Introduction to Signal Processing

Lecture 2

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Useful signals in continuous and discrete-time

We will look at some important signals, that we will often come across and are useful in the analysis of signals and systems.

- Exponential signals
- (Complex) Sinusoids
- Exponential sinusoids
- Impulse/Dirac delta function
- Step function

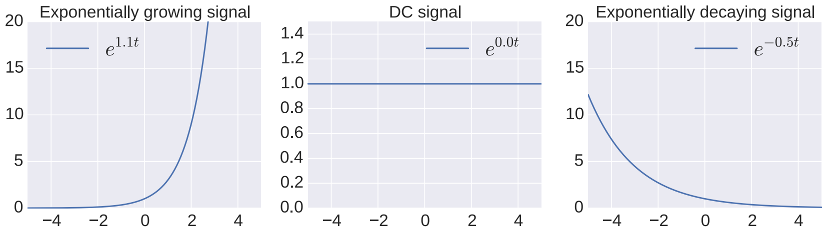
There are some important differences between the corresponding continuous and discrete-time signals.

Real Exponentials

Continuous-time version

$$x(t) = be^{at}$$

where, $a, b, t \in \mathbb{R}$. b is the amplitude and a is the exponential growth or decay rate.

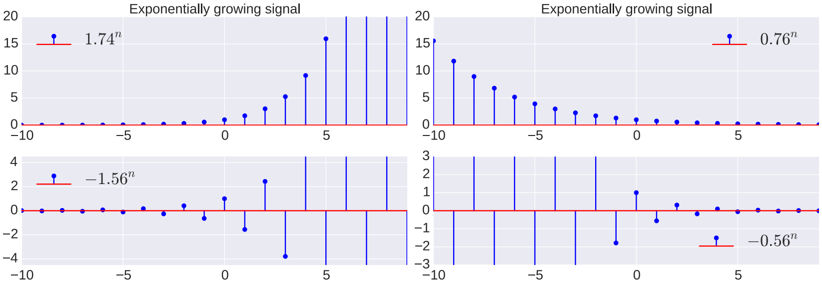


Real Exponentials (Contd ...)

Discrete-time version

$$x[n] = b\,(a)^n$$

where, $a, b \in \mathbb{R}$ and $n \in \mathbb{Z}$. b is the amplitude and a is the exponential growth or decay rate.



Real Exponentials (Contd ...)

These are encountered as solution to first order differential and difference equations.

$$\frac{d}{dt}x(t) = kx(t) \implies x(t) = Ce^{kt}$$

$$x[n] = kx[n-1] \implies x(t) = C(k)^n$$

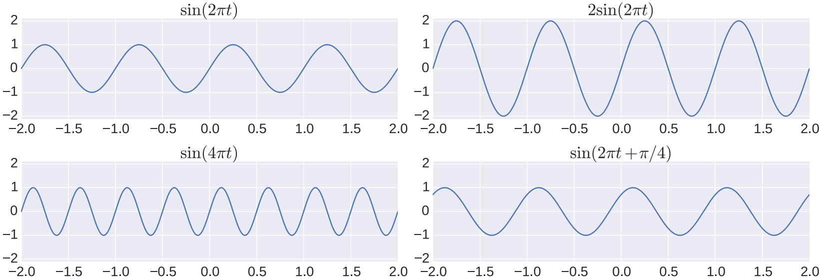
Can you think of practical examples of systems that result in such signals?

Sinusoidal signals

Continuous-time version

$$x(t) = A \sin\left(\omega t + \phi\right)$$

where, A is the amplitude, ω is the angular frequency $\left(\text{rad.sec}^{-1}\right)$, and ϕ is the phase angle.



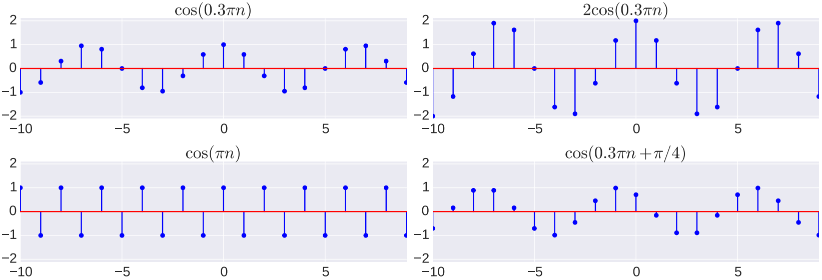
What is the fundamental period of sinusoid?

Sinusoidal signals (Contd ...)

Discrete-time version

$$x[n] = A \sin(\Omega n + \phi)$$

where, A is the amplitude, Ω is the digital frequency (rad.sample^{-1}), and ϕ is the phase angle.



What is the fundamental period?

Sinusoidal signals (Contd ...)

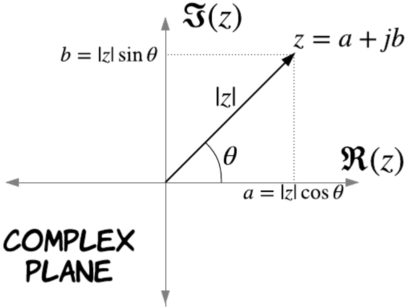
There are some peculiarities to the discrete sinusoid:

- Not all sinusoids are periodic! e.g. $\sin(n)$
- There is a maximum frequency for discrete sinusoids. What is it?
- Two sinusoids that differ by a discrete frequency of 2π are the same sinusoids.

Sinusoidal signals (Contd ...)

Complex exponential representation of sinusoids

$$z = a + jb = |z|\, e^{j\theta} = |z| \cos\theta + j\, |z| \sin\theta$$



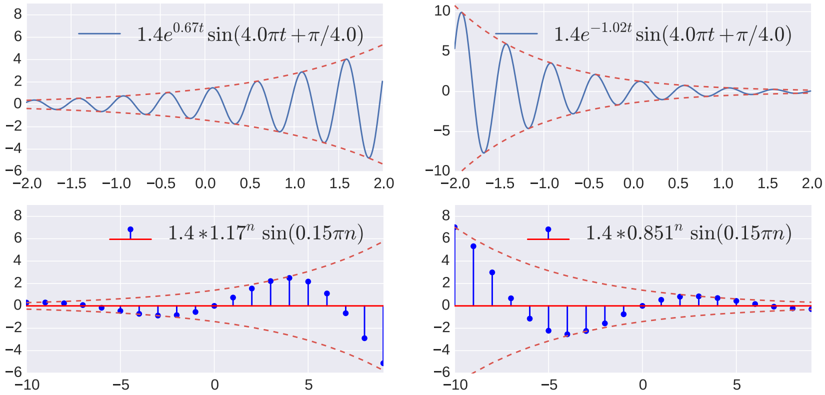
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Exponential sinusoids

Continuous-time version

Amplitude modulated sinusoids

$$x(t) = ae^{bt} \sin\left(\omega t + \phi\right), \quad a, b, \omega, \phi \in \mathbb{R}$$



Impulse function $\delta(t)$, $\delta[n]$

Dirac delta function $\delta(t)$

- This is **NOT** a conventional function.
- It makes sense only when it is used in an integral.
- It is not characterized by the exact values it takes as a function of the independent variable, but by the following important property.

$$\int_a^b \delta(t) = \begin{cases} 1, & 0 \in [a, b] \\ 0, & 0 \notin [a, b] \end{cases}$$

- It operates like a value selector.

$$\int_{-\infty}^{\infty} f(t)\delta(t) = f(0), , \text{ where } f \text{ is continuous at } t = 0.$$

- Impulse function is a very useful theoretical tool for representing: point charges or masses, forces in instantaneous collisions, derivatives of jump discontinuities etc.

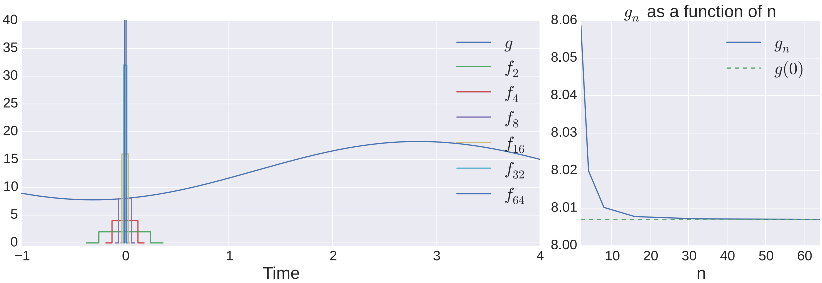
Impulse function $\delta(t)$, $\delta[n]$ (Contd ...)

$\delta(t)$ can be understood through a limiting operation. Let

$$f_n(t) = \begin{cases} n, & -\frac{1}{2n} \leq t \leq \frac{1}{2n} \\ 0, & \text{Otherwise} \end{cases} \quad \text{and} \int_{-\infty}^{\infty} f_n(t)dt = 1$$

$$\int_{-\infty}^{\infty} f_n(t)\, g(t)\, dt = \int_{-\frac{1}{2n}}^{\frac{1}{2n}} n g(t)\, dt = g_n$$

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(t)\, g(t)\, dt = \lim_{n\to\infty} g_n = g(0) = \int_{-\infty}^{\infty} g(t)\delta(t)dt$$

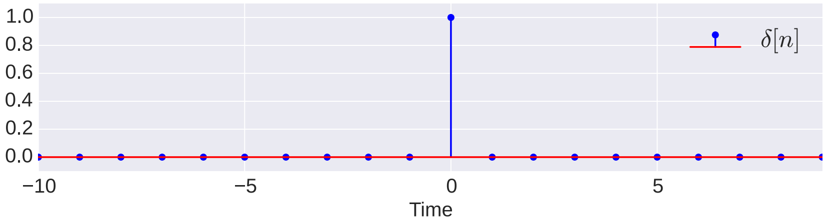


Impulse function $\delta(t)$, $\delta[n]$ (Contd ...)

Kronecker delta function or sequence $\delta[n]$

- Very easy to understand unlike the continuous-time version.

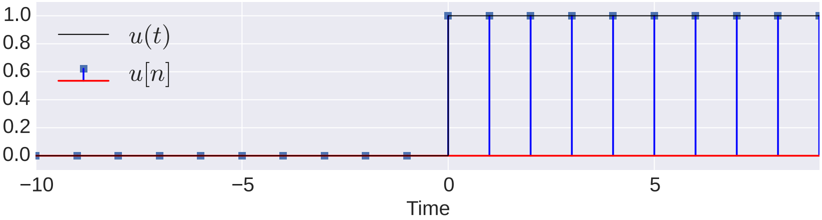
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{Otherwise} \end{cases}$$



Step function $u(t), u[n]$

Definition of **continuous-time** unit step function,

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$



What is the corresponding definition of the discrete-time unit step function $u[n]$?