Introduction to Digital Signal Processing

Sampling theorem revisited

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Sampling theorem

$$x(t) \longrightarrow x[n] \longrightarrow x(t)$$

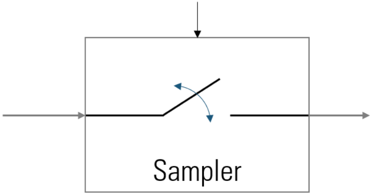
This is possible if the **signal is bandlimited** $\implies$ limit on the how fast the signal varies.

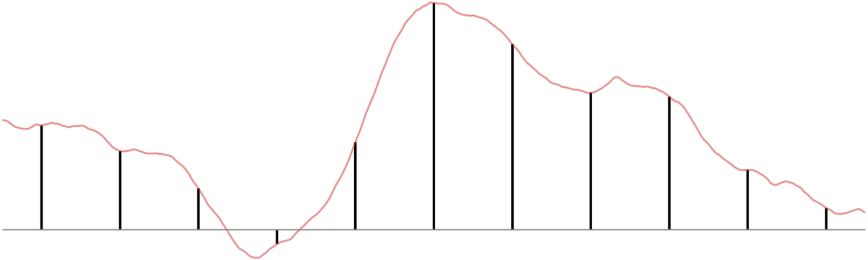
A measure of how fast the signal varies $\longrightarrow$ Max. frequency component.

Nyquist-Shanon Sampling Theorem.
If a signal $x(t)$ contains no frequencies higher than f_{sig} Hz, then it is completely determined by its values at time points spaced less than $1/(2f_{sig})$ seconds apart.

$$\implies \text{Sampling rate} = F_s > 2f_{sig}$$

Sampling process





Fourier transform of an Impulse train

$$\sum_{n=-\infty}^{\infty} \delta\left(t - n \cdot T_s\right) \stackrel{\text{FT}}{\longleftrightarrow} \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n \cdot \omega_s\right)$$

Let's say we have a discrete-time signal $x[n]$ that was obtained from sampling a continuous-time signal $x(t)$, such that

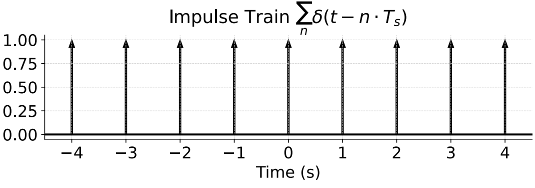
$$x[n] = x(n \cdot T_s), \quad \forall n \in \mathbb{Z}$$

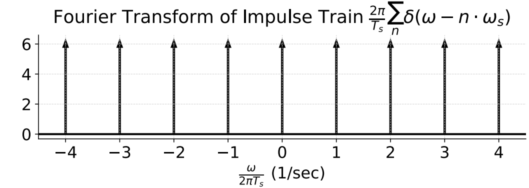
One way to reconstruct $x(t)$ from $x[n]$ is to generate an impulse train first,

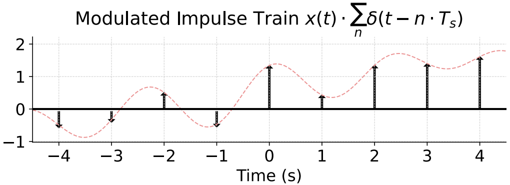
$$x_\delta(t) = \sum_n x[n] \cdot \delta(t - n \cdot T_s) = x(t) \cdot \sum_n \delta(t - n \cdot T_s)$$

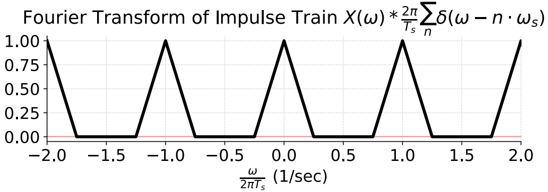
$$x(t) \cdot \sum_n \delta(t - n \cdot T_s) \stackrel{\text{FT}}{\longleftrightarrow} X(\omega) * \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n \cdot \omega_s\right)$$

Fourier transform of a Modulated Impulse train

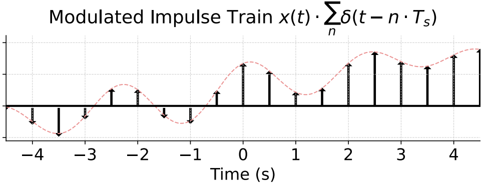


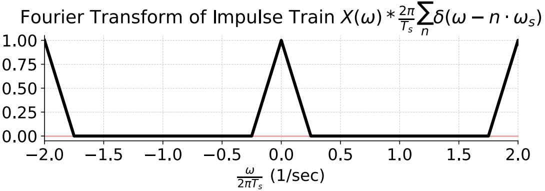




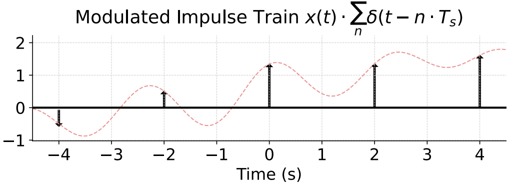


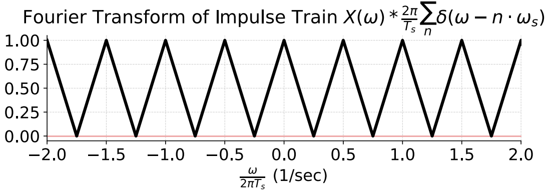
Fourier transform of a Modulated Impulse train





Sampling at the Nyquist rate





What happens when the signal is not bandlimited?