

LDA Classifier Using Maple

1. Load the Linear Algebra package.

```
> with(LinearAlgebra)
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]
```

(above are all the commands that support linear algebra)

2. Create your matrices

```
> X1 := Matrix([[1, 2], [4, 6], [5, 8], [7, 2], [5, 5], [6, 8], [9, 1], [3, 3]])
```

$$X1 := \begin{bmatrix} 1 & 2 \\ 4 & 6 \\ 5 & 8 \\ 7 & 2 \\ 5 & 5 \\ 6 & 8 \\ 9 & 1 \\ 3 & 3 \end{bmatrix}$$

```
> X2 := Matrix([[-1, -3], [-4, -5], [-5, -8], [-7, -2], [-5, -6], [-6, -8], [-9, -1]])
```

$$X2 := \begin{bmatrix} -1 & -3 \\ -4 & -5 \\ -5 & -8 \\ -7 & -2 \\ -5 & -6 \\ -6 & -8 \\ -9 & -1 \end{bmatrix}$$

3. Calculate the mean of each class

$$\left[\begin{array}{l} > \text{mu1} := \text{Matrix}\left(\left[\frac{(1+4+5+7+5+6+9+3)}{8}, \frac{(2+6+8+2+5+8+1+3)}{8}\right]\right) \end{array} \right]$$

$$\mu 1 := \left[\begin{array}{c} 5 \quad \frac{35}{8} \end{array} \right]$$

(4)

$$\left[\begin{array}{l} > \text{mu2} := \text{Matrix}\left(\left[\frac{(-1-4-5-7-5-6-9)}{7}, \frac{(-3-5-8-2-6-8-1)}{7}\right]\right) \end{array} \right]$$

$$\mu 2 := \left[\begin{array}{cc} -\frac{37}{7} & -\frac{33}{7} \end{array} \right]$$

(5)

4. Find the overall mean

$$\frac{8}{15} \cdot \text{mu1}[1, 1] + \frac{7}{15} \cdot \text{mu2}[1, 1]$$

$$\frac{1}{5}$$

(6)

$$\left[\begin{array}{l} > \frac{8}{15} \cdot \text{mu1}[1, 2] + \frac{7}{15} \cdot \text{mu2}[1, 2] \end{array} \right]$$

$$\frac{2}{15}$$

(7)

$$\left[\begin{array}{l} > \mu := \text{Matrix}\left(\left[\frac{1}{5}, \frac{2}{15}\right]\right) \end{array} \right]$$

$$\mu := \left[\begin{array}{cc} \frac{1}{5} & \frac{2}{15} \end{array} \right]$$

(8)

5. Normalize the data by subtracting the mean from each.

> $X1o := \text{Matrix}([[1 - \mu[1, 1], 2 - \mu[1, 2]], [4 - \mu[1, 1], 6 - \mu[1, 2]], [5 - \mu[1, 1], 8 - \mu[1, 2]], [7 - \mu[1, 1], 2 - \mu[1, 2]], [5 - \mu[1, 1], 5 - \mu[1, 2]], [6 - \mu[1, 1], 8 - \mu[1, 2]], [9 - \mu[1, 1], 1 - \mu[1, 2]], [3 - \mu[1, 1], 3 - \mu[1, 2]]])$

$$X1o := \begin{bmatrix} \frac{4}{5} & \frac{28}{15} \\ \frac{19}{5} & \frac{88}{15} \\ \frac{24}{5} & \frac{118}{15} \\ \frac{34}{5} & \frac{28}{15} \\ \frac{24}{5} & \frac{73}{15} \\ \frac{29}{5} & \frac{118}{15} \\ \frac{44}{5} & \frac{13}{15} \\ \frac{14}{5} & \frac{43}{15} \end{bmatrix}$$

(9)

> $X2o := \text{Matrix}([[-1 - \mu[1, 1], -3 - \mu[1, 2]], [-4 - \mu[1, 1], -5 - \mu[1, 2]], [-5 - \mu[1, 1], -8 - \mu[1, 2]], [-7 - \mu[1, 1], -2 - \mu[1, 2]], [-5 - \mu[1, 1], -6 - \mu[1, 2]], [-6 - \mu[1, 1], -8 - \mu[1, 2]], [-9 - \mu[1, 1], -1 - \mu[1, 2]]])$

$$X2o := \begin{bmatrix} -\frac{6}{5} & -\frac{47}{15} \\ -\frac{21}{5} & -\frac{77}{15} \\ -\frac{26}{5} & -\frac{122}{15} \\ -\frac{36}{5} & -\frac{32}{15} \\ -\frac{26}{5} & -\frac{92}{15} \\ -\frac{31}{5} & -\frac{122}{15} \\ -\frac{46}{5} & -\frac{17}{15} \end{bmatrix}$$

(10)

6. Make the covariance matrix for X1. Note that $X1^{++}$ is the shortcut for transpose and $X1.X2$ is the shortcut for matrix multiplication.

$$> Cov1 := \frac{X1o^{+}.X1o}{8}$$

$$Cov1 := \begin{bmatrix} \frac{2829}{100} & \frac{993}{50} \\ \frac{993}{50} & \frac{44507}{1800} \end{bmatrix}$$

(11)

$$> Cov2 := \frac{X2o^{+}.X2o}{7}$$

$$Cov2 := \begin{bmatrix} \frac{886}{25} & \frac{4393}{175} \\ \frac{4393}{175} & \frac{47683}{1575} \end{bmatrix}$$

(12)

7. Add the two covariance matrices

$$> Cov := \frac{8}{15} \cdot Cov1 + \frac{7}{15} \cdot Cov2$$

$$Cov := \begin{bmatrix} \frac{2372}{75} & \frac{1673}{75} \\ \frac{1673}{75} & \frac{6146}{225} \end{bmatrix}$$

(13)

8. Take the inverse of the Covariance

$$> Covinv := (Cov)^{-1}$$

$$Covinv := \begin{bmatrix} \frac{2634}{35323} & -\frac{2151}{35323} \\ -\frac{2151}{35323} & \frac{21348}{247261} \end{bmatrix}$$

(14)

9. Now build and use your classifier.

```
> x1 := Matrix([3, 5]);
```

$$x1 := \begin{bmatrix} 3 & 5 \end{bmatrix}$$

(15)

```
> f1 := mu1 • Covinv • x1 + - 1/2 • mu1.Covinv.mu1 + evalf(ln(8/15));
```

$$f1 := \begin{bmatrix} -0.369371583090861 \end{bmatrix}$$

(16)

```
> f2 := mu2 • Covinv • x1 + - 1/2 • mu2.Covinv.mu2 + evalf(ln(7/15));
```

$$f2 := \begin{bmatrix} -1.99277802365830 \end{bmatrix}$$

(17)

10. Have fun!

```
> x1 := Matrix([-4, -7]);
```

$$x1 := \begin{bmatrix} -4 & -7 \end{bmatrix}$$

(18)

```
> f1 := mu1 • Covinv • x1 + - 1/2 • mu1.Covinv.mu1 + evalf(ln(8/15));
```

$$f1 := \begin{bmatrix} -1.99340195989917 \end{bmatrix}$$

(19)

```
> f2 := mu2 • Covinv • x1 + - 1/2 • mu2.Covinv.mu2 + evalf(ln(7/15));
```

$$f2 := \begin{bmatrix} -0.221500478299930 \end{bmatrix}$$

(20)

```
> x1 := Matrix([-4, 3]);
```

$$x1 := \begin{bmatrix} -4 & 3 \end{bmatrix}$$

(21)

```
> f1 := mu1 • Covinv • x1 + - 1/2 • mu1.Covinv.mu1 + evalf(ln(8/15));
```

$$f1 := \begin{bmatrix} -1.26087640997420 \end{bmatrix}$$

(22)

```
> f2 := mu2 • Covinv • x1 + - 1/2 • mu2.Covinv.mu2 + evalf(ln(7/15));
```

$$f2 := \begin{bmatrix} -1.07297205807076 \end{bmatrix}$$

(23)