Piezoelectricity

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Lecture - Outline

- Millimetre size actuation and sensing
- Resonance for actuation and sensing
    - Focused energy delivery
    - Inherent filtering
- Piezoresistivity and piezoelectricity
- Electrical characterisitics
- Applications

Resistivity – Ohm's law

- Isotropic material

$$\frac{E}{J} = \rho$$

$$\frac{V/l}{I/A} = \rho \;\;, \frac{V}{I} = \rho \frac{l}{A} = R$$

Resistivity – non-isotropic material

- General non-isotropic material
  - e.g., biological tissue

$$E_1 = \rho_{11} J_1 + \rho_{12} J_2 + \rho_{13} J_3$$
$$E_2 = \rho_{21} J_1 + \rho_{22} J_2 + \rho_{23} J_3$$
$$E_3 = \rho_{31} J_1 + \rho_{32} J_2 + \rho_{33} J_3$$

$$E_i = \sum_{j=1,2,3} \rho_{ij} J_j$$

Piezoresistivity

- In material like silicon there is a mechanical stress related component of resistivity
- If there is stress (force/area) in direction *k* and a piezoresistive coefficient corresponding to normal and shear stresses, the effective electric field is:

$$E_i = \sum_{j=1,2,3} \rho_{ij} J_j + \sum_{j=1,2,3} \sum_{k=1,\ldots,6} \pi_{ijk} J_i \phi_k$$

A. Polarization of charge in a dielectric
B. Mechanical stress in solids

- Charge per unit area depends on the permittivity and the electric field

$$\frac{Q}{A} = \epsilon E$$

- Capacitance

$$C = \frac{Q}{V} = \epsilon \frac{A}{l}$$

- Stress in solids

$$\phi = \frac{\Delta f}{A} = Y \frac{\Delta l}{l}$$

$$\frac{\Delta l}{l} = \frac{\phi}{Y}$$

Piezoelectricity

- Stress causes additional charge separation

$$\frac{Q}{A} = \epsilon E + d_p \phi$$

- Electric field causes additional strain

$$\frac{\Delta l}{l} = \frac{\phi}{Y} - d_p E$$

For general non-isotropic piezoelectric material

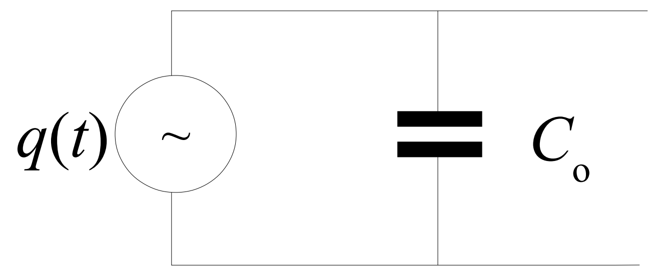
- Charge separation is affected by directional stress

$$\frac{Q_i}{A_i} = P_i = \sum_{j=1\ldots3} \epsilon_{ij} E_j + \sum_{m=1\ldots6} d_{im} \phi_m$$

- Strain is affected by directional electric field

$$\frac{\Delta l_i}{l_i} = \sum_{m=1\ldots6} \phi_i \cdot \frac{1}{Y_{im}} - \sum_{j=1\ldots3} d_{ij} E_i$$

Electrical Equivalent of Piezoelectric crystal



$$\Delta Q(t) = d_p \cdot F(t) \quad , \qquad C_o = \epsilon \frac{bl}{h}$$

Example 1

Ammonium Dihydrogen Phosphate (ADP): $d_{p-36} = 24 \times 10^{-12}$ Coulombs/Newton

(i) dimensions: l=10mm, b=10mm, h=10mm

Applied voltage of 1000V.

$$Strain = \frac{\phi}{Y} + d_p E$$

Applied stress is zero, and the electric potential is applied in z-direction (direction 3).Deflection is a shear about the z-axis (direction 6)

The resulting strain is:

$$(i)\, strain = 24 \cdot 10^{-12} \frac{1000}{10^{-2}}$$

$$= 24 \cdot 10^{-7}\, radians$$

(ii) dimensions: l=1mm, b=10mm, h=10mm

$$(ii)\, strain = 24 \cdot 10^{-6}\, radians$$

Mechanical resonance

- Beam with mass, elasticity and damping
- Exchange of energy between mass and elastic spring

$$\frac{X(s)}{F(s)}=\frac{A}{ms^2+Bs+K}$$

$$\frac{X(s)}{F(s)}=\frac{A}{\frac{s^2}{\omega_c^2}+\frac{2\zeta}{\omega_c}s+1}$$

- Velocity of sound in a solid

$$u=\sqrt{\frac{Y}{\rho}}$$

- Oscillation of a beam with a half-wave (both ends fixed)

$$l=\frac{\lambda}{2}=\frac{u}{2f_r}\ ,\qquad f_r=\frac{u}{2l}$$

Example 2

- Quartz:
    - Speed of sound=5800m/s
- Beam of length:
    - L=1mm for full wavelength
    - L=0.5mm for half wave
    - L=0.25 for quarter wave
- Frequency of oscillation
    - 5.8MHz

Resonance

- Mechanical resonance
  - Exchange of energy between mass and spring
- Electrical resonance
  - Exchange of energy between inductor and capacitor
- Mathematical resonance
  - Solution of second order differential equation

Oscillating Beam

Output Impedance of a piezoelectric crystal

- Mechanical impedance of the crystal

$$Z_o = \frac{Y}{u} = \sqrt{\rho Y} = \rho u$$

$$Stress = k\, Z_o u$$

- If the end has zero force, then the mechanical impedance is zero (short circuit)

$$Z_o = \frac{Stress}{(strain) \cdot u}$$

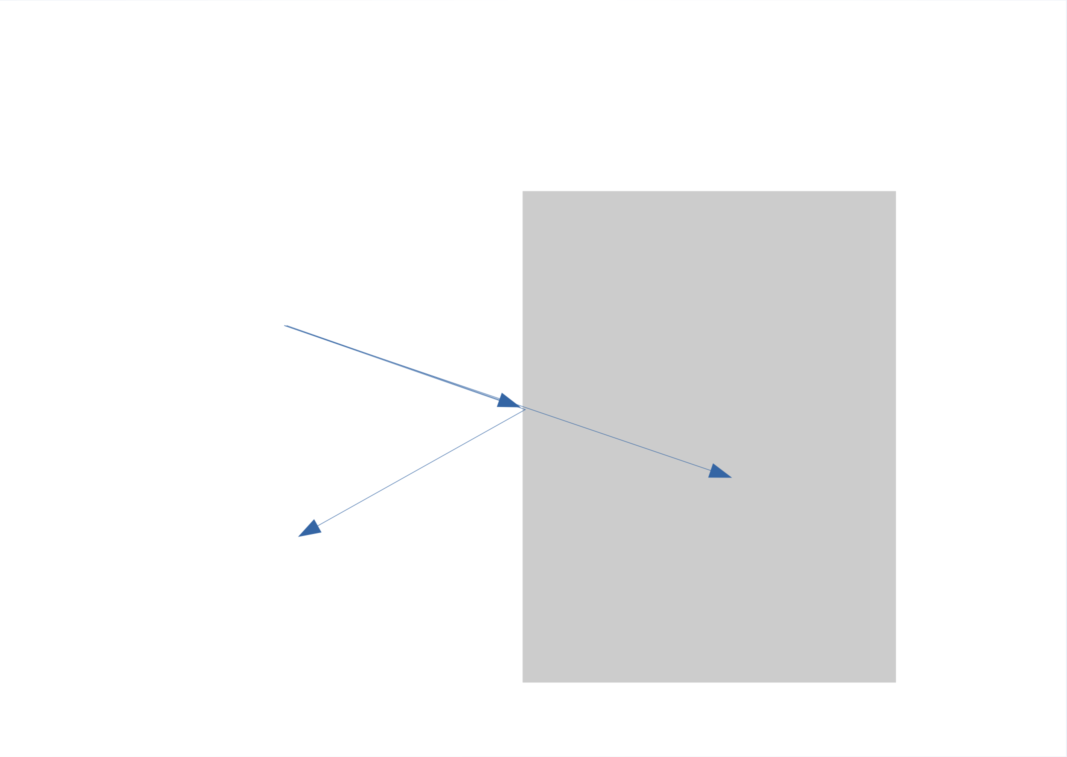
- If the end has zero velocity, then the mechanical impedance is infinity (open circuit)

$$Z_o = \rho u$$

Reflection

- Reflectivity

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$



Applications of Piezoelectric crystals

- Ultrasound
  - Transit time measurement of distance
  - Doppler measurement of motion

Doppler shift

- Source frequency f_o
- Source-target velocity u
- Propagation in medium with velocity c
- Doppler shift f_d

$$\frac{f_d}{f_o} = \frac{u}{c}$$

End of Lecture