Introduction to Digital Signal Processing

Linear Time-Invariant Systems

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Input-Output Relationship of System

Input-Output Relationship of Linear System

Linearity:

$$x_i[n] \mapsto y_i[n] \implies \sum_i \alpha_i x_i[n] \mapsto \sum_i \alpha_i y_i[n]$$

Input-Output Relationship of Time-Invariant System

Time-invariance:

$$x_i[n] \mapsto y_i[n] \implies x_i[n-k] \mapsto y_i[n-k]$$

Linear Time Invariant (LTI) System

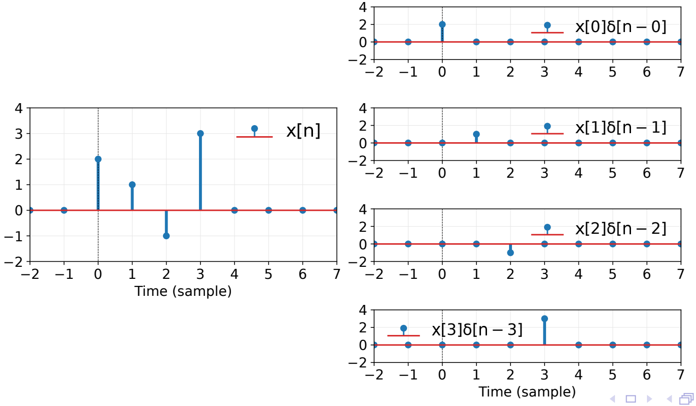
Input-Output Relationship

$$x_i[n] \mapsto y_i[n] \implies \sum_i \alpha_i x_i[n-k_i] \mapsto \sum_i \alpha_i y_i[n-k_i]$$

Importance of the Impulse Signal

Any signal $x[n]$ can be represented as a linear combinration of time-shifted impulse signals.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$



Impulse Response of an LTI System

Impulse Response: The response of an LTI system to an impulse input.

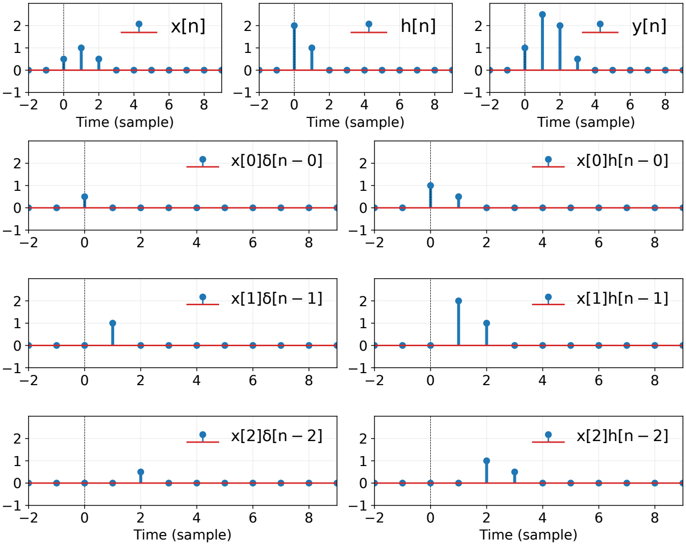
$$h[n] = \mathcal{H}\left(\delta[n]\right)$$

If we know this, then we know the following for an LTI system:

$$\delta[n] \mapsto h[n] \implies \begin{cases} \delta[n-k] & \mapsto h[n-k] \\ \alpha_k \cdot \delta[n-k] & \mapsto \alpha_k \cdot h[n-k] \\ \sum_k \alpha_k \cdot \delta[n-k] & \mapsto \sum_k \alpha_k \cdot h[n-k] \end{cases}$$

$$x[n] = \sum_k x[k] \cdot \delta[n-k] \xrightarrow{\mathcal{H}} \sum_k x[k] \cdot h[n-k] = x[n] * h[n]$$

Output of an LTI System



Convolution Sum

$$y[n] = x[n] * h[n] = \sum_{k} x[k] \cdot h[n-k]$$

Alternative View of the Convolution Sum

$$y[n] = x[n] * h[n] = \sum_{k} x[k] \cdot h[n-k]$$

k	$\cdots$	-3	-2	-1	0	1	2	3	4	5	6	7	$\cdots$
$x[k]$	$\cdots$	0	0	0	0.5	1	0.5	0	0	0	0	$\cdots$	
$h[\quad -k]$	$\cdots$												$\cdots$
$h[\quad -k]$	$\cdots$												$\cdots$
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$h[\quad -k]$	$\cdots$												$\cdots$

What does the impulse response tell us?

$$
\begin{aligned}
y[n] = x[n] * h[n] &= \sum_k x[k] \cdot h[n-k] \\
&= h[n] * x[n] = \sum_k h[k] \cdot x[n-k] \\
&= \cdots + h[2] \cdot x[n-2] + h[1] \cdot x[n-1] \\
&\qquad + h[0] \cdot x[n] \\
&\qquad + h[-1] \cdot x[n+1] + h[-2] \cdot x[n+2] + \cdots
\end{aligned}
$$