Dual Lagrangian

$$L(\lambda, r) = \lambda_1 + \lambda_2 + \lambda_1 \lambda_2 \langle \bar{x}_1, \bar{x}_2 \rangle - \frac{1}{2}\lambda_1^2 \langle \bar{x}_1, \bar{x}_1 \rangle$$
$$- \frac{1}{2}\lambda_2^2 \langle \bar{x}_2, \bar{x}_2 \rangle - r\lambda_1 + r\lambda_2$$

The gradient of the Dual Lagrangian is:

$$\frac{\partial}{\partial \lambda_1} L(\lambda, r) = 1 + \lambda_2 \langle \bar{x}_1, \bar{x}_2 \rangle - \lambda_1 \langle \bar{x}_1, \bar{x}_1 \rangle - r = 0$$

$$\frac{\partial}{\partial \lambda_2} L(\lambda, r) = 1 + \lambda_1 \langle \bar{x}_1, \bar{x}_2 \rangle - \lambda_2 \langle \bar{x}_2, \bar{x}_2 \rangle + r = 0$$

$$\frac{\partial}{\partial r} L(\lambda, r) = -\lambda_1 + \lambda_2 = 0$$

Note that our textbook denotes λ_1 and λ_2 for the dual as a_1 and a_2

Suppose $\bar{x}_1 = (1, 1)$
$\bar{x}_2 = (2, 4)$

We can write

$$1 + \lambda_2 \langle (1,1),(2,4) \rangle - \lambda_1 \langle (1,1),(1,1) \rangle - r = 0$$

$$1 + \lambda_1 \langle (1,1),(2,4) \rangle - \lambda_2 \langle (2,4),(2,4) \rangle + r = 0$$

$$-\lambda_1 + \lambda_2 = 0$$

Note that $\langle \bar{x}_1, \bar{x}_2 \rangle$ is the dot product of $\bar{x}_1$ and $\bar{x}_2$

From $-\lambda_1 + \lambda_2 = 0$ we get $\lambda_1 = \lambda_2$

So we can replace λ_1 and λ_2 with λ

$$1 + \lambda\langle(1,1),(2,4)\rangle - \lambda\langle(1,1),(1,1)\rangle - r = 0$$
$$(1\cdot 2 + 1\cdot 4) = 6 \qquad (1\cdot 1 + 1\cdot 1) = 2$$

$$1 + 6\lambda - 2\lambda - r = 0$$

$$1 + \lambda\langle(1,1),(2,4)\rangle - \lambda\langle(2,4),(2,4)\rangle + r = 0$$
$$(1\cdot 2 + 1\cdot 4) = 6 \qquad (2\cdot 2 + 4\cdot 4) = 20$$

$$1 + 6\lambda - 20\lambda + r = 0$$

Solve for λ

$$1 + 4\lambda - r = 0$$
$$1 - 14\lambda + r = 0$$

$$2 - 10\lambda = 0 \quad \therefore \lambda = \frac{1}{5}$$

Now that we have $\lambda = \frac{1}{5}$, we can solve for w and b.

Recall that

$$\overline{w} - \lambda_1 \overline{x}_1 + \lambda_2 \overline{x}_2 = 0$$

We can use this to solve for w

Recall that equation 81 in the Tutorial is

$$\bar{w} - \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 = 0$$

$$\bar{w} = \lambda_1 \bar{x}_1 - \lambda_2 \bar{x}_2$$

$$\bar{w} = \frac{1}{5}(1,1) - \frac{1}{5}(2,4) = \left(\frac{1}{5} - \frac{2}{5}, \frac{1}{5} - \frac{4}{5}\right)$$
$$= \left(-\frac{1}{5}, -\frac{3}{5}\right)$$

~~So~~

So $\bar{w} = \left(-\frac{1}{5}, -\frac{3}{5}\right)$

Now solve for b

$$b = 1 - \left\langle \left(-\frac{1}{5}, -\frac{3}{5}\right), (1,1) \right\rangle = 1 + \frac{4}{5} = \frac{9}{5}$$

also

$$b = -1 - \left\langle \left(-\frac{1}{5}, -\frac{3}{5}\right), (2,4) \right\rangle = -1 + \frac{14}{5} = \frac{9}{5}$$

Try it for the points.

$(-2, 2)$, $(1, 1)$

$(1, 1)$, $(4, 3)$

$(2, -2)$, $(-1, 1)$