Unit-III

① notes

②

③ 95% confidence

$(\bar{x} - E_{max}, \bar{x} + E_{max})$

$E_{max} = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Given

95% confidence

$1 - \alpha = 0.95$

$1 - 0.95 = \alpha$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$0.5 - 0.025 = 0.475$ for this $Z_{\alpha/2} = 1.96$

$\sigma^2 = 0.25$

$\sigma = 0.5$

$Z_{\alpha/2} = 1.96$

$n = 100$

$E_{max} = 1.96\left(\frac{0.5}{\sqrt{100}}\right)$

$\Rightarrow 0.098$

$(212.3 - 0.098, 212.3 + 0.098)$

④ $n = 11$

$\bar{x} = 3.92$

$s = 0.61$

$(\bar{x} - E_{max}, \bar{x} + E_{max})$

$E_{max} = t_{\alpha/2} \frac{s}{}$

$\alpha = 0.01$

$\alpha/2 = 0.005$

$\nu = 10$

$t_{\alpha/2} = 3.$

$E_{max} = 3.169$

$= 3.16$

$(\bar{x} - E_{max},$

$(3.92 -$

$(3.35$

⑤ $\mu = 0.1$

$\sigma = 2.1$

$n = 900$

$Z_1 = \frac{\bar{x} - \mu}{\sigma/\sqrt{}}$

$Z_1 = \frac{\bar{x} -}{2.}$

$P(0.$

$P(0$

$E_{max} = t_{\alpha/2} \, s/\sqrt{n}$

$\alpha = 0.01$

$\alpha/2 = 0.005$

$\nu = 10$

$t_{\alpha/2} = 3.169$

$E_{max} = 3.169 \times \left(\frac{0.61}{\sqrt{11}}\right)$

$= 3.169 \times 0.18 = 0.570$

$(\bar{x} - E_{max}, \ \bar{x} + E_{max})$

$(3.92 - 0.57, \ 3.92 + 0.57)$

$(3.35, \ 4.49)$

$Z_{\alpha/2} = 1.96$

$\left(\frac{0.5}{\sqrt{100}}\right)$

8

(5) $\mu_{\bar{x}} = 0.1$

$\sigma = 2.1$

$n = 900$

$Z_1 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$Z_1 = \frac{\bar{x} - 0.1}{2.1/\sqrt{900}} = Z_1 = \frac{\bar{x} - 0.1}{0.07}$

$0.07 Z_1 = \bar{x} - 0.1$

$\bar{x} = 0.07 Z_1 + 0.1$

$P(0.1 + 0.07 Z_1 < 0)$

$P(0.07 Z_1 < -0.1) = P\left(Z_1 < \left(\frac{-0.1}{0.07}\right)\right)$

$P(z_1 < (-1.42))$

$0.5 - A(-1.42)$

$0.5 - A(1.42)$

$\Rightarrow 0.5 - 0.4222$

$\rightarrow 0.0778$

(6) Standard error of mean is

$$SE = \frac{\sigma}{\sqrt{n}}$$

If $n = 800$ then $SE = \frac{\sigma}{\sqrt{800}} = \frac{\sigma}{20\sqrt{2}}$

If $n = 200$ then $SE = \frac{\sigma}{\sqrt{200}} = \frac{\sigma}{10\sqrt{2}}$

$SE_1 = \frac{1}{2} SE_2$

$$\boxed{SE_2 = 2SE_1}$$

If size is decreased from 800 to 200 then multiply standard error by 2

(7) $E_{max} = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$E_{max} = 0.06$

95 % confidence means $= Z_{\alpha/2} = \cancel{0.06}\ 1.96$

σ is unknown.

So $E_{max} = Z_{\alpha/2}\sqrt{\frac{PQ}{n}}$

here consider equal proportions for P & Q

so $p = \frac{1}{2}$, $q = \frac{1}{2}$

is

$\frac{\sigma}{\sqrt{800}} = \frac{\sigma}{20\sqrt{2}}$

$\frac{\sigma}{\sqrt{200}} = \frac{\sigma}{10\sqrt{2}}$

800 to 200 then

by 2

$= Z_{\alpha/2} = \frac{\cancel{0.06}}{1.96}$

$\frac{\overline{PQ}}{n}$

for p & q

$$0.06 = 1.96\sqrt{\frac{(1/2)(1/2)}{n}}$$

$$0.06 = 1.96\frac{\sqrt{1/4}}{\sqrt{n}}$$

$$\frac{1.96}{2\sqrt{n}}$$

$$\sqrt{n} = \frac{1.96}{2 \times 0.06}$$

$\cancel{\sqrt{n} = 16.333}$

$\sqrt{n} = 16.33$

$\cancel{n = 9.69}$ $n = 266.66689$

$$\boxed{n \approx 267}$$

Unit-IV

①

②
$n = 900$

$\bar{x} = 3.4$

$\mu = 3.25$

$\sigma = 1.61$

step 1: $H_0: \mu = 3.25$

step 2: $H_1: \mu \neq 3.25$

step 3: 5%

$\alpha = 0.05$

$\alpha/2 = 0.025$

$\boxed{Z_{\alpha/2} = 1.96}$

test statistic

$$Z_{cal} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{3.4 - 3.25}{1.61/\sqrt{900}}$$

③ $n = 10$ t-test

70, 120, 110, 101, 88, 83, 98, 107, 100

$\mu = 100$

$$\bar{x} = \frac{877}{9} = 97.44$$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$$

$$\Rightarrow \frac{(70-97.44)^2 + (120-97.44)^2 + (110-97.44)^2 + (101-97.44)^2 + (88-97.44)^2 + (83-97.44)^2 + (98-97.44)^2 + (107-97.44)^2 + (100-97.44)^2}{9}$$

$$\Rightarrow \frac{1828.2224}{9} \Rightarrow 203.13$$

$$s = \sqrt{203.13} \Rightarrow 14.25$$

different

same $\sigma_1 \quad \sigma_2$

σ on S_1, S_2

$Z_{tab} = 1.96$

$$Z_{cal} = \frac{97.44 - 100}{14.25/\sqrt{10}}$$

(4) Town A

$$p_1 = \frac{400}{1000} \Rightarrow 0.4$$

$$\frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Town 2

$$p_2 = \frac{\cancel{800}\,400}{800} = 0.5$$

$$\Rightarrow p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$p = \frac{(1000)(0.4) + (800)(0.5)}{1000 + 800}$$

$p \Rightarrow 0.4444$

and $q = 1 - p$

$q = 1 - 0.4444$

$q = 0.5556$

$\boxed{Z_{tab} = 1.96}$

5%

$$Z_{cal} = \frac{0.4 - 0.5}{\sqrt{(0.444)(0.556)\left(\frac{1}{1000} + \frac{1}{800}\right)}}$$

(5)

$n_1 = 10$ | $n_2 = 17$

$\bar{x} = 1456$ | $\bar{y} = 1280$

$s_1 = 432$ | $s_2 = 398$

$$\frac{\bar{x}-\bar{y}}{s\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2}$$

$$\Rightarrow \frac{(10)(432)^2 + (17)(398)^2}{10+17}$$

$s^2 = 168855.85$

$s = 410.92$

$t_{tab} = ?$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$v = ?\ n_1+n_2-2$

$10+17-2$

$= 25$

$\boxed{Z_{tab} = 2.060}$

$$\Rightarrow \frac{1456-1280}{410.92\sqrt{\frac{1}{10}+\frac{1}{17}}}$$

$$\Rightarrow \frac{176}{410.92\sqrt{0.1588}}$$

$$\Rightarrow \frac{176}{163.75}$$

$\boxed{Z_{cal} = 1.074}$

(6)

Researcher	BA	A	AA	Genius	Total
X	86	60	44	10	200
Y	40	33	25	2	100
Total	126	93	69	12	300

for 86 =

60 =

44 =

10 =

40 =

33

2

Oi	Ei
86	84
60	6[illegible]
44	4[illegible]
10	8
40	4[illegible]
33	3
25	2
2	[illegible]

from t[illegible]

for $86 = \frac{200 \times 126}{300} = 84$

$60 = \frac{200 \times 93}{300} = 62$

$44 = \frac{200 \times 69}{300} = 46$

$10 = \frac{200 \times 12}{300} = 8$

$40 = \frac{100 \times 126}{300} = 42$

$33 = \frac{100 \times 93}{300} = 31$

$25 = \frac{100 \times 69}{300} = 23$

$2 = \frac{100 \times 12}{300} = 4$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
86	84	2	4	0.0476
60	62	-2	4	0.0645
44	46	-2	4	0.0869
10	8	2	4	0.5
40	42	-2	4	0.0952
33	31	2	4	0.129
25	23	2	4	0.1739
2	4	-2	4	1
				2.0971

$\boxed{\chi^2 = 2.0971}$

from table 5% LOS & $v = (2-1)(4-1) = 1(3) = 3$

(7) Sample 1

$n_1 = 8$

$x = 11, 11, 13, 11, 15, 9, 12, 14$

$\bar{x} = 96/8 = 12$

t test for diff of means

$$\frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$s_1^2 = \frac{\Sigma (x_i - \bar{x})^2}{n - 1}$$

$$s_1^2 = \frac{1+1+1+1+9+9+0+4}{7} \Rightarrow 3.714$$

$s_1 = \sqrt{3.714}$ $\quad \Sigma(x_i - \bar{x})^2 =$

$$x_2 = \frac{9+11+10+13+9+8+10}{7}$$

$x_2 = 10$

$$s_2^2 = \frac{1+1+0+9+1+4+0}{6}$$

$\Rightarrow 16/6 = 2.667$

$s_2^2 = \sqrt{2.667}$

$t_{tab} = 5\%$

$= 0.05$

$t_{\alpha/2} = 0.025$

degrees of freedom $n_1 + n_2 - 2$

$8 + 7 - 2$

$\Rightarrow 13.$

$\boxed{t_{tab} = 2.160}$

$$t_{cal} = \frac{12-10}{1.797\sqrt{\frac{1}{8}+\frac{1}{7}}}$$

$$s = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$$

$$\Rightarrow s^2 = \frac{\sum(x_i-\bar{x})^2 + \sum(y_i-\bar{y})^2}{n_1+n_2-2}$$

$$\Rightarrow \frac{2}{1.797\sqrt{0.2678}}$$

$$\Rightarrow \frac{26+16}{8+7-2} \Rightarrow \frac{42}{13}$$

$$\Rightarrow 3.230$$

$$\Rightarrow \frac{2}{0.9299}$$

$$\Rightarrow 2.150$$

$$\boxed{s = \sqrt{3.230} = 1.797}$$

$t_{cal} < t_{tab}$

(8)

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i-E_i)^2/E_i$
12	10	2	4	0.4
8	10	-2	4	0.4
20	10	10	100	10
2	10	-8	64	6.4
14	10	4	16	1.6
10	10	0	0	0
15	10	5	25	2.5
6	10	-4	16	1.6
9	10	-1	1	0.1
4	10	-6	36	3.6
				26.6

$\chi^2_{cal} = 26.6$

$\chi^2_{tab} = 16.9$ for 5% LOS & $\nu = n-1 = 9$

Unit-V

1

x	y	X $X-\bar{x}$ $x-68$	Y $Y-\bar{y}$ $y-155$	XY	X^2	Y^2
62	126	-6	-29	174	36	841
64	125	-4	-30	120	16	900
65	139	-3	-16	48	9	256
69	145	1	-10	-10	1	100
70	165	2	10	20	4	100
71	152	3	-3	-9	9	9
72	180	4	25	100	16	625
74	208	6	53	318	36	2809
		3	0	761	127	5640

$\bar{x} = \frac{547}{8} = 68.375 \approx 68$

$\bar{y} = 155$

$$r = \frac{\Sigma XY - \frac{\Sigma X \Sigma Y}{N}}{\sqrt{\Sigma X^2 - \frac{(\Sigma X)^2}{N}} \times \sqrt{\Sigma Y^2 - \frac{(\Sigma Y)^2}{N}}}$$

$$= \frac{761 - \frac{(3)(0)}{8}}{\sqrt{127 - \frac{(3)^2}{8}} \times \sqrt{5640 - \frac{0}{8}}} \Rightarrow \frac{761}{\sqrt{125.8}\sqrt{5640}}$$

$$\frac{761}{11.21 \times 75.09}$$

$$\boxed{r = 0.904}$$

Regression line x on y is

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$r \frac{\sigma_x}{\sigma_y} = \frac{\Sigma dx\, dy - \frac{\Sigma dx \Sigma dy}{N}}{\Sigma dy^2 - \frac{(\Sigma dy)^2}{N}}$$

$$\Rightarrow \frac{761 - \frac{0}{8}}{5640 - 0} \Rightarrow$$

$$\Rightarrow 0.134$$

$$x - 68.3 = 0.134\,(y - 155)$$

similarly y on x is

(2)

$y = ae^{bx}$

exponential curve

$Y = \ln y$ | $MA + B\Sigma X = \Sigma Y$

$X = x$ | $A\Sigma X + B\Sigma X^2 = \Sigma XY$

$a = e^A$

$B = b$

x	y	X = x	Y = ln y	X^2	XY
0.0	0.10	0	-2.302	0	0
0.5	0.45	0.5	-0.798	0.25	0.399
1.0	2.15	1	0.765	1	0.765
1.5	9.15	1.5	2.213	2.25	3.3195
2.0	40.35	2	3.697	4	7.394
2.5	180.75	2.5	5.197	6.25	12.9925
		7.5	8.772	13.75	24.87

$6A + 7.5B = 8.77$

$7.5A + 13.75B = 24.8$

$A = -2.49$

$B = 3.162$

$a = e^A$, $b = 3.162$

$a = 0.082$ $b = 3.162$

$y = ae^{bx}$

$y = 0.082\, e^{(3.162)x}$

(3)

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
10	12.9	30	100	354	37.1	130.3

$$ma + b\Sigma x + c\Sigma x^2 = \Sigma y$$

$$a\Sigma x + b\Sigma x^2 + c\Sigma x^3 = \Sigma xy$$

$$a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 = \Sigma x^2 y$$

$$5a + 10b + 30c = 12.9$$

$$10a + 30b + 100c = 37.1$$

$$30a + 100b + 354c = 130.3$$

$$a = 1.42, \quad b = -1.07, \quad c = 0.55$$

$$y = ax^2 + bx + c$$

(4)

(4)

x	y	dx $x-\bar{x}$ $x-8$	dy $y-\bar{y}$ $y-10$	dx^2	$dx \cdot dy$
2	2.4	−6	−7.6	36	45.6
4	5.6	−4	−4.4	16	17.6
5	5.8	−3	−4.2	9	12.6
7	8.9	−1	−1.1	1	1.1
8	9.0	0	−1	0	0
13	17.0	5	7	25	35
16	21.2	8	11.2	64	89.6
		−1	−0.1	151	201.5

$\bar{x} = \frac{55}{7} = 7.857 \simeq 8$

$\bar{y} = \frac{69.9}{7} = 9.985 \simeq 10$

y on x

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$$r\frac{\sigma_y}{\sigma_x} = \frac{\Sigma dx\,dy - \frac{\Sigma dx \cdot \Sigma dy}{N}}{\Sigma dx^2 - \frac{(\Sigma dx)^2}{N}}$$

$$\Rightarrow \frac{201.5 - \frac{(-1)(-0.1)}{7}}{151 - \frac{(-1)^2}{7}} \Rightarrow \frac{201.5 - 0.0142}{151 - 0.1428}$$

$r\frac{\sigma_y}{\sigma_x} =$

Now

$y - 9.98$

$y = 9.$

$\boxed{y =}$

(5)

(6)

x	
100	9
101	
102	
102	
100	
99	
97	
98	9
96	9
95	

$\bar{x} = 9$

$\bar{y} =$

$r\frac{\sigma_y}{\sigma_x} = 1.335$

Now

$y - 9.985 = r\frac{\sigma_y}{\sigma_x}(x - 7.85)$

$y = 9.985 + 1.335(12 - 7.85)$

$\boxed{y = 15.52}$

(5)

(6) Regression

x	y	xy	x^2	y^2
100	98	9800	10000	9604
101	99	9999	10201	9801
102	99	10098	10404	9801
102	97	9894	10404	9409
100	95	9500	10000	9025
99	92	9108	9801	8464
97	95	9215	9409	9025
98	94	9212	9604	8836
96	90	8640	9216	8100
95	91	8645	9025	8281
		94111	98064	90341

correlation

$X = x - 99$ $x - \bar{x}$	$Y = y - 95$ $y - \bar{y}$	X^2	Y^2
1	3	1	9
2	4	4	16
3	4	9	16
3	2	9	4
1	0	1	0
0	-3	0	9
-2	0	4	0
-1	-1	1	1
-3	-5	9	25
-4	-4	16	16
		54	96

$\bar{x} = \frac{990}{10} = 99$

$\bar{y} = \frac{950}{10} = 95$

$5 - 0.0142$

$1 - 0.1428$

Regression lines

x on y

$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

$r\frac{\sigma_x}{\sigma_y} = \frac{\Sigma xy}{\Sigma y^2}$

$\Rightarrow \frac{94111}{90341}$

$x - 99 = 1.041(y-95)$ ✓

y on x

$y - 95 = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$

$r\frac{\sigma_y}{\sigma_x} = \frac{94111}{98064} \Rightarrow 0.959$

$y - 95 = 0.959(x-99)$

correlation coeff

$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} \Rightarrow \frac{61}{\sqrt{(54)(96)}}$

$\Rightarrow \frac{61}{\sqrt{5184}} \Rightarrow \frac{61}{72}$

$\boxed{r = 0.847}$

xy
3
8
12
6
0
0
0
1
15
16
61

(7) rank correlation

same ranks / distinct

x	Rank of x
82	2
-68	6
75	3
61	8
-68	6
73	4
85	1
-68	6

68 rep

71 rep

x	Rank of x	y	Rank of y	D	D^2
83	2	81	1	1	1
68	6	71	3.5	2.5	6.25
75	3	71	3.5	-0.5	0.25
61	8	68	7	1	1
68	6	62	8	-2	4
73	4	67	6	-2	4
85	1	80	2	-1	1
68	6	70	5	1	1
					18.5

68 repeated 3 times $\frac{1}{12}(3^3-3) = 2$

71 repeated 2 times $\frac{1}{12}(2^3-2) = 0.5$

$m=2$

$$1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}(m^3-m) + \frac{1}{12}(m^3-m)\right]}{N(N^2-1)}$$

$$1 - \frac{6\left[18.5 + 2 + 0.5\right]}{8(64-1)}$$

$$\Rightarrow 1 - \frac{126}{504} \Rightarrow 1 - 0.25$$

$$\boxed{\rho = 0.75}$$