5 i, Mean

Weight of earhead in gms	No of earhead (f)	Mid value (x)	fx
40-60	6	50	300
60-80	8	70	560
80-100	35	90	3150
100-120	55	110	6050
120-140	30	130	3900
140-160	15	150	2250
160-180	12	170	2040
180-200	9	190	1710
	N = 170		$\Sigma fx = 19960$

$$\text{Mean}(\bar{x}) = \frac{\Sigma fx}{N}$$

from the table $\Sigma fx = 19400$ and $N = 170$

$$= \frac{19960}{170} = \cancel{114.7176}$$
$$= 117.411$$

ii, Median

Weight	f	commulative frequency
40 - 60	6	6
60 - 80	8	14
80 - 100	35	49
100 - 120	55	104 → median class
120 - 140	30	134
140 - 160	15	149
160 - 180	12	161
180 - 200	9	170

from the table $N = 170$

$$\frac{N}{2} = \frac{170}{2} = 85$$

$l = 100$, $h = 20$, $f = 55$, $c = 49$

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right) = 100 + \frac{20}{55}(85 - 49) = 113.0909$$

(iii) Mode

Weight	f
40 – 60	6
60 – 80	8
80 – 100	35
$\boxed{100 - 120}$	$\boxed{55}$ → modal class
120 – 140	30
140 – 160	15
160 – 180	12
180 – 200	9

from the table,

$l = 100$,

$f_m = 55$

$f_1 = 35$

$f_2 = 30$

$h = 20$

$$\text{mode} = l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 100 + \frac{55 - 35}{(55 - 35) + (55 - 30)} \times 20$$

$$= 100 + 8.889$$

$$= 108.889$$

(iv) Harmonic Mean.

Weight	f	Mid value (x)	$\frac{1}{x_i}$	$f_i \cdot \frac{1}{x_i}$
40-60	6	50	0.02	0.12
60-80	8	70	0.0143	0.1144
80-100	35	90	0.0111	0.3885
100-120	55	110	0.0091	0.5005
120-140	30	130	0.0077	0.231
140-160	15	150	0.0067 ~~0.0059~~	0.1005
160-180	12	170	0.0059	0.0708
180-200	9	190	0.0053	0.0477
	N = 170			1.5734

from the table $N = 170$

$$\sum_{i=1}^{n} f_i \cdot \frac{1}{x_i} = 1.5734$$

$$\text{Harmonic mean} = \frac{N}{\sum_{i=1}^{n} f_i \cdot \frac{1}{x_i}} = \frac{170}{1.5734}$$

$$= 108.0462$$

(v) Geometric

weight

40 - 60

60 - 80

80 - 100

100 - 120

120 - 140

140 - 160

160 - 180

180 - 200

from the

Geometric

6. Mean

weight

60 - 64

65 - 69

70 - 74

75 - 79

80 - 84

Mean

(v) Geometric Mean

weight	f	midvalue(x)	$\log x_i$	$f_i \cdot \log x_i$
40 - 60	6	50	1.6989	10.1934
60 - 80	8	70	1.8451	14.7608
80 - 100	35	90	1.9542	68.397
100 - 120	55	110	2.0414	112.277
120 - 140	30	130	2.1139	63.417
140 - 160	15	150	2.1761	32.6415
160 - 180	12	170	2.2304	26.7648
180 - 200	9	190	2.2787	20.5083
	170			348.9598

from the table $N = 170$

$$\Sigma f_i \cdot \log x_i = 348.9598$$

$$\text{Geometric mean} = \text{Antilog}\left(\frac{\sum_{i=1}^{n} f_i \times \log x_i}{N}\right)$$

$$= \text{Antilog}\left(\frac{348.9598}{170}\right)$$

$$= \text{Antilog}(2.0527) = 112.9016$$

6. Mean

weight	No.of students (f)	Mid value (x)	fx
60 - 64	5	62	310
65 - 69	9	67	603
70 - 74	16	72	1152
75 - 79	12	77	924
80 - 84	8	82	656
	$N = 50$		$\Sigma fx = 3645$

$$\text{Mean}(\bar{x}) = \frac{\Sigma fx}{N} = \frac{3645}{50} = 72.9$$

Median

weight	No. of students (f)	cummulative frequency.
60 - 64	5	5
65 - 69	9	14
70 - 74	16	30 → median class
75 - 79	12	42
80 - 84	8	50
	50	

From the table $N = 50$

$$\frac{N}{2} = \frac{50}{2} = 25$$

$l = 69.5$, $h = 5$, $f = 16$, $c = 14$

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$$= 69.5 + \frac{5}{16}(25 - 14) = 72.9375$$

Mode

weight	No. of students	
60 - 64	5	
65 - 69	9	
70 - 74	16	→ modal class
75 - 79	12	
80 - 84	8	

$l = 69.5$

$f_m = 16$

$f_1 = 9$

$f_2 = 12$

$h = 5$

$$\text{mode} = l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times 5 = 72.6818$$

Variance and

weight N

60 - 64

65 - 69

70 - 74

75 - 79

80 - 84

From

Va

stand

UNIT - II

1. let P(A

and bag

∴ P(

let R d

P(R/A) =

=

P(R|B) =

=

probability

P(B|R)

Variance and standard deviation.

weight	No of students	Mid values (x)	$d = x_i - A$ $= x_i - 72$	d^2	fd^2
60 - 64	5	62	-10	100	500
65 - 69	9	67	-5	25	225
70 - 74	16	72 A	0	0	0
75 - 79	12	77	5	25	300
80 - 84	8	82	10	100	800
	50				1825

from the table $N = 50$

$$\Sigma f d^2 = 1825$$

$$\text{variance} = \frac{\sum_{i=1}^{n} f_i d_i^2}{N} = \frac{1825}{50} = 36.5$$

$$\text{standard deviation} = \sqrt{\text{variance}}$$

$$= \sqrt{36.5} = 6.0415$$

UNIT – II

1. let $P(A)$, $P(B)$ be the probabilities selecting bag A and bag B respectively.

$\therefore P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{2}$

let R denote Red ball

$P(R/A)$ = probability that red ball is ~~drawn~~ from bag A

$$= \frac{3c_1}{5c_1} = \frac{3}{5}$$

$P(R|B)$ = probability that red ball is ~~drawn~~ from bag B

$$= \frac{5c_1}{9c_1} = \frac{5}{9}$$

probability that the red ball is drawn from bag B

$$P(B|R) = \frac{P(R/B) \cdot P(B)}{P(R/A) \cdot P(A) + P(R/B) \cdot P(B)}$$

$$= \frac{\frac{5}{9} \cdot \frac{1}{2}}{\frac{3}{5} \cdot \frac{1}{2} + \frac{5}{9} \cdot \frac{1}{2}}$$

$$= \frac{\frac{1}{2}\left(\frac{5}{9}\right)}{\frac{1}{2}\left(\frac{3}{5} + \frac{5}{9}\right)} = \frac{\frac{5}{9}}{\frac{3}{5} + \frac{5}{9}} = 0.4808$$

2. let $p(x), p(y), p(z)$ be the probabilites that the businessman goes to hotel x, y, z respectively

$p(x) = 20\% = \frac{20}{100} = 0.2$

$p(y) = 50\% = \frac{50}{100} = 0.5$

$p(z) = 30\% = \frac{30}{100} = 0.3$

let A be the faulty plumbing

$p(A/x) = 5\% = \frac{5}{100} = 0.05$

$p(A/y) = 4\% = \frac{4}{100} = 0.04$

$p(A/z) = 8\% = \frac{8}{100} = 0.08$

probability that the businessman's room having faulty plumbing is assigned to hotel z

$$p(z/A) = \frac{p(A/z) \cdot p(z)}{p(A/x) \cdot p(x) + p(A/y) \cdot p(y) + p(A/z) \cdot p(z)}$$

$$= \frac{0.08 \times 0.3}{0.05 \times 0.2 + 0.04 \times 0.5 + 0.08 \times 0.3}$$

$= 0.4445$

3.

X	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i, k

We know that the sum of probabilities $= 1$

$$\sum_{i=0}^{7} p(x_i) = 1$$

$$= p(0)+p(1)+p(2)+p(3)+p(4)+p(5)+p(6)+p(7)$$

$$= 0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$$

$$10k^2+9k = 1$$

$$10k^2+9k-1 = 0$$

$$10k^2+10k-k-1=0$$

$$10k(k+1)-(k+1)=0$$

$$(10k-1)(k+1)=0$$

$$k = \frac{1}{10},\ k=-1.$$

since $0 \le p(x) \le 1$ $\boxed{k = \frac{1}{10}}$

ii, Evaluate $p(x<6)$, $p(x\geq 6)$ and $p(0<x<5)$

$$p(x<6) = p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)+p(x=5)$$

$$= 0+k+2k+2k+3k+k^2$$

$$= k^2+8k$$

$$= \left(\frac{1}{10}\right)^2 + \frac{8}{10} = \frac{1}{100}+\frac{8}{10} = \frac{81}{100} = 0.81$$

$$p(x\geq 6) = 1-p(x<6) \qquad [\because p(x<a)+p(x\geq a)=1$$

$$= 1-0.81$$

$$= 0.19$$

$$p(0<x<5) = p(x=1)+p(x=2)+p(x=3)+p(x=4)$$

$$= k+2k+2k+3k$$

$$= 8k = \frac{8}{10} = 0.8$$

iii, If $p(X \le k) > 1/2$, find the minimum value of k

Given, $p(X \le k) > 1/2$

$k = 1$, $p(X \le 1) = p(X=0) + p(X=1)$

$= 0 + k = \frac{1}{10} = 0.1 < 0.5$

$k = 2$, $p(X \le 2) = p(X=0) + p(X=1) + p(X=2)$

$= 0 + k + 2k = 3k = \frac{3}{10} = 0.3 < 0.5$

$k = 3$, $p(X \le 3) = p(X=0) + p(X=1) + p(X=2) + p(X=3)$

$= 0 + k + 2k + 2k = 5k = \frac{5}{10} = 0.5 = 0.5$

$k = 4$, $p(X \le 4) = p(X=0) + p(X=1) + p(X=2) + p(X=3) + p(X=4)$

$= 0 + k + 2k + 2k + 3k = 8k = \frac{8}{10} = 0.8 > 0.5$

For $k = 4, 5, 6, ---$ $\quad p(X \le k) > \frac{1}{2}$.

$\therefore$ Minimum value of k that satisfying above condition $= 4$

(iv) Distribution function of X $\quad F(x)$

X	F(x)
0	$F(0) = p(X \le 0) = 0$
1	$F(1) = p(X \le 1) = p(X=0) + p(X=1) = 0 + k = \frac{1}{10}$
2	$F(2) = p(X \le 2) = p(X \le 1) + p(X=2) = \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$
3	$F(3) = p(X \le 3) = p(X \le 2) + p(X=3) = \frac{3}{10} + \frac{2}{10} = \frac{5}{10}$
4	$F(4) = p(X \le 4) = p(X \le 3) + p(X=4) = \frac{5}{10} + \frac{3}{10} = \frac{8}{10}$
5	$F(5) = p(X \le 5) = p(X \le 4) + p(X=5) = \frac{8}{10} + \frac{1}{100} = \frac{81}{100}$
6	$F(6) = p(X \le 6) = p(X \le 5) + p(X=6) = \frac{81}{100} + \frac{2}{100} = \frac{83}{100}$
7	$F(7) = p(X \le 7) = p(X \le 6) + p(X=7) = \frac{83}{100} + \frac{7}{100} + \frac{1}{10} = 1$

(v) Mean, variance and standard deviation.

$$\text{Mean} = \sum_{i=0}^{7} x_i\, P(x_i)$$

$$= x_0 P(x_0) + x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5) + x_6 P(x_6) + x_7 P(x_7)$$

$$= 0\times 0 + 1\times k + 2\times 2k + 3\times 2k + 4\times 3k + 5\times k^2 + 6\times 2k^2 + 7(7k^2+k)$$

$$= 0 + k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$$

$$= 66k^2 + 30k$$

$$= 66\left(\frac{1}{100}\right) + 30\left(\frac{1}{10}\right)$$

$$= \frac{66+300}{100} = \frac{366}{100} = 3.66$$

$$\text{variance}(\sigma^2) = E(x^2) - (E(x))^2$$

$$= \sum_{i=0}^{7} x_i^2 P(x_i) - \mu^2$$

$$= (0^2\times 0 + 1^2\times k + 2^2\times 2k + 3^2\times 2k + 4^2\times 3k + 5^2\times k^2 + 6^2\times 2k^2 + 7^2\times(7k^2+k)) - (3.66)^2$$

$$= (0 + k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2 + 49k) - (3.66)^2$$

$$= 440k^2 + 124k - (3.66)^2$$

$$= \left(\frac{440}{100} + \frac{124}{10}\right) - (3.66)^2 = 16.8 - (3.66)^2 = 3.4044$$

$$\text{standard deviation} = \sqrt{\sigma^2}$$

$$= \sqrt{3.4044} = 1.8451$$

4.

x	0	1	2	3	4	5	6
$p(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

a) k

we know that sum of the probabilities $= 1$.

$$\sum_{i=0}^{6} p(x_i) = 1$$

$$= p(0)+p(1)+p(2)+p(3)+p(4)+p(5)+p(6)$$

$$= k+3k+5k+7k+9k+11k+13k = 1$$

$$= 49k = 1$$

$$\therefore k = \frac{1}{49}$$

b) mean

$$\sum_{i=0}^{6} x_i \cdot p(x_i) = x_0 p(x_0) + x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4) + x_5 p(x_5) + x_6 p(x_6)$$

$$= 0\times k + 1\times 3k + 2\times 5k + 3\times 7k + 4\times 9k + 5\times 11k + 6\times 13k$$

$$= 0 + 3k + 10k + 21k + 36k + 55k + 78k$$

$$= 203k = \frac{203}{49} = 0.7347$$

c) variance $(\sigma^2) = E(x^2) - (E(x))^2$

$$= \sum_{i=0}^{6} x_i^2 p(x) - \mu^2$$

$$= (0^2\times k + 1^2\times 3k + 2^2\times 5k + 3^2\times 7k + 4^2\times 9k + 5^2\times 11k + 6^2\times 13k) - (0.7347)^2$$

$$= (0+3k+20k+63k+144k+275k+468k) - (0.7347)^2$$

$$= 973k - (0.7347)^2$$

$$= \frac{973}{49} - (0.7347)^2$$

$$= 19.8571 - 0.5398 = 19.3173$$

d) $P(0<x<5)$

5. Given

mean, μ

standard

we have

i, $26 \le x \le$

$x = 2$

$x = 4$

$P(26 \le x$

$= P(-0.8$

$= P(0 \le z$

$= 0.28$

$= 0.765$

ii, $P(x \ge$

$x = 45$

$P(x \ge$

$= 0.5$

$= 0.5$

$= 0.$

d) $p(0<x<5) = p(x=1)+p(x=2)+p(x=3)+p(x=4)$

$= 3k+5k+7k+9k$

$= 24k = \frac{24}{49} = 0.4898$

5. Given

mean, $\mu = 30$

standard deviation, $\sigma = 5$

we have $z = \frac{x-\mu}{\sigma}$

i, $26 \le x \le 40$

$x = 26,\ z = \frac{26-30}{5} = \frac{-4}{5} = -0.8$

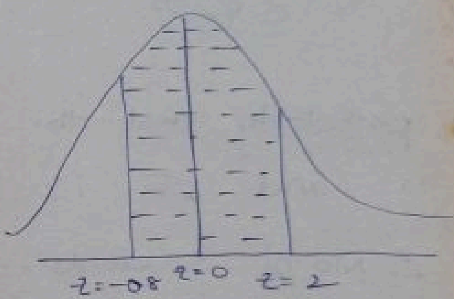
$x = 40,\ z = \frac{40-30}{5} = \frac{10}{5} = 2$

$p(26 \le x \le 40) = p(-0.8 \le z \le 2)$

$= p(-0.8 \le z \le 0) + p(0 \le z \le 2)$

$= p(0 \le z \le 0.8) + p(0 \le z \le 2)$

$[\because p(-z_1 \le z \le 0) = p(0 \le z \le z_1)]$



$= 0.2881 + 0.4772$

$= 0.7653$

ii, $p(x \ge 45)$

$x = 45,\ z = \frac{x-\mu}{\sigma}$

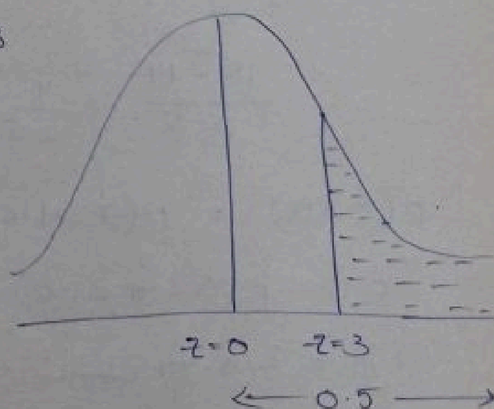
$= \frac{45-30}{5} = \frac{15}{5} = 3$

$p(x \ge 45) = p(z \ge 3)$

$= 0.5 - p(0<z<3)$

$= 0.5 - 0.4987$

$= 0.0013$



6. No. of students, $N = 1000$

mean, $\mu = 14$

standard deviation, $\sigma = 2.5$

i, How many students score between 12 and 15

$p(12 \le x \le 15)$

$$z = \frac{x-\mu}{\sigma}$$

$$x = 12, \quad z = \frac{12-14}{2.5} = \frac{-2}{2.5} = -0.8$$

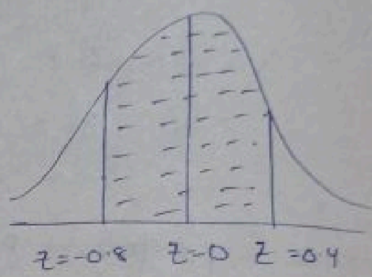
$$x = 15, \quad z = \frac{15-14}{2.5} = \frac{1}{2.5} = 0.4$$

$$p(12 \le x \le 15) = p(-0.8 \le z \le 0.4)$$

$$= p(0 \le z \le 0.8) + p(0 \le z \le 0.4)$$

$$= 0.2881 + 0.1554$$

$$= 0.4435$$



probability that the students got marks between 12 and 15 i.e., $p(12 \le x \le 15) = 0.4435$

$\therefore$ No. of students who score between 12 and 15

$$= N \times p(12 \le x \le 15) = 1000 \times 0.4435$$

$$= 443.5 = 444$$

ii, $p(x > 18)$

$$z = \frac{x-\mu}{\sigma}$$

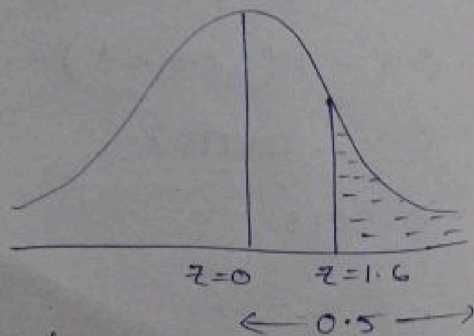
$$= \frac{18-14}{2.5} = \frac{4}{2.5} = 1.6$$

$$p(x > 18) = p(z > 1.6)$$

$$= 0.5 - p(0 < z < 1.6)$$

$$= 0.5 - 0.4452$$

$$= 0.0548$$



probability that the students got marks above 18 i.e., $p(x > 18) = 0.0548$.

No. of students who score above 18

$= N \times p(x > 18)$

$= 1000 \times 0.0548$

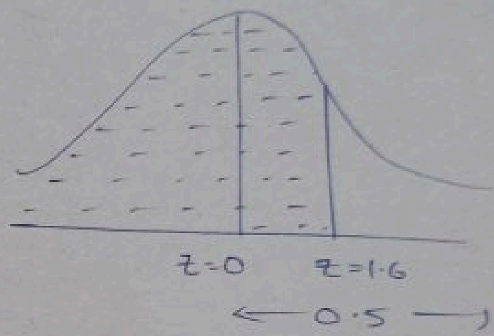
$= 54.8 = 55$

iii, $p(x < 18)$

$z = \dfrac{x - \mu}{\sigma}$

$= \dfrac{18-14}{2.5} = \dfrac{4}{2.5} = 1.6$

$p(x<18) = p(z > 1.6)$



$= p(0 < z < 1.6) + 0.5$

$= 0.5 + 0.4452$

$= 0.9452$

probability that the students got marks below 18

i.e., $p(x < 18) = 0.9452$

No. of students who score below 18

$= N \times p(x<18)$

$= 1000 \times 0.9452 = 945.2 = 945$