**MS Bioengineering**

**Linear Systems – 2023 Jan**

**Course Coordinator**

Sivakumar Balasubramanian

**Course Coordinator**

Ms Monisha Yuvaraj, Mr Diwakar Reddy

**Coarse Information**

Introduction to linear systems theory, in particular state space representation and analysis, state feedback control and state estimation.

The first half of the course focuses on applied linear algebra, and the second half focuses on the theory of linear systems.

The course will cover the following topics:

* Important concepts in applied linear algebra
* A brief introduction to optimization
* State space representation and analysis of physical systems
* Design and analysis of linear state feedback controllers
* Design and analysis of linear state observers

**Student Learning Objectives**

On successful completion of the course, the students will be able to:

* Understand basic linear through a geometric perspective.
* Understanding multi-input, multi-output systems.
* Solve many practical problems which can be posed as linear optimization problems.
* Gain a deeper understanding of linear dynamical systems.

**Assessment**

Assignments (pen-and-paper, programming)

Quiz

Mid-term exam

Final exam.

**Weightage for Assessment**

Assignments (pen-and-paper, programming) - 15%

Quiz - 25%

Mid-term exam - 15%

Final exam. - 45%

**Schedule**

Tuesday: 7:30 – 8:30 AM

Thursday: 4:00 – 5:00 PM

**Course Content**

**Vectors**

1. Vectors; 2. Vector spaces; 3. Subspaces; 4. Linear independence; 5. Span and spanning sets; 6. Inner product; 7. Norm; 8. Angle between vectors; 9. Basis; 10. Dimension of a vector space; 11. Linear functions

**Matrices**

1. Matrices; 2. Matrix Operations; 3. Matrix Multiplication; 4. Properties of Matrix Multiplication; 5. Geometry of Linear Equations; 6. Gaussian Elimination; 7. Gauss-Jordan Method; 8. Row Echelon Forms; 9. Homogenous systems; 10. Non-homogenous systems; 11. LU factorization; 12. Linear transformation; 13. Four fundamental subspaces; 14. Matrix inverse

**Orthogonality**

1. Orthogonality; 2. Orthogonal subspaces; 3. Relationship between the four fundamental subspaces; 4. Gram-Schmidt orthogonalization; 5. QR factorization; 6. Orthogonal projection

**Matrix Inverses**

1. Representation of vectors in a basis; 2. Matrix Inverse; 3. Left Inverse; 4. Right Inverse; 5. Pseudo-inverse; 6. In- verses and QR factorization

**Least squares methods**

1. Overdetermined System of linear equations 2. Least Squares Problem 3. Multi-Objective Least Squares 4. Con- strained Least Squares

**Eigenvectors and eigenvalues**

1. Linear transformation; 2. Representation of linear trasnfor- mations in different basis; 3. Similarity transformation; 4. Complex vectors and matrices; 5. Eigenvectors and Eigen- values; 6. Diagonalization of matrix XΛX−1; 7. Jordan form

**Positive definiteness and matrix norm**

1. Positive definite matrices; 2. Matrix Norm – Frobinius norm and Induced Norm

**Singular Value Decomposition (SVD)**

1. Matrix equivalence; 2. SVD – Diagonalizing any matrix; 3. Geometry of SVD

**Linear dynamical systems (LDS) - Transfer Function View**

1. Important signals; 2. Linear Time-Invariant (LTI) Systems; 3. Unilateral Laplace Transform; 4. Impulse response of continuous-time LTI systems; 5. Convolution Integral; 6. Transfer function of continuous-time LTI systems; 7. z-transform; 8. Impulse response of discrete-time LTI systems; 9. Convolution sum; 10. Transfer function of discrete-time LTI systems

**LDS - State Space View**

1. States of a system 2. State space representation of linear systems 3. Block diagram representation of linear systems 4. State space representation of discrete-time linear systems 5. Block diagram representation of discrete-time linear systems 6. State space visualization

**Solution of LDS**

1. Zero-input solution for x(t) 2. Cayley-Hamilton theorem 3. Laplace transform approach to zero-input response 4. etA and its properties 5. Modes of a system 6. Zero-state solution 7. Complete solution of a linear system.

**Stability**

1. Internal stability 2. Lyapunov stability criteria 3. In- put-Output stability

**Controllability & Observability**

**Textbook & Resources**

1. G Strang, Introduction to linear algebra. Wellesley, MA: Wellesley-Cambridge Press, 1993.
2. CD Meyer, Matrix analysis and applied linear algebra. Siam; 2000 Jun 1.
3. S Boyd and L Vandenberghe, Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares

**Course Schedule**

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| --- | --- | --- | --- |
| **Week** | **Dates** | **Lectures** | **Assignment due date** |
| Week 1 | Jan 23 – Jan 28 | [Lecture 01](https://www.youtube.com/watch?v=ju2OOo9SSDw&list=PL8eY_HDc45x-7XVnCNQGUbgagJAoLdPME&index=1) |  |
| Week 2 | Jan 30 – Feb 4 | [Lecture 02](https://www.youtube.com/watch?v=JmVBRKQi4ns&list=PL8eY_HDc45x-7XVnCNQGUbgagJAoLdPME&index=2)  [Lecture 03](https://youtu.be/eSHx-JXb4KI) |  |
| Week 3 | Feb 6 – Feb 11 | Lecture 04  Lecture 05 | Assignment 1 is due on Feb 7 |
| Week 4 | Feb 13 – Feb 18 | Lecture 06  Lecture 07 |  |
| Week 5 | Feb 20 – Feb 25 | Lecture 08  Lecture 09 | Assignment 2 is due on Feb 21 |
| Week 6 | Feb 27 – Mar 4 | Lecture 10  Lecture 11 |  |
| Week 7 | Mar 6 – Mar 11 | Lecture 12  Lecture 13 | Assignment 3 is due on Mar 07 |
| Week 8 | Mar 13 – Mar 18 | Lecture 14  Lecture 15 | Assignment 4 is due on Mar 14 |
| Week 9 | Mar 20 – Mar 25 | Lecture 16  Lecture 17 | Assignment 5 is due on Mar 21 |
| Week 10 | Mar 27 – Apr 1 | Mid-term |  |
| Week 11 | Apr 3 – Apr 8 | Lecture 18  Lecture 19 | Assignment 6 is due on Mar 21 |
| Week 12 | Apr 10 – Apr 15 | Lecture 20  Lecture 21 | Assignment 7 is due on Mar 21 |
| Week 13 | Apr 17 – Apr 22 | Lecture 22  Lecture 23 | Assignment 8 is due on Mar 18 |
| Week 14 | Apr 24 – Apr 29 | Lecture 24  Lecture 25 | Assignment 9 is due on Mar 25 |
| Week 15 | May 1 – May 6 | Lecture 26  Lecture 27 | Assignment 10 is due on May 2 |
| Week 16 | May 8 – May 13 | Final Exam |  |