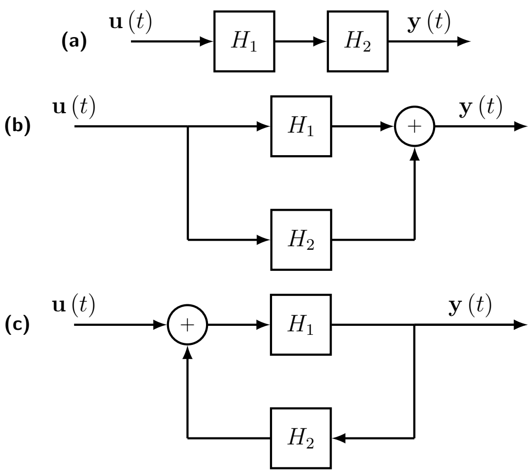
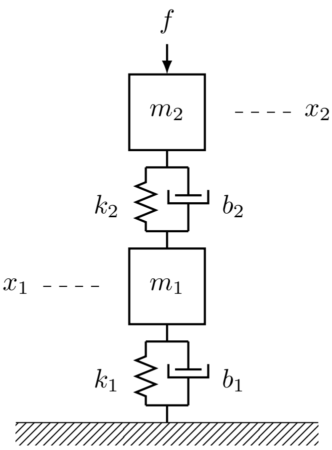
Linear Systems: State Space View Assignment

1. Derive the state and measurement equations for the following composite systems, assuming the system H_i to have the parameters $(\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i, \mathbf{D}_i)$.



2. Derive the state and measurement equation for the following system, where the input is the force f applied to mass m_2, and the output is the acceleration of the mass m_1 and velocity of mass m_2.



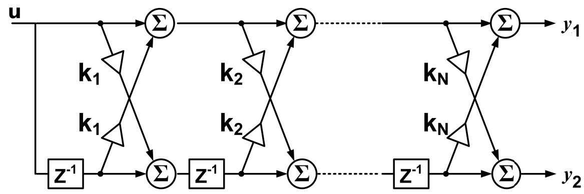
Assume now that instead of f the input to this was the position of the mass m_2 (i.e. x_2) and the output of interest was the acceleration of the mass m_1. What would be correspondoing state and measurement equations in this case?

3. Obtain a state space representation for the following systems:

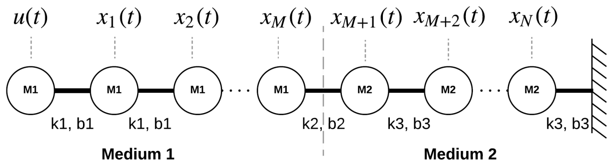
   (a) $\sum_{i=0}^{n} a_i y^{(i)} = \sum_{j=0}^{m} b_j x^{(j)}$, where $x^{(k)} = \frac{d^k}{dt^k} x(t)$

   (b) $\sum_{i=0}^{n} a_i y[k+i] = \sum_{j=0}^{m} b_j x[k+j]$

4. Write down the state and measurement equations for the following system with the scalar input $u[k]$ and output $\mathbf{y}[k] = \begin{bmatrix} y_1[k] \\ y_2[k] \end{bmatrix}$.



5. The following model shows two media and their constituent elements. Medium 1 consists of elements with mass M_1 which are interconnected through a spring and damper in parallel with spring constant k_1 and damping coefficient b_1. Medium 2 consits of elements with mass M_2 connected through k_2 and b_2. At the interface M_1 and M_2 are connected through k_3 nd b_3.



The input to this system is $u(t)$ which is the position imposed on the let most element in medium 1. The output of the system are the successive differences in the positions of the massess.

$$y_i(t) = x_{i+2}(t) - x_{i+1}(t)\ ,\ \ 1 \leq i \leq N-2$$

Derive the state and measurement equations for the system.

6. Write a python program to simulate a continuous-time mass, spring, damper system, described by the follloing differential equation.

$$M\ddot{y}(t) + B\dot{y}(t) + Ky(t) = u(t)$$

Assuming the states of the system to be $\mathbf{x}(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}$, find out the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and $\mathbf{D}$.

Assuming that the input $u(t) = 0,\ \forall t \geq 0$, and assuming an initial condition of $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, numerically solve the state compute the evolution of the state and the output of the system using the following procedure. Let Δ be the time step used for the integration, then the time is divided into discrete time instants $n\Delta$, where $n \in \mathbb{Z}_{\geq 0}$. Assuming that we know the value of the state at time $n\Delta$, the rate of change of the state $\dot{\mathbf{x}}$ and the output $\mathbf{y}(n\Delta)$ at a time $n\Delta$ are given by,

$$\dot{\mathbf{x}}(n\Delta) = \mathbf{A}\mathbf{x}(n\Delta) + \mathbf{B}\mathbf{u}(n\Delta)$$
$$\dot{\mathbf{y}}(n\Delta) = \mathbf{C}\mathbf{x}(n\Delta) + \mathbf{D}\mathbf{u}(n\Delta)$$

We can compute the state at time $(n+1)\Delta$ from $\dot{\mathbf{x}}(n\Delta)$,

$$\mathbf{x}((n+1)\Delta) \approxeq \mathbf{x}(n\Delta) + \mathbf{x}(n\Delta) \cdot \Delta$$

Starting from the value of the start at time 0, $\mathbf{x}(0)$, we can numerically compute the evolution of the state for a given input $\mathbf{u}(t)$.

Compute the states and the output of the system from time $t = 0s$ to $t = 10s$ for following values of the parameters M, B, K,

(a) M = 1, B = 3, K = 1
(b) M = 1, B = 1, K = 1
(c) M = 0, B = 0, K = 1

Carry out the simulations for different values of $\Delta = 0.1, 0.01, 0.001$. Compute the states and plot them as function of time.

What differences do you find for the three systems for the different parameters and when using different step times.