Introduction to DSP: Mathematical Preliminaries - Assignment

1. Consider a complex number, $z = x + jy$. We know that this complex number can also be written as $z = re^{j\theta}$, which is the called the Euler representation of the complex number. Find the Euler representation for the following complex numbers:

   (a) $z = -1 - 1j$

   (b) $z = -100j$

   (c) $z = 10\cos(\frac{\pi}{4}) + j10\sin(\frac{\pi}{4})$

   Plot these complex number in the complex plane.

2. The previous two complex numbers are fixed complex numbers. Let us now consider a complex number that changes with time $t \in \mathbb{R}$, and we write this complex numbers as $z(t)$ to indicate that it is a function of time t. $z(t)$ can be thought of as a mathematical function, that maps from the set of real number ($\mathbb{R}$) (time) to the set of complex numbers ($\mathbb{C}$). Find the Euler representation of the following time-varying complex numbers. Note that the r and θ will be time-varying as well.

   (a) $z(t) = 1 + t + j2t$

   (b) $z(t) = \cos(\pi t) + j\sin(\pi t)$

   (c) $z(t) = \cos(\pi t) + j2\sin(\pi t)$

   (d) $z(t) = t\cos(\pi t) + jt\sin(\pi t)$

3. Plot the trajectory of the complex number $z = t\,e^{j2\pi t}$ for time $0 \leq t \leq 5$.

4. For each complex number $z = x + jy$, we can define a complex conjugate representd by $\overline{z}$, which is defined as $\overline{z} = x - jy$, i.e the sign of the imaginary component is changed, while the real component remains unchanged. For the complex number $z = 3 + 4j$, plot the z and $\overline{z}$ on the complex plane.

5. If a complex number $z = re^{j\theta}$, what is the correponding Euler representation for its complex conjugate $\overline{z}$.

6. For any complex number $z = x + jy$, what can you say about the outcomes of the following operations:

   (a) $z + \overline{z}$

   (b) $z - \overline{z}$

   (c) $j(z - \overline{z})$

   (d) $z\overline{z}$

7. Consider two complex numbers $z_1 = 1 - i$ and $z_2 = 2 + i$. Using the complex plane, explain geometrically what happens we add the two complex numbers $(z_1 + z_2)$, and multipy the two complex numbers $(z_1 z_2)$.

8. Write down the domain and range of the following functions:

   (a) $y = \log(x)$

   (b) $y = \sqrt{x - 5}$

   (c) $y = \sin(x)$