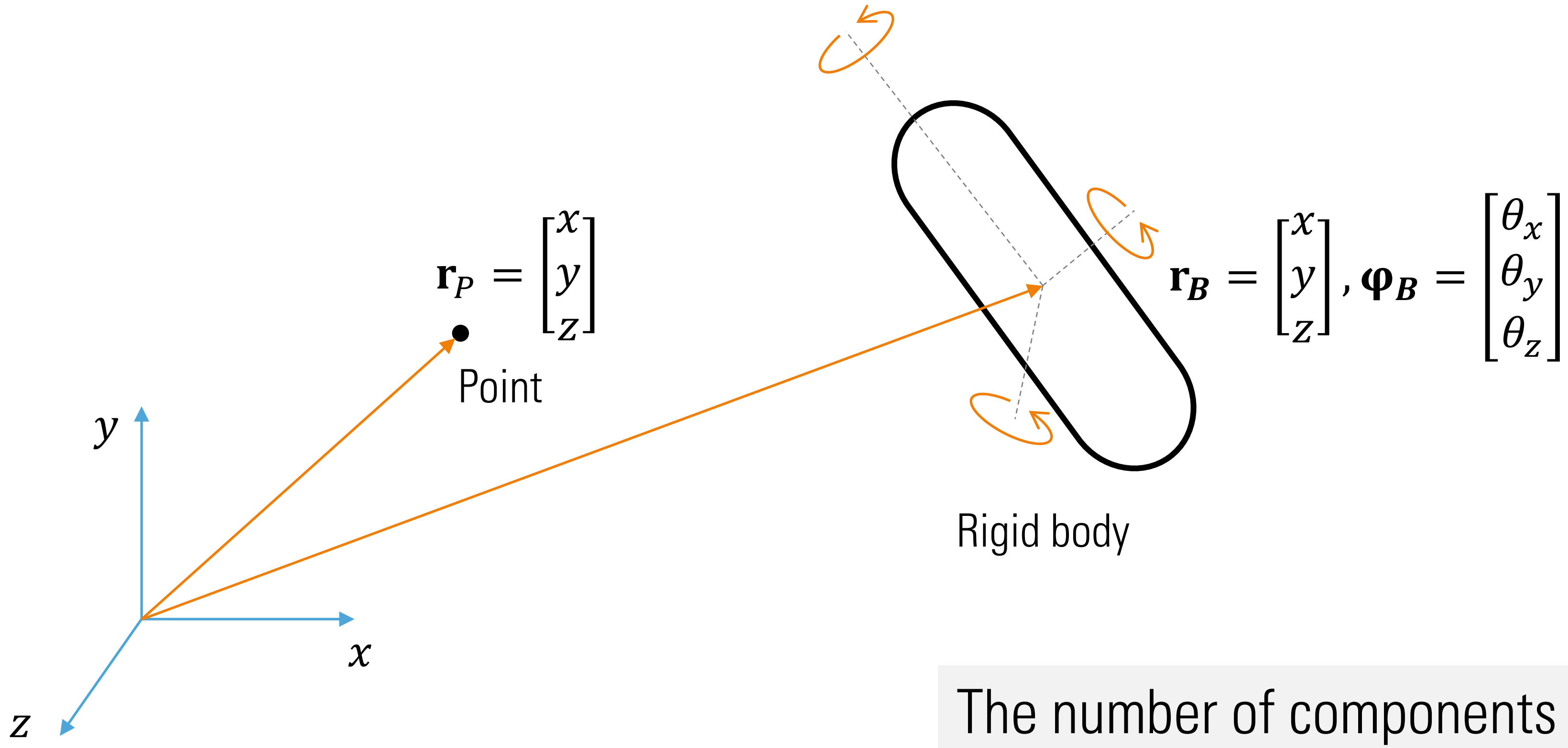
Transducers & Instrumentation

Module 03 - 01

Measuring Movements

Kinematics

- Study of motion without considering the forces/torques driving the motion.



The number of components required depends on the number of degrees of freedom.

Kinematics

- Derivatives of position and orientation measurements

Linear velocity $\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix}$

Linear acceleration $\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \frac{dv_x}{dt} \\ \frac{dv_y}{dt} \\ \frac{dv_z}{dt} \end{bmatrix}$

Linear Jerk $\mathbf{j} = \begin{bmatrix} j_x \\ j_y \\ j_z \end{bmatrix} = \begin{bmatrix} \frac{da_x}{dt} \\ \frac{da_y}{dt} \\ \frac{da_z}{dt} \end{bmatrix}$

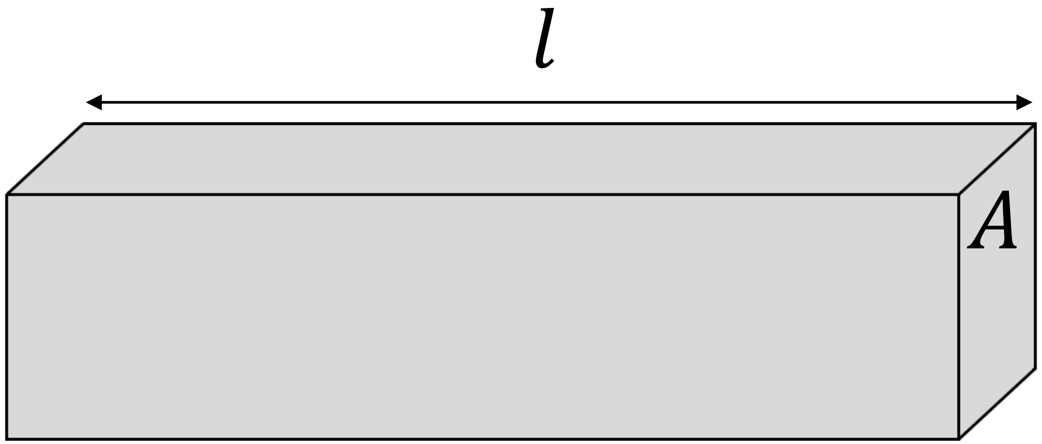
Angular velocity $\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \frac{d\theta_x}{dt} \\ \frac{d\theta_y}{dt} \\ \frac{d\theta_z}{dt} \end{bmatrix}$

Angular Acceleration $\boldsymbol{\alpha} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} = \begin{bmatrix} \frac{d\omega_x}{dt} \\ \frac{d\omega_y}{dt} \\ \frac{d\omega_z}{dt} \end{bmatrix}$

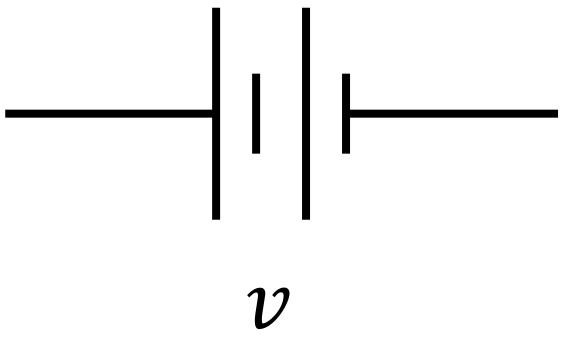
Position measurement

- Measurement are made with respect to a reference.
- Different approaches:
  - Resistive sensors
  - Inductive sensors
  - Capacitive sensors
  - Camera-based sensors

Resistance of a material



$$v = \rho \frac{l}{A} i \quad \Longrightarrow \quad R = \rho \frac{l}{A}$$

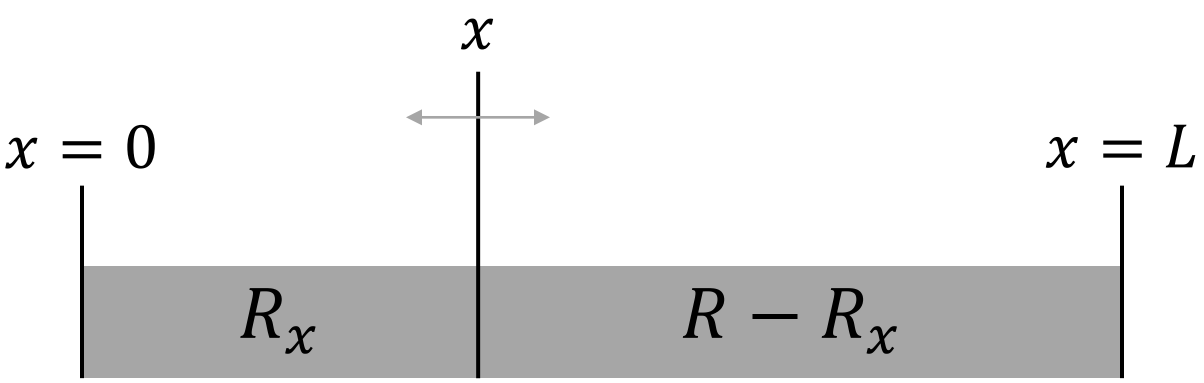


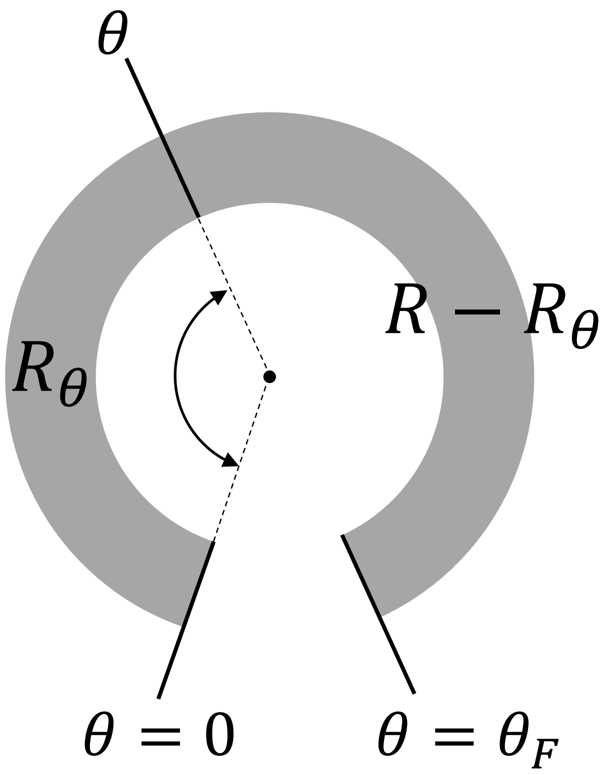
Specific Resistivity = $\rho = \dfrac{m}{ne^2\tau}$

Potentiometer for measuring position

- Change in length leads to change in resistance.

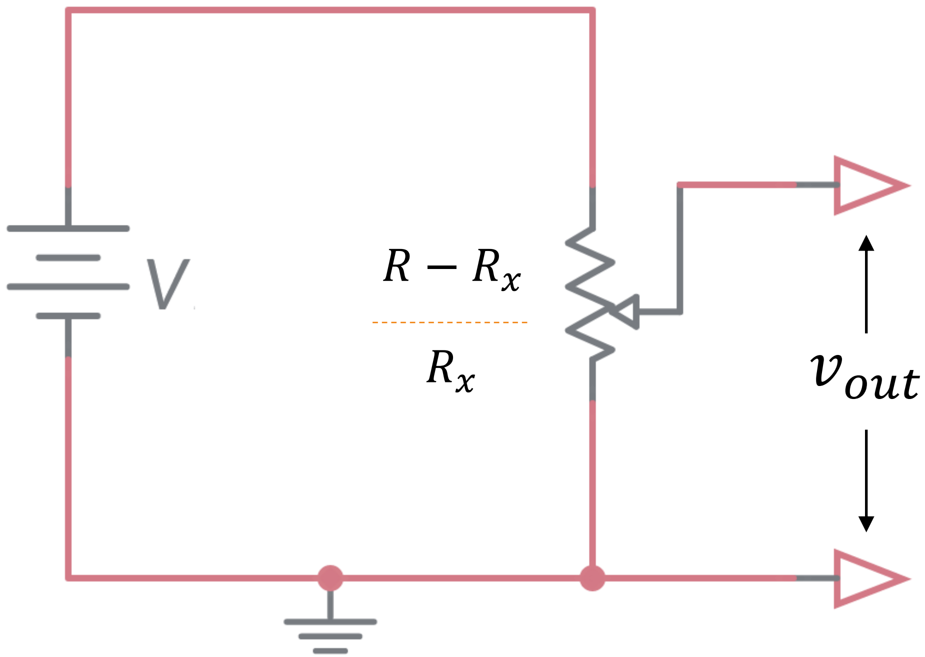
$$R = \rho \frac{l}{A}$$





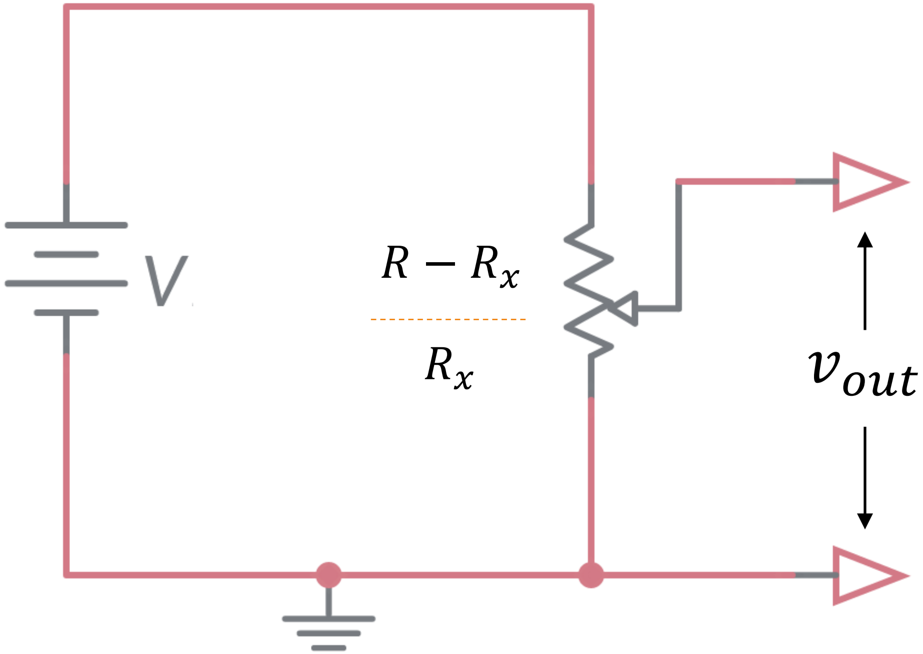
Potentiometer for measuring position

What happens when there is loading?



$$v_{out} = \frac{R_x}{R} V = \left(\frac{V}{R \cdot L}\right) \cdot x$$

Potentiometer for measuring position



$$v_{out} = \frac{R_x}{R} V = \left(\frac{V}{L}\right) \cdot x$$

Interfering Inputs

- Electrical noise in the circuit.
- Noise due to changes in wiper contact resistance.
- Wiper bounce over the resistor during movement.

Modifying Inputs

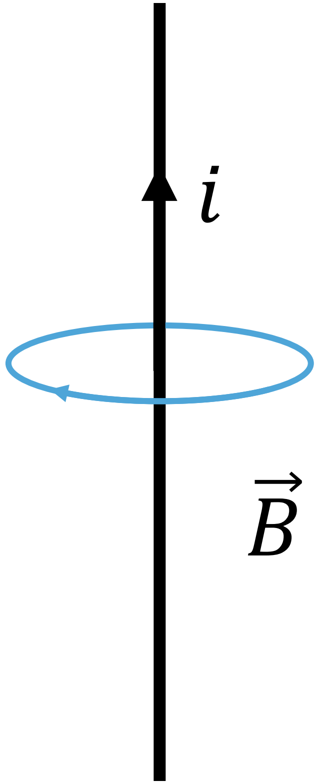
- Supply voltage V.

Potentiometer for measuring position

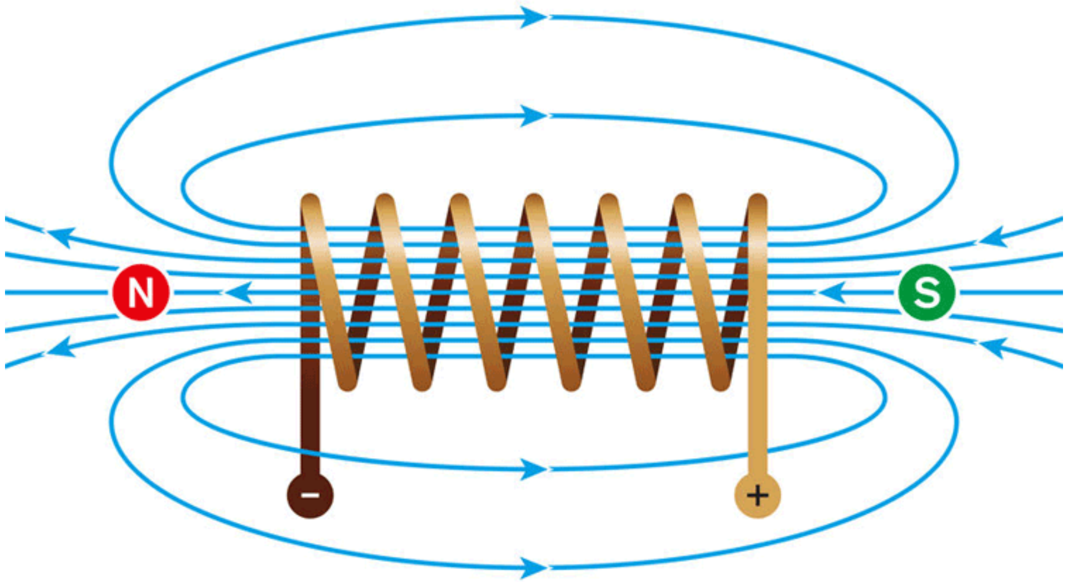
- Problems:
  - Movement of the wiper against friction.
  - Mechanical wear
  - High frequency measurements are problematic (friction, slider bounce, etc.)

Inductance

- Moving changes produce magnetic fields.



$$\vec{B} = \frac{\mu_0}{2} \oint \frac{i \cdot \overrightarrow{dl} \times \vec{r}}{|\vec{r}|^3}$$



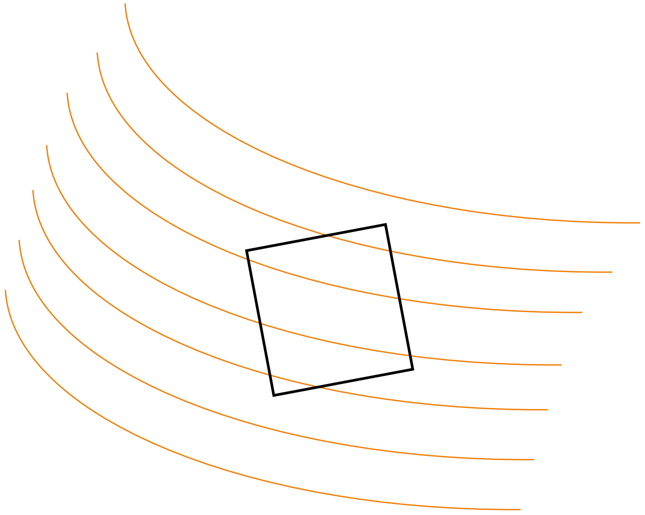
Source: https://circuitdigest.com/sites/default/files/field/image/What-is-Solenoid.png

$$B = \mu_0 \cdot i \cdot n$$

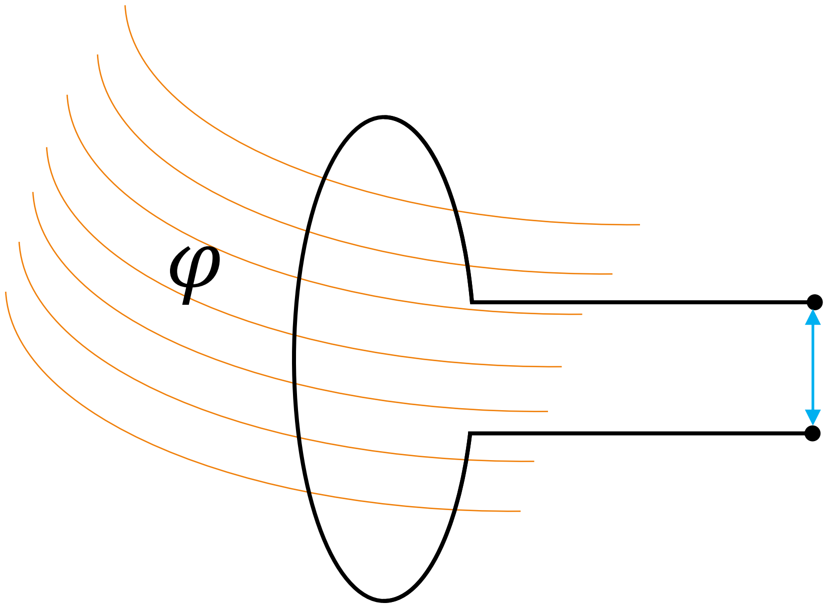
Inductance

Magnetic flux

$$\varphi = \int \vec{B} \cdot \overrightarrow{dS}$$

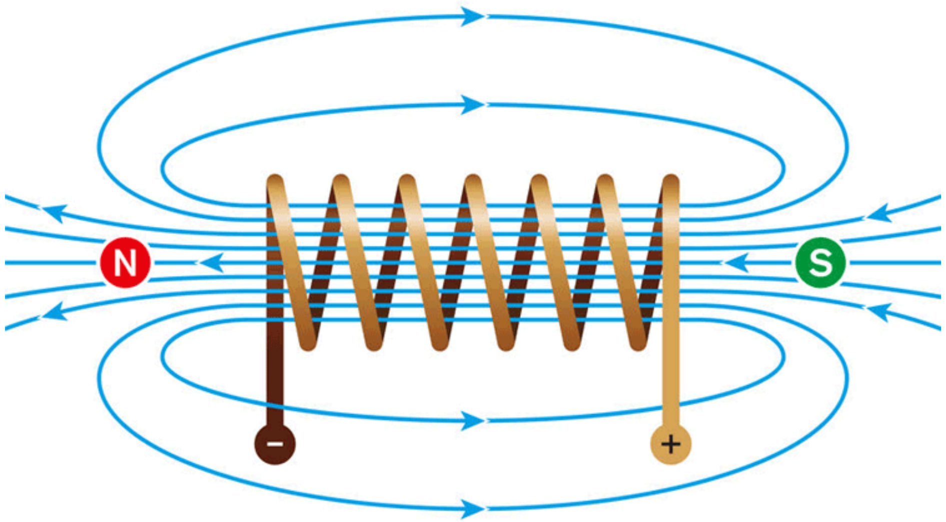


Magnetic induction



$$e = -\frac{d\varphi}{dt}$$

Inductance



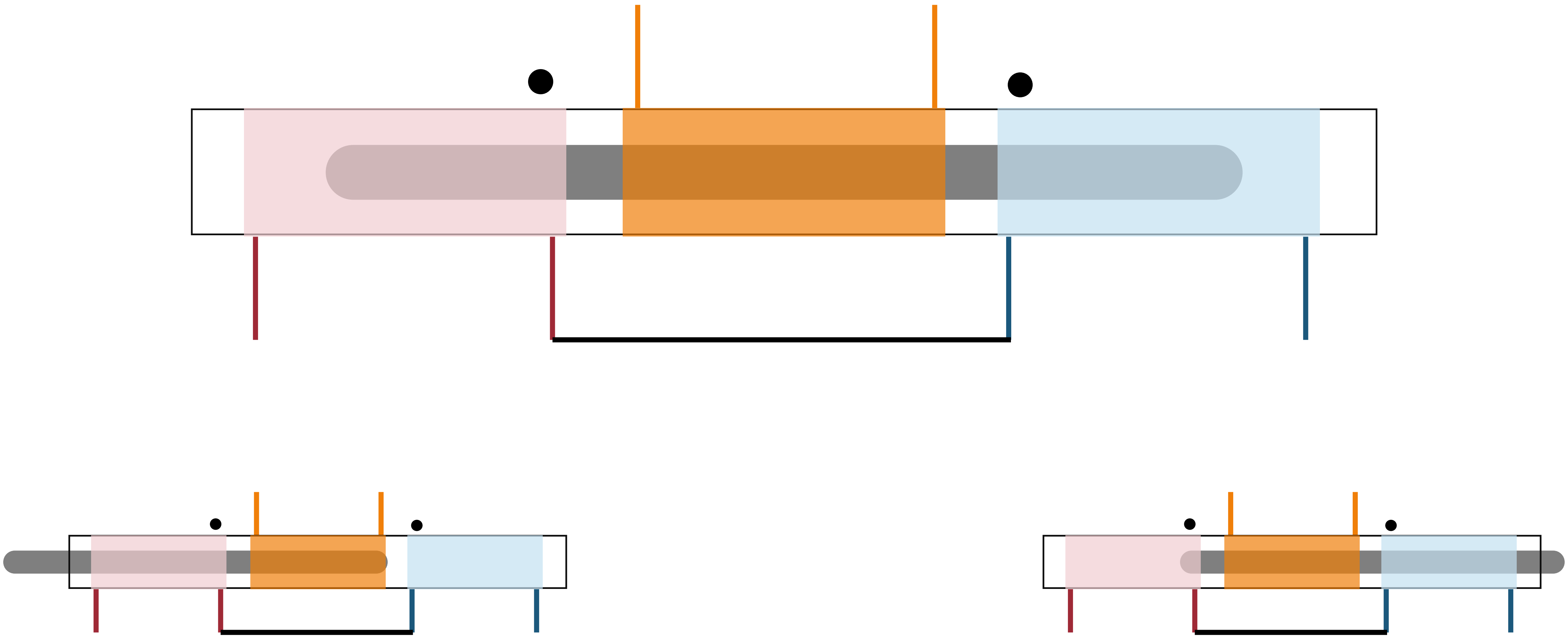
Source: https://circuitdigest.com/sites/default/files/field/image/What-is-Solenoid.png

$$v = -N\frac{d(BA)}{dt} = -\mu_0 n^2 lA\frac{di}{dt}$$

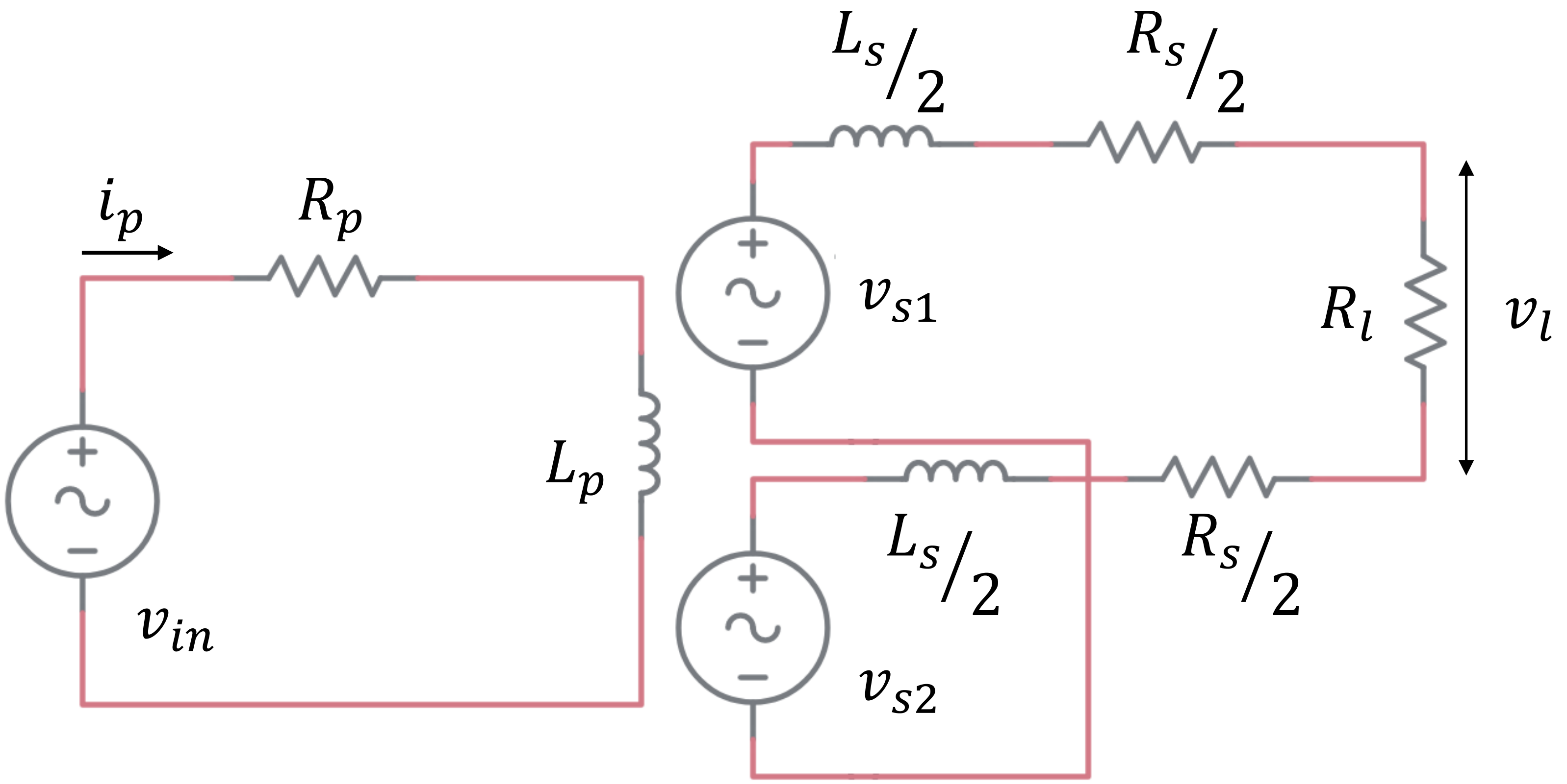
$$v = -L\frac{di}{dt}$$

$$L = \mu_0 n^2 lA$$

Linear Variable Differential Transformer (LVDT)



LVDT



$$i_p R_p + L_p \frac{di_p}{dt} = v_{in}$$

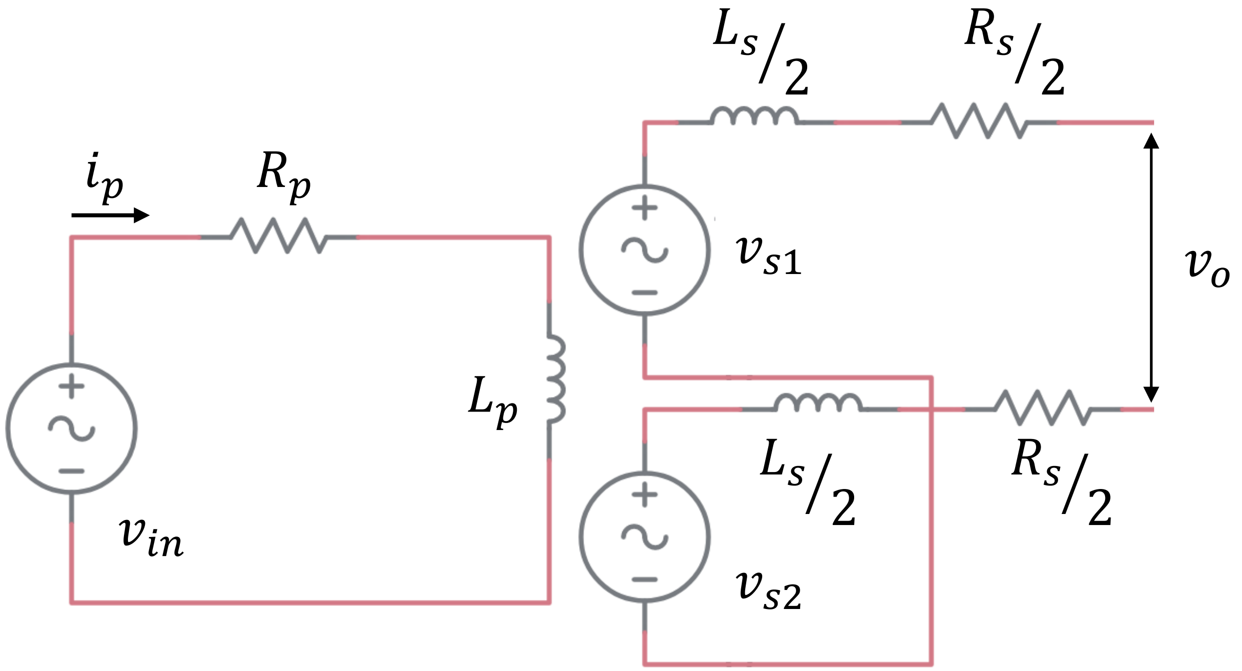
$$v_{s1} = M_1 \frac{di_p}{dt}$$

$$v_{s2} = M_2 \frac{di_p}{dt}$$

$$I_p(\omega) = \frac{V_{in}(\omega)}{R_p + j\omega L_p}$$

$$V_l(\omega) = \frac{R_l}{R_l + R_s + j\omega L_s} \frac{M_1 - M_2}{R_p + j\omega L_p} V_{in}(\omega)$$

LVDT



$$V_o(\omega) = \frac{j\omega}{R_p + j\omega L_p} \cdot (M_1 - M_2) \cdot V_{in}(\omega)$$

Let's assume the core is moved by x units

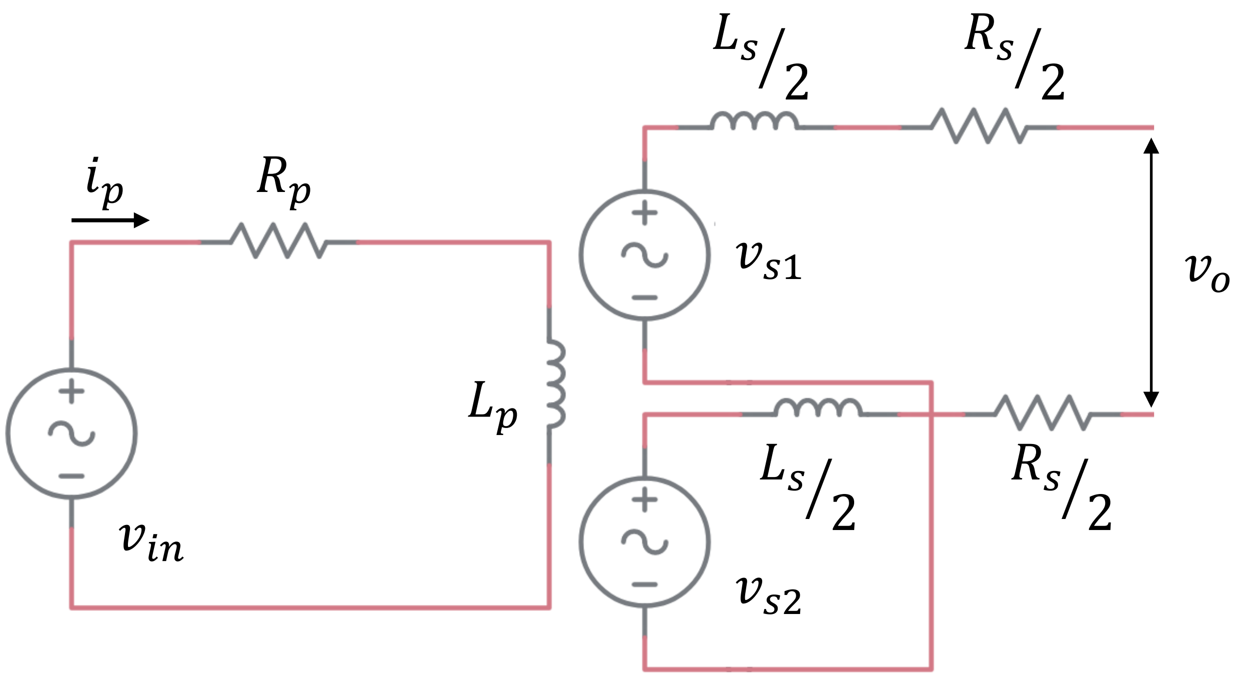
$$V_{s1}(\omega) = j\omega \cdot M_1(x)V_{in}(\omega) = j\omega \cdot M\left(\frac{L}{2} + x\right)V_{in}(\omega)$$

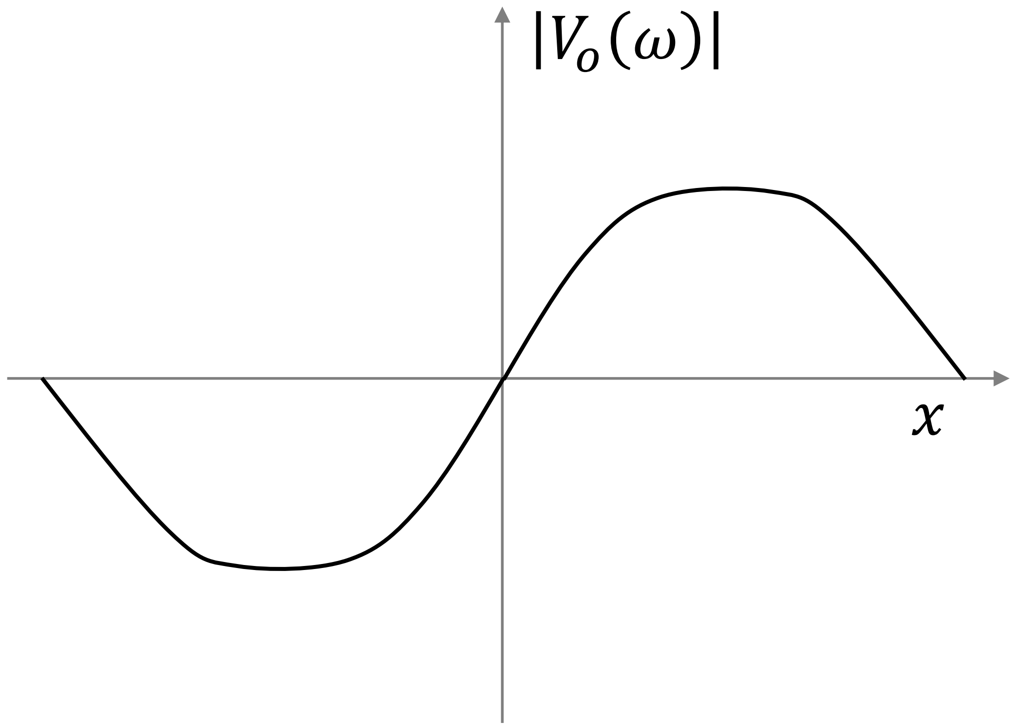
$$V_{s2}(\omega) = j\omega \cdot M_2(x)V_{in}(\omega) = j\omega \cdot M\left(\frac{L}{2} - x\right)V_{in}(\omega)$$

$$V_o(\omega) \propto j\omega \cdot \left(M\left(\frac{L}{2} + x\right) - M\left(\frac{L}{2} - x\right)\right)V_{in}(\omega)$$

$$V_o(\omega) \propto 2V_{in}(\omega) \cdot \left.\frac{\partial M}{\partial x}\right|_{x={}^{L}/_{2}} \cdot x$$

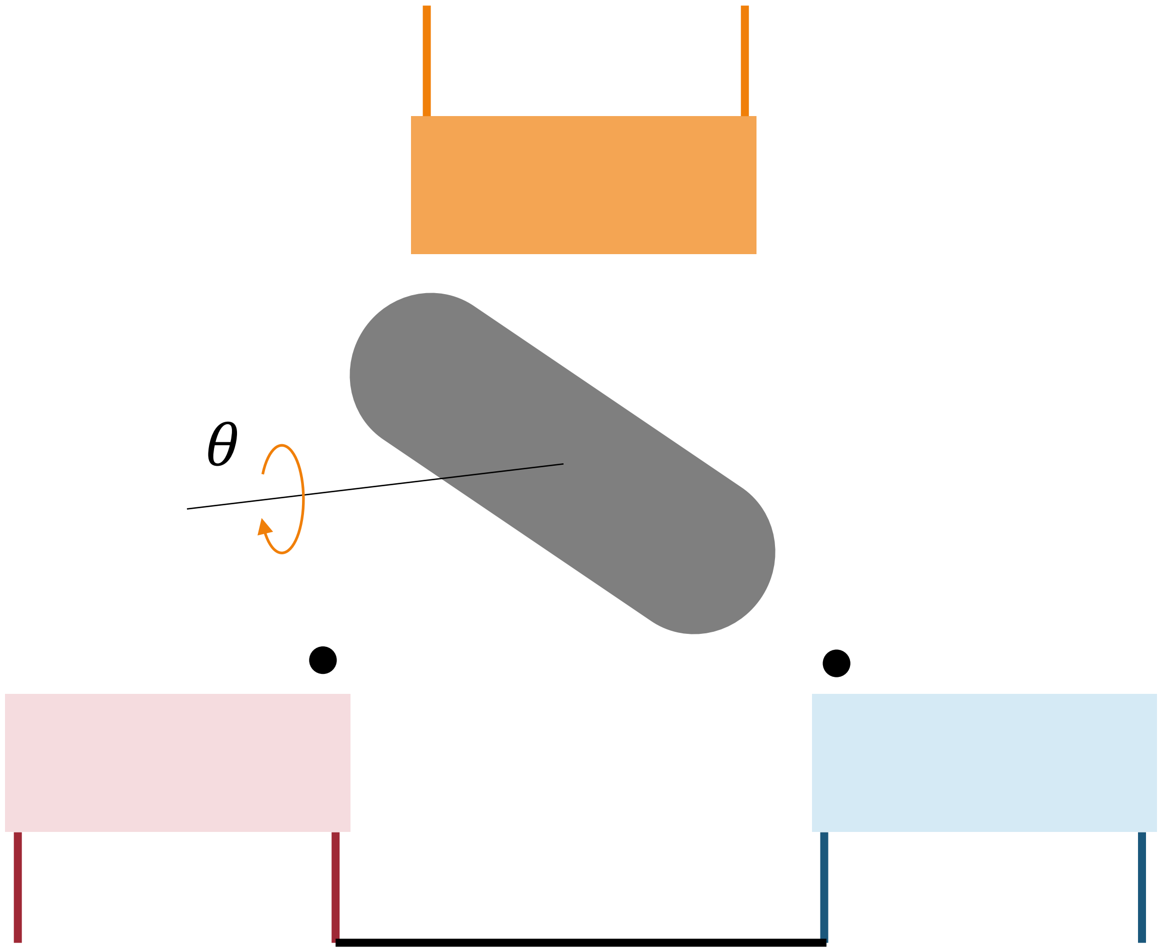
LVDT

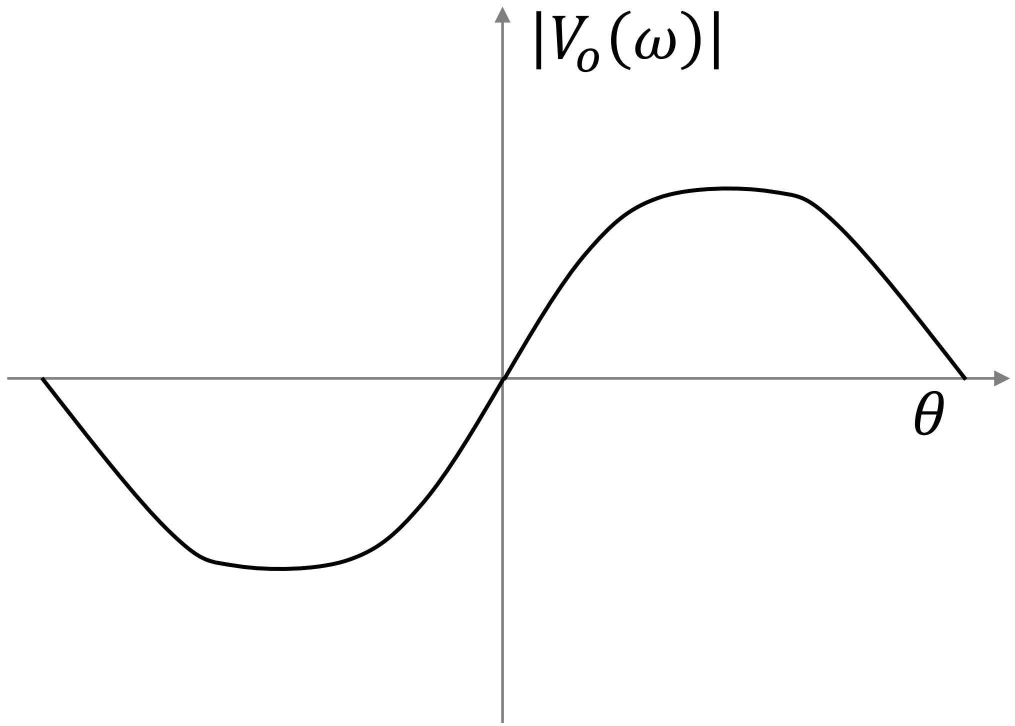




$$V_o(\omega) \propto 2V_{in}(\omega) \cdot \left.\frac{\partial M}{\partial x}\right|_{x=L/2} \cdot x$$

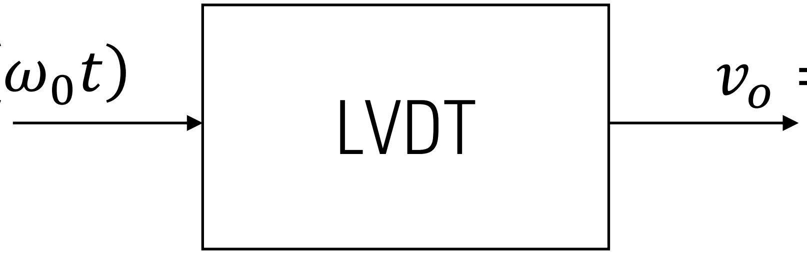
Rotary Variable Differential Transformer





LVDT Demodulation

$$v_p = \sin(\omega_0 t)$$



$$v_o = k \cdot x \cdot \sin(\omega_0 t + \varphi)$$

$$\begin{aligned}
v_p \cdot v_o &= k \cdot x \cdot \sin(\omega_0 t + \varphi)\sin(\omega_0 t) \\
&= k \cdot x \cdot (\cos(\varphi) - \cos(\omega_0 t + \varphi)) \\
&= k \cdot x \cdot \cos(\varphi) - k \cdot x \cdot \cos(\omega_0 t + \varphi)
\end{aligned}$$

LVDT: Interfering and Modifying Inputs

Interfering inputs

- Electrical noise in the circuit.
- External time-varying magnetic fields affecting the two secondary coils differently.

Modifying inputs

- Primary voltage
- Temperature dependence on magnetic permeability of the moving core.

LVDT

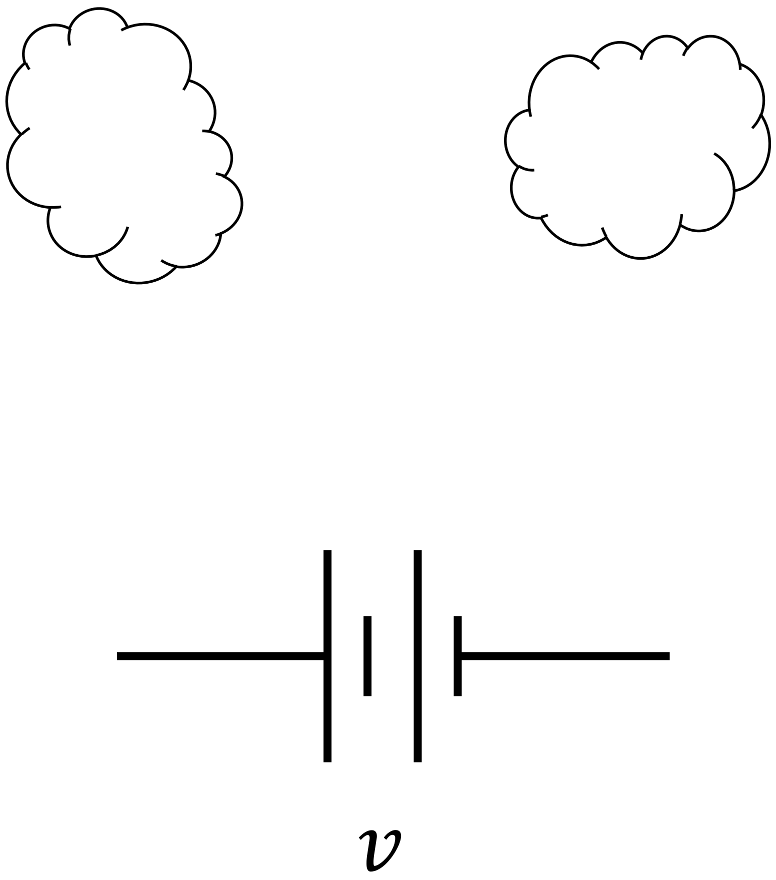
Advantages

- Good for measuring sub-mm to a few cm displacement.
- Reliable and durable; no direct surface contact with the sensing element.

Disadvantages

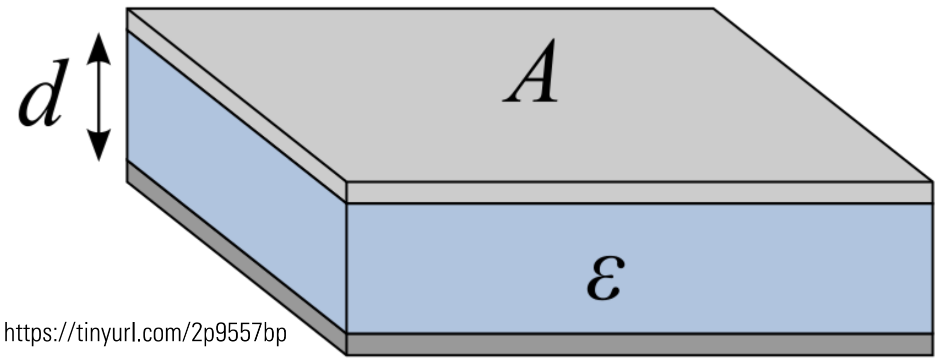
- Not suitable for frequencies greater than 1/10 of the primary frequency.
- Inertia of the core can be a problem for dynamic measurements.

Capacitance



$$C = \frac{q}{v}$$

Capacitance

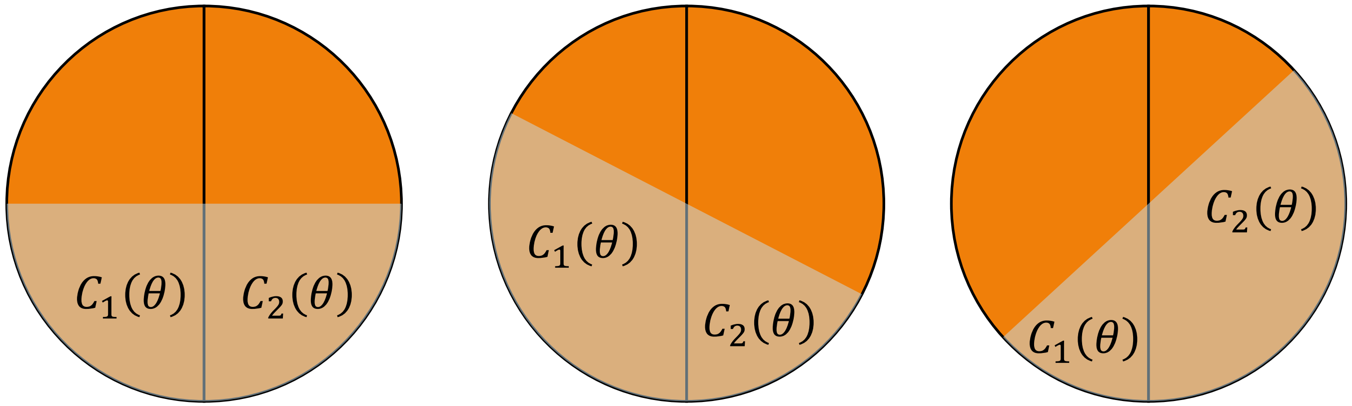


Source: https://tinyurl.com/2p9557bp

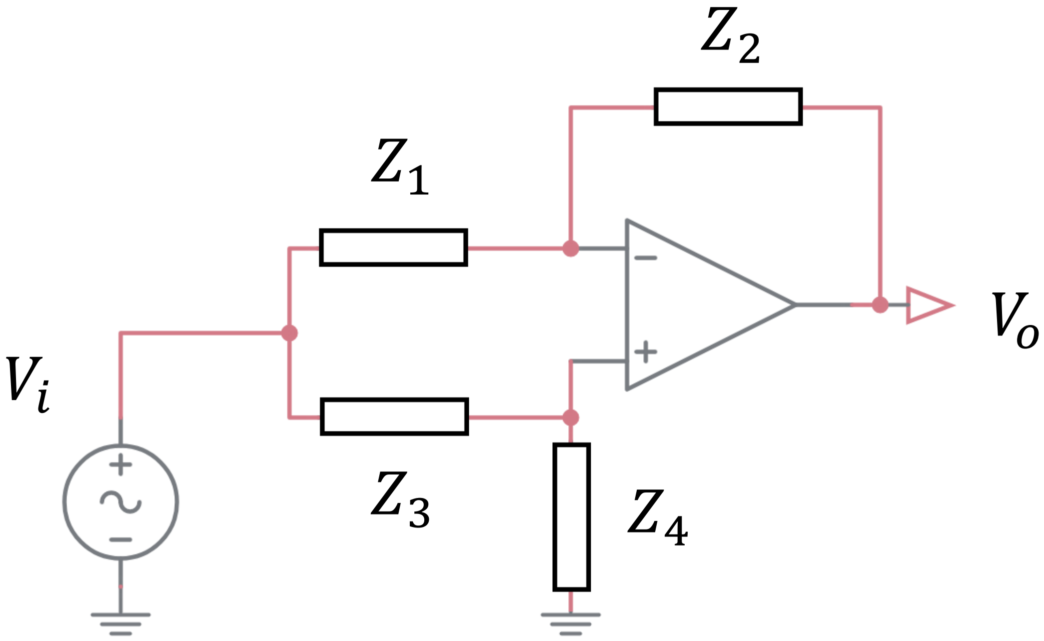
$$C = \varepsilon \frac{A}{d}$$

Capacitive sensors

Sensing rotational motion

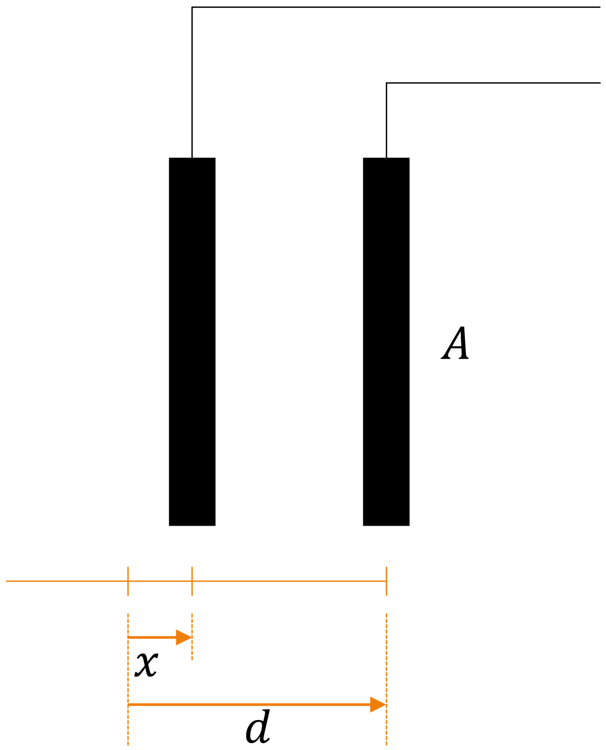


$C_1(\theta) = C_2(\theta)$ $\quad$ $C_1(\theta) > C_2(\theta)$ $\quad$ $C_1(\theta) < C_2(\theta)$

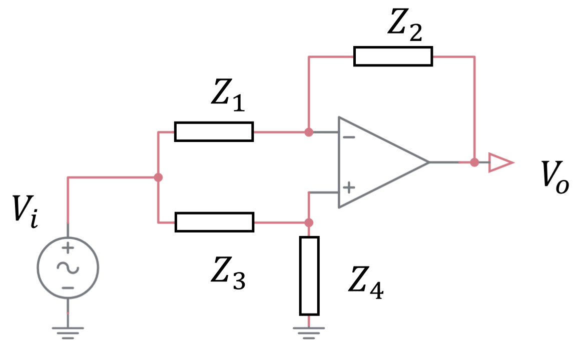


$$V_o = \frac{\left(\frac{Z_4}{Z_3} - \frac{Z_2}{Z_1}\right)}{1 + \frac{Z_4}{Z_3}} V_i$$

Capacitive sensors



$$C(x) = \frac{\epsilon_0 A}{d - x}$$



$$V_o = \frac{\left(\frac{Z_4}{Z_3} - \frac{Z_2}{Z_1}\right)}{1 + \frac{Z_4}{Z_3}} V_i$$

$$Z_4 = Z_3$$

$$Z_1 = \frac{1}{j\omega C(0)} = \frac{d}{j\omega \epsilon_0 A}$$

$$Z_2 = \frac{1}{j\omega C(x)} = \frac{d - x}{j\omega \epsilon_0 A}$$

$$V_o = \frac{\left(1 - \frac{d - x}{d}\right)}{2} V_i = \frac{x}{2} V_i$$

Capacitive sensors: Interfering and Modifying Inputs

Interfering inputs

- Electrical noise.
- Unwanted signal coupling through stray capacitance.

Modifying inputs

- Primary voltage.
- Temperature dependence of reluctance of the core.

Capacitive sensors

Advantages

- For very small displacement in the order of sub-mm.
- Reliable and durable; no direct surface contact with the sensing element.

Disadvantages

- Not suitable for frequencies greater than 1/10 of the primary frequency.
- Inertia of the core can be a problem for dynamics measurements.

Digital Encoder

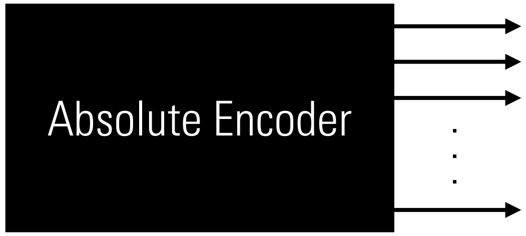
- Produce digital output for measuring translational and rotational motion.
- Information produced is in the form of pulses or in coded form.
- Interfaced to digital hardware for reading and further processing.

Shaft Encoder

Incremental Encoder

Pulse output, which is counted to measure change in position (or) width is used to measure velocity.

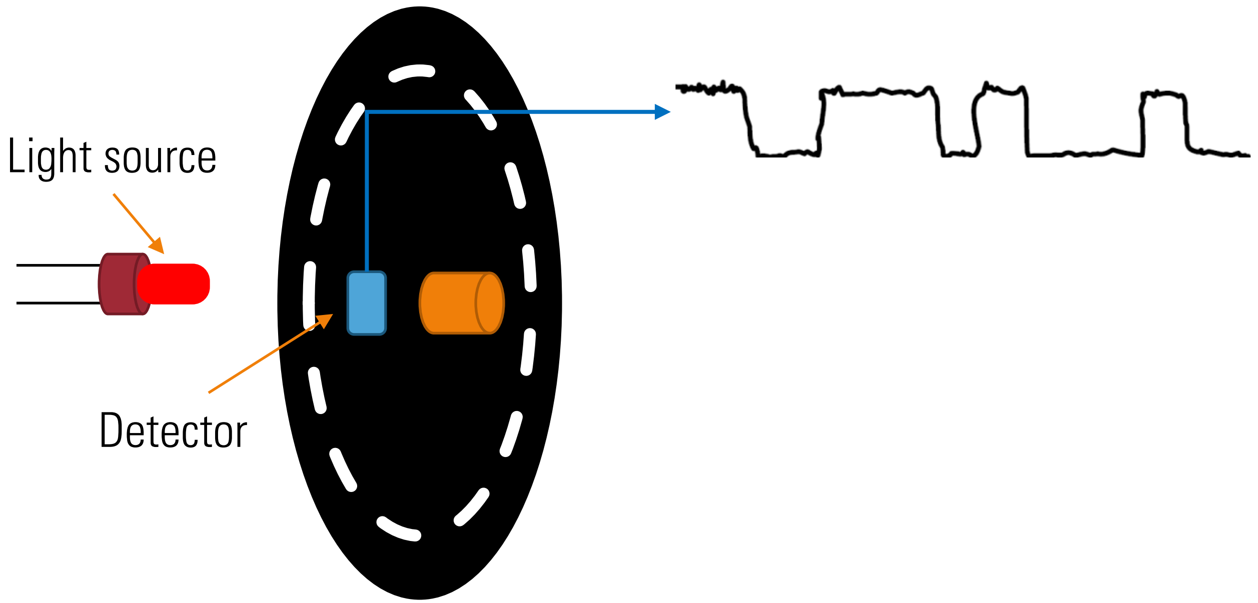
Absolute Encoder



Series of pulses where the position is already coded.

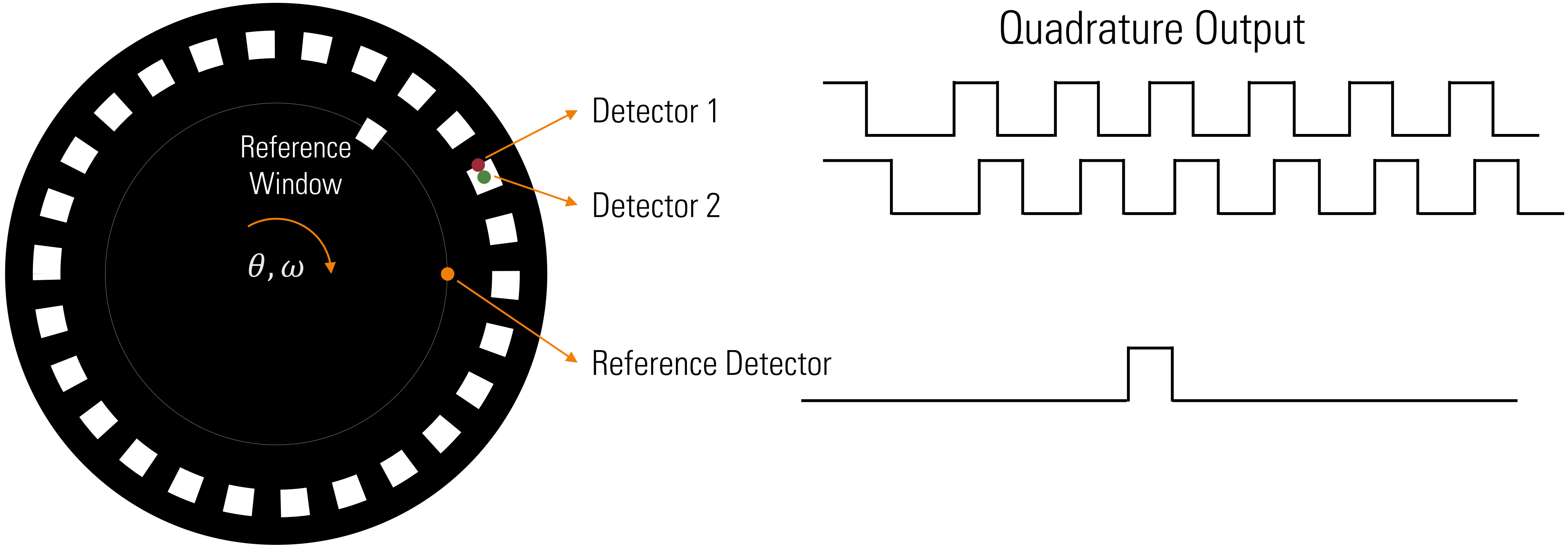
Shaft encoder

- Four different ways of transducer signal generation:
  1. Optical method
  2. Sliding contact method
  3. Magnetic saturation method
  4. Proximity sensor method



Shaft Encoder: Incremental Encoder

- Offset Sensor or Offset Track configurations.



Shaft Encoder: Incremental Encoder

- Resolution of the encoder:
  - Single Channel Output.

$$\Delta\theta = \frac{360}{N}$$

  - Quadrature Output.

$$\Delta\theta = \frac{360}{4N}$$