Linear Systems

Dept. of Bioengineering, CMC Vellore

Instructor: Sivakumar Balasubramanian
TA: Prem Kumar, Tanya Subash
Lecture timings: Mon, Thur 7:30-9:00AM
Course Web-page: Please refer to this page for the up-to-date lecture notes and assignments.
https://siva82kb.github.io/teaching/ls/ls.html

What is the course about?

- Introduction to linear systems theory, in particular state space representation and analysis, state feedback control and state estimation.
- First half of the course focuses on applied linear algebra, and the second half focuses on the theory of linear systems.

What to expect from the course?

- Important concepts in applied linear algebra
- Brief introduction to optimization
- State space representation and analysis of physical systems
- Design and analysis of linear state feedback controllers
- Design and analysis of linear state observers

Course Scoring and Grading

Course Activities

- **Homework assignment** 15%
  Assignments will be provided to the student by the instructor and will be due one week after the assignments are provided. Late submissions will not be evaluated. Assignments will include both regular paper-and-pencil and programming problems. The student is free to use any programming language to solve the problems. You are encourged to work in groups to solves these problems, and learn from each other. But write down your own solutions and do not copy.
- **Surprize Quiz** 25%
  These will be given throughout the duration of the course. They will be short 20-30 min open book, in-class quizes.
- **Mid-term** 15%
  Take home exam, due the next day. This can include both paper-and-pencil and programming problems. Students are not allowed to discuss among themselves in solving these problems.
- **Final** 45%
  Take home exam, due the two days after it is given. This can include both paper-and-pencil and programming problems. Students are not allowed to discuss among themselves in solving these problems.

Grading policy:No relative grading

A+ : $90 \leq$ Score; **A** : $80 \leq$ Score < 90; **B** : $70 \leq$ Score < 80; **C** : $60 \leq$ Score < 70; **D** : $50 \leq$ Score < 60; **E** : $40 \leq$ Score < 50; **F** : Score < 40;

Policy for academic dishonesty

There will be zero tolerance towards academic dishonesty, and anyone found carrying out such activities will receive an '**F**' grade in the course. Activities such as copying assignments, submitting some else's code, cheating on quizzes and exams etc. are considered academically dishonest behavior.

References

1. G Strang, *Introduction to linear algebra*. Wellesley, MA: Wellesley-Cambridge Press, 1993.
2. CD Meyer, *Matrix analysis and applied linear algebra*. Siam; 2000 Jun 1.
3. S Boyd and L Vandenberghe, *Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares* . Online book

Other Resources

1. Online course on *Linear Algebra* by G Straing at MIT OCW. Course Link
2. Online course on *Linear Dynamical Systems* by S Boyd. Course Link

Course Content

1. Vectors
2. Matrices
3. Orthogonality
4. Matrix Inverses
5. Least squares methods
6. Eigenvectors and eigenvalues
7. Positive definiteness and matrix norm
8. Singular Value Decomposition
9. Optimization - A brief introduction
10. Linear dynamical systems (LDS) - Transfer Function View
11. LDS - State Space View
12. Solution of LDS
13. Stability
14. Controllability
15. Observability
16. State feedback control
17. Linear observers

Course content details

Vectors

1. Vectors; 2. Vector spaces; 3. Suspaces; 4. Linear independence; 5. Span and spanning sets; 6. Inner product; 7. Norm; 8. Angle between vectors; 9. Basis; 10. Dimension of a vector space; 11. Linear functions

Matrices

1. Matrices; 2. Matrix Operations; 3. Matrix Multiplication; 4. Properties of Matrix Multipilication; 5. Geometry of Linear Equations; 6. Gaussian Elimination; 7. Gauss-Jordan Method; 8. Row Echelon Forms; 9. Homogenous systems; 10. Non-homogenous systems; 11. $\mathbf{LU}$ factorization; 12. Linear transformation; 13. Four fundamental subspace; 14. Matrix inverse

Orthogonality

1. Orthogonality; 2. Orthogonal subspaces; 3. Relationship between the four fundamental subspaces; 4. Gram-Schmidt orthogonalization; 5. $\mathbf{QR}$ factorization; 6. Orthogonal projection

Matrix Inverses

1. Representation of vectors in a basis; 2. Matrix Inverse; 3. Left Inverse; 4. Right Inverse; 5. Pseudo-inverse; 6. Inverses and $\mathbf{QR}$ factorization

Least squares methods

1. Overdetermined System of linear equations 2. Least Squares Problem 3. Multi-Objective Least Squares 4. Constrained Least Squares

Eigenvectors and eigenvalues

1. Linear transformation; 2. Representation of linear trasnformations in different basis; 3. Similarity transformation; 4. Complex vectors and matrices; 5. Eigenvectors and Eigenvalues; 6. Diagonalization of matrix $\mathbf{X\Lambda X}^{-1}$; 7. Jordan form

Positive definiteness and matrix norm

1. Positive definite matrices; 2. Matrix Norm – Frobinius norm and Induced Norm

Singular Value Decomposition (SVD)

1. Matrix equivalence; 2. SVD – Diagonalizing any matrix; 3. Geometry of SVD

Optimization - A brief introduction

1. Linear optimization 2. Non-linear optimization 3. Linear programming

Singular Value Decomposition (SVD)

1. Matrix equivalence; 2. SVD – Diagonalizing any matrix; 3. Geometry of SVD

Linear dynamical systems (LDS) - Transfer Function View

1. Important signals; 2. Linear Time-Invariant (LTI) Systems; 3. Unilateral Laplace Transform; 4. Impulse response of continous-time LTI systems; 5. Convolution Integral; 6. Transfer function of continous-time LTI systems; 7. z-transform; 8. Impulse response of discrete-time LTI systems; 9. Convolution sum; 10. Transfer function of discrete-time LTI systems

LDS - State Space View

1. States of a system 2. State space representation of linear systems 3. Block diagram representation of linear systems 4. State space representation of discrete-time linear systems 5. Block diagram representation of discrete-time linear systems 6. State space visualization

Solution of LDS

1. Zero-input solution for $\mathbf{x}(t)$ 2. Cayley-Hamilton theorem 3. Laplace transform approach to zero-input response 4. $e^{t\mathbf{A}}$ and its properties 5. Modes of a system 6. Zero-state solution 7. Complete solution of a linear system.

Stability

1. Internal stability 2. Lyapunov stability criteria 3. Input-Output stability

Controllability

Observability

State feedback control

Linear observers