LDA for 2 Variables

We have looked at how to find the projection for the LDA problem, but we need some measure to make the decision as to which class a sample belongs. A good explanation can be found at the following link:
https://people.revoledu.com/kardi/tutorial/LDA/

- Using Bayes' rule we can express the LDA formula as:

$$f_i = \boldsymbol{\mu}_i \boldsymbol{C}^{-1} \boldsymbol{x}_k^T - \frac{1}{2} \boldsymbol{\mu}_i \boldsymbol{C}^{-1} \boldsymbol{\mu}_i^T + \log(p_i)$$

  Note: This is equivalent to equations (4.13) and (4.19) on pages 140 and 143 in our textbook.
  $\mu_{\mathbf{i}}$ is the vector of means for class i. This is a row vector
  $\mathbf{C^{-1}}$ is the inverse of the covariance matrix. Its dimension is the number of classes we have.
  $\mathbf{x_k^T}$ is transpose of the vector of data (predictors). $\log(p)$ is the log of the prior probabilities (number of samples of each class)

- We assign object k to group i that has maximum f_i.

- Lets look at the data from the example from the Elhabian (see link)
  We have a set of data from two classes. Since the data from each sample point is organized in a row. We will use row vectors instead of column vectors.

  - Let our class 1 samples be represented in the following matrix:

$$\mathbf{X_1} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \\ 2 & 3 \\ 3 & 6 \\ 4 & 4 \end{bmatrix}$$

    with

$$\mu_{\mathbf{1}} = \begin{bmatrix} \frac{15}{5} & \frac{19}{5} \end{bmatrix}$$

  - Let our class 2 samples be represented in the following matrix:

$$\mathbf{X_2} = \begin{bmatrix} 9 & 10 \\ 6 & 8 \\ 9 & 5 \\ 8 & 7 \\ 10 & 8 \end{bmatrix}$$

    with

$$\mu_{\mathbf{2}} = \begin{bmatrix} \frac{42}{5} & \frac{38}{5} \end{bmatrix}$$

  - Our overall mean is

$$\mu = \begin{bmatrix} \frac{57}{10} & \frac{57}{10} \end{bmatrix}$$

    which is $p_1\boldsymbol{\mu_1} + p_2\boldsymbol{\mu_2}$ where $p_1 = p_2 = 1/2$

– We normalize everything to have a mean of 0 by subtracting the mean of each class.

$$\mathbf{X_1^0} = \begin{bmatrix} -\frac{17}{10} & -\frac{37}{10} \\ -\frac{37}{10} & -\frac{17}{10} \\ -\frac{37}{10} & -\frac{27}{10} \\ -\frac{27}{10} & \frac{3}{10} \\ -\frac{17}{10} & -\frac{17}{10} \end{bmatrix}$$

$$\mathbf{X_2^0} = \begin{bmatrix} \frac{33}{10} & \frac{43}{10} \\ \frac{3}{10} & \frac{23}{10} \\ \frac{33}{10} & -\frac{7}{10} \\ \frac{23}{10} & \frac{13}{10} \\ \frac{43}{10} & \frac{23}{10} \end{bmatrix}$$

– The covariance matrix for each class is:

$$\boldsymbol{C}_i = \frac{\left(\boldsymbol{X}_i^0\right)^T \boldsymbol{X}_i^0}{n_i}$$

– The covariance matrix for class 1 is

$$\boldsymbol{C}_1 = \begin{bmatrix} \frac{809}{100} & \frac{493}{100} \\ \frac{493}{100} & \frac{537}{100} \end{bmatrix}$$

– The covariance matrix for class 2 is

$$\boldsymbol{C}_1 = \begin{bmatrix} \frac{913}{100} & \frac{509}{100} \\ \frac{509}{100} & \frac{25}{4} \end{bmatrix}$$

– The overall Covariance is given as

$$C(r,s) = \frac{1}{n}\sum_{i=1}^{g} n_i \cdot c_i(r,s)$$

$$\boldsymbol{C} = \begin{bmatrix} \frac{861}{100} & \frac{501}{100} \\ \frac{501}{100} & \frac{581}{100} \end{bmatrix}$$

– The inverse of the $\boldsymbol{C}$ matrix is:

$$\boldsymbol{C}^{-1} = \begin{bmatrix} \frac{2905}{12462} & -\frac{835}{4154} \\ -\frac{835}{4154} & \frac{1435}{4154} \end{bmatrix}$$

– Our p matrix is:

$$p = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

- The model is now trained. Lets look at some examples

$$x_1 = \begin{bmatrix} 1 & 3 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 0.119756045206451445 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} -2.82006099860674064 \end{bmatrix}$$

This is class 1

$$x_2 = \begin{bmatrix} 5 & 5 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 1.28104636778709646 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 0.775509535817910489 \end{bmatrix}$$

This is class 1

$$x_3 = \begin{bmatrix} 7 & 7 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 2.57136894843225861 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 3.51022306494646052 \end{bmatrix}$$

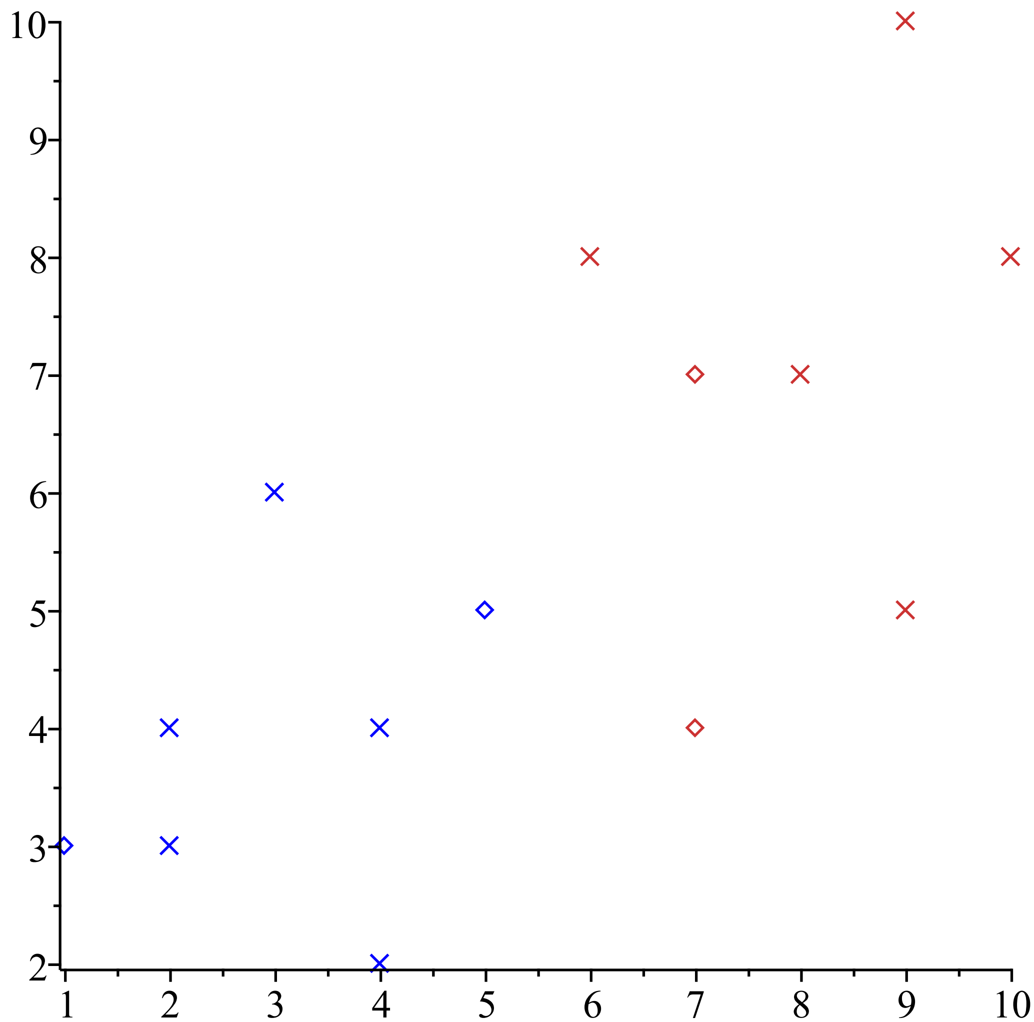
This is class 2

$$x_4 = \begin{bmatrix} 7 & 4 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 0.442336690367741925 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 0.699438279197785562 \end{bmatrix}$$

This is class 2



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