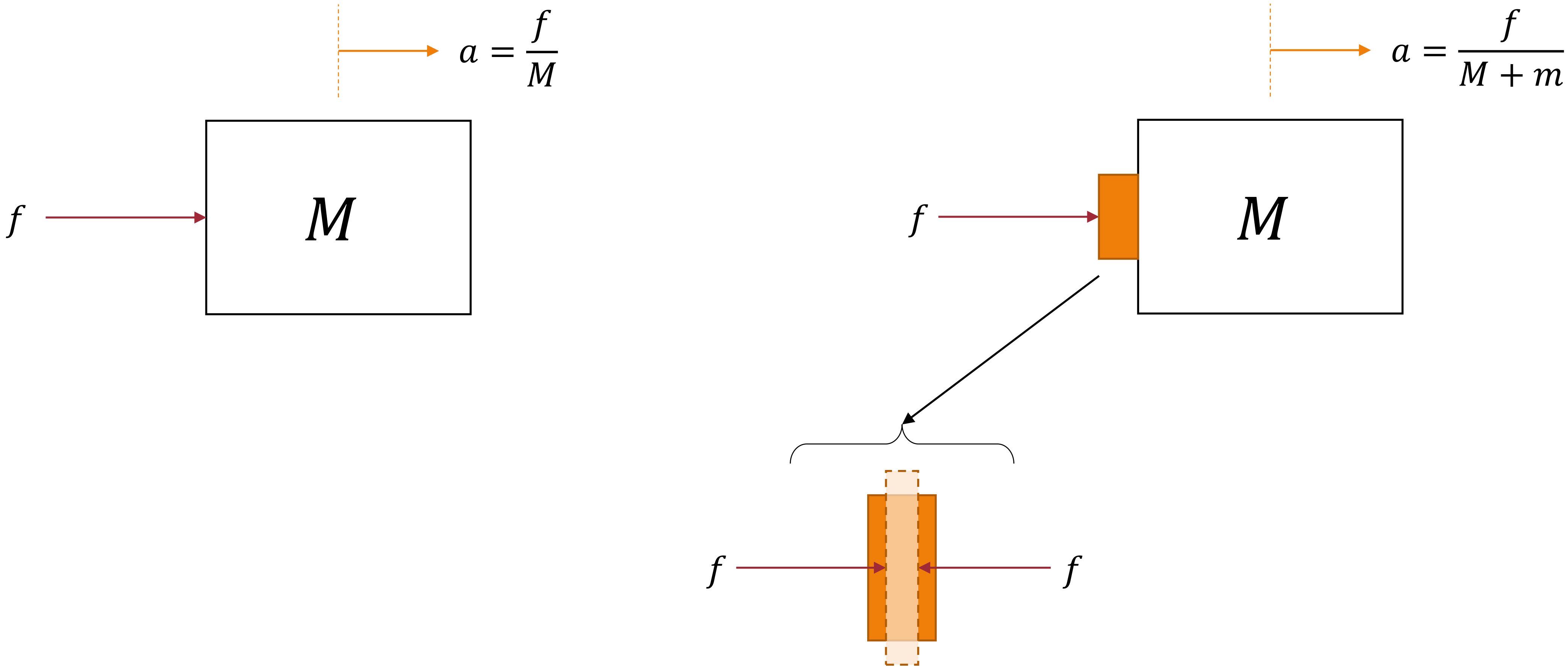
Transducers & Instrumentation

Module 04

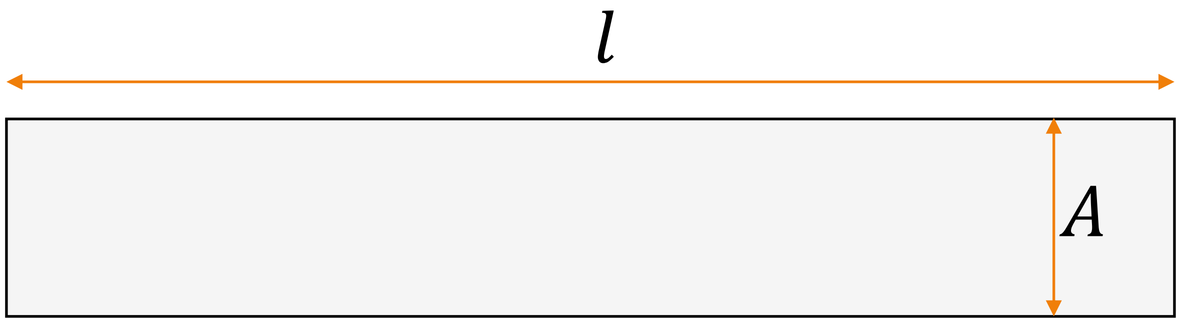
Measuring Forces and Torques

Forces and Torques

- Forces and torques cause bodies to undergo linear or rotational acceleration.
- Cause deformation.



Resistance of a material



$$R = \rho \frac{l}{A}$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l}(1 + 2\nu)$$

Piezoresistivity

Geometric factor

$$\frac{dR}{R} = S_s \cdot \varepsilon$$

Gauge factor

Strain $\left(\varepsilon = \frac{dl}{l}\right)$

$$S_s = \frac{d\rho/\rho}{\varepsilon} + (1 + 2\nu)$$

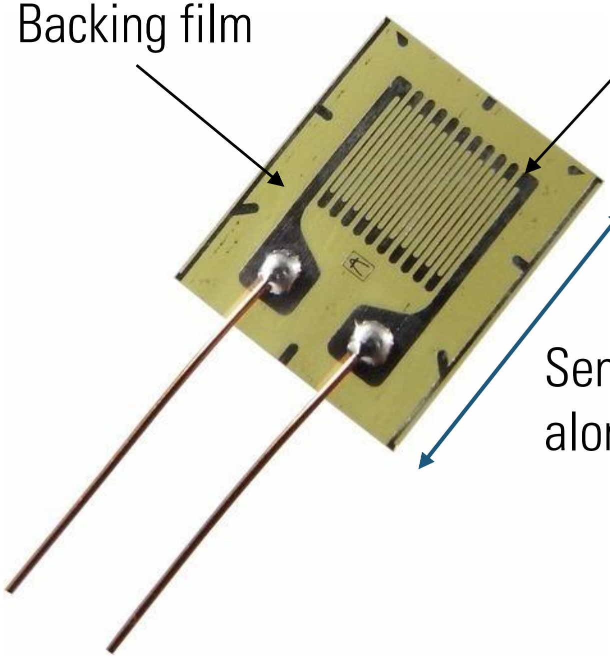
Gauge factors of common metals

Material	Sensitivity (S_s)
Platinum (Pt 100%)	6.1
Platinum-Iridium (Pt 95%, Ir 5%)	5.1
Platinum-Tungsten (Pt 92%, W 8%)	4.0
Isoelastic (Fe 55.5%, Ni 36% Cr 8%, Mn 0.5%) *	3.6
Constantan / Advance / Copel (Ni 45%, Cu 55%) *	2.1
Nichrome V (Ni 80%, Cr 20%) *	2.1
Karma (Ni 74%, Cr 20%, Al 3%, Fe 3%) *	2.0
Armour D (Fe 70%, Cr 20%, Al 10%) *	2.0
Monel (Ni 67%, Cu 33%) *	1.9
Manganin (Cu 84%, Mn 12%, Ni 4%) *	0.47
Nickel (Ni 100%)	-12.1

Source: https://www.efunda.com/designstandards/sensors/strain_gages/strain_gage_sensitivity.cfm

Strain gauges

Source: https://5.imimg.com/data5/BG/MR/MY-41643329/strain-gauge-500x500.jpg

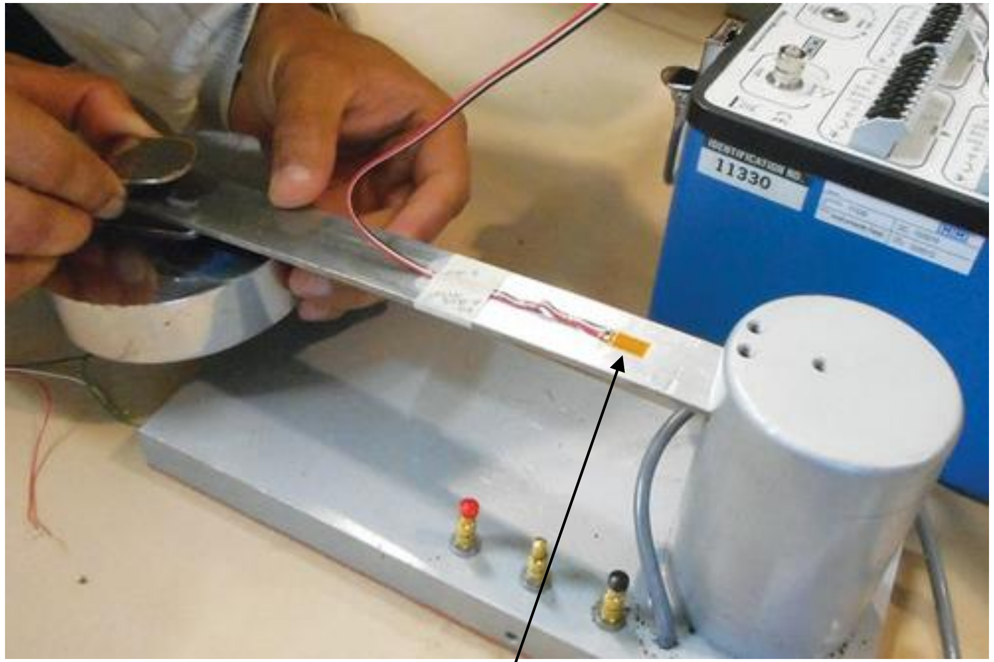


Backing film

Strain gauge made from metal foil.

Sensitive to length changes along this direction.

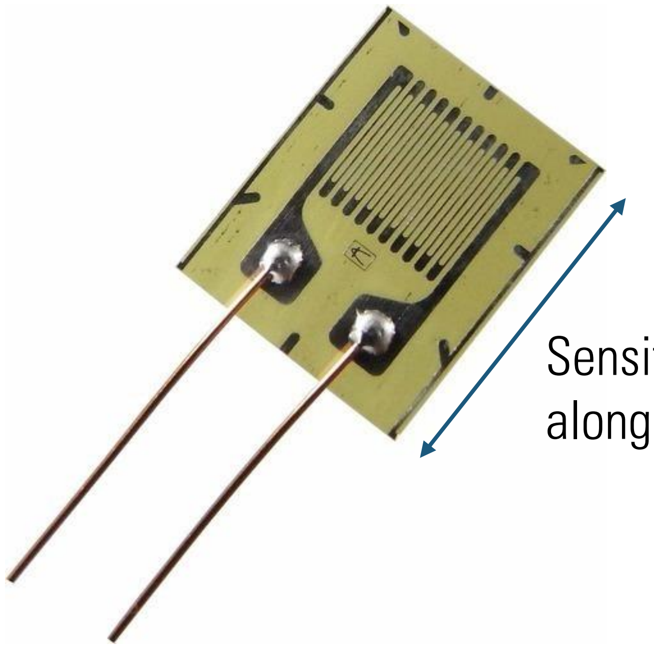
Source: https://resourcespace-prod.s3.amazonaws.com/rs/filestore/1/1/6/3/9_981464e59f794c3/116391ai_b5fc4cbf03762f5.jpg



Strain gauge bonded to a cantilever.

Strain gauges

Source: https://5.imimg.com/data5/BG/MR/MY-41643329/strain-gauge-500x500.jpg



Sensitive to length changes along this direction.

Problem:

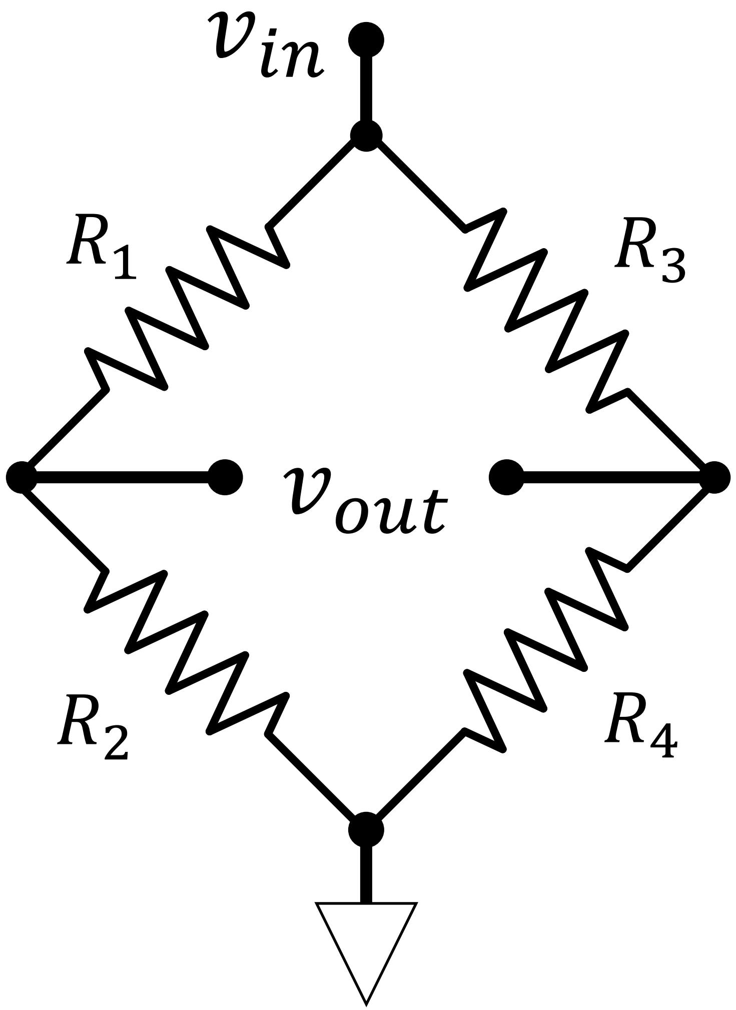
Strain gauge with GF 2.5 and resistance 150 ohms.

Experiences a 400 micro-strain

What is the change in resistance of the strain gauge?

Measuring resistance change of strain gauges

- Wheatstone Bridge

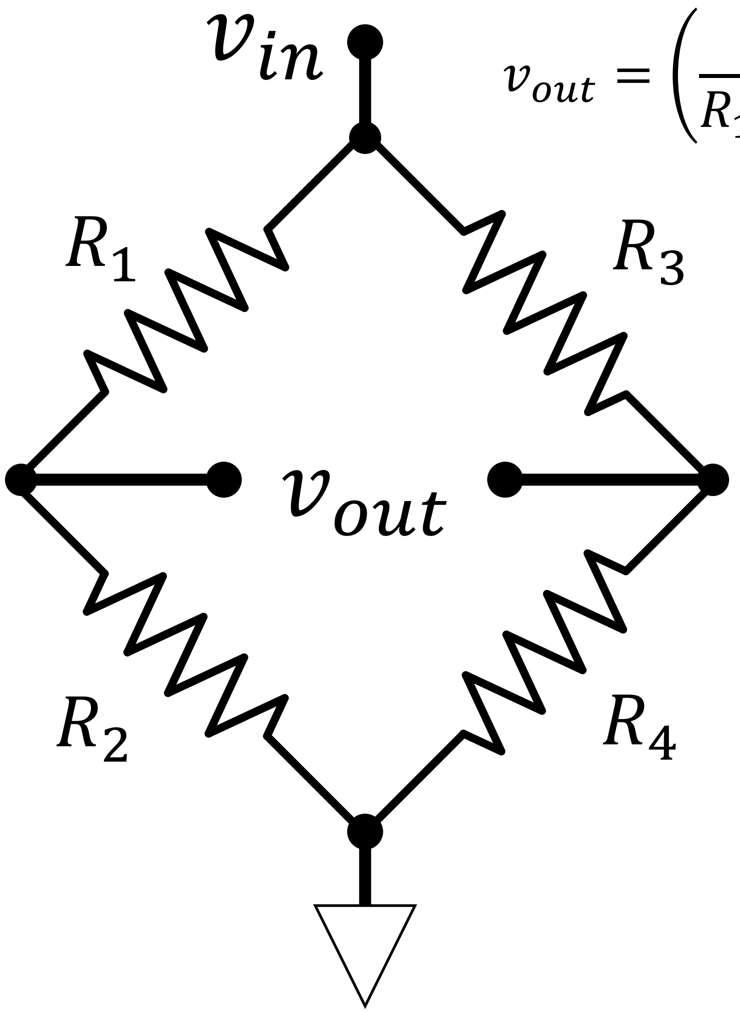


$$v_{out} = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) v_{in}$$

$$v_{out} = \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_2)(R_3 + R_4)} v_{in}$$

The bridge is balanced when, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$.

Wheatstone bridge with a single strain gauge



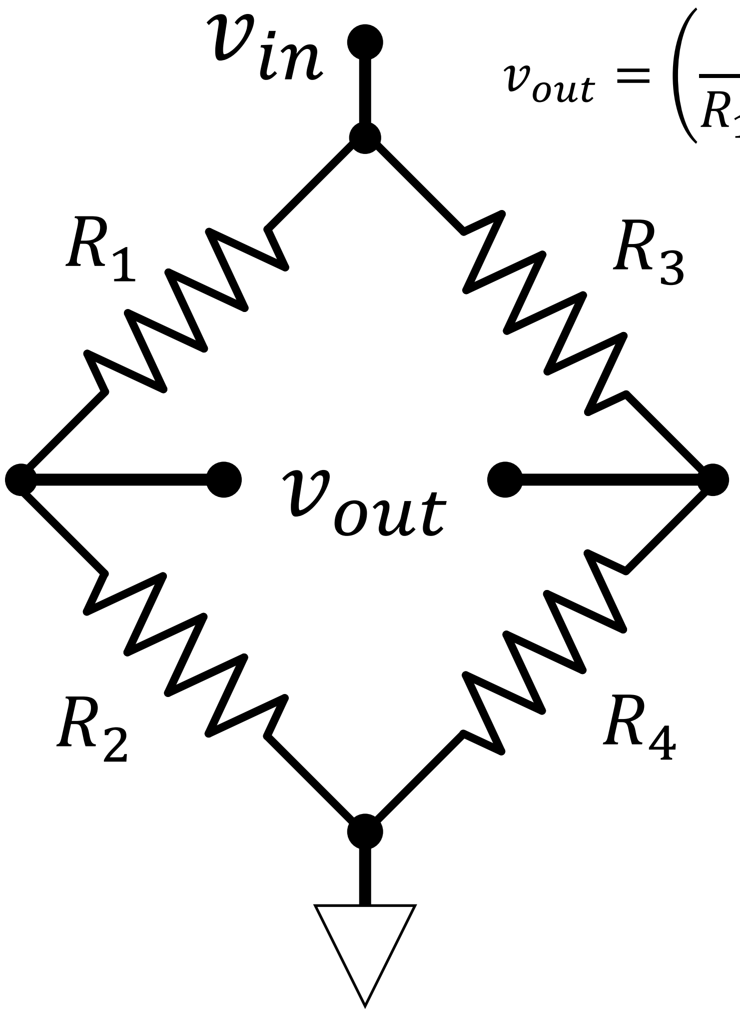
$$v_{out} = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) v_{in}$$

$$R_1 = R_2 = R_3 = R_4 = R$$

Let's assume R_2 changes by ΔR.

$$\frac{\Delta v_o}{v_i} = \frac{\Delta R}{4\left(R + \frac{\Delta R}{2}\right)}$$

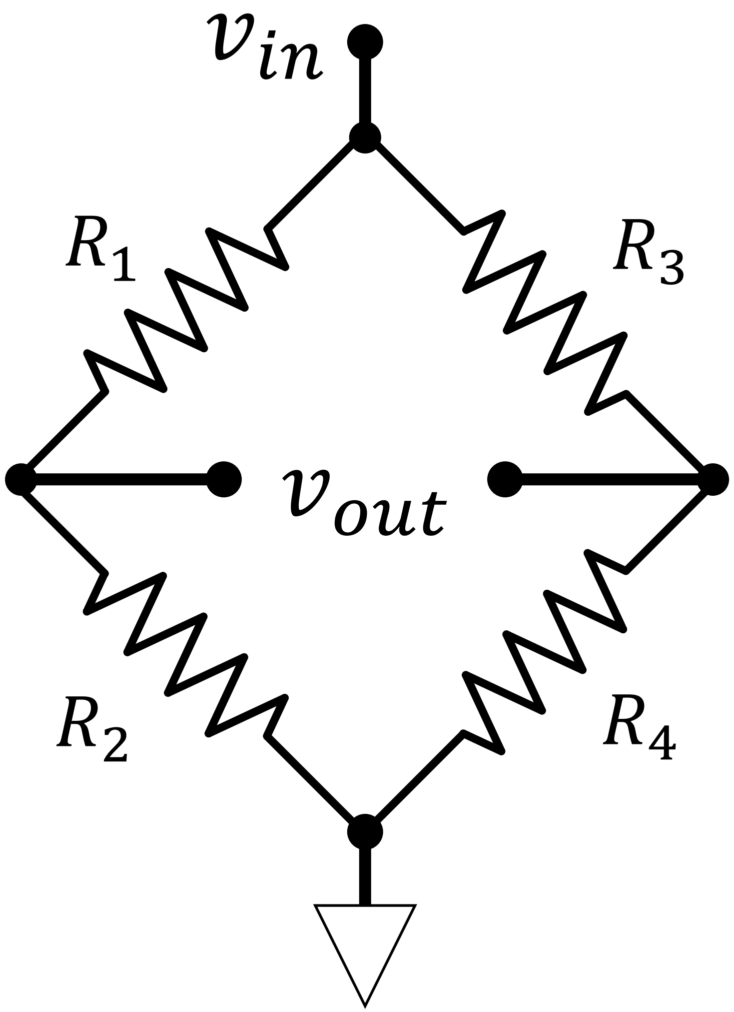
Wheatstone bridge with a single strain gauge



$$v_{out} = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) v_{in}$$

$$R_1 = R_2 = R_3 = R_4 = R$$

Bridge Constant (k) & Calibration Constant (C)

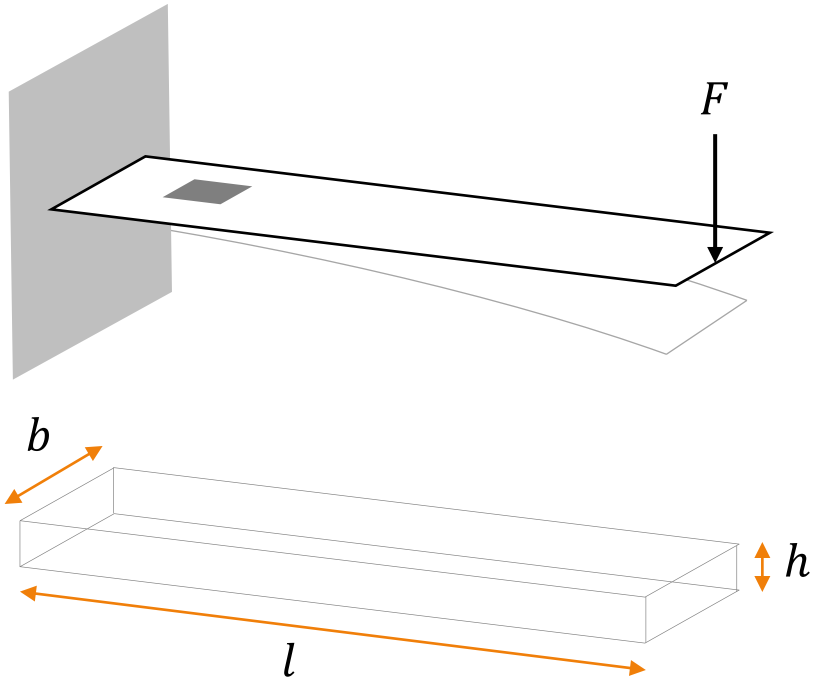


$$\frac{\Delta v_o}{v_i} = k\,\frac{\Delta R}{4R'}$$

$$\frac{\Delta v_o}{v_i} = C \cdot \varepsilon = \frac{k}{4} S_s$$

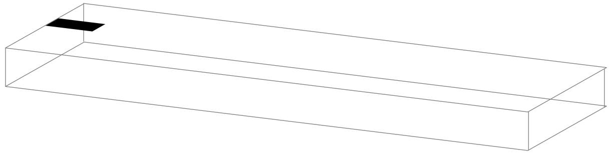
Some load-cell configurations

Cantilever based load-cell

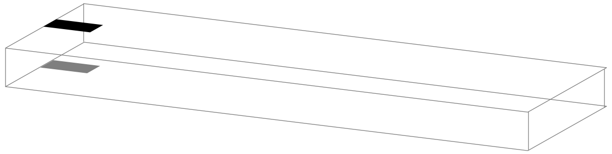


$$\sigma = \frac{6Fl}{bh^2} \qquad \varepsilon = E\sigma$$

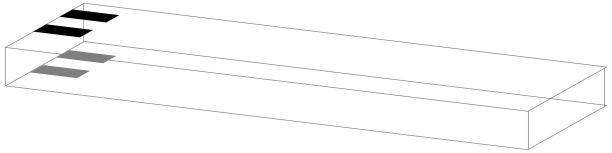
Single strain gauge



Double strain gauges

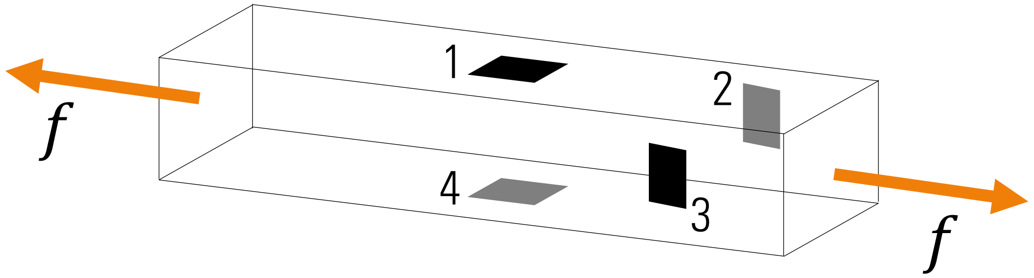


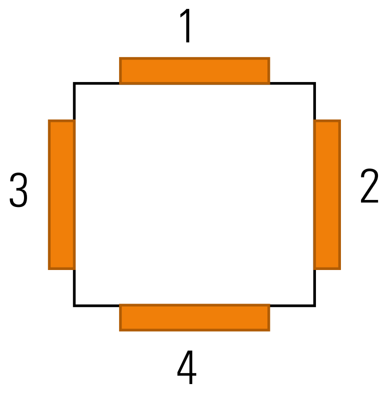
Four strain gauges

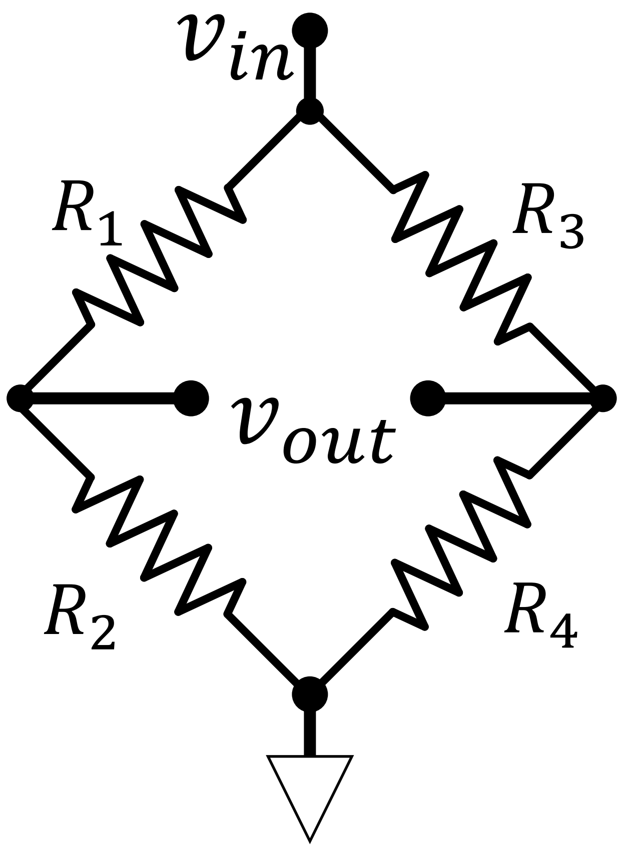


Some load-cell configurations

Tension/compression load-cell

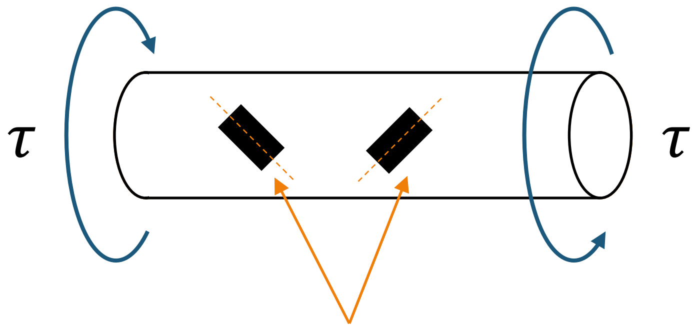






$$\frac{\delta v_{out}}{v_{in}} = 2(1+\nu)\frac{\delta R}{R'}$$

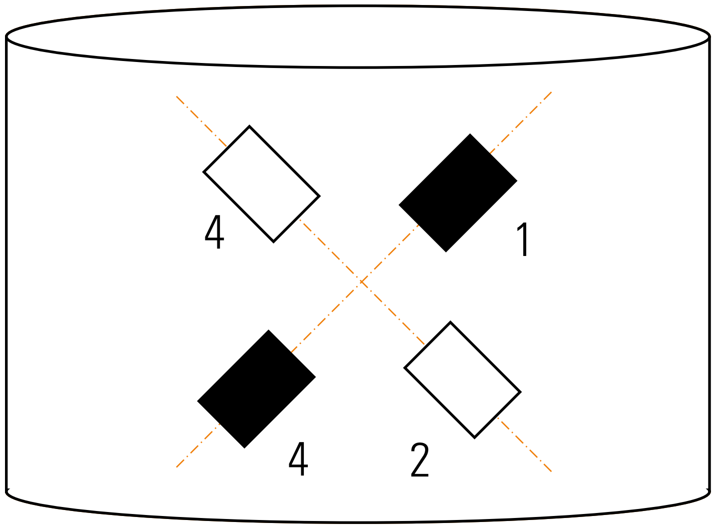
Strain gauge torque sensors



Directions of maximum tensile stresses.

$$T = \frac{\tau r}{J}$$

$$\varepsilon = \frac{r}{2GJ}\tau$$



Torque computed from

$$\tau = \frac{8GJ}{JS_s r}\frac{\delta v_{out}}{v_{in}}$$

Semiconductor strain gauges

- Highly sensitive to small strains compared to metallic foil strain gauges.

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l}(1 + 2\nu)$$

- Change of resistivity due to a mechanical strain: **Piezo-resistivity**.

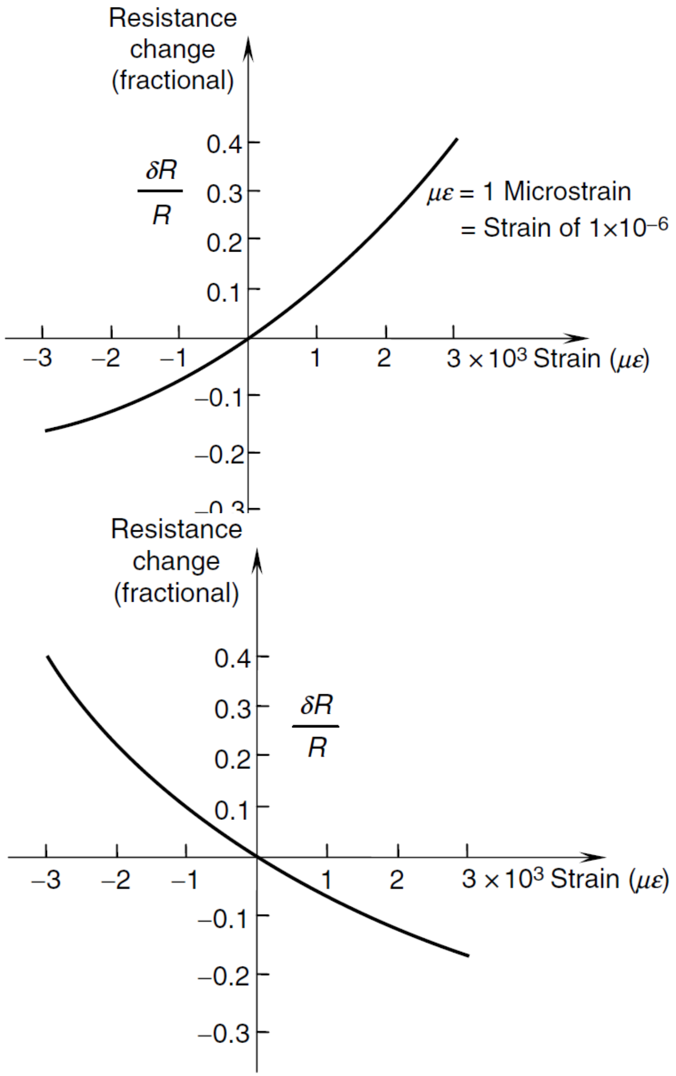
$$\rho_\varepsilon = \frac{d\rho/\rho}{\varepsilon}$$

- Gauge factor: 100 to 170 for p-type Silicon, -140 to -100 for n-type Silicon.
- Higher resistivity, lower power consumption, lower heat dissipation.
- Metal foil strain gauge: 150-350 ohms, SC strain gauge: 5000 ohms

Semiconductor strain gauges

- Some disadvantages:
  - Non-linear strain-resistance relationship.
  - Brittle and difficult to mount on curved surfaces.
  - Max. measurable strain is an order of magnitude lower.
  - Costly
  - More sensitive to temperature.
- Normalized resistance change of SC strain gauges:

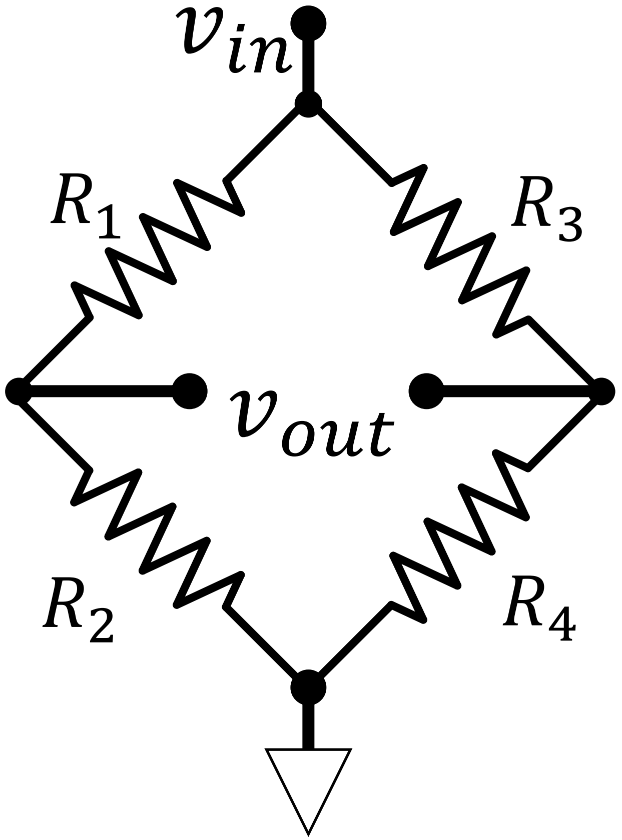
$$\frac{dR}{R} = S_1\varepsilon + S_2\varepsilon^2$$



Source: De Silva, C W, Sensors and Actuators, CRC Press (2007)

Temperature compensation

$$\Delta R = \Delta R_s + \Delta R_T$$



Temperature compensation

- Resistance of materials are sensitive to temperature.

- Resistivity changes with temperature.

$$\rho_T = \rho_0 \cdot (1 + \alpha \cdot \Delta T)$$

This linear approximation works when $\alpha \cdot \Delta T \ll 1$.

- Semiconductor strain gauges are more sensitive than metal foil strain gauges.

- For semiconductor strain gauges the resistance change is larger, and the gauge factor is also temperature dependent.

$$S_{s_T} = S_{s_0} \cdot (1 + \beta \cdot \Delta T)$$