Introduction to DSP: Systems - Assignment: Frequency Anaysis of LTI Systems

1. Determine the frequency response of the LTI system with impulse response, $h[n] = \left(\frac{1}{2}\right)^n 1[n]$. Plot the magnitude and phase response by writing a python program. Determine the magnitude and phase spectra of the following inputs and their corresponding outputs for the given LTI system.

   (a) $x[n] = \cos \frac{2\pi n}{10}, -\infty < n < \infty$

   (b) $x[n] = \frac{1}{2}\left(\delta[n] + \delta[n-1]\right)$

2. Consider the following LTI system,

$$y[n] = x[n] + x[n-3]$$

   Compute the output of this system for the following input, $x[n] = \cos \frac{\pi n}{2} + \cos \frac{\pi n}{4}$.

   Can you explain the results in terms of the magnitude and phase responses the LTI system?

3. A frequency components in the output of a LTI system can only be the ones that are at its input. An LTI system does not generate new frequency components, whereas a non-linear system can. For the input $x[n] = \cos\left(\frac{\pi}{4}n\right)$, find the frequency components of the output for the following systems,

   (a) $y[n] = x[n] + x[n-1]$

   (b) $y[n] = x[2n]$

   (c) $y[n] = x^2[n]$

   (d) $y[n] = \cos(\Omega n) \cdot x[n]$

   (e) $y[n] = x^3[n]$

4. Consider the following LTI system,

$$y[n] = -a \cdot y[n-1] + x[n]$$

   Find the frequency response $H(\Omega)$ of this system. Find the value of a such that $|H(0.25\pi)| = \frac{1}{\sqrt{H(0)}}$.

5. Consdier the following LTI system,

$$y[n] = a \cdot x[n] + b \cdot x[n-1] + c \cdot x[n-2]$$

   Let the frequency response of this system be $H(\Omega)$. Find the values of a, b, c such that $H\left(\frac{\pi}{3}\right) = 0$.