Introduction to Digital Signal Processing

Mathematical Preliminaries

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Sets

- A set is a collection of distinct objects or elements.
- The definition of a set must make it clear to find out if an element belongs or does not belong to a set.
- Sets allow us to establish the universe of things that we are dealing with.
- Elements of a set are unique.
- Set are often represented by captial letters.

$$A = \{1, 2, \pi, Orange\}$$

$$B = \{n \mid n \text{ is an even non-zero integer}\}$$

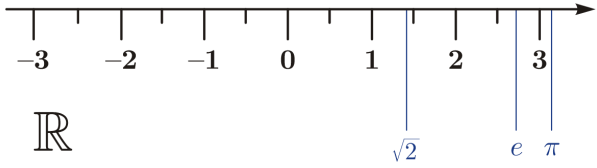
Sets

Notations for some standard sets:

- Set of natural numbers.
$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$
- Set of integers.
$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$$
- Set of rational numbers.
$$\mathbb{Q} = \left\{ \frac{n}{m} \;\middle|\; n, m \in \mathbb{Z} \right\}$$
- Set of real numbers. $\mathbb{R}$
- Set of complex numbers. $\mathbb{C}$

Real numbers

The value of a continuous quantity, which can be represented as a distance on a line. This the familiar idea of a **real number line**.



What type of a number would we use for the following purposes?

1. The age of a person in years.
2. Cost of 3Kgs of banana (assuming we do not have fractions of a rupee).
3. Solution of the equation: $x^2 = 2$

Functions

- A function is a relationship that associates elements from one set to exactly one element in another set.
- Let f be a function from set A to set B. We write, $f : A \mapsto B$.

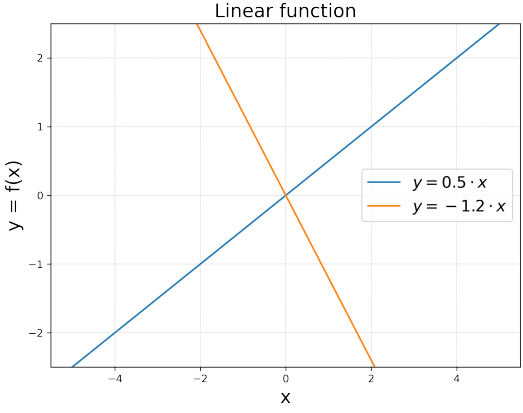
$$y = f(x), \text{ where, } x \in A, \ y \in B$$

- Every element of A is mapped to an element in B
- Every element of A is only mapped to one element in B.
- A is called the **domain** of f, and B is called the **range** of f.

Functions

Linear function $f : \mathbb{R} \mapsto \mathbb{R}$

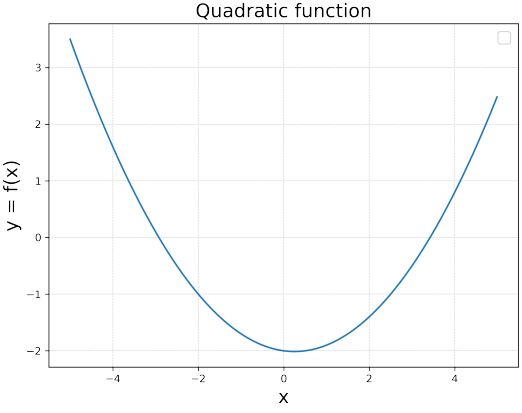
$$y = f(x) = k \cdot x, \ k, x \in \mathbb{R}$$



Functions

Quadratic function $f : \mathbb{R} \mapsto \mathbb{R}_{\geq 0}$

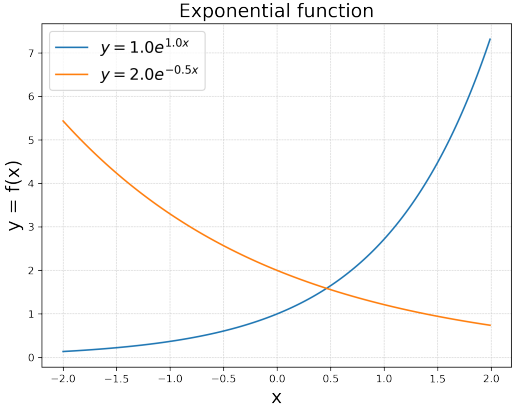
$$y = f(x) = ax^2 + bx + c, \quad a, b, c \in \mathbb{R}$$



Functions

Exponential function $f : \mathbb{R} \mapsto \mathbb{R}_{\geq 0}$

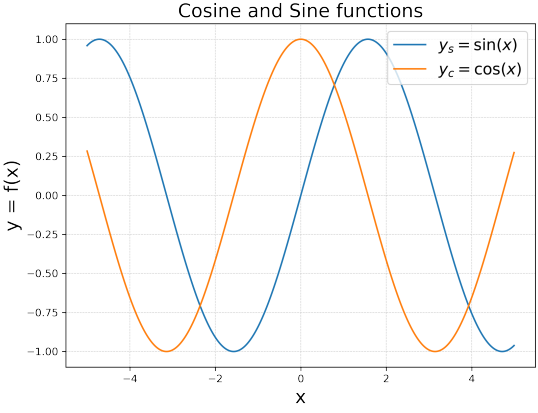
$$y = f(x) = ae^{kx}, \quad a, k, e \in \mathbb{R}$$



Functions

Sine and Cosine function $f : \mathbb{R} \mapsto \mathbb{R}$

$$y_s = \sin(x) \quad \text{and} \quad y_c = \cos(x)$$



Complex numbers

$$\mathbb{C} = \left\{ a + ib \;\middle|\; a, b \in \mathbb{R} \right\}$$

where, $i = \sqrt{-1} \implies i^2 = -1$.

- A complex number $x = a + ib$ consist of two components:
  - **real** part $- a = \text{Re}(z)$
  - **imaginary** part $- b = \text{Im}(z)$
- Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$, then,
- **Complex Addition**

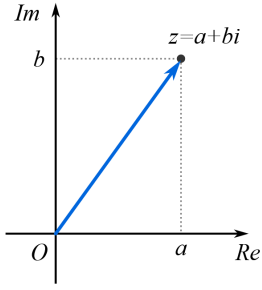
$$z_3 = z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$$

- **Complex Multiplication**

$$z_3 = z_1 \times z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_1 b_2)$$

Complex numbers

- **Complex conjugate of a complex number** $\overline{z} = \overline{a + ib} = a - ib$
- **Length of a complex number** $|z|^2 = z\overline{z} = (a + ib)(a - ib) = a^2 + b^2$
- **Geometry of complex numbers**



- **Euler forumla** $z = a + ib = re^{i\theta} = r\cos(\theta) + ir\sin(\theta) = |z|e^{i\arg(z)}$
  where, $r = |z| = \sqrt{a^2 + b^2}$, and $\theta = \arg(z) = \text{atan2}(b, a)$.