Introduction to Digital Signal Processing

Frequency Domain Analysis of LTI Systems

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Response of an LTI system to a complex exponential

- Consider an LTI system with impulse response $h[n]$.

$$Ae^{j\Omega n} \longrightarrow A\left(\sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}\right)e^{j\Omega n}$$

$$H(\Omega) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}$$

$H(\Omega)$ is called the frequency response of the LTI system. It exists if the LTI system is BIBO stable.

$$H(\Omega) = H(z)\Big|_{z=e^{j\Omega}}$$

Response of an LTI system to a complex exponential

$$H(\Omega) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\Omega k} = \left|H\left(\Omega\right)\right| e^{j\Theta(\Omega)}$$

where,

- $\left|H\left(\Omega\right)\right|$ is the mangitude response.
- $\Theta\left(\Omega\right) = \arg\{H\left(\Omega\right)\}$ is the phase response.

The output to $Ae^{j\Omega n}$ is given by,

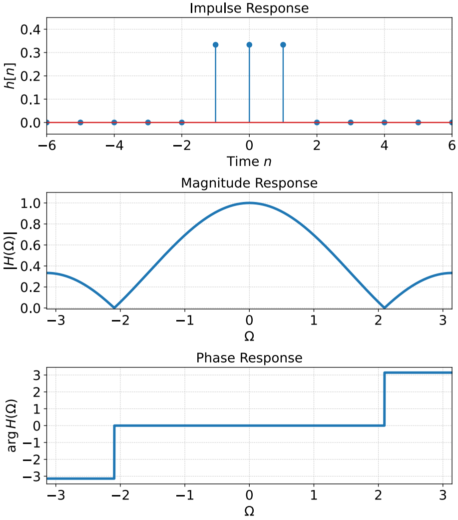
$$Ae^{j\Omega n} \longrightarrow A\left|H\left(\Omega\right)\right| e^{j\{\Omega n + \Theta(\Omega)\}}$$

Response of an LTI system to a complex exponential

Moving average filter

$$y[n] = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$$

Response of an LTI system to a complex exponential



Response of an LTI system to a complex exponential

$$x[n] \to X(\Omega) \longrightarrow H(\Omega)\, X(\Omega) \to y[n]$$

$$\left|Y(\Omega)\right| = \left|H(\Omega)\right|\left|X(\Omega)\right|$$

$$\arg\{Y(\Omega)\} = \arg\{H(\Omega)\} + \arg\{X(\Omega)\}$$

$$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(\Omega)\, e^{j\Omega n} d\Omega$$

Ideal filters