LDA Example for 3 Variables

We have looked at how to find the projection for the LDA problem, here is an example for 3 variables. Using Maple, Mathematica, Matlab etc., it is relatively easy to generalize the process to many variables. Below is an example in 3 variables and 3 classes. (Note: the number of variables and number of classes do not need to be the same.)

- Using Bayes' rule we can express the LDA formula as:

  $$f_i = \boldsymbol{\mu}_i \boldsymbol{C}^{-1} \boldsymbol{x}_k^T - \frac{1}{2} \boldsymbol{\mu}_i \boldsymbol{C}^{-1} \boldsymbol{\mu}_i^T + \log(p_i)$$

  Note: This is equivalent to equations (4.13) and (4.19) on pages 140 and 143 in our textbook.

  $\mu_\mathbf{i}$ is the vector of means for class i. This is a row vector
  $\mathbf{C^{-1}}$ is the inverse of the covariance matrix. Its dimension is the number of classes we have.
  $\mathbf{x_k^T}$ is transpose of the vector of data (predictors). $\ln(p)$ is the log of the prior probabilities (number of samples of each class)

- We assign object k to group i that has maximum f_i.

- We have a set of data from three classes. Since the data from each sample point is organized in a row. We will use row vectors instead of column vectors to represent each sample.

  - Let our class 1 samples be represented in the following matrix:

    $$\mathbf{X_1} = \begin{bmatrix} 2 & 7 & 12 \\ 8 & 1 & 15 \\ 3 & 5 & 11 \\ 9 & 7 & 18 \\ 11 & 9 & 16 \\ 5 & 8 & 17 \end{bmatrix}$$

  - Let our class 2 samples be represented in the following matrix:

    $$\mathbf{X_2} \begin{bmatrix} 12 & 4 & 8 \\ 17 & 9 & 4 \\ 8 & 4 & 9 \\ 19 & 8 & 7 \\ 14 & 3 & 3 \\ 18 & 5 & 6 \end{bmatrix}$$

  - Let our class 3 samples be represented in the following matrix:

    $$\mathbf{X_3} \begin{bmatrix} 12 & 8 & 11 \\ 10 & 2 & 16 \\ 14 & 9 & 17 \\ 18 & 8 & 14 \\ 17 & 12 & 12 \\ 15 & 4 & 9 \end{bmatrix}$$

- The mean for each variable in each class is:

$$\mu_{\mathbf{1}} = \begin{bmatrix} \frac{19}{3} & \frac{37}{6} & \frac{89}{6} \end{bmatrix}$$

  with

$$\mu_{\mathbf{2}} = \begin{bmatrix} \frac{44}{3} & \frac{11}{2} & \frac{37}{6} \end{bmatrix}$$

  and

$$\mu_{\mathbf{3}} = \begin{bmatrix} \frac{43}{3} & \frac{43}{6} & \frac{79}{6} \end{bmatrix}$$

- Our overall mean is

$$\mu = \begin{bmatrix} \frac{106}{9} & \frac{113}{18} & \frac{205}{18} \end{bmatrix}$$

  where $p_1 = p_2 = p_3 = 1/3$
- We normalize everything to have a mean of 0 by subtracting the mean of each class.

$$\mathbf{X_1^0} = \begin{bmatrix} -\frac{88}{9} & \frac{13}{18} & \frac{11}{18} \\ -\frac{34}{9} & -\frac{95}{18} & \frac{65}{18} \\ -\frac{79}{9} & -\frac{23}{18} & -\frac{7}{18} \\ -\frac{25}{9} & \frac{13}{18} & \frac{119}{18} \\ -\frac{7}{9} & \frac{49}{18} & \frac{83}{18} \\ -\frac{61}{9} & \frac{31}{18} & \frac{101}{18} \end{bmatrix}$$

$$\mathbf{X_2^0} = \begin{bmatrix} \frac{2}{9} & -\frac{41}{18} & -\frac{61}{18} \\ \frac{47}{9} & \frac{49}{18} & -\frac{133}{18} \\ -\frac{34}{9} & -\frac{41}{18} & -\frac{43}{18} \\ \frac{65}{9} & \frac{31}{18} & -\frac{79}{18} \\ \frac{20}{9} & -\frac{59}{18} & -\frac{151}{18} \\ \frac{56}{9} & -\frac{23}{18} & -\frac{97}{18} \end{bmatrix}$$

$$\mathbf{X_3^0} = \begin{bmatrix} \frac{2}{9} & \frac{31}{18} & -\frac{7}{18} \\ -\frac{16}{9} & -\frac{77}{18} & \frac{83}{18} \\ \frac{20}{9} & \frac{49}{18} & \frac{101}{18} \\ \frac{56}{9} & \frac{31}{18} & \frac{47}{18} \\ \frac{47}{9} & \frac{103}{18} & \frac{11}{18} \\ \frac{29}{9} & -\frac{41}{18} & -\frac{43}{18} \end{bmatrix}$$

- The covariance matrix for each class is given as:

$$\boldsymbol{C}_i = \frac{\left(\boldsymbol{X}_i^0\right)^T \boldsymbol{X}_i^0}{n_i}$$

- The covariance matrix for class 1 is

$$C_1 = \begin{bmatrix} \frac{3256}{81} & \frac{112}{81} & -\frac{2057}{162} \\ \frac{112}{81} & \frac{2209}{324} & \frac{479}{324} \\ -\frac{2057}{162} & \frac{479}{324} & \frac{5941}{324} \end{bmatrix}$$

- The covariance matrix for class 2 is

$$C_2 = \begin{bmatrix} \frac{1855}{81} & \frac{527}{162} & -\frac{3083}{162} \\ \frac{527}{162} & \frac{1789}{324} & \frac{1073}{324} \\ -\frac{3083}{162} & \frac{1073}{324} & \frac{10285}{324} \end{bmatrix}$$

- The covariance matrix for class 3 is

$$C_3 = \begin{bmatrix} \frac{1141}{81} & \frac{1277}{162} & \frac{215}{81} \\ \frac{1277}{162} & \frac{3757}{324} & \frac{449}{324} \\ \frac{215}{81} & \frac{449}{324} & \frac{3553}{324} \end{bmatrix}$$

- The overall Covariance is given as

$$C(r,s) = \frac{1}{n}\sum_{i=1}^{g} n_i \cdot c_i(r,s)$$

$$C = \begin{bmatrix} \frac{2084}{81} & \frac{338}{81} & -\frac{785}{81} \\ \frac{338}{81} & \frac{2585}{324} & \frac{667}{324} \\ -\frac{785}{81} & \frac{667}{324} & \frac{6593}{324} \end{bmatrix}$$

- The inverse of the C matrix is:

$$\begin{bmatrix} \frac{4149504}{73425479} & -\frac{2752029}{73425479} & \frac{2254671}{73425479} \\ -\frac{2752029}{73425479} & \frac{11274912}{73425479} & -\frac{2451348}{73425479} \\ \frac{2254671}{73425479} & -\frac{2451348}{73425479} & \frac{4930164}{73425479} \end{bmatrix}$$

- Our p matrix is:

$$p = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

- The model is now trained. Lets look at some examples

$$x_1 = \begin{bmatrix} 3 & 2 & 12 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 3.08483518295997072 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 1.38622245753260387 \end{bmatrix}$$

$$f_3 = \begin{bmatrix} 0.641855246799121204 \end{bmatrix}$$

This is class 1

$$x_2 = \begin{bmatrix} 15 & 6 & 7 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 6.00650302798689850 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 8.08294003699844410 \end{bmatrix}$$

$$f_3 = \begin{bmatrix} 7.06048708166941275 \end{bmatrix}$$

This is class 2

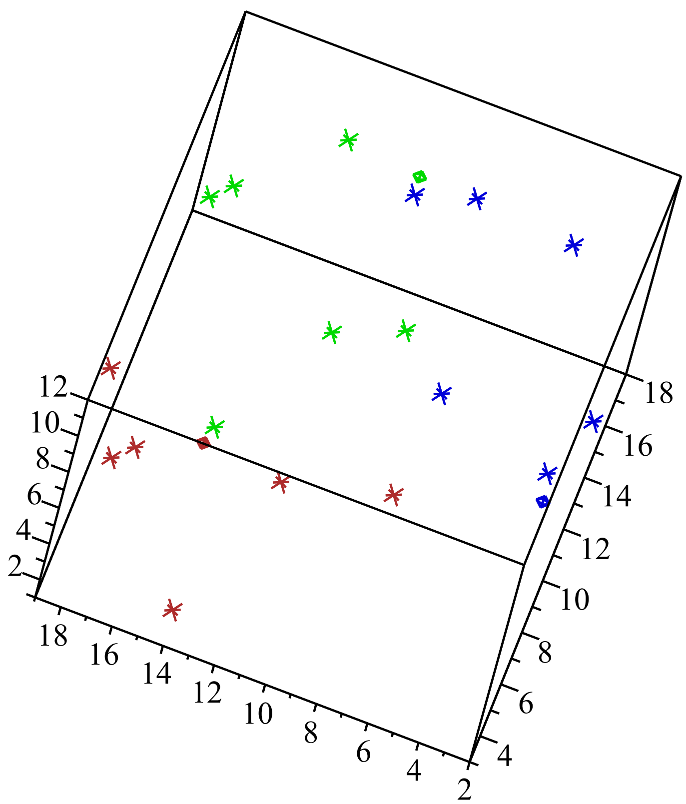
$$x_3 = \begin{bmatrix} 11 & 10 & 16 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} 13.3960166190388588 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 11.3177832444904922 \end{bmatrix}$$

$$f_3 = \begin{bmatrix} 13.5368828706570437 \end{bmatrix}$$

This is class 3



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