Introduction to DSP: Geometric Signal Theory & Sampling Theorem - Tutorial

1. Which of the following signal pairs are orthogonal? We assume that all signals here are of finite length N, i.e. $0 \leq n < N$.

   (a) $x[n] = 1$ and $y[n] = (-1)^n$.

   (b) $x[n] = 1$ and $y[n] = \cos\left(\frac{2\pi n}{N}\right)$.

   (c) $x[n] = \cos(\Omega_1 n)$ and $y[n] = \cos(\Omega_2 n)$.

   (d) $x[n] = \cos\left(\frac{2\pi k_1 n}{N}\right)$ and $y[n] = \cos\left(\frac{2\pi k_2 n}{N}\right)$.

2. Consider the signals $x_1[n] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^\top$, $x_2[n] = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}^\top$, and $x_3[n] = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top$. Find the representation of the signal $w[n] = \begin{bmatrix} 2 & 3 & -4 \end{bmatrix}^\top$ in terms of $x_1[n]$, $x_2[n]$, and $x_3[n]$.

3. Consider a signal $x(t) = \sin(12\pi t + 0.25\pi)$. Find the digital frequency of the signal when $x(t\ rp$ is sampled at the following sampling frequencies.

   (a) $F_s = 50Hz$

   (b) $F_s = 18Hz$

   (c) $F_s = 1Hz$

   (d) $F_s = \sqrt{2}Hz$