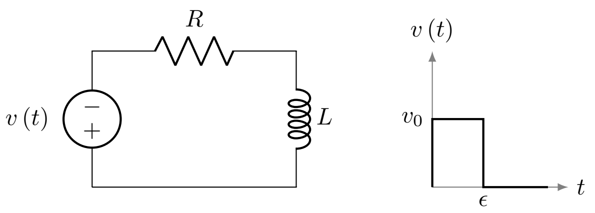
Linear Systems: Single Input Single Output View Assignment

1. Write down the differential equation representing relationship between the loop current $y(t)$ and the input voltage $v(t)$. Assume a initial loop current of $y(0^-) = y_0$.



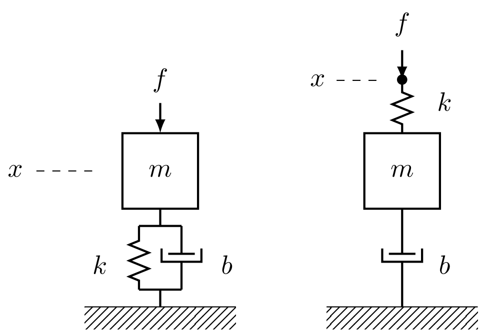
   (a) Find the response of the system for the input $v(\bullet), \forall t \geq 0$ shown in the figure.

   (b) Show that for a suitable choice ϵ, $y(t) = 0, \; \forall t > \epsilon$.

   (c) Assuming $y(0^-) = 0$, what happens to $y(t)$ when, $\epsilon \to 0$ and $v_0 \to \frac{1}{\epsilon}$? Derive the mathematical expression applying this limit. Compare this solution to the input $v(t) = \delta(t)$.

   (d) What will happen when $\epsilon \to 0$ and v_0 is constant?

2. Derive the differential equation governing the following two mechanical systems. The input to both these systems is the force $f(t)$, and the position $x(t)$ is the output; assume the initial conditions to $x(0^-), \dot{x}(0^-)$.

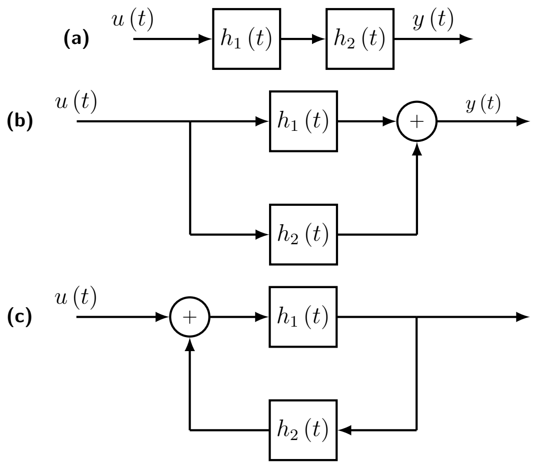


   Find the expression for the step response of these two systems.

3. Consider a continuous-time LTI system with impulse response, $h(t) = e^{-2t}1(t)$. What is the output of this system to the following inputs using the convolution integral? (a) $e^{-2t}1(t)$; (b) e^{-2t}; (c) e^{-1t}; (d) e^{-4t}; and (e) $\cos(\omega t)$.

   Now, obtain the expression for the output of the system for the above inputs using the system's transfer function $H(s)$.

4. Find the impulse response and transfer functions of the following composition of subsystems with individual impulse response $h_i(t)$.



5. Consider the second order system, $\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = u(t)$. Find the impulse response of this system. Plot the impulse response of the system for $\omega_n = 1$ and the following values of the parameter ζ. (a) $\zeta = \sqrt{2}$; (b) $\zeta = 1$; (c) $\zeta = 0.5$; (d) $\zeta = 0$; (e) $\zeta = -0.5$; and (f) $\zeta = -1.0$; For each of these parameter values show the location of the poles of the corresponding transfer function.