Linear Regression Derivation

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\begin{aligned}
\hat{y}_i &= \beta_0 + \beta_1 x_i \\
\text{RSS} &= \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \\
\text{RSS} &= \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \\
0 &= \frac{\partial \text{RSS}}{\partial \beta_0} \left[\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \right] \\
0 &= -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \\
0 &= \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \\
0 &= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \beta_0 - \beta_1 \sum_{i=1}^{n} x_i \\
0 &= \sum_{i=1}^{n} y_i - n\beta_0 - \beta_1 \sum_{i=1}^{n} x_i \\
0 &= \frac{\sum_{i=1}^{n} y_i - n\beta_0 - \beta_1 \sum_{i=1}^{n} x_i}{n} \\
0 &= \frac{\sum_{i=1}^{n} y_i}{n} - \frac{n\beta_0}{n} - \frac{\beta_1 \sum_{i=1}^{n} x_i}{n} \\
0 &= \bar{y} - \beta_0 - \beta_1 \bar{x} \\
\beta_0 &= \bar{y} - \beta_1 \bar{x} \qquad (1) \\
0 &= \frac{\partial \text{RSS}}{\partial \beta_1} \left[\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \right] \\
0 &= -2x_i \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \\
0 &= \sum_{i=1}^{n} \left(x_i y_i - \beta_0 x_i - \beta_1 x_i^2 \right)^2 \\
0 &= \sum_{i=1}^{n} \left(x_i y_i - (\bar{y} - \beta_1 \bar{x})\, x_i - \beta_1 x_i^2 \right) \\
0 &= \sum_{i=1}^{n} \left(x_i y_i - \bar{y} x_i + \beta_1 \bar{x} x_i - \beta_1 x_i^2 \right) \\
0 &= \sum_{i=1}^{n} (x_i y_i - \bar{y} x_i) + \sum_{i=1}^{n} \left(\beta_1 \bar{x} x_i - \beta_1 x_i^2 \right) \\
\beta_1 &= \frac{\sum_{i=1}^{n} (x_i y_i - \bar{y} x_i)}{\sum_{i=1}^{n} \left(x_i^2 - \bar{x} x_i \right)} \qquad (2)
\end{aligned}
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