \land (\neg P \lor Q)$.

5) Test the validity of following Arguments.
"If you work hard, you will pass the exam. You did not pass. Therefore you did not work hard.

Sol:- Premises:- Statements

p : you work hard

q : you will pass the exam

$\neg q$: you did not pass.

Conclusion:- You did not work hard

Statements → Premises

1) $p \to q$

2) $\neg q$

Conclusion : $\neg p$.

$\therefore$ {1} $p \to q$ Rule P

{2} $\neg q$ Rule P

{1, 2} $\neg p$ Rule T. $(p \to q, \neg q \Rightarrow \neg p)$

$\therefore$ Given Argument is valid.

6) Prove that the following arguments are valid.
"All men are falliable, All kings are men. Therefore, all Kings are falliable.

Sol:- Arguments

$M(x) = x$ is men

Kings = K

$F(x) = x$ is falliable

Premises:

1) $(M(x) \longrightarrow F(x))$

2) All kings are men : $M(k)$

conclusion : $F(k)$

$\{1\}$	1. $\forall(M(x) \to F(x))$	Rule P
$\{2\}$	2. $M(K) \to F(K)$	Rule P
$\{3\}$	3. $M(K)$	Rule P
$\{2,3\}$	4. $F(K)$	Rule T $((P, P\to q) \Rightarrow q)$

conclusion

$\therefore$ The given arguments are valid.

7) Explain the warshall's algorithm and find the transitive closure of the relation.
$R = \{(1,4),(2,1),(2,3),(3,1),(3,4),(4,3)\}$ on $A = \{1,2,3,4\}$ using the same.

Sol:- Warshall's Algorithm:-

Warshall's algorithm is used to determine the transitive closure of a directed graph or all paths in a directed graph by using the adjacency matrix.

Warshall $(A[1 \ldots n, 1 \ldots n])$ // A is the adjacency matrix

$R^{(0)} \leftarrow A$

for $K \leftarrow 1$ to n do

for $i \leftarrow 1$ to n do

for $j \leftarrow 1$ to n do

$R^{(K)}[i,j] \leftarrow R^{(K-1)}[i,j]$ or $(R^{(K-1)}[i,K]$ and $R^{(K-1)}[K,j])$

return $R^{(n)}$

$$W = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

W_1 = Take 1 in first column $\{2,3\}$.

Take 1 in first row $= \{4\}$

Cartesian Product $= ((2,4),(3,4))$

$$W_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$\to W_2 \to$ second column there is no cartesian product.

$W_1 - W_2$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

The '1' in 3^{rd} column $= \{2,4\}$
The '1' in 3^{rd} row $= \{1,4\}$
Cartesian product $= \{(2,1),(2,4),(4,1),(4,4)\}$

$$W_3 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow$ Take '1' in fourth column $= \{1,2,3,4\}$
Take '1' in fourth row $= \{1,3,4\}$
$\Rightarrow$ Cartesian product $=$
$\{(1,1),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)\}$

$$W_4 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Take 1 in fourth column $= \{1,2,3,4\}$
Take 1 in 4^{th} row $= \{1,3,4\}$
$\Rightarrow$ Cartesian product $=$
$\{(1,1),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)\}$

$$W_4 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$\therefore$ This is the transitive closure

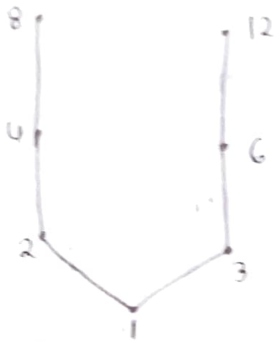
Draw the hasse diagram representing the partial ordering $\{(a,b) \mid a \text{ divides } b\}$ on $\{1,2,3,4,6,8,12\}$

(The hasse diagram is):

The partial ordered set is

$\{(1,1),(1,2),(1,3),(1,6),(1,4),(1,8),(1,12),(2,4),(2,6),(2,8),(2,12),$
$(3,6),(3,12),(4,8),(4,12),(6,12),(8,8),(12,12),(6,6),(4,4),(3,3),(2,2)\}$

Cover $=\{(1,2),(1,3),(2,4),(2,6),(3,6),(4,8),(4,12),(6,12)\}$



Hasse Diagram

q) If R be a relation in the set of integers Z defined by $R=\{(x,y): x\in Z, y\in Z, (x-y) \text{ is divisible by } 6\}$. Then prove that R is an equivalence relation.

Sol:- We say that xRy if $(x-y)$ is divisible by 6.

Reflexive:- Let $(x,x)\in R$ that means

$x-x$ $(\because x-y=0$ is divisible by $6)$

$(x,x)\in R$

R is reflexive.

Symmetric:- Assume $(x,y)\in R \Rightarrow (x-y)$ is divisible by 6

$\Rightarrow -(x-y)$ is divisible by 6

$\Rightarrow (y-x)$ is divisible by 6

$\Rightarrow (y,x)\in R$

$\therefore R$ is symmetric

Transitive:- $(x,y)\in R$ and $(y,z)\in R$.

$\Rightarrow (x-y)$ is divisible by $6 \longrightarrow ①$

and $(y-z)$ is divisible by $6 \longrightarrow ②$

$① + ② \Rightarrow$

$(x-y)+(y-z)$

$\Rightarrow (x-z)$ is divisible by 6

$\Rightarrow (x,z)\in R$.

$\therefore$ R is transitive.

$\therefore$ R is equivalence Relation.

10) List the ordered pairs in the relation on $[1,2,3,4]$ corresponding to the following matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

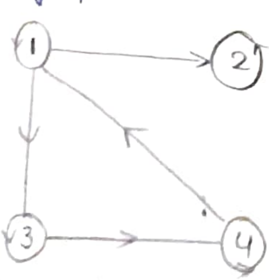
Also draw the digraph representing this relation. Use the graph to find if the relation is reflexive, Symmetric and transitive.

Sol:- Ordered pairs for

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 \\ 4 & 1 & 0 & 0 & 1 \end{array}$$

$\Rightarrow R = \{(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1), (4,4)\}$

Digraph



The relation is reflexive since all vertices has loop.

This

11) The relation matrices of two relations R and S from A to B is given below.

Determine the relation matrices of $R\cup S$, $R\cap S$, R^c, $\bar{S}$ where

$A=\{1,2,3\}$, $B=\{1,2,3,4\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad M_S = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(1,1), (1,3), (2,4), (3,1), (3,2), (3,3)\}$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,4), (3,2), (3,4)\}$

$R \cup S = \{(1,1), (1,2), (1,3), \cancel{(3,2)}\, (1,4), (2,4), (3,1), (3,2), (3,3), (3,4)\}$

$$M_{R \cup S} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$R \cap S = \{(1,1), (1,3), (2,4), (3,2)\}$

$$M_{R \cap S} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$R^c = \{(1,1), (3,1), (4,2), (1,3), (2,3), (3,3)\}$

$$M_{R^c} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$A \times B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)\}$

$\bar{S} = \{(2,1), (2,2), (2,3), (3,1), (3,3)\}$

$$\bar{S} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

12) let the recurrence relation be $a_n = 2^n + 5 \cdot 3^n$.

i) Find the first five terms of a recurrence relation.

ii) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, $a_4 = 5a_3 - 6a_2$.

Sol:- i) Given $a_n = 2^n + 5 \cdot 3^n$.

First 5 terms,

$n=0 \quad a_0 = 2^0 + 5 \cdot 3^0 = 1+5 = 6.$

$n=1 \quad a_1 = 2^1 + 5 \cdot 3^1 = 2+15 = 17$

$n=2 \quad a_2 = 2^2 + 5 \cdot 3^2 = 4+45 = 49$

$n=3 \quad a_3 = 2^3 + 5 \cdot 3^3 = 8+135 = 143$

$n=4 \quad a_4 = 2^4 + 5 \cdot 3^4 = 16+405 = 421.$

$\therefore$ The first five terms are 6, 17, 49, 143, 421.

ii) $a_2 = 5a_1 - 6a_0$
$= 5(17) - 6(6)$
$= 49$
$= a_2$

$a_3 = 5a_2 - 6a_1$
$= 5(49) - 6(17)$
$= 143$
$= a_3$

$a_4 = 5a_3 - 6a_2$
$= 5(143) - 6(49)$
$= 421$
$= a_4$.

13) Find the first five terms of $a_n = na_{n-1} + n^2 a_{n-2}$; $a_0 = 1$, $a_1 = 1$.

Sol:- Given $a_n = na_{n-1} + n^2 a_{n-2} \longrightarrow (1)$

$a_0 = 1$ & $a_1 = 1$

put $n=2$ in (1),

$a_2 = 2a_1 + 2^2 a_0$
$= 2(1) + 4(1)$
$a_2 = 6$

$n=3, \quad a_3 = 3a_2 + 3^2 a_1 = 3(6) + 9(1) = 27$

$n=4, \quad a_4 = 4a_3 + 4^2 a_2 = 4(27) + 16(6) = 204$

$n=5, \quad a_5 = 5a_4 + 5^2 a_3 = 5(204) + 25(27) = 1491.$

$\therefore$ The first five terms are 1, 1, 6, 27, 204, 1491.