Introduction to DSP: Systems - Assignment: Frequency Anaysis of LTI Systems

1. Determine the frequency response of the LTI system with impulse response, $h[n] = \left(\frac{1}{2}\right)^n 1[n]$. Plot the magnitude and phase response by writing a python program. Determine the magnitude and phase spectra of the following inputs and their corresponding outputs for the given LTI system.
   (a) $x[n] = \cos\frac{2\pi n}{10}, -\infty < n < \infty$
   (b) $x[n] = \frac{1}{2}\left(\delta[n] + \delta[n-1]\right)$
2. Consider the following LTI system,
   $$y[n] = x[n] + x[n-3]$$
   Compute the output of this system for the following input, $x[n] = \cos\frac{\pi n}{2} + \cos\frac{\pi n}{4}$.
   Can you explain the results in terms of the magnitude and phase responses the LTI system?
3. A frequency components in the output of a LTI system can only be the ones that are at its input. An LTI system does not generate new frequency components, whereas a non-linear system can. For the input $x[n] = \cos\left(\frac{\pi}{4}n\right)$, find the frequency components of the output for the following systems,
   (a) $y[n] = x[n] + x[n-1]$
   (b) $y[n] = x[2n]$
   (c) $y[n] = x^2[n]$
   (d) $y[n] = \cos(\Omega n) \cdot x[n]$
   (e) $y[n] = x^3[n]$
4. Consider the following LTI system,
   $$y[n] = -a \cdot y[n-1] + x[n]$$
   Find the frequency response $H\left(\Omega\right)$ of this system. Find the value of a such that $|H\left(0.25\pi\right)| = \frac{1}{\sqrt{2}}|H\left(0\right)|$.
5. Consdier the following LTI system,
   $$y[n] = a \cdot x[n] + b \cdot x[n-1] + c \cdot x[n-2]$$
   Let the frequency response of this system be $H\left(\Omega\right)$. Find the values of a, b, c such that $H\left(\frac{\pi}{3}\right) = 0$.