Introduction to Digital Signal Processing

Signals

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What is signal processing?

"Signal processing is an enabling technology that encompasses the fundamental theory, applications, algorithms, and implementations of **processing or transferring information** *contained in many different physical, symbolic, or abstract formats broadly designated as signals and uses* **mathematical, statistical, computational, heuristic, and/or linguistic representations**, *formalisms, and techniques for* **representation, modeling, analysis, synthesis, discovery, recovery, sensing, acquisition, extraction, learning, security, or forensics**.*"*[1]

[1] Moura, J.M.F. (2009). "What is signal processing?, President's Message". IEEE Signal Processing Magazine 26 (6). doi:10.1109/MSP.2009.934636

What is a signal?

Any physical quantity carrying information that varies with one or more independent variables.

$$s\left(t\right) = 1.23t^2 - 5.11t + 41.5$$

$$s\left(x, y\right) = e^{-\left(x^2 + y^2 + 0.5xy\right)}$$

Mathematical representation will not be possible (e.g. *physiological signals, either because the exact function is not known or is too complicated*.)

What is a signal? (Contd ...)

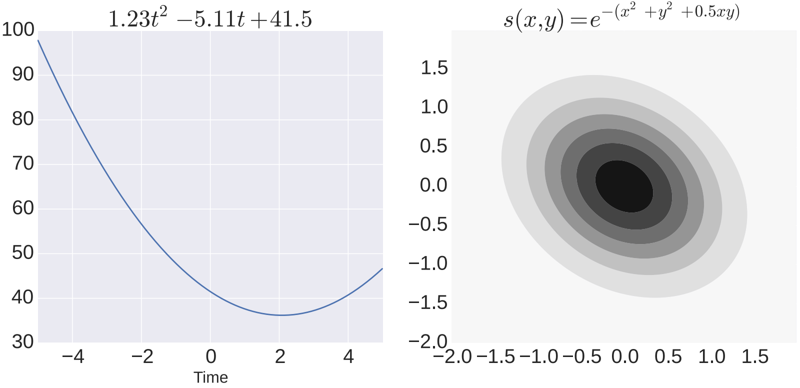


Figure: Example of 1D and 2D signals

Can you think of examples of 3D and 4D signals?

Classification of signals

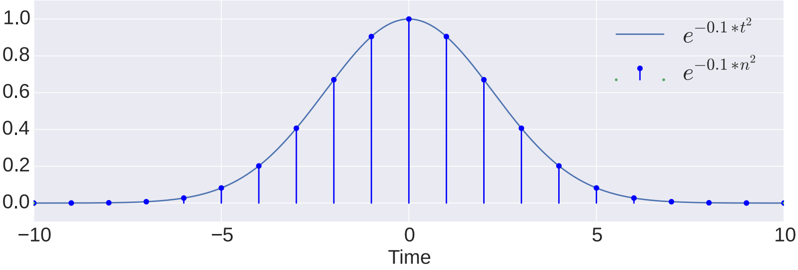
- Based on the signal dimensions. *e.g.* 1D, 2D ...
- **Scalar** vs. **Vector** signals: *e.g. gray scale versus RGB image*

$$I_g\left(x,y\right) \in \mathbb{R} \text{ and } I_{color}\left(x,y\right) \in \mathbb{R}^3$$

Classification of signals

- **Continuous-time** vs. **Discrete-time**: *based on the values assumed by the independent variable.*

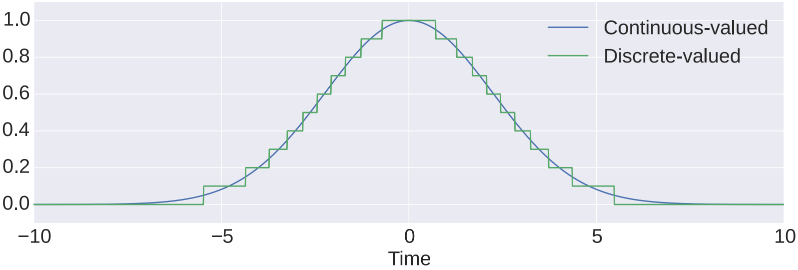
$$\begin{cases} x(t) = e^{-0.1t^2}, \ t \in \mathbb{R} & \text{Continuous-time} \\ x[n] = e^{-0.1n^2}, \ n \in \mathbb{Z} & \text{Discrete-time} \end{cases}$$



Classification of signals (Contd ...)

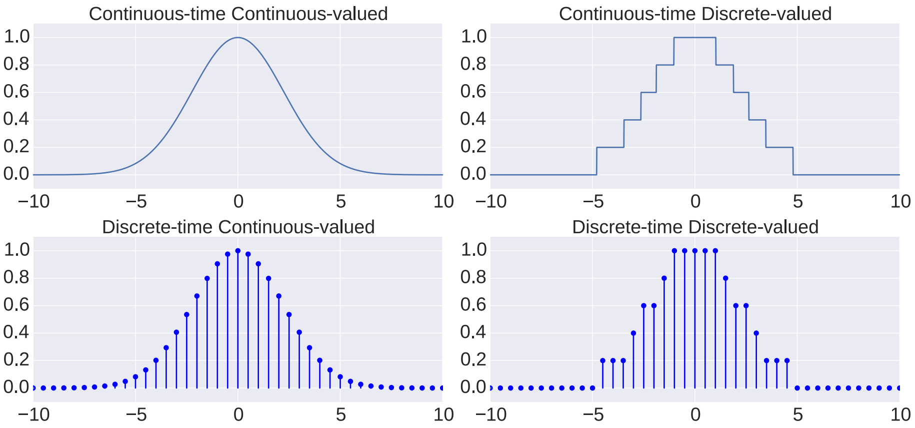
- **Continuous-valued** vs. **Discrete-valued**: *based on the values assumed by the dependent variable.*

$$\begin{cases} x(t) \in [a, b] & \text{Continuous-valued} \\ x(t) \in \{a_1, a_2, \cdots\} & \text{Discrete-valued} \end{cases}$$



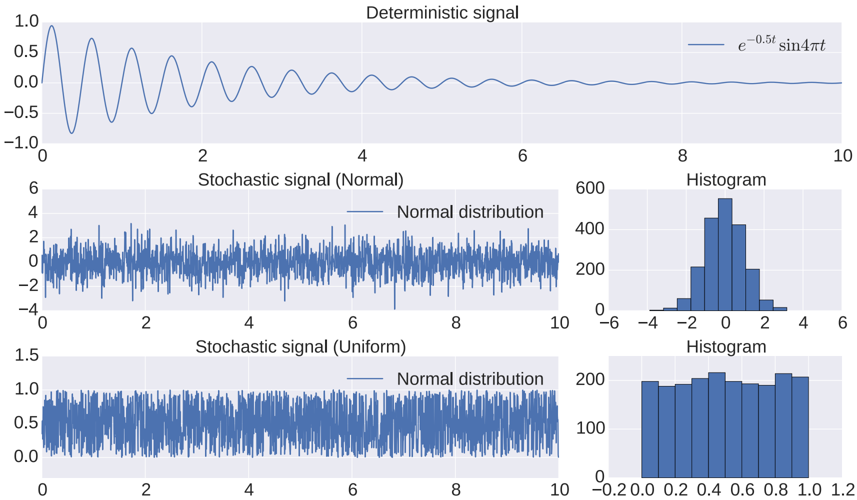
Classification of signals (Contd ...)

Four types of signals



Classification of signals (Contd ...)

- **Deterministic** vs. **Stochastic**: *e.g. EMG is an example of a stochastic signal.*



Classification of signals (Contd ...)

- **Even** vs. **Odd**: *based on the symmetry about the $t = 0$ axis.*

$$\begin{cases} x(t) = x(-t), & \text{Even signal} \\ x(t) = -x(-t), & \text{Odd signal} \end{cases}$$

Most signals are neither even or odd?

Theorem

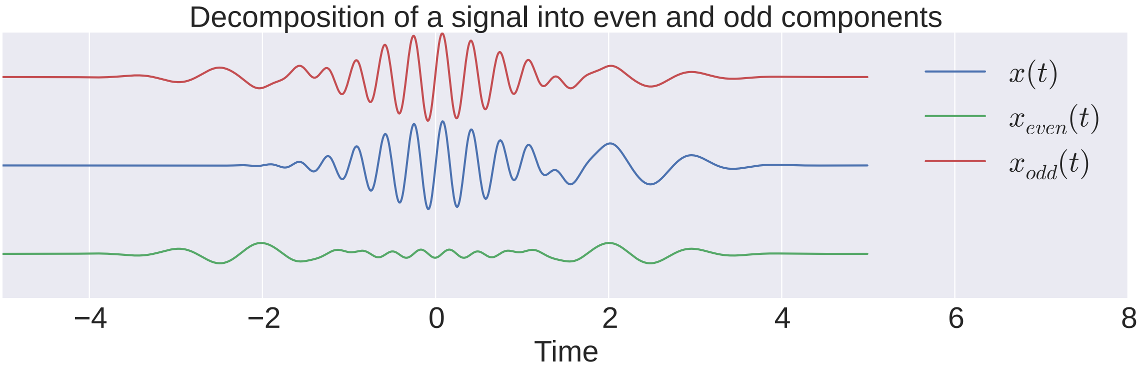
Any arbitrary function can be represented as a sum of an odd and even function.

$$x(t) = x_{even}(t) + x_{odd}(t)$$

where, $x_{even}(t) = \frac{x(t)+x(-t)}{2}$ and $x_{odd}(t) = \frac{x(t)-x(-t)}{2}$.

Classification of signals (Contd ...)

Decomposition of an arbitrary signal into even and odd components



Classification of signals (Contd ...)

- **Periodic** vs. **Non-periodic**: *a signal is periodic, if and only if*

$$x\left(t\right) = x\left(t + T\right), \forall t$$

where, T is the fundamental period.

Useful signals in continuous and discrete-time

We will look at some important signals, that we will often come across and are useful in the analysis of signals and systems.

- Exponential signals
- (Complex) Sinusoids
- Exponential sinusoids
- Impulse/Dirac delta function
- Step function

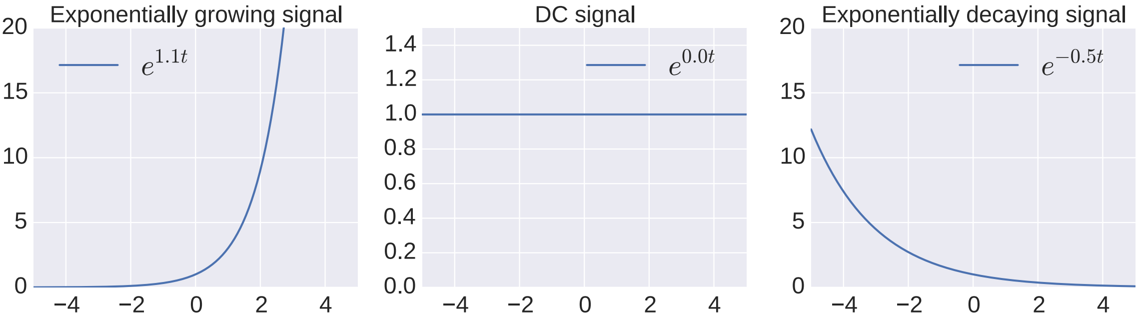
There are some important differences between the corresponding continuous and discrete-time signals.

Real Exponentials

Continuous-time version

$$x(t) = be^{at}$$

where, $a, b, t \in \mathbb{R}$. b is the amplitude and a is the exponential growth or decay rate.

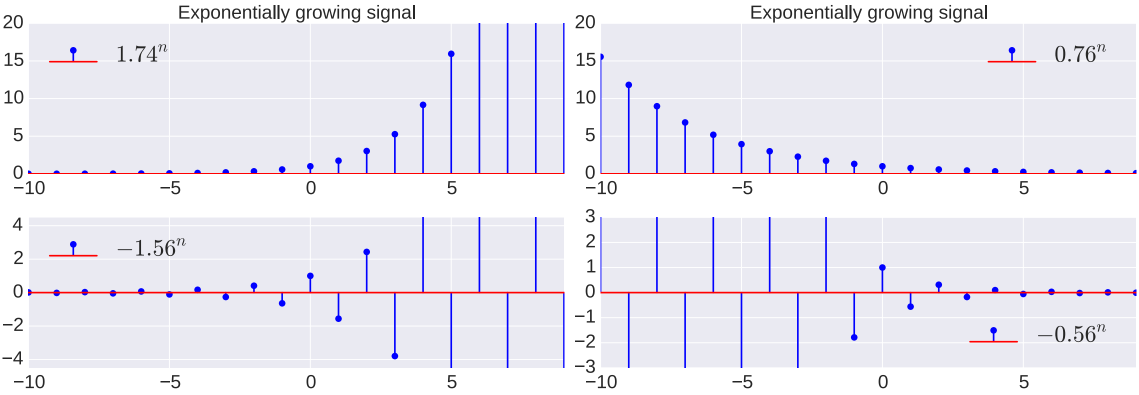


Real Exponentials (Contd ...)

Discrete-time version

$$x[n] = b\,(a)^n$$

where, $a, b \in \mathbb{R}$ and $n \in \mathbb{Z}$. b is the amplitude and a is the exponential growth or decay rate.



Real Exponentials (Contd ...)

These are encountered as solution to first order differential and difference equations.

$$\frac{d}{dt}x(t) = kx(t) \implies x(t) = Ce^{kt}$$

$$x[n] = kx[n-1] \implies x(t) = C(k)^{n}$$

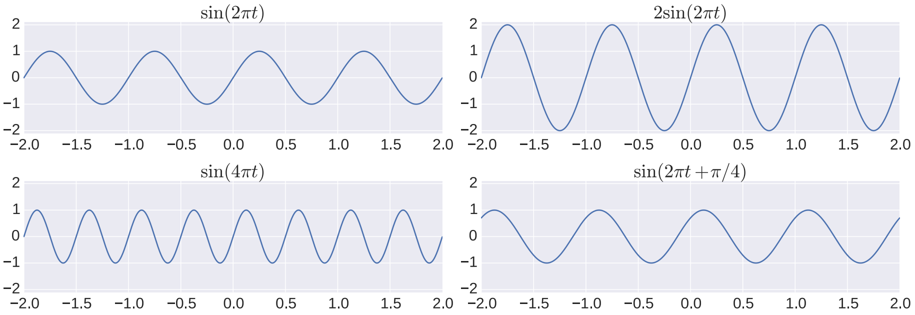
Can you think of practical examples of systems that result in such signals?

Sinusoidal signals

Continuous-time version

$$x(t) = A\sin\left(\omega t + \phi\right)$$

where, A is the amplitude, ω is the angular frequency $\left(\mathrm{rad.sec}^{-1}\right)$, and ϕ is the phase angle.



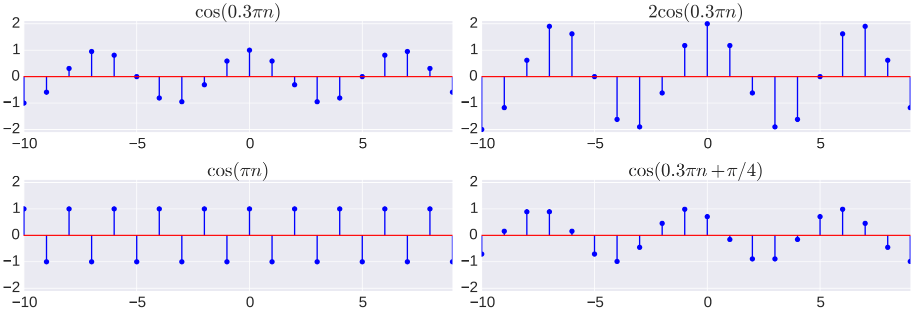
What is the fundamental period of sinusoid?

Sinusoidal signals (Contd ...)

Discrete-time version

$$x[n] = A\sin\left(\Omega n + \phi\right)$$

where, A is the amplitude, Ω is the digital frequency $\left(\text{rad.sample}^{-1}\right)$, and ϕ is the phase angle.



What is the fundamental period?

Sinusoidal signals (Contd ...)

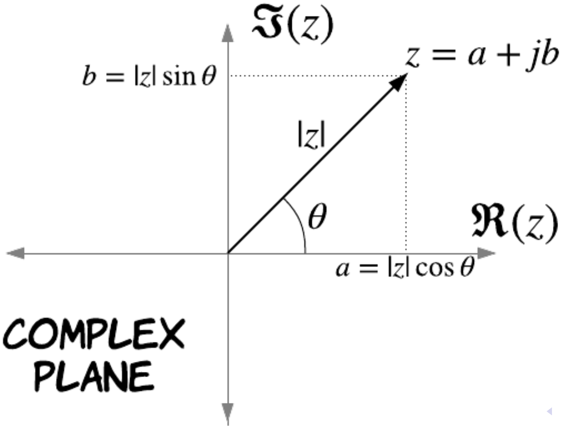
There are some peculiarities to the discrete sinusoid:

- Not all sinusoids are periodic! e.g. $\sin(n)$
- There is a maximum frequency for discrete sinusoids. What is it?
- Two sinusoids that differ by a discrete frequency of 2π are the same sinusoids.

Sinusoidal signals (Contd ...)

Complex exponential representation of sinusoids

$$z = a + jb = |z|\, e^{j\theta} = |z| \cos\theta + j\, |z| \sin\theta$$

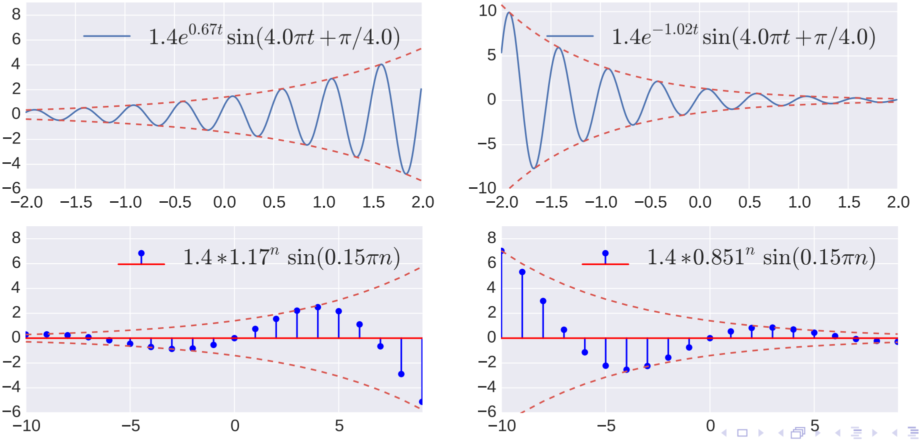


Exponential sinusoids

Continuous-time version

Amplitude modulated sinusoids

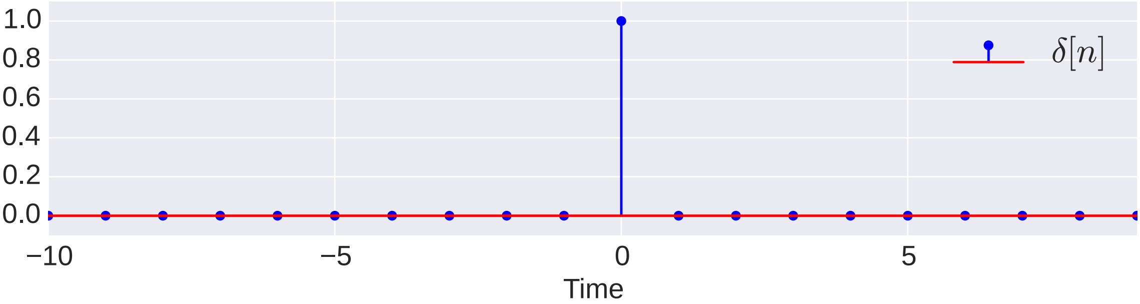
$$x(t) = ae^{bt}\sin\left(\omega t + \phi\right), \quad a, b, \omega, \phi \in \mathbb{R}$$



Impulse function $\delta[n]$ (Contd ...)

Kronecker delta function or sequence $\delta[n]$

- $$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{Otherwise} \end{cases}$$



Step function $u(t), u[n]$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$