

**Department of Bioengineering, CMC-Vellore**

**Transducers and Instrumentation for Physiological Measurement**  
**Mid-Term Test**

1<sup>st</sup> March, 2019  
1 hour

1. The following table gives the resistance of a thermistor measured at 3 different temperatures. Calculate the value of the thermistor constant  $\beta$ .

$T(^{\circ}\text{C})$	20	30	40
$R_T(\Omega)$	2492	1611	1060

The Resistance versus Temperature relation is approximated by the expression:  $R_T = 1611 - 55.1(T-30)$  where the temperature is in Celsius. This thermistor is to be used in the range  $35^{\circ}$  to  $42^{\circ}\text{C}$ . What will be the maximum error in this range, and at what temperature?

2. The resistance variation of a strain gauge is given by:  $\frac{\Delta R}{R} = G \frac{\Delta l}{l} + \alpha(T - T_0)$

The gauge factor of strain gauges made of Nichrome and Silicon are 2.5 and 170 respectively, while the temperature coefficients of resistivity are 0.0009 and 0.007, with  $20^{\circ}\text{C}$  being the reference temperature.

(a) Derive the expression for the output of a Wheatstone bridge for a load cell with (i) one strain gauge, and (ii) two strain gauges. (b) If 0.1% strain is present and  $5^{\circ}\text{C}$  temperature variation exists, calculate the following. Use an excitation voltage of 5V and  $R=100\ \Omega$ .

	Output due to 0.1% strain	Output due to $5^{\circ}\text{C}$	Total Output	Apparent strain
One active arm bridge				
Two active arm bridge				

3. We have two piezoelectric crystals: (a) quartz crystal of dimension  $x=5\text{mm}$ ,  $y=5\text{mm}$ ,  $z=2\text{mm}$ , and (b) barium titanate ceramic of dimension  $x=2\text{mm}$ ,  $y=2\text{mm}$ ,  $z=1\text{mm}$ . If a force of 100N is applied to each of them in the  $x$  direction for quartz and in the  $z$  direction for the ceramic crystal, what is the voltage in the preferred direction? Permittivity of free space,  $8.8542 \times 10^{-12}\ \text{F}\cdot\text{m}^{-1}$

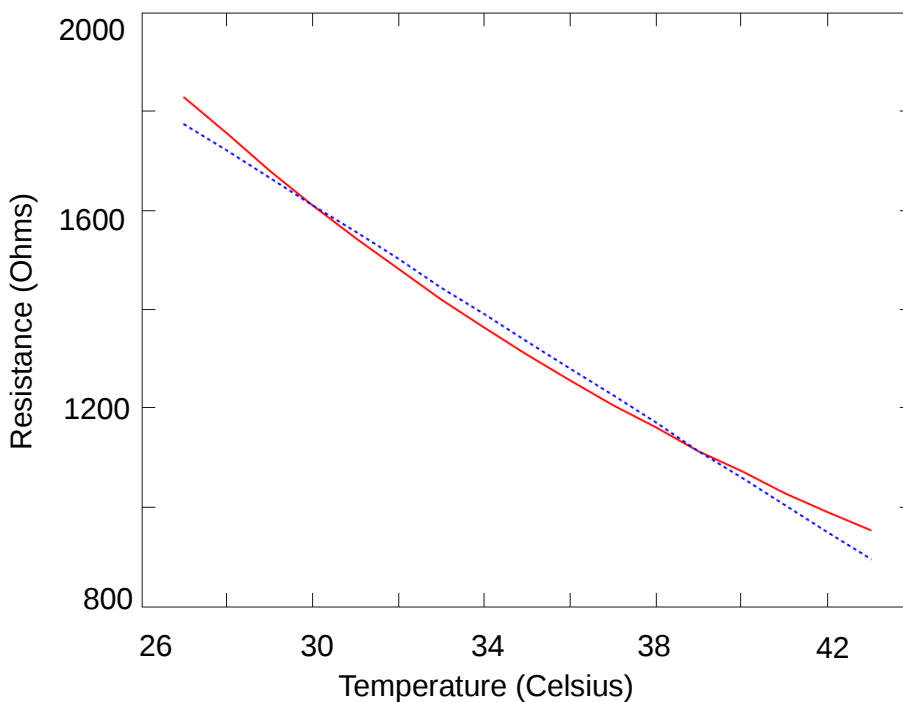
	Relative permittivity (Dielectric constant)	Piezoelectric constant
Quartz	4.5	$d_{p(11)} = 2.3 \times 10^{-12}\ \text{Coulombs/N}$
Barium Titanate Ceramic	1800	$d_{p(33)} = 78 \times 10^{-12}\ \text{Coulombs/N}$

## Solutions

$$1. \quad R_T = R_o e^{\beta(\frac{1}{T} - \frac{1}{T_o})}, \quad \ln \left[ \frac{R_T}{R_o} \right] = \beta \left( \frac{1}{T} - \frac{1}{T_o} \right), \quad \frac{R_T}{2492} = \beta \left( \frac{1}{T} - \frac{1}{293} \right)$$

$$\text{Using } T_o = 20^\circ\text{C} = 293\text{K}, \quad \ln \left( \frac{R_T}{2492} \right) = \beta \left( \frac{1}{T} - \frac{1}{293} \right)$$

$$\text{and } T = 30^\circ\text{C} = 303\text{K}, \quad -0.4362 = \beta \left( \frac{1}{303} - \frac{1}{293} \right) = \beta (-0.00011264) \quad \beta = 3872$$



The actual resistance curve and the linear approximation curve plotted against temperature are shown here. The two graphs can be drawn quickly using the numbers given. The local maximum of the error has to be found, not the global maximum. The maximum error in the range 35-40 occurs around 35°C. In the range 35-42, the maximum is at 42°C.

2. Let the resistance variation due to strain and temperature be denoted by  $\Delta R_s$  and  $\Delta R_T$  respectively.

Strain component:  $\Delta_s = G \frac{\Delta l}{l}$  , Temperature component:  $\Delta_T = \alpha \Delta T$

Output of bridge with one strain gauge  $R \pm \Delta R_s \pm \Delta R_T$ , and three equal resistors, R is:

$$V_o = V_i \left( \frac{+ \Delta_s + \Delta_T}{4 + \Delta_s + \Delta_T} \right) \approx V_i \left( \frac{\pm \Delta_s \pm \Delta_T}{4} \right)$$

Two strain gauges are placed on a beam to experience strain in opposite directions so that the resistances are  $R + \Delta R_s + \Delta R_T$  and  $R - \Delta R_s + \Delta R_T$  respectively. A bridge made with these two gauges and two equal resistors, R will have output:

$$V_o = V_i \left( \frac{2 \Delta_s}{(2 + \Delta_T)^2 - \Delta_s^2} \right) \approx V_i \left( \frac{2 \Delta_s}{4} \right)$$

Therefore, in the two strain gauge configuration with the gauges experiencing opposite strains, the effect of temperature is cancelled.

**Nichrome** (  $G=2.5, \alpha=0.0009$  ,  $V_e=5V$ )

	Output due to 0.1% strain	Output due to 5°C	Total Output	Apparent strain
One active arm bridge	3.125 mV	5.625 mV	8.75 mV	0.28%
Two active arm bridge	6.25 mV	0	6.25mV	0.1%

**Silicon** (  $G=170, \alpha=0.007$  ,  $V_e=5V$ )

	Output due to 0.1% strain	Output due to 5°C	Total Output	Apparent strain
One active arm bridge	0.2125V	0.04375 V	0.25625 V	0.12%
Two active arm bridge	0.425 V	0	0.425V	0.1%

$$3. \quad \frac{Q}{A} = \sum_{j=1 \dots 3} \epsilon_{ij} E_j + \sum_{m=1 \dots 6} d_{im} \phi_m, \quad E_j = 0, \quad \frac{Q}{A} = d_p \frac{F}{A}, \quad Q = d_p F$$

$$V = \frac{Q}{C}, \quad C = \epsilon \frac{A}{l}$$

**Quartz:**

$$Q = 2.3 \times 10^{-12} \times 100 = 230 \times 10^{-12}$$

$$C = 4.5 \times 8.8542 \times 10^{-12} \times (5 \times 2) \times 10^{-6} / 5 \times 10^{-3} = 79.6878 \times 10^{-15}, \quad V = 2886 \text{ V}$$

**Barium Titanate Ceramic:**

$$Q = 78 \times 10^{-12} \times 100, \quad Q = 7800 \times 10^{-12}$$

$$C = 1800 \times 8.8542 \times 10^{-12} \times 4 \times 10^{-6} / 1 \times 10^{-3} = 63750 \times 10^{-15}, \quad V = 122 \text{ V}$$