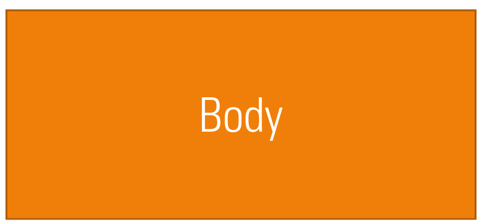
# Transducers & Instrumentation

Module 06

Measuring Temperature

# Temperature

- A macroscopic property of a body → It's a measure of the average kinetic energy of the molecules of an object.




- Thermometer – Produces a physical response to the temperature of a body through physical contact or EM radiation.

# Temperature scales

- Three common scales:
  - Kelvin scale $(K)$
  - Celsius scale $(°\mathrm{C})$
  - Fahrenheit scale $(°\mathrm{F})$

- **Kelvin scale**: Based on the *triple point* of water, where all three states exist at a pressure of 4.58mmHg. This is defined at 273.16K.
  - 0 K is the absolute lowest temperature where all particles have zero kinetic energy

- **Celsius scale:** Based on the melting and boiling point of water at atmospheric pressure at sea level.
  - $1°\mathrm{C} = 1K$

# Modes of heat transfer

## Conduction

$$\dot{Q} = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$k$ : Thermal conductivity

## Convection

Requires and intermediate fluid to take heat from the warmer body to the cooler body.

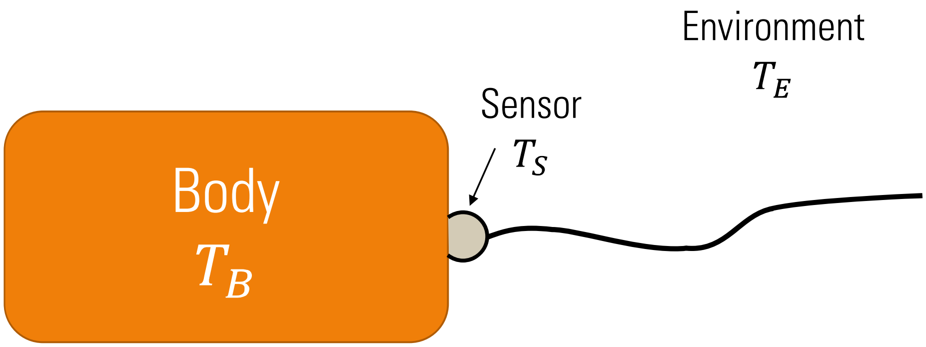
## Radiation

Vibrating atoms and molecules generate electromagnetic radiation.

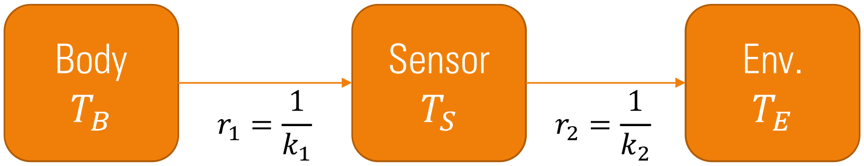
Temperature of the body determines the wavelength of the emitted radiation.

Total energy emitted will also depend on the emissivity.

# Temperature measurement – Static case

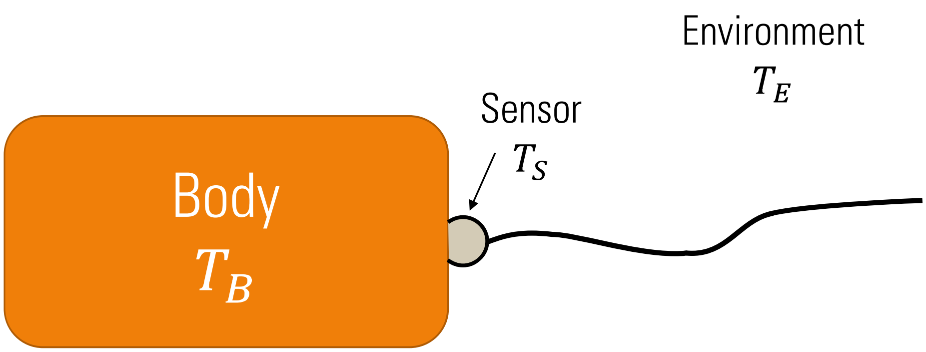



- Contact-based sensors.
- Requires flow of heat between the sensor and the body to measure $T_B$.
- Static measurement of temperature



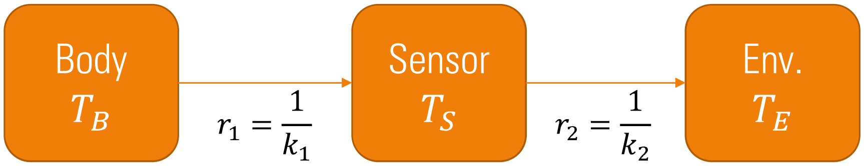

$$T_s = \left(\frac{r_2}{r_1 + r_2}\right) T_s + \left(\frac{r_1}{r_1 + r_2}\right) T_E$$

# Temperature measurement – Dynamic case




- Dynamic measurement of temperature.
- Assuming infinite capacity for the body, and infinite resistance for $r_2$.

$$T_s(t) = T_B - \left(T_B - T_S(0)\right)e^{-t/\tau}$$



# Resistance temperature detectors (RTD)

- Resistance of materials change with temperature.
- Variation of resistance of metals can be approximated through the following relationship:

$$R_{T_1} = R_{T_0}[1 + \alpha_0(T_1 - T_0)]$$

- $\alpha_0$ is the temperature coefficient of resistivity of the material.
- Most metals have 0.3-0.5% change in resistance per degree Celsius.
- Platinum is often preferred due to is wide linear resistance-temperature relationship.

# Resistance Temperature Detector (RTD)

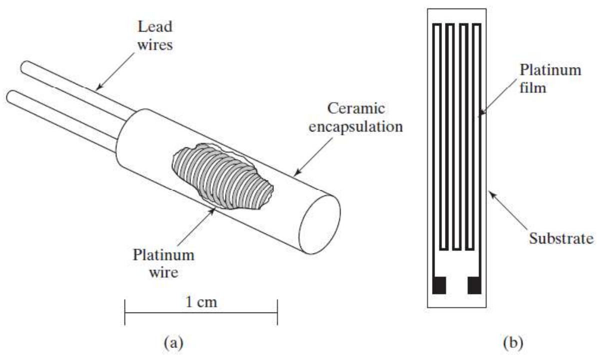



FIGURE 9.20

Resistance temperature detectors: (a) platinum wire; (b) thin film.

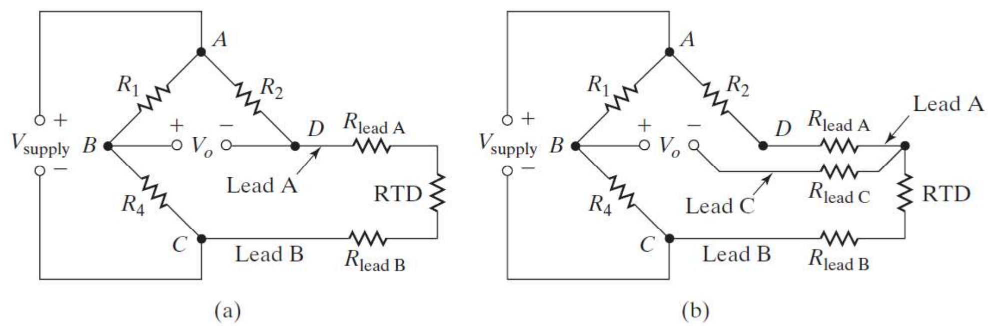



FIGURE 9.21

Wheatstone bridge circuits for RTD: (a) two-wire; (b) three-wire.

Source: Wheeler, Anthony J., et al. *Introduction to engineering experimentation*. Vol. 480. Pearson, 2010.

# Thermistors

- Made from ceramics, composed of a mixture of metal oxides.
- Shaped into different forms depending on the application.
- Temperature coefficient is negative, and its absolute value is larger.

$$R_{T_1} = R_{T_0} \cdot e^{\beta\left({}^{1}/_{T_1} - {}^{1}/_{T_0}\right)}$$

- Temperature coefficients are around 4-6% change in resistance per degree Celsius.
- Non-linear temperature-resistance relationship.
- Typical resistance values are a few hundred ohms to a megaohm.
- Care must be taken to minimize errors due to self-heating.

# Thermistor

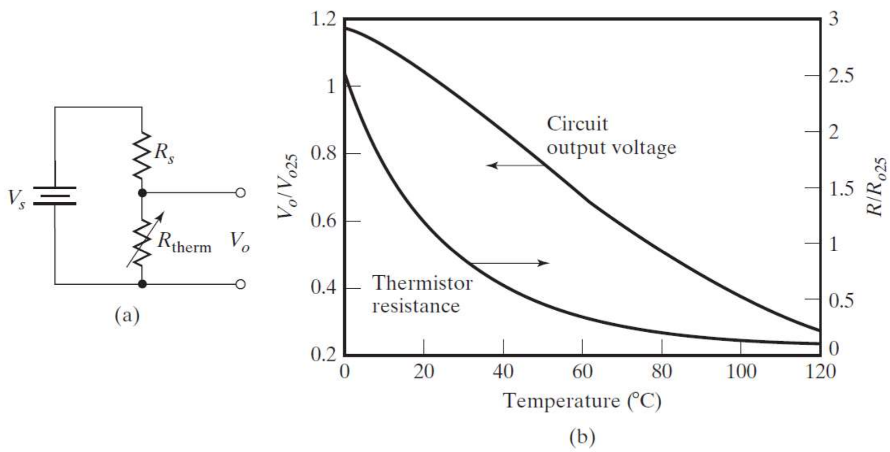
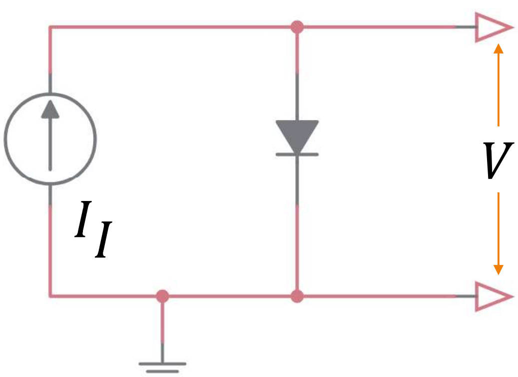



FIGURE 9.24

Linearization circuit for thermistor.

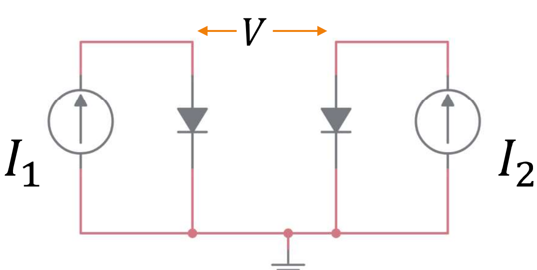
Source: Wheeler, Anthony J., et al. *Introduction to engineering experimentation*. Vol. 480. Pearson, 2010.

# p-n Junction Temperature Sensor




$$I = I_0 e^{\left(\frac{qV - E_g}{kT}\right)}$$

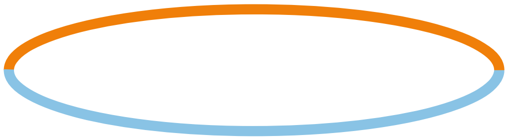
When the current I is kept constant, the voltage across the diode will be a function of the temperature




$$V = V_1 - V_2 = \left(\frac{kT}{q}\right) \ln\left(\frac{I_1}{I_2}\right)$$

# Thermocouple

- Seebeck Effect



# Thermocouple

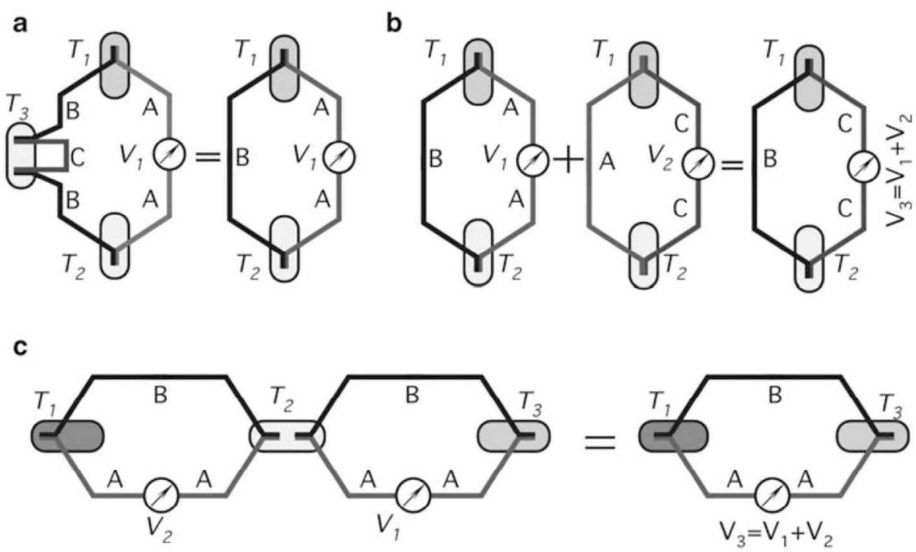


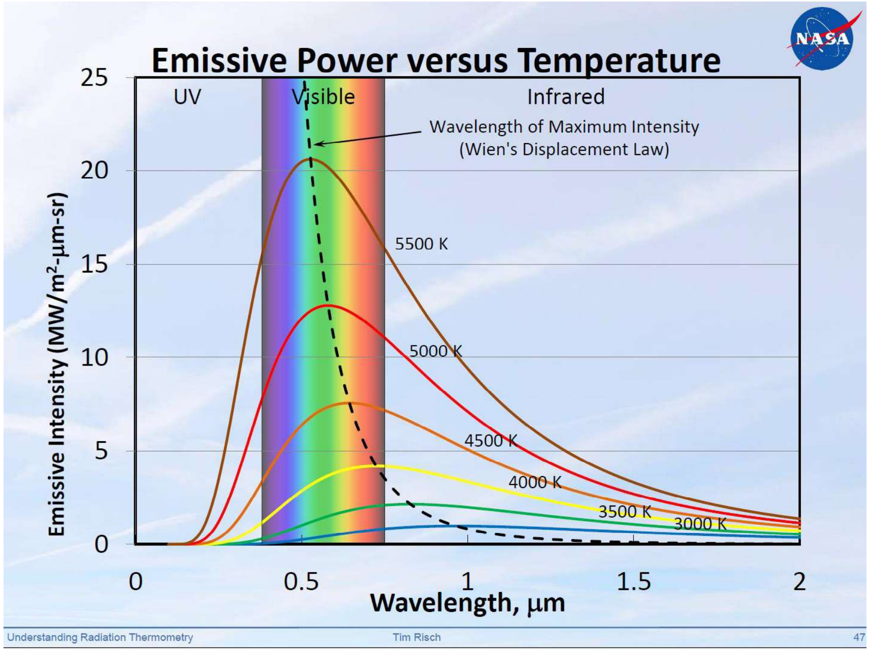

**Fig. 17.27** Illustrations for the thermoelectric laws

# Radiation thermometry

- All bodies with temperature above 0 Kelvin will emit electromagnetic radiation.
- The spectrum density of black body radiation is given by Planck's law:

$$L_\lambda(\lambda, T) = \frac{hc}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

$$W/_{m^2 \cdot sr \cdot \mu m}$$



# Radiation thermometry

- Wein's displacement law:

$$\lambda_{max} T = 2897.77 \mu m \cdot K$$

- Stefan-Boltzmann law: Total thermal radiation power.

$$\int_0^{\infty} L_\lambda(\lambda, T) d\lambda = \frac{\sigma T^4}{\pi} = L_b$$

- For a real body,

$$L_r = \epsilon L_b = \epsilon \frac{\sigma T^4}{\pi}$$

# Radiation thermometry

- Measurement system

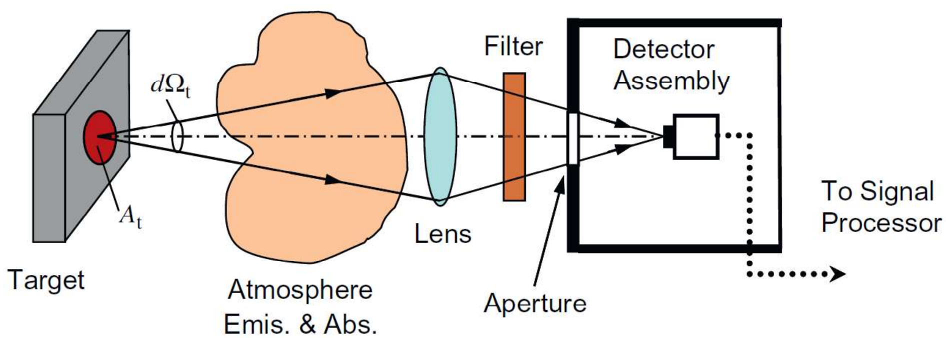



**Figure 1** Schematic diagram of radiometric temperature measurement.

$$L_\lambda(\lambda) = \tau_{env}\left[\varepsilon_\lambda L_{b,\lambda}(\lambda, T_b) + L_{ref,\lambda}(\lambda)\right] + L_{em,\lambda}(\lambda, T_{env})$$

# Radiation thermometry methods

- "True" surface temperature: $T$
- Equivalent black body temperature. $T_b$
- **Wideband measurement**: IF $L_r$ is the measured power, then the estimated temperature of the body is given by,

$$T = \frac{1}{\epsilon^{1/4}} \cdot T_b \Longrightarrow \frac{\delta T}{T} = -\frac{1}{4}\frac{\delta \epsilon}{\epsilon}$$

# Radiation thermometry methods

- Single wavelength measurement:

$$L_\lambda(\lambda, T) = \epsilon \frac{hc}{\lambda^5(e^{hc/\lambda kT} - 1)} = \frac{hc}{\lambda^5(e^{hc/\lambda kT_b} - 1)}$$

$$\frac{1}{T} = \frac{1}{T_b} + \frac{\lambda k}{hc}\ln(\epsilon)$$

# Radiation thermometry methods

- Dual wavelength measurement:

$$L_{\lambda_1}(\lambda_1, T) = \epsilon_{\lambda_1} \frac{hc}{\lambda_1^5(e^{hc/\lambda_1 kT} - 1)} = \frac{hc}{\lambda_1^5\left(e^{hc/\lambda_1 kT_{b,\lambda_1}} - 1\right)}$$

$$L_{\lambda_2}(\lambda_2, T) = \epsilon_{\lambda_2} \frac{hc}{\lambda_2^5(e^{hc/\lambda_2 kT} - 1)} = \frac{hc}{\lambda_2^5\left(e^{hc/\lambda_2 kT_{b,\lambda_2}} - 1\right)}$$

$$\frac{1}{T} = \frac{1}{\tilde{T}} + \Lambda \frac{k}{hc} \ln\left(\frac{\epsilon_{\lambda_1}}{\epsilon_{\lambda_2}}\right)$$

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \qquad \tilde{T} = \frac{\Lambda}{\lambda_1 T_{b,\lambda_1}} + \frac{\Lambda}{\lambda_1 T_{b,\lambda_2}}$$

# Radiation thermometry methods

- **Multiwavelength measurement**:
  - A functional form is assumed for emissivity as a function of wavelength.
  - Measurement at N different wavelengths can is used to estimate emissivity function and the unknown temperature.