DAA Exam #2 Online (100 points)

Last Name: _____ First Name: _____ ID: _____

1. [20]

a) [5] Find an optimal parenthesization of a matrix chain product whose sequence of dimensions is (6, 11, 7, 15, 3, 21). Use the given table.

	A_1	A_2	A_3	A_4	A_5
A_1	0	462/1	1092/2	744/1	1122/4
A_2		0	1155/2	546/2	1239/4
A_3			0	315/3	756/4
A_4				0	945/4
A_5					0

b) [15] Suppose we would like to compute the product of five matrices, A_1, A_2, A_3, A_4, A_5, whose dimensions are (30, B, 20, C, 20, 10) given below:

	A_1	A_2	A_3	A_4	A_5
Dimensions	30 × B	B × 20	20 × C	C × 20	20 x 10

- B value should be your student ID's last one digit (i.e., If your student ID is 16301378, the B value should be "8". If it ends with "0", use the last none zero digit. If your student ID is 16301300, the B value for you should be "3".)
- C value should be your student ID's first and last digits (i.e., If your student ID is 16301370, the B value should be "10".)

What is the fastest way to compute the product $A_1\ A_2\ A_3\ A_4\ A_5$? <u>Fill the given table</u> and <u>find an optimal parenthesization on your answer sheet</u>.

i \ j	A_1	A_2	A_3	A_4	A_5
A_1	0				
A_2		0			
A_3			0		
A_4				0	
A_5					0

2. [15]

Find Longest Common Subsequence (LCS) for strings.

- **X has 8 letters starting with your first name. If your first name is less than 8 letters, fill up from "abc234". (i.e., if your first name is "sejun", your X string is "sejunabc". If your first name is "sun", your X string is "sunabc23".)**
- **Y has 7 letters starting with your last name. If your last name is less than 7 letters, fill up from "a1b2c3". (i.e., if your last name is "song", your Y string is "songa1b". If your last name is "yu", your Y string is "yua1b2c".)**

Find LCS and its length by using the given table (make sure to use your own X and Y strings).

		s	**e**	**j**	**u**	**n**	**a**	**b**	**e**
	0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
s	*0*								
o	*0*								
n	*0*								
g	*0*								
a	*0*								
1	*0*								
b	*0*								

3. [20]

a) [5] Using the given 5 keys of Catalan sequence, find the number of possible unique Binary Search Trees (BSTs) for 6 keys. Show your step by step process to come up with the answer.

The number of keys: n = 1, # BST 1; **n = 2**, # BST 2; **n = 3**, # BST 5 (2 + 1 + 2); **n = 4**, # BST 14 (5 + 4 + 5); **n = 5**, # BST 42 (14 + 5 + 4 + 5 + 14)

b) [15] Consider the following 4 keys with the search frequencies. What is Optimal Binary Search Tree? Show your work sequence using the given table and show the optimal BST.

Key	1	2	3	4
Key Value	10	20	30	40
Frequency	A	B	C	D

- **A value should be your student ID's last (none zero) digit (i.e., If your student ID is 16701300, the A value should be "3".)**
- **B value should be your student ID's second last (none zero) digit (i.e., If your student ID is 16701300, the B value should be "1".)**
- **C value should be your student ID's third last (none zero) digit (i.e., If your student ID is 16701300, the C value should be "7".)**
- **D value should be your student ID's fourth last (none zero) digit (i.e., If your student ID is 16701300, the D value should be "6".)**

i/j	0	1	2	3	4
1	0				
2		0			
3			0		
4				0	
5					0

4. [15]

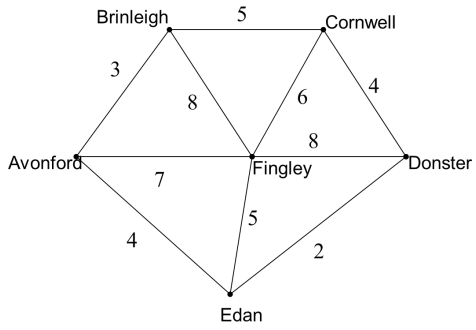
For the given n = 4 (# of elements), W = 6 (max weight), and elements (weight, benefit): (3, A), (2, B), (4, C), (1, D),

- **A value should be your student ID's first digit (none zero) * 10 (i.e., If your student ID is 16301378, the A value should be "10".)**
- **B value should be your student ID's second digit (none zero) * 10 (i.e., If your student ID is 16301378, the B value should be "60".)**
- **C value should be your student ID's third digit (none zero) * 10 (i.e., If your student ID is 16301378, the C value should be "30".)**
- **D value should be your student ID's fourth digit (none zero) * 10 (i.e., If your student ID is 16301378 (the fourth digit is zero, so take the fifth digit), the D value should be "10".)**

Find the maximum possible value that can be carried in the knapsack and identify the items (each item must be entirely accepted or rejected: 0-1 knapsack algorithm).

i\w	0	1	2	3	4	5	6
0							
1							
2							
3							
4							

5. [5] A cable company want to connect five villages to their network which currently extends to the market town of Avonford. What is the minimum length of cable needed? Show your work by using Kruskal's Algorithm.



6. [10] Design a Huffman encoding to send the following 20 characters. Show the Huffman encoding design and state the number of bits necessary to send.

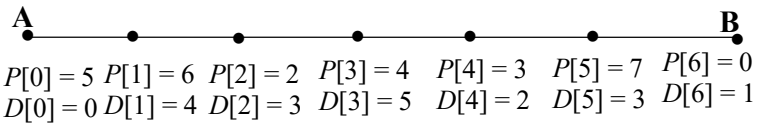
aaaaa(X)(Y) (n = 20, using ASCII code (8 bits))

- **X has 8 letters starting with your first name. If your first name is less than 8 letters, fill up from "aaaaaa". (i.e., if your first name is "sejun", your X string is "sejunaaa". If your first name is "sun", your X string is "sunaaaaa".)**
- **Y has 7 letters starting with your last name. If your last name is less than 7 letters, fill up from "bbbbbb". (i.e., if your last name is "song", your Y string is "songbbb". If your last name is "yu", your Y string is "yubbbbb".)**
- **So, for example, Sejun Song's 20 char string is "aaaaasejunaaasongbbb" and Sun Yu's 20 char string is "aaaaasunaaaaayubbbbb"**

7. [5] For the given items with a jar with 8 ml jar, solve a fractional knapsack problem. What is the maximum benefit and are the items to take?

	1	2	3	4	5
Weight	4 ml	8 ml	2 ml	6 ml	1 ml
Benefit	$12	$32	$40	$30	$50

8. [5] Let L (miles) be the distance a car can run with full tank of gas. It is assumed that in any L miles, there is at least one station and at most k stations, where k is a constant. P[i] stands for the gas cost and D[i] is the distance. We want to compute the amount of gas that needs be added (in terms of miles) at each station such that the total cost is minimized to drive the car from city A to city B, assuming the gas tank is empty initially. When **L** is **6**, **<u>find the minimum total cost from A to B</u>**. **<u>Show your work</u>**.



9. [5] Show the Utilization Bound test for the above tasks. State if the RM schedulability is conclusive or incon clusive.

Scheduling for two periodic tasks:

	First Arrival	Deadline	Period	Execution Time
T1	0	5	5	2
T2	3	7	7	4

Hints: $2^{1/2} = 1.41$, $2^{1/3} = 1.25$, $2^{1/4} = 1.19$

Necessary and sufficient conditions for RM scheduling on a single processor and n tasks:

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \leq 1 \qquad \mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \leq n(2^{1/n} - 1)$$