THE OTHER CONNECTIVES

◆ Exclusive OR

Let P and Q be any two formulas. Then the formula $P \overline{\vee} Q$, in which the connective $\overline{\vee}$ is called an exclusive OR, is true whenever either P or Q, but not both, is true.

Table

Exclusive OR		
P	Q	$P \overline{\vee} Q$
T	T	F
T	F	T
F	T	T
F	F	F

♦ NAND (↑)

"NAND" is a word which is a combination of the words "NOT" and "AND" when. NOT stands for negation and "AND" stands for conjunction. It is denoted by the symbol "$\uparrow$".

Let P and Q be any two statement formulas, then "P NAND Q" is denoted by "$P \uparrow Q$" and is defined as $P \uparrow Q \Leftrightarrow \neg(P \wedge Q)$

♦ NOR (↓)

The word "NOR" is a combination of "NOT" and "OR" where NOT stands for negation and "OR" stands for disjunction. It is denoted by "$\downarrow$".

Let P and Q be any two statement formulas, then "P NOR Q" is denoted by "$P \downarrow Q$" and is defined as $P \downarrow Q \Leftrightarrow \neg(P \vee Q)$.

Some basic properties of NAND and NOR.

1. $P \uparrow Q \Leftrightarrow Q \uparrow P$, $P \downarrow Q \Leftrightarrow Q \downarrow P$ (commutative)

2. $P \uparrow (Q \uparrow R) \Leftrightarrow P \uparrow \neg(Q \wedge R)$

$$\Leftrightarrow \neg(P \wedge \neg(Q \wedge R))$$

$$\Leftrightarrow \neg P \vee (Q \wedge R)$$

$(P \uparrow Q) \uparrow R \Leftrightarrow (P \wedge Q) \vee \neg R$ (not associative)

Similarly, $P \downarrow (Q \downarrow R)$ also not associative.

Example 7 : Prove that $\{\uparrow\}$ and $\downarrow$ are functionally complete sets of co$\wedge$ectives. They are also known as minimal functionally complete set.