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A = 16 - 6 = 10 C = 12 - 6 = 6

B = 14 - 8 = 6 D = 20 - 4 = 16

Q1) (1) 5 (2) NO (3) Yes

(4) NO (since ~~the~~, there is an Existence of loops & multiple edges)

Q2) A graph is having 6 vertices & 40 edges.

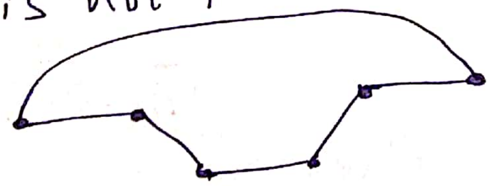
It is not possible.

In the worst case a graph is having $nC_2 \left(\frac{n*(n-1)}{2}\right)$

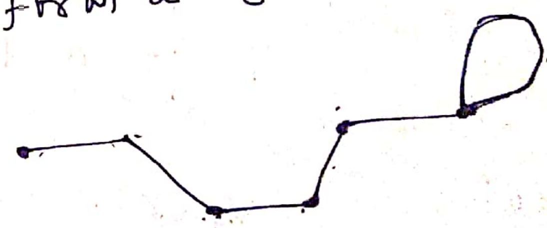
if it is a complete graph. [15]

Q3) Can a tree graph have 6 vertices and 6 edges.

It is not possible since it form a cycle or loop



(or)



Q4) $K_B = K_6$, $K_C = K_6$.

Exactly equal edges in both graphs.

Since $\boxed{\underset{\uparrow\ K_B}{K_6} = 6C_2 = 15 = K_6 (K_C)}$

Q5)

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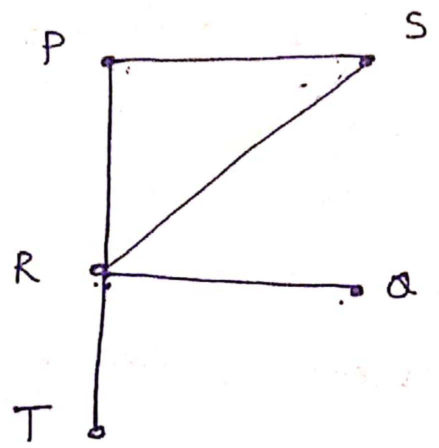
Q-5

(a)



(b) Adjacency matrix:

$$\begin{array}{c|c} & \begin{matrix} P & Q & R & S & T \end{matrix} \\ \begin{matrix} P \\ Q \\ R \\ S \\ T \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Q-6:

$\boxed{\text{No. of vertices} = 15}$

Using Handshaking theorm

$$2 * e = \sum_{v \in V} \deg(v)$$

$$2 * e = 2 * 6 + 1 * 3 + 1 * 2 + (x - 4) * 1$$

$$2 * e = 12 + 3 + 2 + x - 4 = 13 + x$$

$\boxed{x = 2 * e - 13}$ $\boxed{e = x - 1}$

$e = \dfrac{x + 13}{2}$ where $x = \{1, 3, 5, \dots\}$ i.e $\underline{2n-1}$

$x - 1 = \dfrac{x + 13}{2} = 2x - 2 = x + 13$

$\boxed{x = 15}$

Q7) $c = 6$, edges $= 15$

As per Euler theorem,

$R = E - V + 2 = 15 - 6 + 2 = 11.$

$=$ ~~6C ✓ 15 ⊕ 2~~ ~~= 8 = 15~~

$\boxed{\text{No. of. bounded face} = R - 1 = 10}$

Q8) $K_{m,n}$ $m = 5,\ n = 5$

Ans $K_{5,5}$

$\boxed{K_{5,5}\ (\text{max. no. of. edges}) = 25}$

$K_{6,4}$ (") $= 24$

$K_{7,3}$ (") $= 21$

$K_{8,2}$ (") $= 16$

$K_{9,1}$ (") $= 9$

Q9) Complete graph with 16 vertices $K_{16} = {}^{16}C_2 = \underline{\underline{120}}$

K_{16} is having 120 edges

It is not possible to design Eulerian circuit with K_{16}.

Reason: Euler circuit requires even degree vertices. In K_{16} it is not possible

Q10) ① no. of. vertices are 5 in each.

② Degree is 2 for each vertex in both graph.

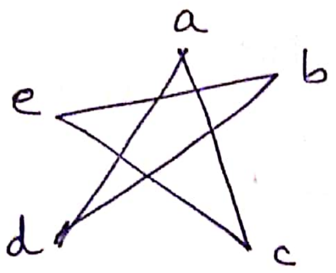
③ It ~~it~~ is a regular graph (both)

④ circuit length 5 in each graph.

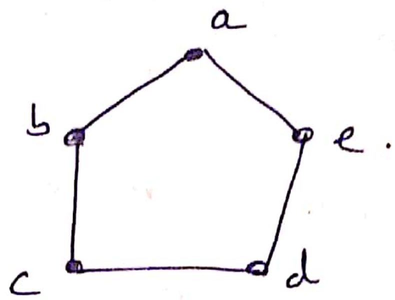
⑤ No. of. edges in the both graphs is 5
From above all, we conclude

It is isomorphic to each other.

Q 11)



is isomorphic to



There are no cross over edges in the graph. So we conclude given graph is planar.

Q 12) There is no Euler cycle since it is not to cover every edge exactly once in the given graph and reaches to the starting vertex. So there is no Euler cycle.

Q 13) Consider 'c' as source node. It is possible to visit all the vertices exactly once.

① c — a — b — d — e ['c' source vertex]

② e — d — b — a — c ['e' "]

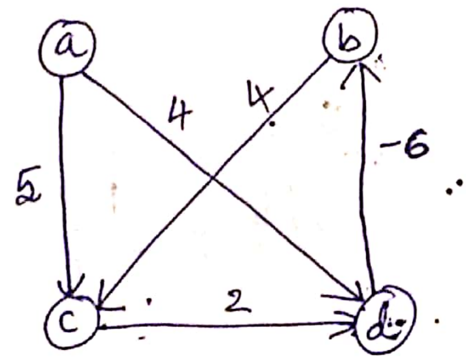
Q.14) D = 16

T – TRUE, F – FALSE

Q	2	4	6	8	10	11	12	13	14	16	18	X
A	F	T	T	T	T	F	T	T	T	T	T	X

C = 6

Q 15)

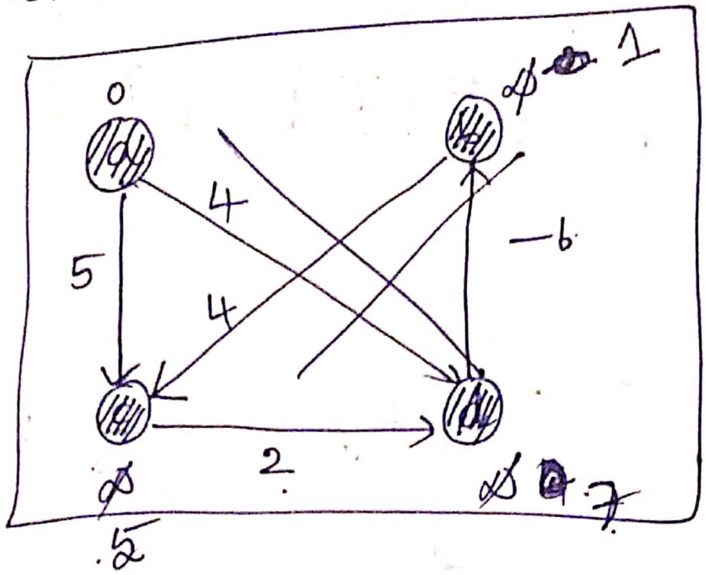


4 - 6 + 4 = 8 - 6 = 2

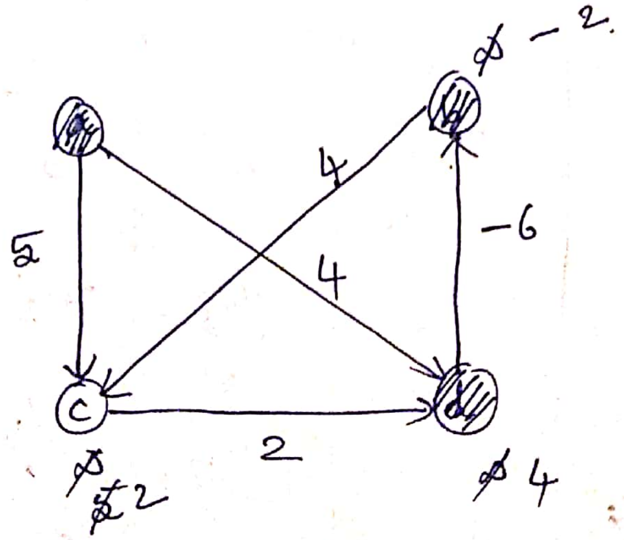
	a	b	c	d
a	0/a	∞	5/a	4/a
d	–	-6/d	5/a	$\boxed{4/a}$
b	–	$\boxed{-6/d}$	4/b	–
c	–	–	$\boxed{4/b}$	–

Path is : a — d — b — c [cost : 2]

It is not possible. Since $c \rightarrow d \rightarrow b \rightarrow c$ creates cycle/circuit. So Dijkstra's is not possible to construct



After visiting all nodes



After visiting a $\rightarrow$ $c = 5, d = 4$

d $\rightarrow$ $c = 5; d = 4, b = -2$

b $\rightarrow$ $c = 2, d = 4, b = -2.$

After visting all the nodes from a → c

Cost is +2

~~Second time visit if we do,~~

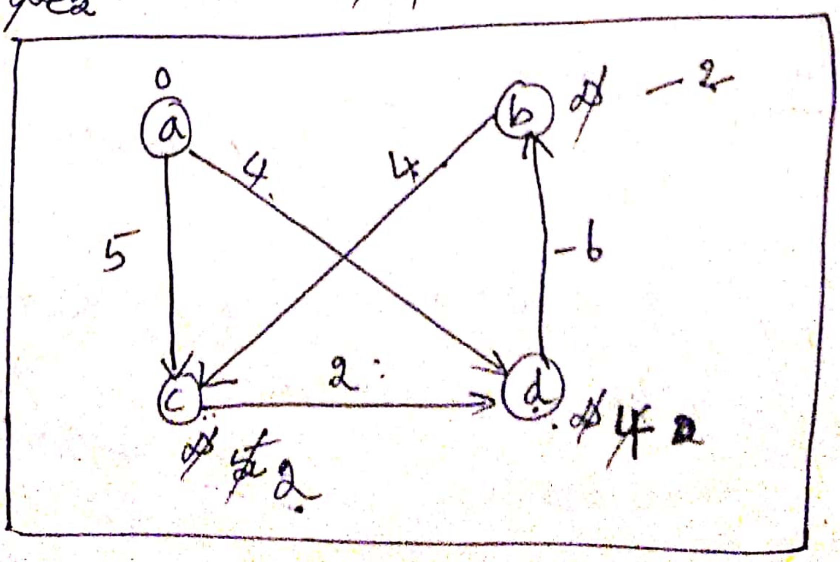
	a	b	c	d
a	0/a	∞	5/a	4/a
d	–	-2/d	5/a	4/a
b	–	-2/d	2/b	–
c	–	–	2/b	–

total cost is 2 (a → d → b → c)

$$\underset{(a,c)}{5}\quad \underset{(a,d)}{4}\quad \underset{(c,d)}{2}\quad \underset{(d,b),}{-6}\quad \underset{(b,c)}{4}$$

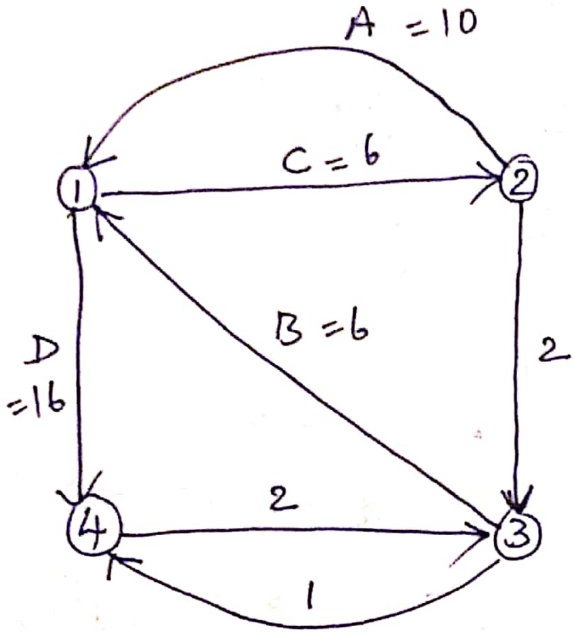
15) 2

Solution



	b	c	d
1^{st}	-2	2	4
2^{nd}	-2	2	4
3^{rd}	-2	2	4

(16)



$$A_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 6 & \infty & 16 \\ 2 & 10 & 0 & 2 & \infty \\ 3 & 6 & \infty & 0 & 1 \\ 4 & \infty & \infty & 2 & 0 \end{array}$$

$$A_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 6 & \infty & 16 \\ 2 & 10 & 0 & 2 & 26 \\ 3 & 6 & 12 & 0 & 1 \\ 4 & \infty & \infty & 2 & 0 \end{array}$$

$A_1[2,3] = A_0[2,1] + A_0[1,3]$

$2 < 10 + \infty$

$A_1[2,4] = A_0[2,1] + A_0[1,4]$

$\infty \ngtr \underline{10 + 16}$

$A_1[3,2] = A_0[3,1] + A_0[1,2]$

$\underline{\underline{\infty}} > \underline{6 + 6}$

$A_1[3,4] = A_0[3,1] + A_0[1,4]$

$1 < 6 + 16$

$A_1[4,2] = A_0[4,1] + A_0[1,2]$

$\infty = \underline{\underline{\infty + 6}}$

$A[4,3] = 2 < \infty + \infty$

$A[4,2] = \infty + 6 \leq \infty \; (\infty < \infty)$

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$$A_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 6 & 8 & 16 \\ 2 & 10 & 0 & 2 & 26 \\ 3 & 6 & 12 & 0 & 1 \\ 4 & \infty & \infty & 2 & 0 \end{array} \leftarrow 16(2)$$

Extra Work

$A_2[1,3] = A_1[1,2] + A_1[2,3]$

$\infty > \quad 6 + 2$

$A_2[1,4] = A_1[1,2] + A_1[2,4]$

$16 = \quad 6 + 26$

$A_2[2,3] = A_1[2,2] + A_1[2,3] = 0 + 2$

$A_2[3,1] = A_1[3,2] + A_1[2,1] = 12 + 10 = \underline{22} \quad \infty > 6$

$A_2[3,4] = A_1[3,2] + A_1[2,4] = 12 + 26$

$1 < 38$

$A_2[4,1] = A_1[4,2] + A_1[2,1] = \infty + 10$

$\infty \leq \infty$

$A_2[4,3] = A_1[4,2] + A_1[2,3] = \infty + 2 > 2$

$$A_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 6 & 8 & 9 \\ 2 & 8 & 0 & 2 & 3 \\ 3 & 6 & 12 & 0 & 1 \\ 4 & 8 & 14 & 2 & 0 \end{array}$$

$A[1,2] = 8 + 12$

$6 < 20$

$A[1,3] = 8$

$A[1,4] = 8 + 1$

$16 > 9$

$A[2,1] = 2 + 6$

$10 > \underline{2 + 6}$

$A[2,4] = 2 + 1$

$26 > 3$

$A[4,1] = 2 + 6$

$\infty > 8$

$A[4,2] = 2 + 12$

∞

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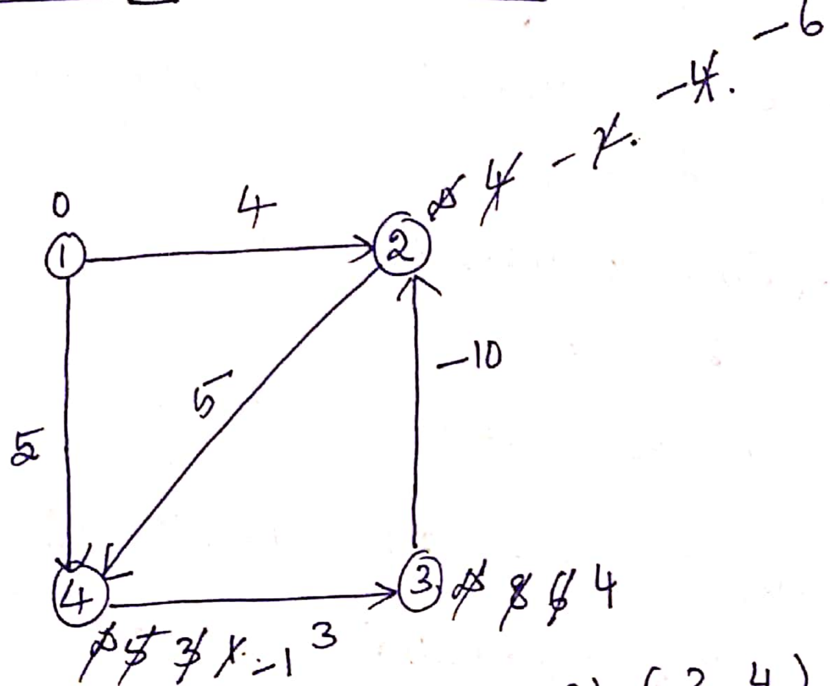
$$
A_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & & & \\ 2 & & 0 & & \\ 3 & & & 0 & \\ 4 & & & & 0 \end{array}
$$

Q.17)



(1, 4) (1, 2) (4, 3) (3, 2) (2, 4)

1st loop: ② → −2, ③ → 8, ④ → 3

2nd loop: ② → −4, ③ → 6, ④ → 1

3rd loop: ② → −6, ③ → 4, ④ → −1

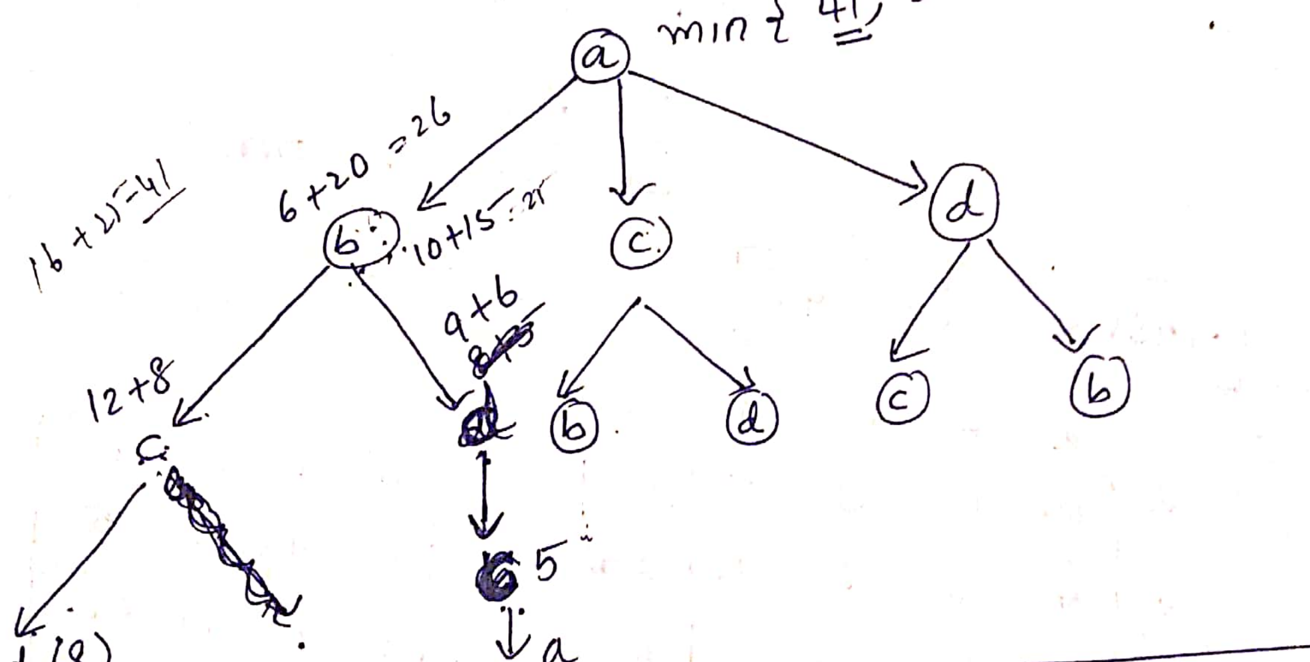
After 3rd loop values of vertices remaining same. But it is not true in this problem since it is generating −ve weight cycle (4 → 3 → 2 → 4)

18) 2

6

Dynamic programming

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 16 & 15 & 10 \\ b & 5 & 0 & 6 & 10 \\ c & 6 & 6 & 0 & 12 \\ d & 8 & 8 & 9 & 0 \end{array}$$



$$\boxed{g(a, \{b, c, d\}) = \min\{ c_{ak} + g(k, S - \{k\}) }$$

① $g(d, \phi) = 8$, $g(b, \phi) = 5$, $g(c, \phi) = 6$

② $g(c, \{d\}) = \min\{ c_{cd} + g(d, \phi)$

$= 12 + 8 = 20$

③ $g(d, \{c\}) = \min\{ c_{dc} + g(c, \phi)$

$= \cancel{8 + 5}$

$= 9 + 6 = 15$

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$$g(b, \{c, d\}) = \min \begin{cases} c_{bc} + g(c, \{d\}) = 6 + 20 \\ c_{bd} + g(d, \{c\}) = 10 + 15 \end{cases}$$

$$g(a, \{b, c, d\}) = \min \begin{cases} c_{ab} + g(b, \{c, d\}) \quad 16 + 25 = 41 \\ \boxed{\begin{matrix} c_{ac} + g(c, \{b, d\}) \\ c_{ad} + g(d, \{b, c\}) \end{matrix}} \end{cases}$$

Out of Question

$\boxed{\text{Answer is 4)}}$

18 (3)

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} \infty & 16 & 15 & 10 \\ 5 & \infty & 6 & 10 \\ 6 & 6 & \infty & 12 \\ 8 & 8 & 9 & \infty \end{bmatrix} \begin{matrix} 10 \\ 5 \\ 6 \\ 8 \end{matrix} \end{array} = \begin{bmatrix} \infty & 6 & 5 & 0 \\ 0 & \infty & 1 & 5 \\ 0 & 0 & \infty & 6 \\ 0 & 0 & 1 & \infty \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 1 & 0 \end{matrix}$$

(a)

$$= \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} \infty & 6 & 4 & 0 \\ 0 & \infty & 0 & 5 \\ 0 & 0 & \infty & 6 \\ 0 & 0 & 0 & \infty \end{bmatrix} \end{array} \leftarrow \underline{r = 30}$$

$$\text{cost}(a, b) = c(a, b) + r + \hat{r}$$

$$\begin{array}{c|cccc|c} & a & b & c & d & \\ \hline a & \infty & \infty & \infty & \infty & 0 \\ b & \infty & \infty & 0 & 5 & 0 \\ c & 0 & \infty & \infty & 6 & 0 \\ d & 0 & \infty & 0 & \infty & 0 \\ \hline & 0 & 0 & 0 & 5 & \end{array} = \begin{array}{c|cccc|} & a & b & c & d \\ \hline a & \infty & \infty & \infty & \infty \\ b & \infty & \infty & 0 & 0 \\ c & 0 & \infty & \infty & 1 \\ d & 0 & \infty & 0 & \infty \\ \hline \end{array} \quad (b)$$

$\hat{\gamma} = 5$

$\text{Cost}(a, b) = c(a, b) + \gamma + \hat{\gamma} =$

$= 6 + 30 + 5 = 41$

$\text{Cost}(a, c) = c(a, c) + \gamma + \hat{\gamma}$

$$\begin{array}{c|cccc|c} & a & b & c & d & \\ \hline a & \infty & \infty & \infty & \infty & 0 \\ b & 0 & \infty & \infty & 5 & 0 \\ c & \infty & 0 & \infty & 6 & 0 \\ d & 0 & 0 & \infty & \infty & 0 \\ \hline & 0 & 0 & 0 & 5 & \end{array} = \begin{array}{c|cccc|} & a & b & c & d \\ \hline a & \infty & \infty & \infty & \infty \\ b & 0 & \infty & \infty & 0 \\ c & \infty & 0 & \infty & 1 \\ d & 0 & 0 & \infty & \infty \\ \hline \end{array} \quad (c)$$

$$\begin{array}{r} 35 \\ 15 \\ \hline 50 \end{array}$$

$\hat{\gamma} = 5$

$\text{Cost}(a, c) = 4 + 30 + 5 = 39$

$\text{Cost}(a, d) = c(a, d) + \gamma + \hat{\gamma} = 0 + 30 + 0 = 30$

$$\begin{array}{c|cccc|c} & a & b & c & d & \\ \hline a & \infty & \infty & \infty & \infty & 0 \\ b & 0 & \infty & 0 & \infty & 0 \\ c & 0 & 0 & \infty & \infty & 0 \\ d & \infty & 0 & 0 & \infty & 0 \\ \hline & 0 & 0 & 0 & 0 & \end{array} \rightarrow (d)$$

L-3

$cost(d,b) = c(d,b) + r + \hat{r} = 0 + 30 + 0 = 30$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty \\ 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{array}\right] \\ \begin{array}{cccc} 0 & 0 & 0 & 0 \end{array} \end{array} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ 0 \\ \\ \end{array} \rightarrow (b)$$

noreduction

$cost(d,c) = c(d,c) + r + \hat{r} = 0 + 30 + 0 = 30$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty \end{array}\right] \end{array} \rightarrow (a)$$

$\rightarrow$ noreduction

L-4

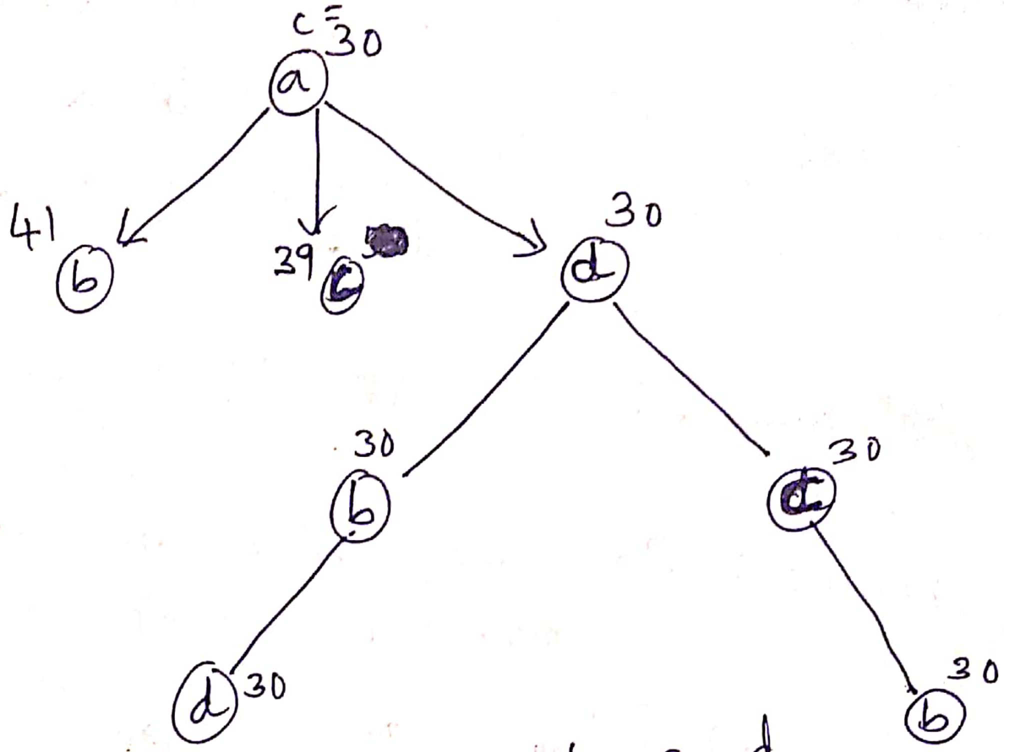
$cost(b,c) = c(b,c) + r + \hat{r} = 0 + 30 + 0 = 30$

there is no reduction on (b) $\hat{r} = 0$

$cost(c,b) = c(c,b) + r + \hat{r} = 0 + 35 + 0 = 30$

there is no reduction on (c)

Final graph using Branch & Bound



18 3 1

Reduced matrix $a = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & \infty & 6 & 4 & 0 \\ b & 0 & \infty & 0 & 5 \\ c & 0 & 0 & \infty & 6 \\ d & 0 & 0 & 0 & \infty \end{array}$

reduced cost = 30.

3 2 Reduced matrix $6 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & \infty & \infty & \infty & \infty \\ b & \infty & \infty & 0 & 0 \\ c & 0 & \infty & \infty & 1 \\ d & 0 & \infty & 0 & \infty \end{array}$

reduced cost = 41

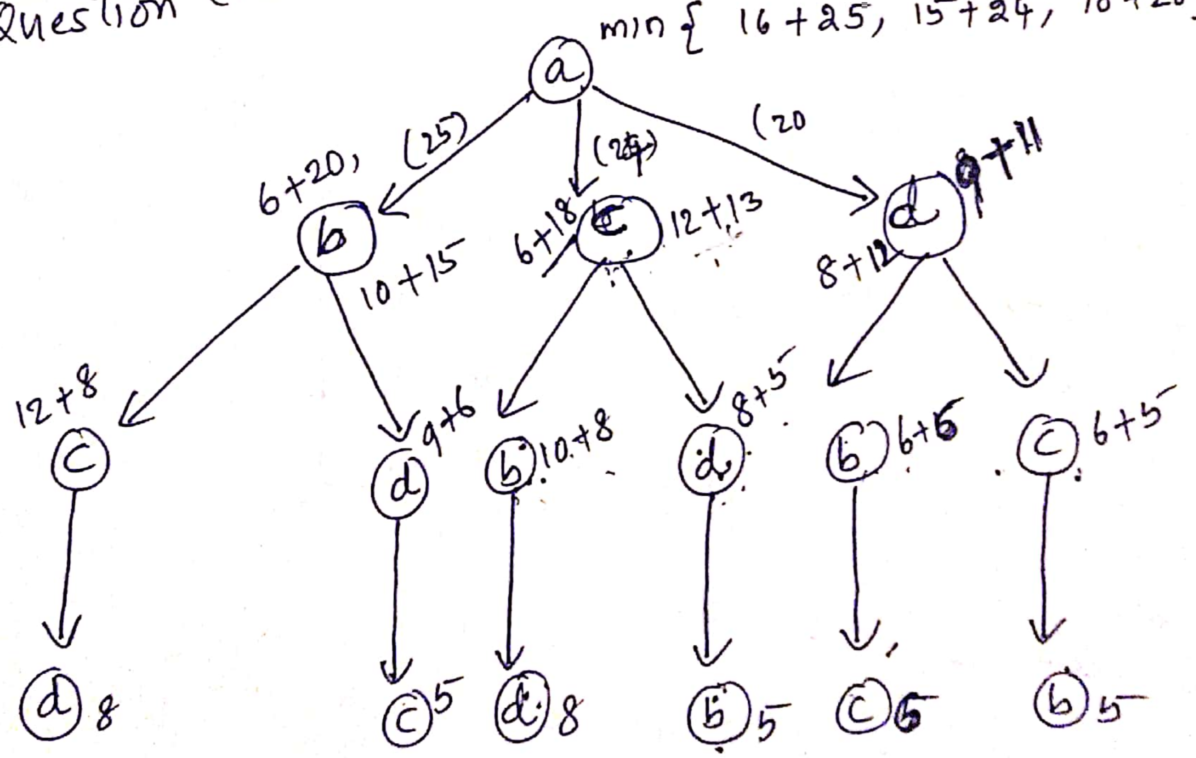
18

③ - ③ Reduced matrix =

$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
a & \infty & \infty & \infty & \infty \\
b & 0 & \infty & \infty & 0 \\
c & \infty & 0 & \infty & 1 \\
d & 0 & 0 & \infty & \infty
\end{array}
$$

~~Cost~~ Reduced Cost = 39.

Question (18 − 1) : Bruteforce Algorithm starting from a



15
24
39

Ans = 30