Introduction to Digital Signal Processing

Systems

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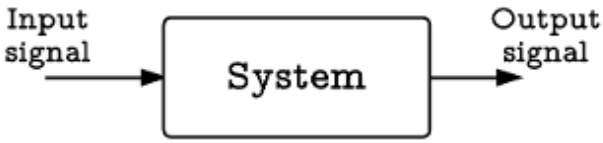
What is a system?

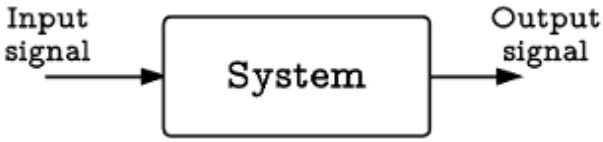
A collection of objects united by some form of interaction or interdependence[1].

From the signal processing point of view, **a system is any physical device or algorithm that performs some operation on a signal to transform it into another signal.** B



[1] Zadeh, Lotfi A., and Charles A. Deoser. *Linear system theory*. New York: McGraw Hill, 1963.

What is a system?



Can be thought of a mapping function. *e.g.*

$$y[n] = \mathcal{H}\left(x[n]\right)$$

Operations on signals

Operations on the dependent variable

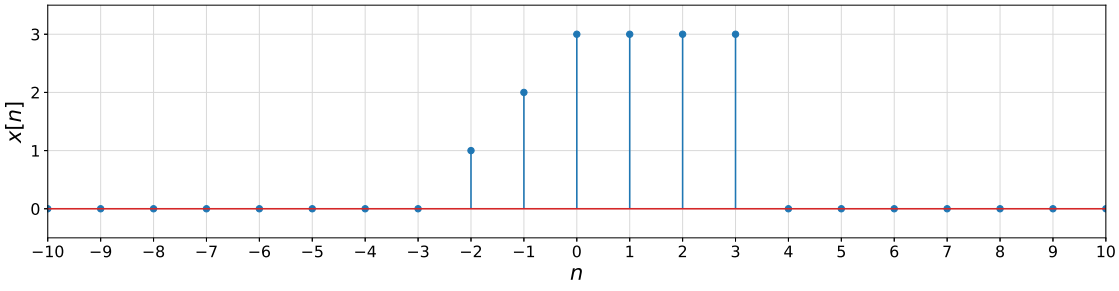
- **Scaling**: $y[n] = a \cdot x[n]$
- **Addition**: $y[n] = x_1[n] + x_2[n]$

Operations on the independent variable

- **Time shifting**: $y[n] = x[n-k], k \in \mathbb{Z}$
- **Time reversal**: $y[n] = x[-n], k \in \mathbb{Z}$

Operation on the independent variable: **Time shifting**

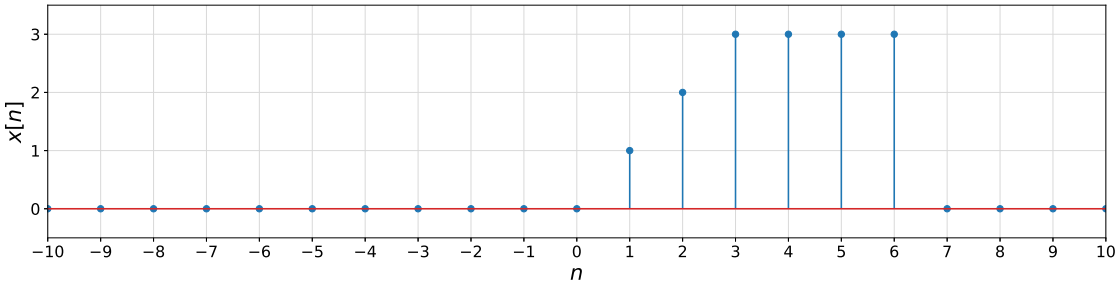
Consider $x[n]$ shown below,



What does $x[n-3]$ look like?

Operation on the independent variable: **Time shifting**

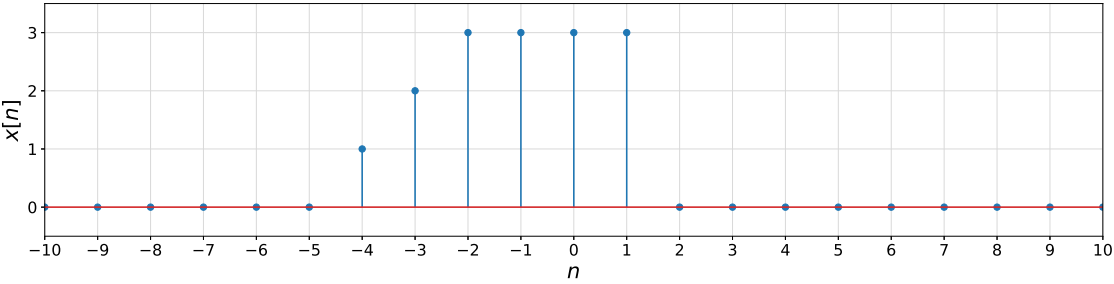
$x\left[n-3\right]$



What about $x\left[n+2\right]$?

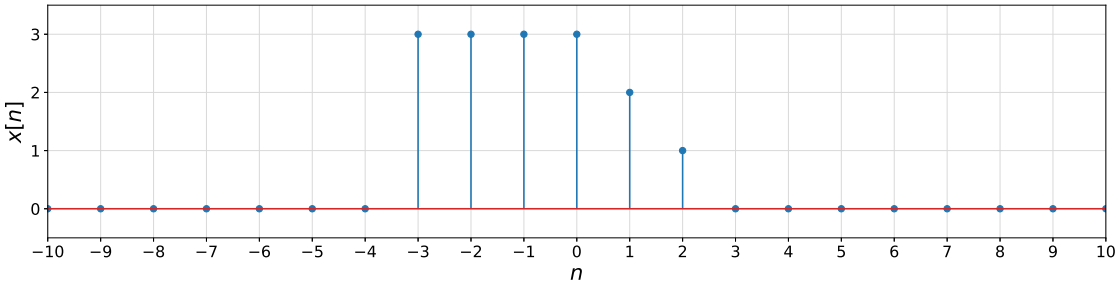
Operation on the independent variable: **Time shifting**

$x\left[n+2\right]$



Operation on the independent variable: **Time reversal**

$x\left[-n\right]$



And $x\left[-n\right]$?

Classification of systems

Based on the properties of the system.

- **Linearity**: *satisfies the properties of* **scaling** *and* **superposition**.
  Let us assume,

$$\mathcal{H} : x_i[n] \mapsto y_i[n]$$

  The system $\mathcal{H}$ is linear, if and only if,

$$\mathcal{H} : \sum_i a_i x_i[n] \mapsto \sum_i a_i y_i[n]$$

  Which of these systems are linear?

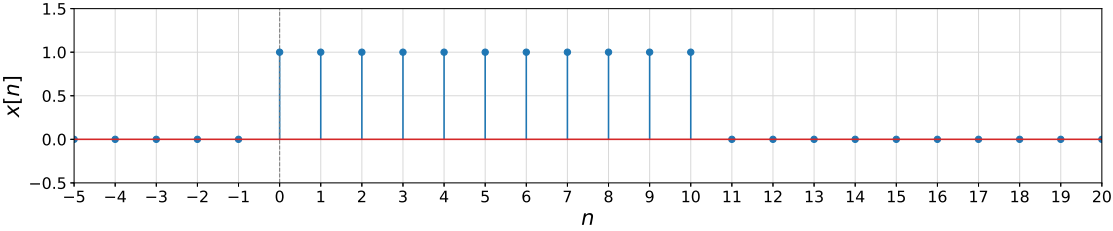
  1. $y[n] = k_1 x[n] + k_2 x[t-2]$
  2. $y[n] = \sum_{k=n-N}^{n} x[k]$
  3. $y[n] = 0.5x[n] + 1.5$

Classification of systems

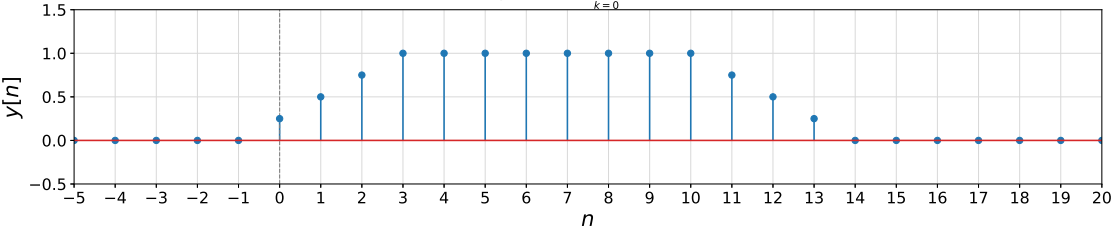
- **Memory**: *a system whose output depends on past or future values of its input is a system with memory, else the system is memoryless.*
  Note: the system may or may not depends on its present.

$$\begin{cases} y[n] = 0.5x[n] & \textbf{Memoryless system} \\ y[n] = \sum_{k=m_1}^{m_2} x[n-k] & \textbf{System with memory} \end{cases}$$

Classification of systems



$$y[n] = (1/4) \cdot \sum_{k=0}^{3} x[n-k]$$

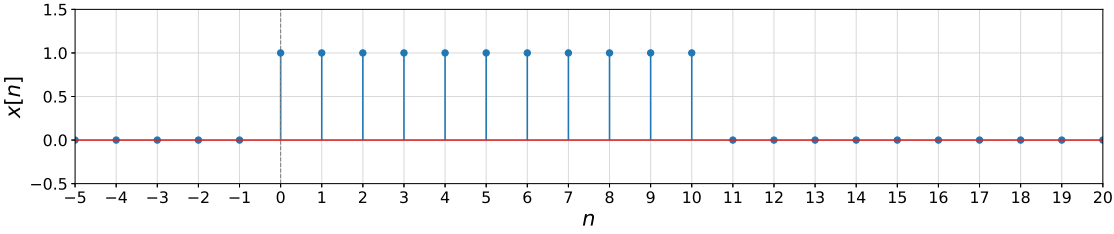


Classification of systems

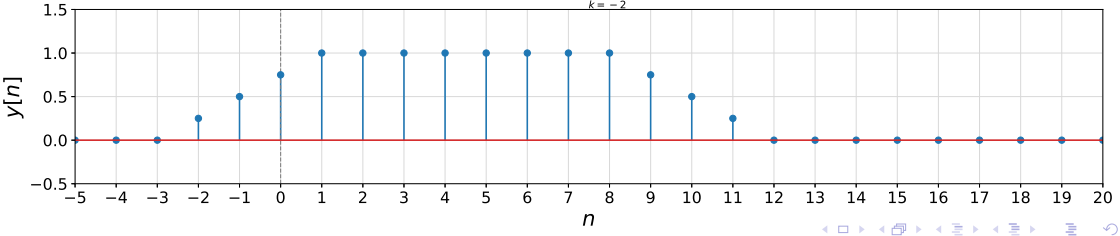
- **Causality**: *a system whose output depends on the past and present only values of the input is a causal system.*

Classification of systems

Causality



$$y[n] = (1/4) \cdot \sum_{k=-2}^{1} x[n-k]$$

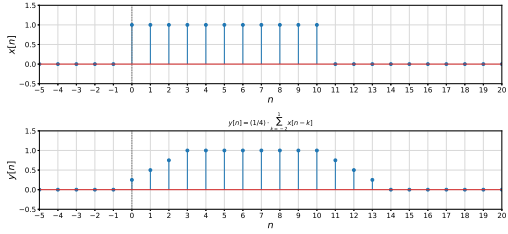


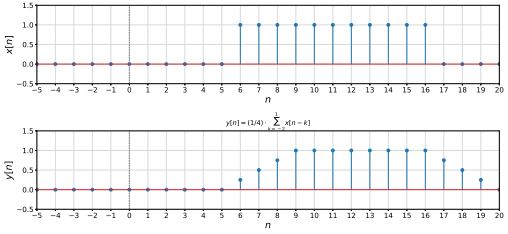
Classification of systems

- **Time invariance**: *system remains the same with time.*
  If a system is time-invariant, then

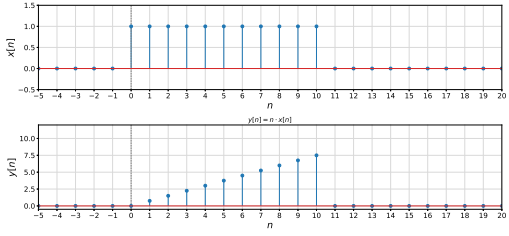
$$\mathcal{H} : x[n] \mapsto y[n] \iff f : x[n-k] \mapsto y[n-k]$$

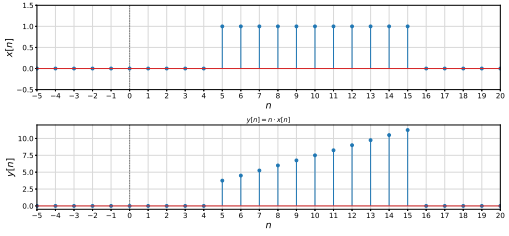
Classification of systems





Classification of systems





Classification of systems

- **Stability**: *bounded input produces bounded output*.

$$|x[n]| < M_x < \infty \mapsto |y[n]| < M_y < \infty$$