Resistive Displacement & Force Sensors
Strain Gauges

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Lecture - Outline

- Strain gauges
  - Load cells
  - Force measurement
  - Pressure measurement
- Thermistors
  - Temperature measurement
  - Self-heating

Strain Gauge

- Resistance
- Taking log
- Differentiating

$$R = \rho \frac{l}{A}$$

$$\ln R = \ln \rho + \ln l - \ln A$$

$$\frac{d \ln R}{dR} = \frac{1}{R}, Thus: d \ln R = \frac{dR}{R}$$

- Fractional change of resistance

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{d\,l}{l} - \frac{dA}{A}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}$$

- Can calculate for a cylinder, $A=\pi r^2$
- Poisson's ratio σ is 0.5 for incompressible solids (constant volume), 0.3 for metals

$$\frac{\Delta r}{r} = -\sigma \frac{\Delta l}{l} \qquad \frac{\Delta A}{A} \approx 2 \frac{\Delta r}{r}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} + 2\sigma \frac{\Delta l}{l} + \frac{\Delta \rho}{\rho}$$

Strain Gauge (contd)

- Temperature dependence of resistivity
- Temperature dependence of length
-

$$\rho=\rho_o[1+\alpha_R(T-T_o)]=\rho_o+\Delta\rho_T$$

$$l=l_o[1+\alpha_l(T-T_o)]$$

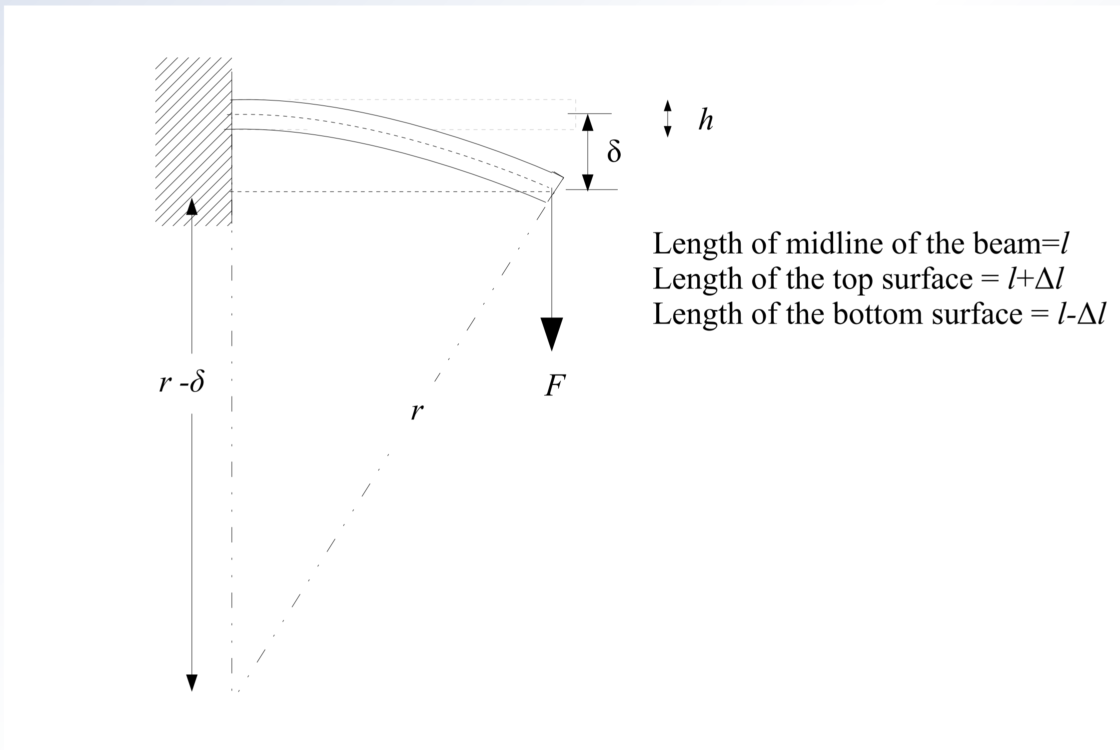
$$\frac{\Delta R}{R}=\frac{\Delta l}{l}(1+2\sigma)+\frac{\Delta\rho_T}{\rho}=G\frac{\Delta l}{l}+\alpha_R\Delta T$$

Strain Gauge Examples

Material	Temperature Coeff α_E	Maximum strain $\Delta l/l$	Modulus of Elasticity Y	Gauge factor G
Platinum	0.0038	0.0023	1.5×10^{10}	6
Nichrome	0.0004	0.0036	19×10^{10}	2.5
Silicon	0.007	0.0033	19×10^{10}	170

Load Cell

Load Cell – cantilever beam



Bending of the cantilever beam

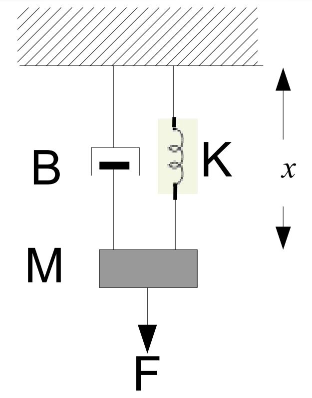
- Deflection of beam

$$\delta = \frac{W\,l^3}{3\,Y\,I_{xx}} \qquad where \quad I_{xx} = \frac{1}{12}bh^3$$

- Horizontal distance: $l^2 - \delta^2 = r^2 - (r-\delta)^2$
- Similar Sectors: $\frac{\Delta l}{l} = \frac{h/2}{r}$
- Substiting for r: $\frac{\Delta l}{l} = \frac{h\,\delta}{l^2}$
- Strain in surface fibres: $\frac{\Delta l}{l} = \frac{h\,\delta}{l^2} = \frac{4\,W}{Ybh^2}$

Mechanical Equivalent

$$F(t) = K\,x(t) + B\,\frac{dx(t)}{dt} + M\,\frac{d^2x(t)}{dt^2}$$

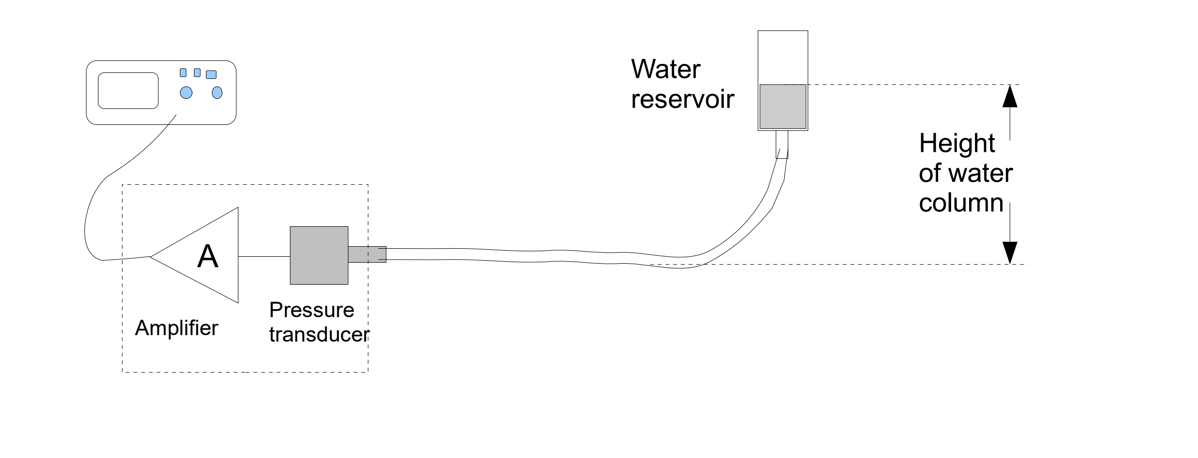


$$X(s) = \frac{F(s)}{K + Bs + Ms^2}$$

$$X_o(s) = \frac{A\,F(s)}{\frac{s^2}{\omega_n^2} + 2\frac{\zeta}{\omega_n}s + 1}$$

Pressure Transducer

Pressure Measurement



Equivalent of catheter fluid system

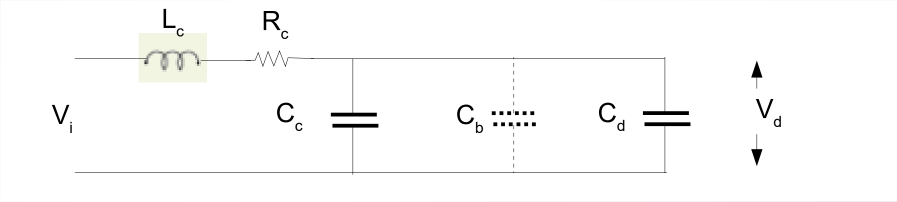
$$L_c = \frac{\rho l}{A}$$

$$C_c = \frac{Ald}{Eh}$$

$$R_c = \frac{8\pi\eta l}{A^2}$$

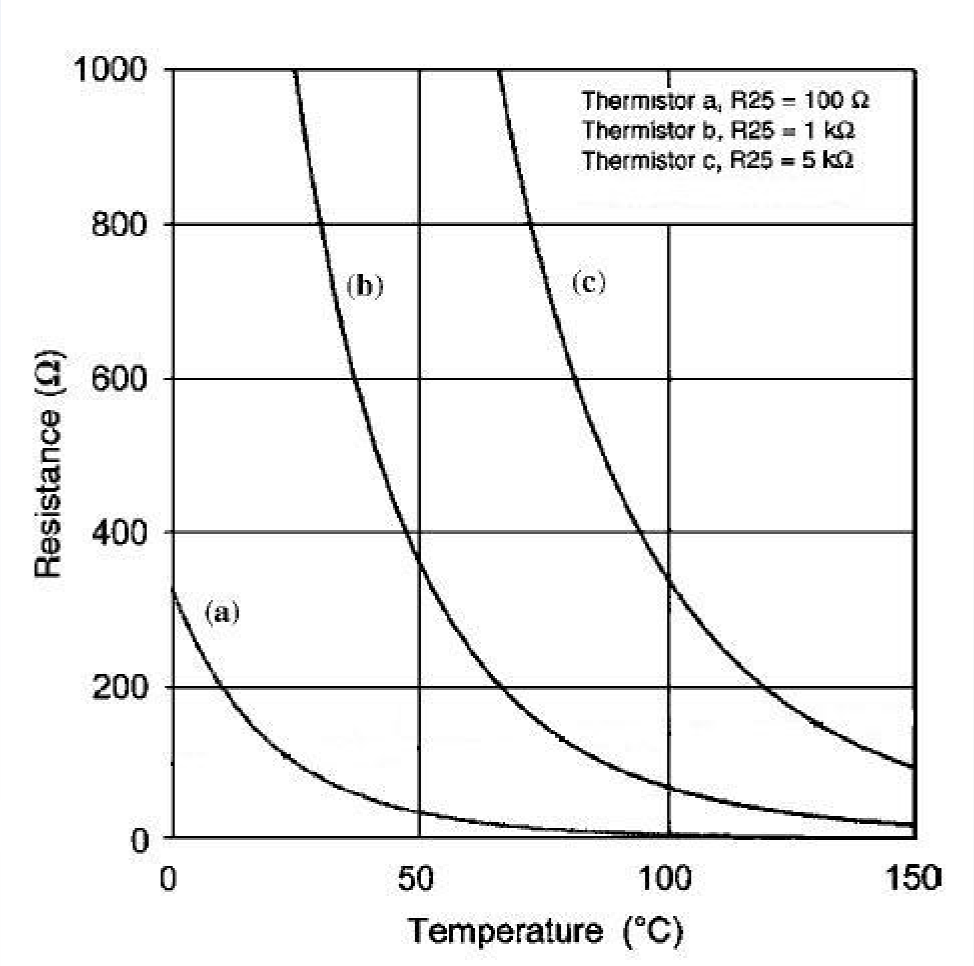
$$C_d = \frac{1}{E_d}$$

$$C_b = \frac{\Delta V_b}{\Delta P_b}$$



Thermistors –
negative temperature coefficient sensors

Thermistors



Thermistors

- Thermistor characteristics

$$R_T = R_o e^{\beta\left(\frac{1}{T} - \frac{1}{T_o}\right)}$$

- Sensitivity:

$$\ln R_T - \ln R_o = \frac{\beta}{T} - \frac{\beta}{T_o}$$

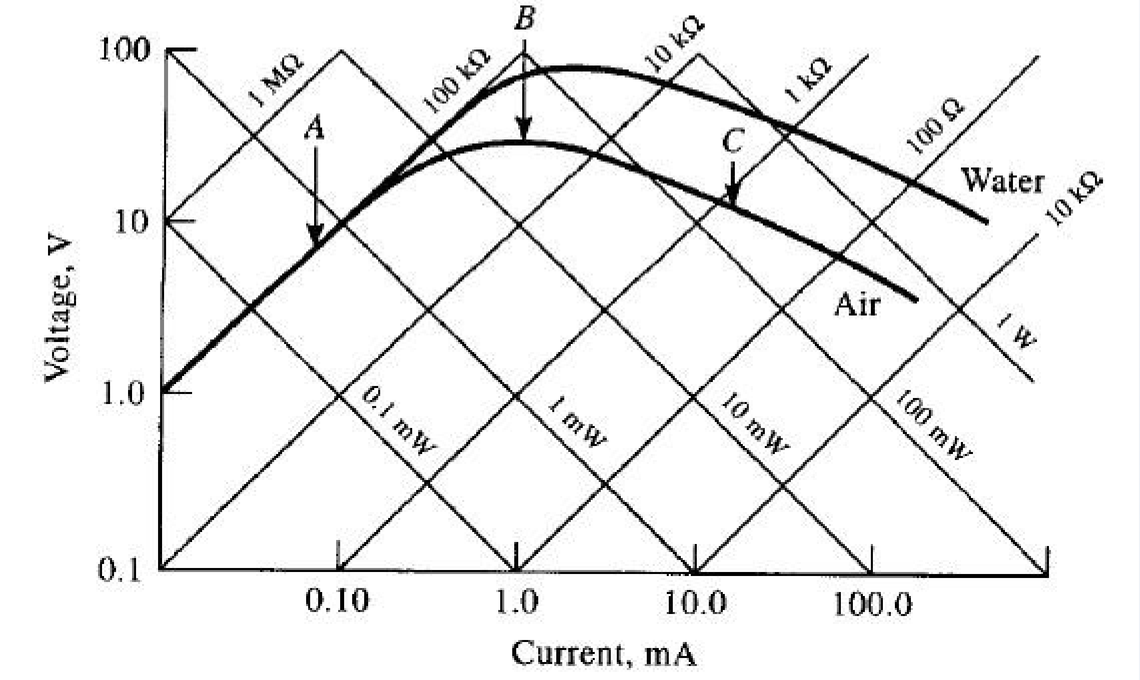
$$\frac{1}{R_T}\frac{dR_T}{dT} - 0 = \beta\frac{-1}{T^2} - 0$$

$$\frac{1}{R_T}\frac{dR_T}{dT} = \frac{-\beta}{T^2}$$

- Non-linear sensitivity:

$$\frac{\Delta R_T}{R_T} = \Delta T\left(\frac{-\beta}{T^2}\right)$$

Voltage-Current characteristics



Thermistor self-heating

- Power dissipation (due to electric current) results in heating of thermistor
- As temperature changes, resistance decreases
- Equilibrium is reached when the electrical power is equal to the thermal power dissipated

$$P = CM\frac{dT}{dt} + D\left(T - T_a\right)$$

$$where \quad C = sp.\ heat, \quad M = mass, \quad D = dissipation \quad constant$$

$$T = T_a + \frac{P}{D}\left[1 - e^{\frac{D}{CM}t}\right]$$

$$steady \quad state \quad : \quad D = \frac{P}{T - T_a}$$

Application of Thermistor in Self-Heating

- Measurement of fluid flow – changing dissipation constant
- Mass flow meters
    - gas and liquid flow sensors