Introduction to DSP: Systems - Assignment: Digital Filters

1. **All pass filter**. Show that a filter of the following transfer function has a cosntant magnitude response for all frequencies.

$$H(z) = \frac{z^{-1} + p}{1 - pz^{-1}}, \;\; p \in \mathbb{R}$$

Write python program to the plot the mangitude and phase response of this filter.

2. Consider a bandlimited the time domain signal $x_c(t)$ that is sampled satisfying the Nyquist criterion.

$$x_d[n] = x_c(n \cdot T_s), \;\; T_s \text{ is the sampling time}$$

Write down the frequency response of a filter H that delays an input signal $x_d[n]$ by 2 samples, such that

$$y[n] = H\{x_d[n]\} = x_d[n-2] = x_c(n \cdot T_s - 2 \cdot T_s)$$

Find the impulse response of this filter using the inverse DTFT formula.

It is possible to design a filter $\hat{H}$ that delays the input signal by a fraction of a sample, i.e

$$\hat{y}[n] = \hat{H}\{x_d[n]\} = x_c(n \cdot T_s - 0.5 \cdot T_s)$$

Write down the frequenct response of such a filter, and derive it impulse response.

3. **Notch filter**. A notch filter is filter that selectively remove on more frequency components in the incoming singal. These filters have a magnitude response of 0 at specific frequencies.

In a system of interest, continous-time signal are sampled at 500 Hz, and are processed using a computer. The incoming data is corrupted by the 50Hz power line interfernce. We are interest in designing a simple FIR filter for removing the 50Hz interference. We can accomplish this using the following filter,

$$H(z) = 1 - 2\cos(\Omega)\, z^{-1} + z^{-2}$$

Where are the poles and zeros of this transfer function? Choose the value of Ω such that the digital filter $|H(\Omega) = 0$, such that 50Hz interference in the sampled data is removed.

Write a python program to plot the magnitude and phase response of this filter as a function of real frequency.

You will find that this filter introduces signifiant attenuation to the frequencies around 50Hz. One approach to address this issue is to develop a FIR filter using the frequency sampling method, by defining a desired frequency response for a set N discrete frequencies, obtain the impulse response for such a filter. Write a python program to design a FIR notch filter of length $N = 100$ to remove the 50Hz powerline interference.

Plot the magnitude and phase response of this filter as a function of real frequency.