DAA Exam #3 online (100 points)

Last Name: _____ First Name: _____ ID: _____

Q. [1] make your seed values A, B, C, D:

a) Find an **A** value by subtracting your student ID's last one digit from the first two digits (i.e., If your student ID is 16301378, the A value should be "16 – 8 = 8".) [ ]

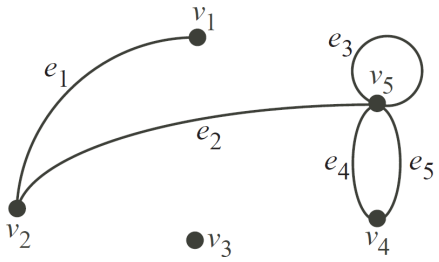
b) Find a **B** value by subtracting your student ID's last one digit from 14 (i.e., If your student ID is 16301378, the B value should be "14 – 8 = 6".) [ ]

c) Find a **C** value by subtracting your student ID's last one digit from 12 (i.e., If your student ID is 16301378, the C value should be "12 – 8 = 4".) [ ]

d) Find a **D** value by subtracting your student ID's second last one digit from 20 (i.e., If your student ID is 16301378, the D value should be "20 – 7 = 13".) [ ]

Graph Problems (45 points)

Q1. [4] For the given graph, answer the graph type definition:



Q1-1) [1] What is the degree of v5? [ ]

Q1-2) [1] Are e3 and e4 parallel edges? [ ]

Q1-3) [1] Is v1 a pendant vertex? [ ]

Q1-4) [1] Is the above graph a simple graph? [ ]

Q2. [4] Can a simple graph have C vertices and 40 edges? Explain and justify your answer.

Q3. [4] Can a tree graph have C vertices and B edges? Explain and justify your answer.

Q4. [3] How many *more* edges are there in the complete graph with B vertices (K_B) than in the complete graph with C vertices (K_C)? Explain and justify your answer.

Q5. [4] Suppose that in a group of 5 people: P, Q, R, S, and T, the following pairs of people are acquainted with each other (P and R, P and S, Q and R, R and S, R and T)

a) Draw a graph G to represent this situation.

b) Draw an adjacency matrix for G (0 = no edge; 1 = edge).

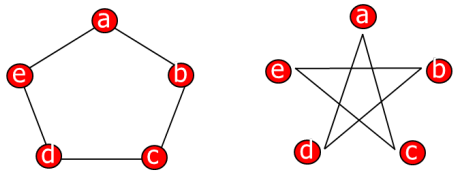
Q6. [4] A certain tree has two vertices of degree C, one vertex of degree 3 and one vertex of degree 2. If the other vertices have degree 1, how many vertices are there in the graph? Explain and justify your answer.

Q7. [4] Let G be a simple undirected planar graph on C vertices with 15 edges. If G is a connected graph, then what is the number of bounded regions in any embedding of G on the plane? Explain and justify your answer.

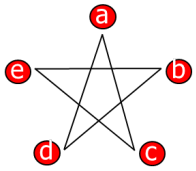
Q8. [4] What is the maximum number of edges in a bipartite graph having A vertices? Explain and justify your answer.

Q9. [4] Does a complete graph on D vertices has an Eulerian circuit? Explain and justify your answer.

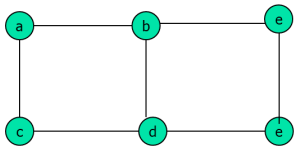
Q10. [4] Show if the two graphs shown below is isomorphic. Is it isomorphic or not? Clearly, explain and justify your answer. Simple yes or no answer will result 0 score.



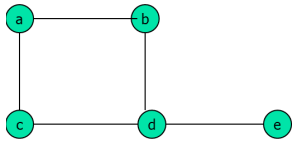
Q11. [2] Show if the following graph is a planar graph. Clearly, explain and justify your answer.



Q12. [2] Does the following graph has any Euler Cycle? Clearly, explain and justify your answer.



Q13. [2] Does the following graph have any Hamiltonian path? Clearly, explain and justify your answer.



P, NP, NP-Hard, and NP-Complete (NPC) Problems (24 points)

Q14 [24]. Answer if the following statement is TRUE or FALSE.

If your D number is 20 ~ 18 only solve 12 questions including 1, 3, 5, 7, 8, 9, 11, 13, 15, 17, 19, and 21.
If your D number is 17 ~ 14 only solve 12 questions including 2, 4, 6, 8, 10, 11, 12, 13, 14, 16, 18, and 20.
If your D number is 13 ~ 11 only solve 12 questions including 1, 2, 5, 6, 8, 10, 11, 12, 15, 16, 19, and 21.

Your D value:												
Q #												
Ans												

1. [2] $O(n^{1,000,000,000})$ algorithm complexity is intractable.
2. [2] $O(n^{\log n})$ algorithm complexity is intractable.
3. [2] $O(2^n)$ algorithm complexity is intractable.
4. [2] NP, NPC, and NP-Hard are intractable optimization problems.
5. [2] NPC ∈ NP-Hard.
6. [2] NP problems are always harder than P problems, iff P != NP
7. [2] NP-hard problems are always harder than NPC problems, iff P != NP
8. [2] NPC problems are always harder than P problems, iff P != NP
9. [2] NPC ∈ NP
10. [2] If 3-SAT problem can be solved in polynomial time, then P = NP.
11. [2] If we want to prove that a problem X is NPC, it is sufficient to take a known NP-Hard problem Y and reduce Y to X.

Let X be a problem that belongs to the class NP (for 12 ~ 14):

12. [2] if X is NP-complete, then it is NP-hard.
13. [2] there is a polynomial time decision algorithm for X.
14. [2] if X can be solved deterministically in polynomial time, then P = NP.

Consider two decision problems X1 and X2 such that X1 reduces in polynomial time to 3-SAT and 3-SAT reduces in polynomial time to X2 (for 15 ~ 19):

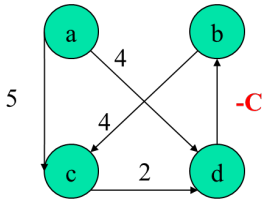
15. [2] X1 is NP-complete
16. [2] X1 is in NP
17. [2] X2 is NP-complete
18. [2] X2 is NP-hard
19. [2] X1 is NP-hard

Ram and Shyam have been asked to show that a certain problem Z is NP-complete. Ram shows a polynomial time reduction from the 3-SAT problem to Z, and Shyam shows a polynomial time reduction from Z to 3-SAT (for 20 ~ 21):

20. [2] Z is NP-hard.
21. [2] Z is in NP.

Algorithm Solving Problems (30 points)

Q15 [6]. For the following graph (d to b edge cost is – C value) solve both 15-1 and 15-2:



Q15-1. [3] Find the shortest route and cost from "a" to "c" using **Dijkstra's Algorithm**. Show your work on the given table and write the algorithm. What is the shortest route and cost from "a" to "c"?

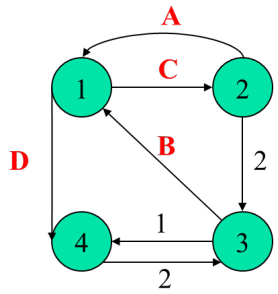
	a	b	c	d
a				

Q15-2. [3] Find the shortest route and cost from "a" to "c" using **Bellman-Ford Algorithm**. Show your work on the given table by using the edge sequence. What is the shortest route and cost from "a" to "c"?

(a,c)	(a,d)	(c,d)	(d,b)	(b,c)

	b	c	d
1st			
2nd			
3rd			

Q16. [6] All pairs shortest path problem:



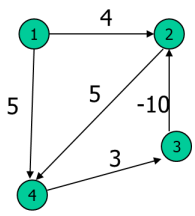
A_0

	1	2	3	4
1	0	C	∞	D
2	A	0	2	∞
3	B	∞	0	1
4	∞	∞	2	0

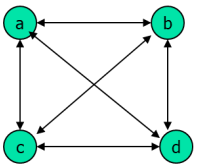
Q16-1. [3] show the step process from A_0 to A_1 using a dynamic programming (DP) algorithm.

Q16-2. [3] show the step process from A_1 to A_2 using a dynamic programming (DP) algorithm.

Q17. [2] Does Bellman-Ford (BF) Algorithm find shortest path from 1 to 2 for the following graph? If yes, what is the shortest path? If not, what is the reason?



Q18. [16] Solve the following Traveling Salesman Problem



Adjacency Matrix

	a	b	c	d
a	0	**D**	15	**A**
b	5	0	**C**	10
c	6	**B**	0	12
d	8	8	9	0

Q18-1. [3] Show the Brute Force algorithm design starting from **a**.

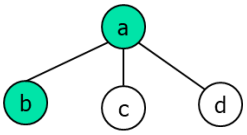
Q18-2. [4] Show the Dynamic Programming (DP) algorithm for solving the given TSP, g(i,S), where i: source, S: a set of cities to visit. Specifically, show the TSP traveling a→b→c→d→a route and its cost by using the Dynamic Programming (DP) algorithm.

Q18-3. [9] Solve the given TSP using a branch and bound algorithm starting from a. Use the given table to show the process of finding the reduced cost on the each node and specify the reduced cost.

Q18-3-1. [3] Show the reduced cost process at **a** and its reduced cost:

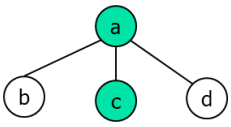
	a	b	c	d	
a					
b					
c					
d					

Q18-3-2. [3] Show the reduced cost process at **b** and its reduced cost:



	a	b	c	d	
a					
b					
c					
d					

Q18-3-3. [3] Show the reduced cost process at **c** and its reduced cost:



	a	b	c	d	
a					
b					
c					
d					