Introduction to DSP: Systems - Assignment: Fourier Representation of Signals

1. Find the Fourier series or transform of the following continuous-time signals.

   (a) $x(t) = e^{-at} \cdot 1(t), \quad a > 0$

   (b) $x(t) = e^{-a|t|}, \quad a > 0$

   (c) $x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ -e^{at}, & t < 0 \end{cases}, \quad a > 0$

2. Show that the inverse Fourier transform of $X(\omega) = \pi \left(\delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right)$ is $\cos(\omega_0 t)$.

3. Show that when a time-domain signal $x(t)$ is real, then $|X(\omega)|$ is an even function of ω and $\arg X(\omega)$ is an odd function of ω.

4. Show that the Fourier transform of a even function of time $x(t)$ is real, and that of a odd function of time is purely imaginary.

5. Find the DTFT of the following two signals, and plot their magnitude and phase responses:

   (a) $x[n] = \frac{1}{2} \left(\delta[n] + \delta[n-1] \right)$

   (b) $x[n] = \frac{1}{2} \left(\delta[n] - \delta[n-1] \right)$