Electrical Impedance

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Lecture - Outline

- Resistive and Reactive Elements
- Laplace and Fourier Transforms
- Measuring Impedance
- Transfer Function
- Amplifiers
- Operational Amplifiers

Laplace Transform and Fourier Transform

$$L\{x(t)\}=\int x(t)e^{-st}dt=X(s)$$

$$F\{x(t)\}=\int x(t)e^{-j\omega t}dt=X(j\omega)$$

- Properties of the Laplace Transform:
- Differentiation in time domain => multiply by s (or jω) in Transform domain

  Integration in time domain => multiply by 1/s (or 1/jω) in Transform domain

Resistive and Reactive Elements – Time and Laplace

Resistance $$V = R \cdot I$$

Inductance $$V(t) = L \frac{dI(t)}{dt}$$

Capacitance $$V(t) = \frac{1}{C} \int I(t)\, dt$$

Resistance $$V(s) = R \cdot I(s)$$

Inductance $$V(s) = L\, s \cdot I(s)$$

Capacitance $$V(t) = \frac{1}{C\, s} \cdot I(s)$$

Impedance

- Impedance is the ratio of voltage to current in the Laplace domain or Fourier domain

$$Z(s) = \frac{V(s)}{I(s)}$$

$$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)}$$

Calculating Impedance

- Resistor and capacitor in series

$$Z(s) = R + \frac{1}{Cs}$$

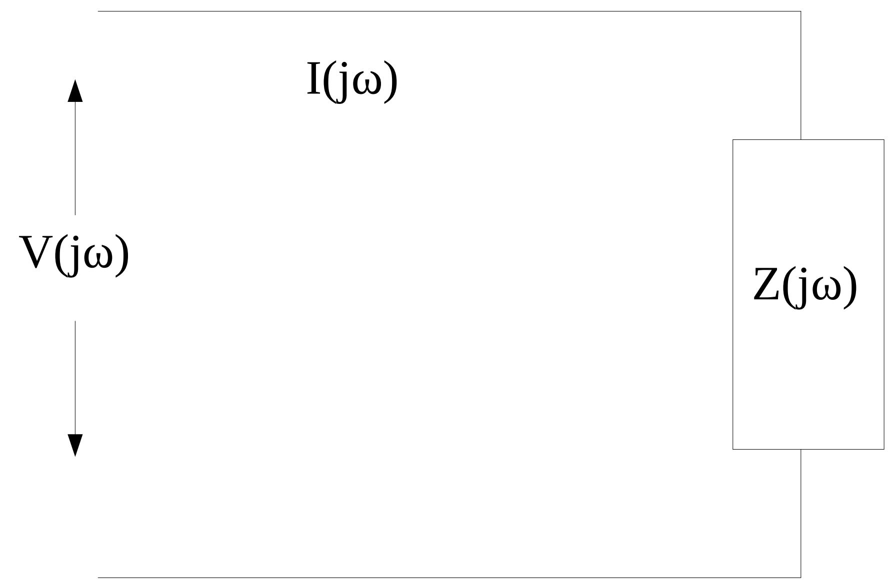
- R and C in parallel

$$Z(s) = \frac{R}{1 + RCs}$$

Measuring Impedance

- Simple method is to Measure with sinusoidal V, I

$$\frac{V(j\omega)}{I(j\omega)} = Z(j\omega)$$



Input-output Transfer ratio and Transfer function

- Input-Output relation
  - Purely real terms (e.g., resistance)
  - Includes Reactive terms
- Transfer function:

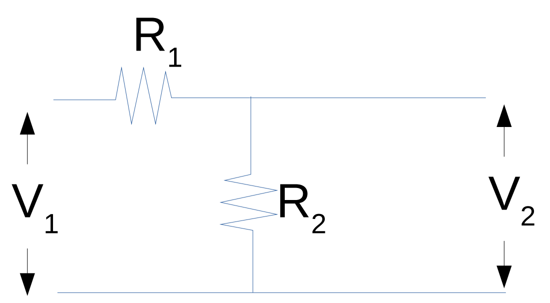
$$H(s) = \frac{Output}{Input}$$

- Measuring Transfer ratio

Input-output Transfer ratio

- Input-Output relation
  - Purely real terms (i.e., only resistance)

$$H = \frac{Output}{Input}$$



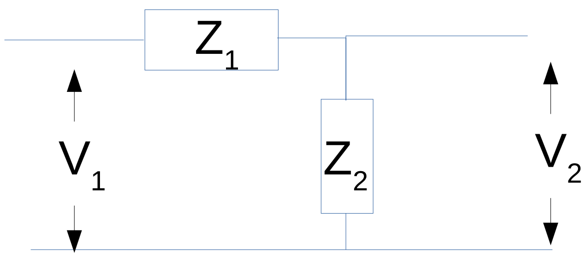
- Measuring Transfer ratio:

$$\frac{V_2}{V_1} = \frac{R_2}{R_1 + R_2}$$

Input-output Transfer Function

- Input-Output relation
  - Complex terms (i.e., R, L, C)

$$H(s) = \frac{Output}{Input}$$

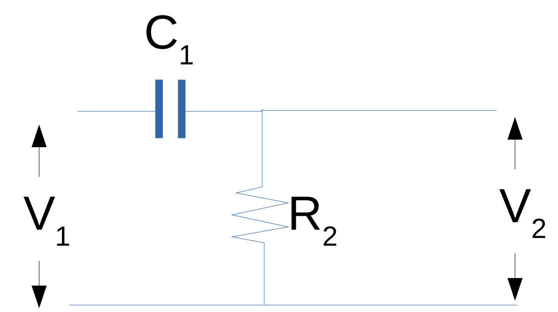


- Measuring Transfer ratio:

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

Input-output Transfer Function – High Pass Filter

- Transfer function of an RC filter – High Pass



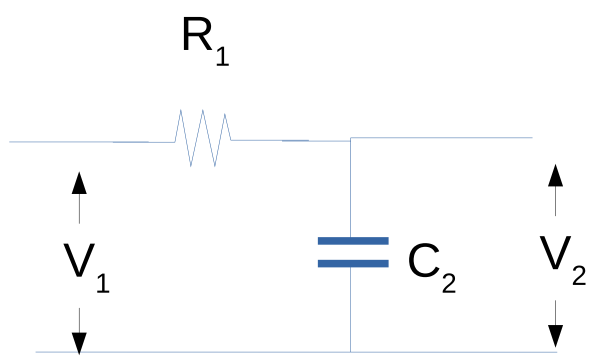
$$\frac{V_2(s)}{V_1(s)} = \frac{R_2}{1/(C_1 s) + R_2} = \frac{R_2 C_1 s}{1 + R_2 C_1 s}$$

$$H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{R_2}{1/(C_1 j\omega) + R_2} = \frac{R_2 C_1 j\omega}{1 + R_2 C_1 j\omega}$$

$$|H(j\omega)| = \frac{R_2 C_1 \omega}{\sqrt{1 + (R_2 C_1 \omega)^2}}$$

Input-output Transfer Function

- Transfer function of RC filter – Low Pass:



$$\frac{V_2(s)}{V_1(s)} = \frac{1/(C_2 s)}{R_1 + 1/(C_2 s)} = \frac{1}{1 + R_1 C_2 s}$$

$$H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{1/(C_2 j\omega)}{R_1 + 1/(C_2 j\omega)} = \frac{1}{1 + R_1 C_2 j\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (R_1 C_2 \omega)^2}}$$

Attenuators and Amplifiers