Introduction to DSP: Systems - Assignment: Z transform

1. Find the inverse z-transform of the following $X(z)$ assuming the time doman signal to be causal.
   (a) $x[n] = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}$
   (b) $x[n] = \frac{z^{-5}+z^{-4}}{1+z^{-2}}$
2. Determine all the possible signals $x[n]$ associated with the z-transform $X(z) = \frac{5z^{-1}}{3-7z^{-1}+2z^{-2}}$.
3. Compute the convolution of the following two signals by means of the z-transform.

   $x_1[n] = \left(\frac{1}{4}\right)^n \cdot 1[n-1]$ and $x_2[n] = \left(1 + \left(\frac{1}{2}\right)^n\right) \cdot 1[n]$

4. Consider the following LTI system.

   $$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - \frac{1}{3}x[n-2]$$

   Find the complete output of the system starting from time $n = 0$, assuming the input is $x[n] = 1[n]$, with initial conditions $y[-1] = -1, y[-2] = 3$. Compute the natural response, and the forced response. What are the zero-input and zero-state responses of the system?