## Dendrogram Examples

1. Single Linkage Euclidean

   (a) We start with 6 clusters, each consisting of a single point. For single linkage, we calculate minimum distances to the *nearest* point in each cluster

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.23 | 0 | | | | |
| 3 | 0.22 | 0.14 | 0 | | | |
| 4 | 0.37 | 0.19 | 0.16 | 0 | | |
| 5 | 0.34 | 0.14 | 0.28 | 0.28 | 0 | |
| 6 | 0.24 | 0.24 | **0.10** | 0.22 | 0.39 | 0 |

   (b) Clusters 3 and 6 are closest, so they are merged to give us cluster ((3),(6)) at level 0.10.

   (c) For clusters (1),(2),(4),(5) and ((3),(6)) choose the minimum distance to the *nearest* point in every other cluster. This includes checking nearest of (3)or(6) in the cluster ((3),(6)).

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.23 | 0 | | | | |
| 3 | **0.22** | **0.14** | 0 | | | |
| 4 | 0.37 | 0.19 | 0.16 | 0 | | |
| 5 | 0.34 | 0.14 | **0.28** | 0.28 | 0 | |
| 6 | 0.24 | 0.24 | 0.10 | **0.22** | 0.39 | 0 |

   (d) The distances shown above are the distances to the nearest point in cluster ((3),(6)). Since we have recalculated the minimum distances to ((3),(6)), we can now write it as (3,6). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.23 | 0 | | | |
| (3,6) | 0.22 | **0.14** | 0 | | |
| 4 | 0.37 | 0.19 | 0.16 | 0 | |
| 5 | 0.34 | 0.14 | 0.28 | 0.22 | 0 |

   The table shows that (2) is merged with (3,6). The new cluster is ((3,6),(2)) at level 0.14. Note: If there are multiple minimum values, it doesn't matter which one you pick. The next merge will still occur at the exact same height.

   (e) For clusters (1),(4),(5), and ((3,6),(2)), choose the minimum distance to every other cluster including the nearest cluster in ((3,6),(2)).

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.23 | 0 | | | |
| (3,6) | **0.22** | 0.14 | 0 | | |
| 4 | 0.37 | 0.19 | **0.16** | 0 | |
| 5 | 0.34 | **0.14** | 0.28 | 0.22 | 0 |

(f) The distances underlined above are the distances to the nearest point in cluster ((3,6),(2)) so we can now write it as (3,6,2). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | 1 | (3,6,2) | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (3,6,2) | 0.22 | 0 | | |
| 4 | 0.37 | 0.16 | 0 | |
| 5 | 0.34 | **0.14** | 0.22 | 0 |

The table shows that the new cluster is ((3,6,2),(5)) at level 0.14

(g) For clusters (1),(4), and ((3,6,2),(5)) choose the closest distance to any other cluster including either ((3,6,2) or (5)) in the cluster (3,6,2,5)

| | 1 | (2,3,6) | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (2,3,6) | **0.22** | 0 | | |
| 4 | 0.37 | **0.16** | 0 | |
| 5 | 0.34 | 0.14 | 0.22 | 0 |

(h) The distances underlined above are the distances to the nearest point in cluster (3,6,2,5). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | 1 | (3,6,2,5) | 4 |
|---|---|---|---|
| 1 | 0 | | |
| (3,6,2,5) | 0.22 | 0 | |
| 4 | 0.37 | **0.16** | 0 |

The table shows that the new cluster is ((3,6,2,5),(4)) at level 0.16

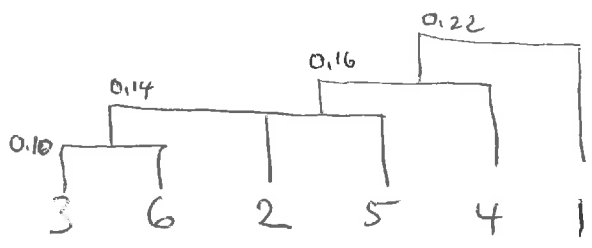
(i) For clusters (1) and ((3,6,2.5),(4)) choose the nearest distance of (1) to either ((3,6,2.5)or (4)).

| | 1 | (3,6,2,5) | 4 |
|---|---|---|---|
| 1 | 0 | | |
| (3,6,2,5) | **0.22** | 0 | |
| 4 | 0.37 | 0.16 | 0 |

(j) The distances underlined above are the distances to the nearest point in cluster (3,6,2,5,4). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | 1 | (3,6,2,5,4) |
|---|---|---|
| 1 | 0 | |
| (3,6,2,5,4) | **0.22** | 0 |

The table shows that the new cluster is ((3,6,2,5,4),(1)) linked at level 0.22



2. Complete Linkage Euclidean

(a) We start with 6 clusters, each consisting of a single point. For single linkage, we calculate distances to the *farthest* point in each cluster

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.23 | 0 | | | | |
| 3 | 0.22 | 0.14 | 0 | | | |
| 4 | 0.37 | 0.19 | 0.16 | 0 | | |
| 5 | 0.34 | 0.14 | 0.28 | 0.28 | 0 | |
| 6 | 0.24 | 0.24 | **0.10** | 0.22 | 0.39 | 0 |

(b) Since all clusters start with a single point (considered both nearest and farthest), we start by merging the nearest clusters. Clusters (3) and (6) are closest, so they are merged to give us cluster ((3),(6)) at level 0.10.

(c) For clusters (1),(2),(4),(5) and ((3),(6)) choose the minimum distance to the *farthest* point in all other clusters including (3) or (6) in the cluster ((3),(6)).

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.23 | 0 | | | | |
| 3 | 0.22 | 0.14 | 0 | | | |
| 4 | 0.37 | 0.19 | 0.16 | 0 | | |
| 5 | 0.34 | 0.14 | 0.28 | 0.28 | 0 | |
| 6 | **0.24** | **0.24** | 0.10 | **0.22** | **0.39** | 0 |

(d) The distances underlined above are the distances to the farthest point in cluster ((3),(6)). Since all distances have been calculated to the farthest point of ((3),(6)), we can notate it as (3,6). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.23 | 0 | | | |
| (3,6) | 0.24 | 0.24 | 0 | | |
| 4 | 0.37 | 0.19 | 0.22 | 0 | |
| 5 | 0.34 | **0.14** | 0.39 | 0.28 | 0 |

The table shows that (2) is merged with (5), the new cluster is ((2),(5)) at level 0.14.

(e) For clusters (1),(4), ((2),(5)), (3,6) choose the min distance to farthest point in any other cluster.

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.23 | 0 | | | |
| (3,6) | 0.24 | 0.24 | 0 | | |
| 4 | 0.37 | 0.19 | 0.22 | 0 | |
| 5 | **0.34** | 0.14 | **0.39** | **0.28** | 0 |

(f) The distances underlined above are the distances to the farthest point in cluster ((2),(5)) which we can now notate as (2,5). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | 1 | (2,5) | (3,6) | 4 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (2,5) | 0.34 | 0 | | |
| (3,6) | 0.24 | 0.39 | 0 | |
| 4 | 0.37 | 0.28 | **0.22** | 0 |

The table shows that the new cluster is ((3,6),(4)) at level 0.22.

(g) For clusters (1),(2,5), and ((3,6),(4)) choose the min distance to farthest point in any other cluster including either ((3,6) or (4)) in the cluster ((3,6),(4))

| | 1 | (2,5) | (3,6) | 4 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (2,5) | 0.34 | 0 | | |
| (3,6) | 0.24 | **0.39** | 0 | |
| 4 | **0.37** | 0.28 | 0.22 | 0 |

(h) The distances underlined above are the distances to the farthest point in cluster ((3,6),(4)), so the cluster is (3,6,4). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | 1 | (2,5) | (3,6,4) |
|---|---|---|---|
| 1 | 0 | | |
| (2,5) | **0.34** | 0 | |
| (3,6,4) | 0.37 | 0.39 | 0 |

The table shows that the new cluster is ((2,5),(1)) at level 0.34.

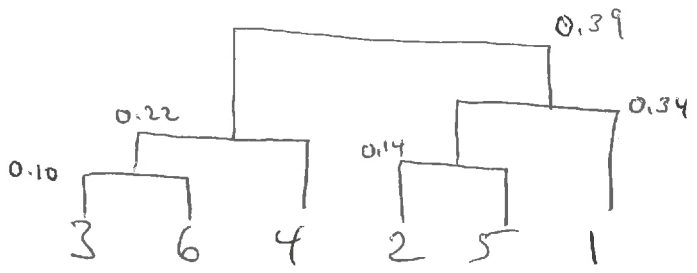
(i) For clusters (3,6,4) and ((2,5),(1)) choose the min distance to the farthest point in any other cluster including either ((2,5)or(1)).

| | 1 | (2,5) | (3,6,4) |
|---|---|---|---|
| 1 | 0 | | |
| (2,5) | 0.34 | 0 | |
| (3,6,4) | 0.37 | **0.39** | 0 |

(j) The distances underlined above are the distances to the farthest point in cluster which we now call (2,5,1). Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | (2,5,1) | (3,6,4) |
|---|---|---|
| (2,5,1) | 0 | |
| (3,6,4) | **0.39** | 0 |

The table shows that the new cluster is ((3,6,4),(2,5,1)) at level 0.39.



3. Single Linkage Manhattan

(a) We start with 6 clusters, each consisting of a single point. For single linkage, we calculate distances to the *nearest* point in each cluster

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.33 | 0 | | | | |
| 3 | 0.26 | 0.19 | 0 | | | |
| 4 | 0.48 | 0.23 | 0.22 | 0 | | |
| 5 | 0.44 | 0.17 | 0.36 | 0.40 | 0 | |
| 6 | 0.28 | 0.31 | **0.12** | 0.30 | 0.48 | 0 |

(b) Clusters 3 and 6 are closest, so they are merged to give us cluster ((3),(6)) at level 0.12.

(c) For clusters 1,2,4,5 and ((3),(6)) choose the minimum distance to the *nearest* point in any other cluster including ((3) or (6)) in the cluster (3,6).

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.33 | 0 | | | | |
| 3 | **0.26** | **0.19** | 0 | | | |
| 4 | 0.48 | 0.23 | **0.22** | 0 | | |
| 5 | 0.44 | 0.17 | **0.36** | 0.40 | 0 | |
| 6 | 0.28 | 0.31 | 0.12 | 0.30 | 0.48 | 0 |

(d) Fill in a new table based on the distances chosen above and merge the two clusters that are closest.

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.33 | 0 | | | |
| (3,6) | 0.26 | 0.19 | 0 | | |
| 4 | 0.48 | 0.23 | 0.22 | 0 | |
| 5 | 0.44 | **0.17** | 0.36 | 0.40 | 0 |

The table shows that 2 is merged with 5. The new cluster is ((2),(5)) at level 0.17.

(e) For clusters 1,4,(3,6), and ((2),(5)) choose the min distance to any other cluster including (2 or (3,6)) in the cluster (2,3,6).

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.33 | 0 | | | |
| (3,6) | **0.26** | 0.19 | 0 | | |
| 4 | 0.48 | 0.23 | **0.16** | 0 | |
| 5 | 0.44 | **0.17** | 0.36 | 0.40 | 0 |

(f) Fill in a new table based on the distances chosen above and merge the two clusters that are closest.

| | 1 | (2,5) | (3,6) | 4 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (2,5) | 0.33 | 0 | | |
| (3,6) | 0.26 | **0.19** | 0 | |
| 4 | 0.48 | 0.23 | 0.22 | 0 |

The table shows that the new cluster is ((3,6),(2,5)) at level 0.19

(g) For clusters 1,4, and ((3,6),(2,5)) choose the closest distance to any other cluster including either ((3,6)or(2,5)) in the cluster (3,6,2,5)

| | 1 | (2,5) | (3,6) | 4 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (2,5) | 0.33 | 0 | | |
| (3,6) | **0.26** | 0.19 | 0 | |
| 4 | 0.48 | 0.23 | 0.22 | 0 |

(h) Fill in a new table based on the distances chosen above and merge the two clusters that are closest.

| | 1 | (3,6,2,5) | 4 |
|---|---|---|---|
| 1 | 0 | | |
| (3,6,2,5) | 0.26 | 0 | |
| 4 | 0.48 | **0.22** | 0 |

The table shows that the new cluster is ((3,6,2,5),4) at level 0.22

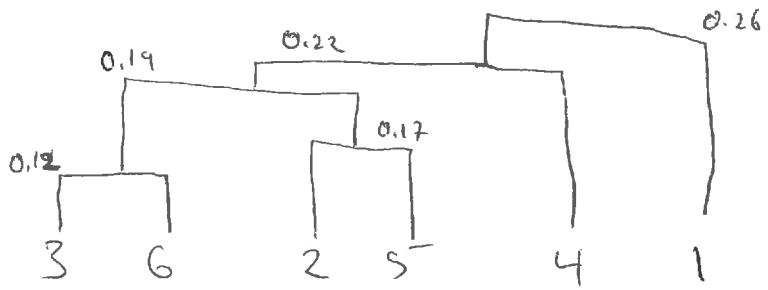
(i) For clusters 1 and ((3,6,2,5),4) choose the closest of the distance from 1 to either ((3,6,2,5)or 4).

| | 1 | (3,6,2,5) | 4 |
|---|---|---|---|
| 1 | 0 | | |
| (3,6,2,5) | **0.26** | 0 | |
| 4 | 0.48 | 0.22 | 0 |

(j) Fill in a new table based on the distances chosen above and merge the two clusters that are closest

| | 1 | (3,6,2,5) |
|---|---|---|
| 1 | 0 | |
| (3,6,2,5) | **0.26** | 0 |

The table shows that the new cluster is ((3,6,2,5,4),(1)) linked at level 0.26



4. Complete Linkage Manhattan

(a) We start with 6 clusters, each consisting of a single point. For single linkage, we calculate distances to the *farthest* point in each cluster

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.33 | 0 | | | | |
| 3 | 0.26 | 0.19 | 0 | | | |
| 4 | 0.48 | 0.23 | 0.22 | 0 | | |
| 5 | 0.44 | 0.17 | 0.36 | 0.40 | 0 | |
| 6 | 0.28 | 0.31 | **0.12** | 0.30 | 0.48 | 0 |

(b) Clusters 3 and 6 are closest, so they are merged to give us cluster (3,6) at level 0.12.

(c) For clusters 1,2,4,5 and (3,6) choose the minimum distance to the *farthest* point in any other cluster including (3 or 6) in the cluster (3,6).

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | | | | | |
| 2 | 0.33 | 0 | | | | |
| 3 | 0.26 | 0.19 | 0 | | | |
| 4 | 0.48 | 0.23 | 0.22 | 0 | | |
| 5 | 0.44 | 0.17 | 0.36 | 0.40 | 0 | |
| 6 | **0.28** | **0.31** | 0.12 | **0.30** | **0.48** | 0 |

(d) Fill in a new table based on the distances chosen above and merge the two clusters that are closest.

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.33 | 0 | | | |
| (3,6) | 0.28 | 0.31 | 0 | | |
| 4 | 0.48 | 0.23 | 0.30 | 0 | |
| 5 | 0.44 | **0.17** | 0.48 | 0.40 | 0 |

The table shows that 2 is merged with (5), the new cluster is (2,5) at level 0.17.

(e) For clusters 1,4, ((2),(5)), (3,6) choose the min distance to farthest point in any other cluster.

| | 1 | 2 | (3,6) | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | | | | |
| 2 | 0.33 | 0 | | | |
| (3,6) | 0.28 | 0.19 | 0 | | |
| 4 | 0.48 | 0.23 | 0.30 | 0 | |
| 5 | **0.44** | 0.17 | **0.48** | **0.40** | 0 |

(f) Fill in a new table based on the distances chosen above and merge the two clusters that are closest.

| | 1 | (2,5) | (3,6) | 4 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (2,5) | 0.44 | 0 | | |
| (3,6) | **0.28** | 0.48 | 0 | |
| 4 | 0.48 | 0.40 | 0.30 | 0 |

The table shows that the new cluster is ((3,6),1) at level 0.28.

(g) For clusters 4, ((3,6),(1)) and (2,5) choose the min distance to farthest point in any other cluster including either ((3,6) or (1)) in the cluster (3,6,1)

| | 1 | (2,5) | (3,6) | 4 |
|---|---|---|---|---|
| 1 | 0 | | | |
| (2,5) | 0.44 | 0 | | |
| (3,6) | 0.28 | **0.48** | 0 | |
| 4 | **0.48** | 0.40 | 0.30 | 0 |

(h) Fill in a new table based on the distances chosen above and choose the two clusters that are closest.

| | (3.6,1) | (2,5) | 4 |
|---|---|---|---|
| (3,6,1) | 0 | | |
| (2,5) | 0.48 | 0 | |
| 4 | 0.48 | **0.40** | 0 |

The table shows that the new cluster is ((2,5),(4)) at level 0.40.

(i) For for clusters (3,6,1)((2,5),(4)) choose the min distance to farthest point in any other cluster including either ((2,5)or (4)).

| | (3.6,1) | (2,5) | 4 |
|---|---|---|---|
| (3,6,1) | 0 | | |
| (2,5) | 0.48 | 0 | |
| 4 | **0.48** | 0.40 | 0 |

(j) Fill in a new table based on the distances chosen above and merge the two clusters that are closest

| | (3,6,1) | (2,5,4) |
|---|---|---|
| (3,6,1) | 0 | |
| (2,5,4) | **0.48** | 0 |

The table shows that the new cluster is ((3,6,1),(2,5,4)) at level 0.48.