Overview

Titan Insurance Company wants to have an early view of the success or failure of the new incentive payment scheme for its lift policy sales force. The scheme is expensive for the company but they are looking for sales increases which more than compensate. It will be abandoned after six months if the scheme does not at least break even for the company.For Titan to break even, the average output must increase by £5000. Titan has collected sales output of 30 sales persons randomly for observing the effectiveness of variation in sales after introducing the new scheme.

Titan is interested in knowing the following:

Titan wants to check if introduction of new scheme has raised the sales significantly. Titan wants to check if average sales output is greater than £5000.

The above output is what Titan wants to achieve. So to check if that can be achieved, the data collected has to be analysed using appropriate statistical methods. Hypothesis Testing method can be used to cross check above requirement.

Preliminary Analysis

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> summary(SchemewiseData)
  SalesPerson      OldScheme         NewScheme    
 Min.   : 1.00   Min.   : 28.00   Min.   : 32.00  
 1st Qu.: 8.25   1st Qu.: 54.00   1st Qu.: 55.00  
 Median :15.50   Median : 67.00   Median : 74.00  
 Mean   :15.50   Mean   : 68.03   Mean   : 72.03  
 3rd Qu.:22.75   3rd Qu.: 81.50   3rd Qu.: 85.75  
 Max.   :30.00   Max.   :110.00   Max.   :122.00  
> mean(OldScheme)
[1] 68.03333
> mean(NewScheme)
[1] 72.03333
> sd(OldScheme)
[1] 20.45598
> sd(NewScheme)
[1] 24.06239
> var(OldScheme)
[1] 418.4471
> var(NewScheme)
[1] 578.9989
> sd(NewScheme - OldScheme)
[1] 14.08105
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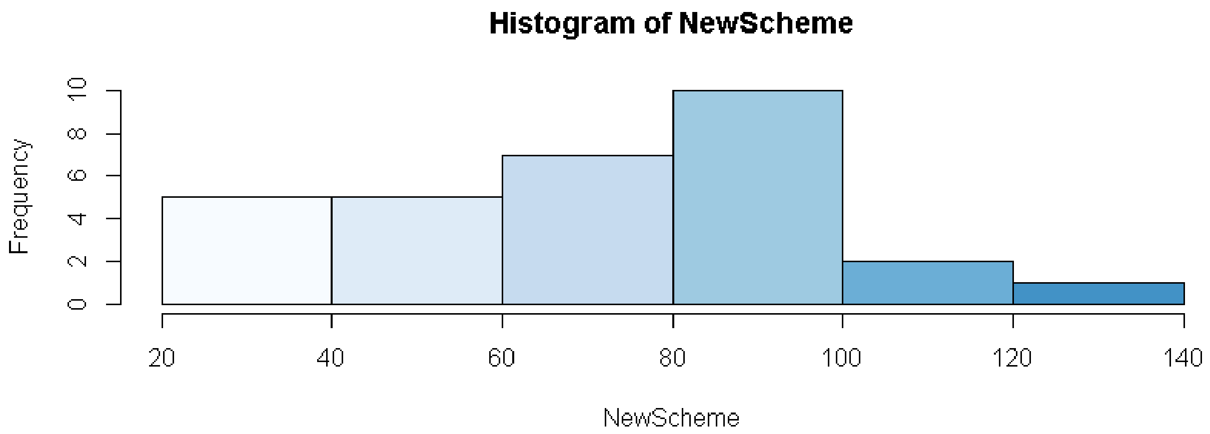
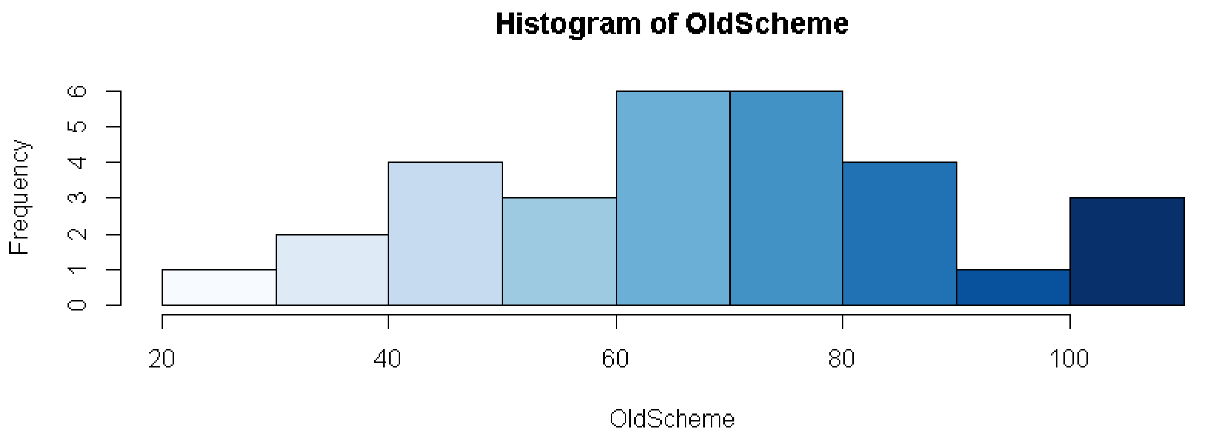
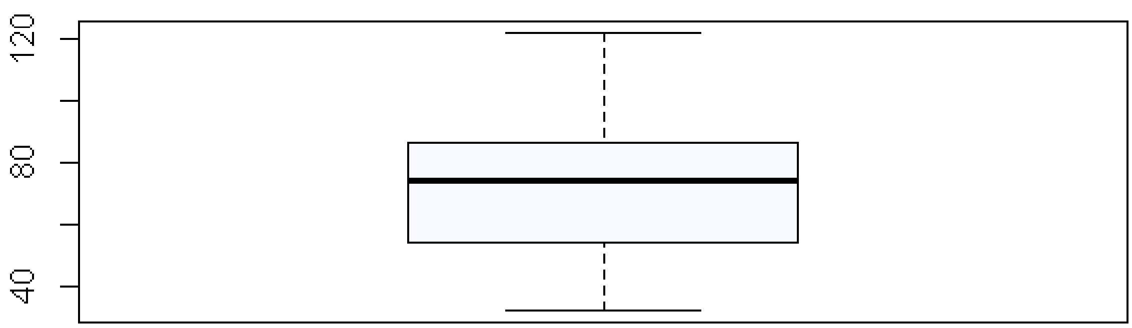
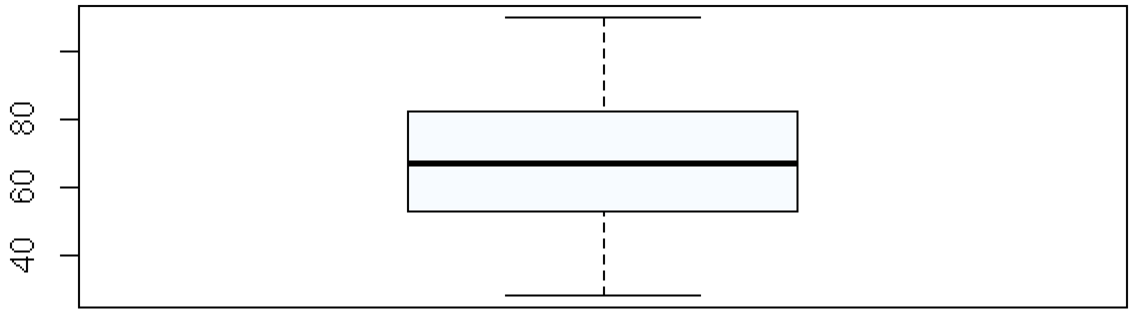

Histogram of NewScheme
Frequency
0
2
4
6
8
10
20
40
60
80
100
120
140
NewScheme


Histogram of OldScheme
Frequency
0
1
2
3
4
5
6
20
40
60
80
100
OldScheme


40
80
120



<u>Observations:</u>

- Both the samples seems to be normally distributed.
- Mean and Median Values are not much different.
- The Old scheme data looks more normally distributed, whereas the New scheme data looks left skewed. there is a dip in the performance.

(a) Describe the five per cent significance test you would apply to these data to determine whether new scheme has significantly raised outputs?

The sample data given is as follows:

Salesperson	Old Scheme	New Scheme
1	57	62
2	103	122
3	59	54
4	75	82
5	84	84
6	73	86
7	35	32
8	110	104
9	44	38
10	82	107
11	67	84
12	64	85
13	78	99
14	53	39
15	41	34
16	39	58
17	80	73
18	87	53
19	73	66
20	65	78
21	28	41
22	62	71
23	49	38
24	84	95
25	63	81
26	77	58
27	67	75
28	101	94
29	91	100
30	50	68

The following values can be calculated by using Excel:

<u>Old Scheme:</u>
Sample Mean (X1 bar) = 68.03333
Sample Standard Deviation (Sigma 1) = 20.456

<u>New Scheme:</u>
Sample Mean (X2 bar) = 72.0333
Sample Standard Deviation (Sigma 2) = 24.0624

It can be obeserved from above data that sample has been collected for 30 sales persons picked randomly. Therefore Sample Size, n = 30

Hypothesis can be defined as below:

Null Hypothesis: Ho: $\mu 1 = \mu 2$
Alternate Hypothesis: H1: $\mu 1 < \mu 2$

Where $\mu 1$, $\mu 2$ = Population means at beginning of the testing period and end of the testing period respectively.
As per null hypothesis $\mu 1 = \mu 2$
That implies $\mu 1 - \mu 2 = 0$

Z-test for two sample means

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2}}}$$

Since we consider **μ1 = μ2**, the above equation will change as given below

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2}}}$$

By substituting the values in above formula and calculating Z
X1 bar = 68.03333
X2 bar = 72.0333
Sigma 1 = 20.456
Sigma 2 = 24.0624
Sample Size is same. So n1 = n2 = 30

Z statistic =(68.03333 – 72.03333)/Sqrt(((20.456 * 20.456) + (24.062 * 24.062)) / 30) = -0.693

Therefore **Z statistic = -0.693**

For Five Percent Significance Test, Alpha = 0.05 and Z critical = -1.645
Z value can be calculated using Excel or from Z tables.

Z statistic is (-0.693) which is greater than Z critical (-1.645)
So Z falls in the Acceptance Region.

If we substitute Z critical as -1.645 and other values in the above formula and compute Xbar for New scheme

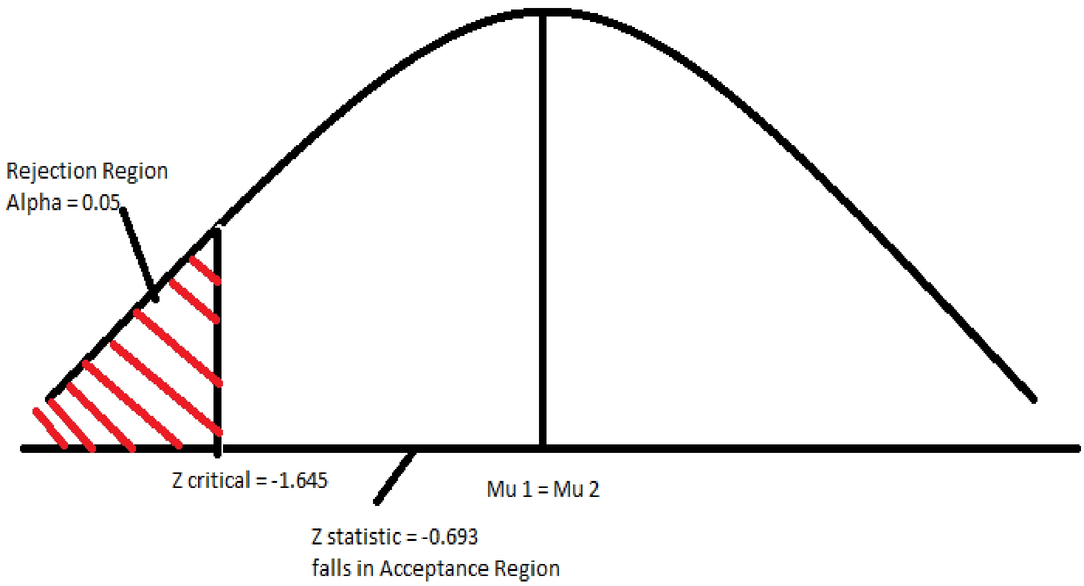
Z critical = -1.645 = X1 bar – X2 bar / 5.7661

That implies X2 bar = X1 bar + 3.9959 = 68.03333 + 3.9959 = 72.02923

Therefore X2 bar critical is approximately 72.03
And X2 bar of sample mean is 72.033

Both are almost the same. So there is no much significant difference observed in the new scheme's average output when compared to the required critical average. So by this it is evident that the new scheme did not raise the outputs significantly.

(b) What conclusion does the test lead to?



By above test and the values computed considering 5% significance, we can conclude that **there is no sufficient evidence to reject Null hypothesis**. So it can be accepted and alternate hypothesis can be rejected. Which means that New scheme output is either less than or equal to Old scheme output. But can not be greater than Old scheme output. **So we can say with 95% confidence level New scheme output is less than old scheme output.**

(c) What reservations do you have about this result?

As the average output of new scheme is almost the same as required, critical and Z statistic computed also does not provide sufficient evidence to reject null hypothesis. Therefore based on above computation, **new scheme is not very effective and the outputs does not vary much. So we cannot suggest to go ahead with the new scheme.** Hence it should be abandoned as it is very expensive and also not increasing sales output significantly.

(d) Suppose it has been calculated that in order for Titan to break even, the average output must increase by £5000. If this figure is alternative hypothesis, what is:
(i) The probability of a type 1 error?
(ii) The probability of a type 2 error?
(iii) The power of test?

Formulating Hypothesis as per the above statements:

Null Hypothesis: H0: $\mu <= 5000$ **Alternate Hypothesis:** H1: $\mu > 5000$

It has been specified that Titan has collected sample for 30 sales people. So the sample size is 30.

Sample size = n = 30

Average output of the new scheme can be calculated from the sample provided. It would be the Xbar.

Xbar = 72.03333 (Average output of New Scheme)

Standard Deviation for New Scheme = sigma = 24.0624 (approximately)

To apply 5% Significance Test, we assume significance as 5%. This is the alpha value.

So alpha = 5% = 0.05

1- alpha = 1 – 0.05 = 0.95 = confidence level

So the confidence level for the test would be 95%.

When alpha is 0.05 Zcritical value(Zc) will be 1.645 approximately. This can be found by using z tables or microsoft excel.

<u>Find Zstatictic by formula:</u>

Zstatistic is the ratio of sample errot to standard error.

Therefore: **Zstatistic = Zs= (Xbar – μ)/(sigma/sqrt(n))**

That implies Zs = (72.03333 – 5000)/(24.0634/sqrt(30)) = -4927.9667/4.2931 = -1121.7515

Zs = -1121.75 (Negative Value)

As we know the Zcritical as 1.645, we can calculate Xbar critical based on the formula.

Zs = (Xbar – μ)/(sigma/sqrt(n))
That implies Xbar = Mu + Zs(sigma/sqrt(n))

Xbar critial is Xbar value when Zs is Zcritical.

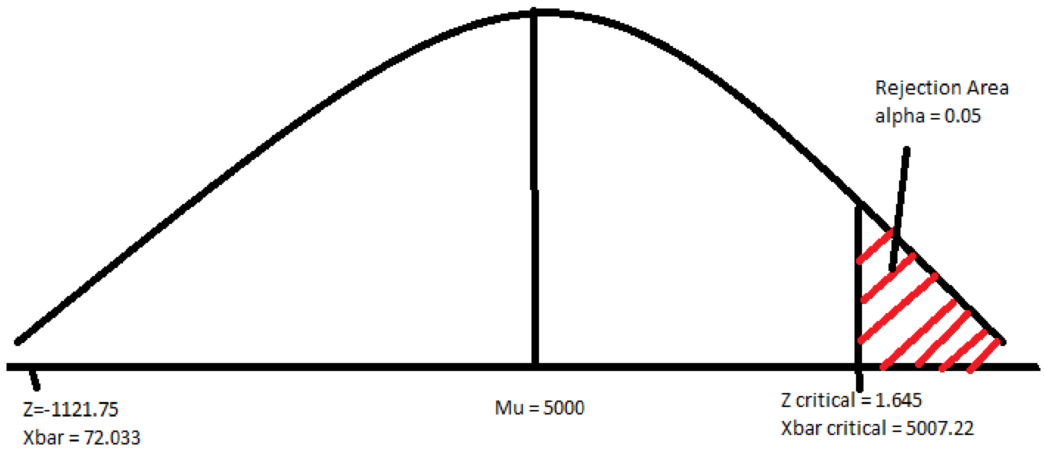
Therefore Xbar critical = μ + Zcritical(sigma/sqrt(n))

In the above formula μ is the hypothesised mean which is equal to 5000.
n is the sample size = 30
And sigma is the computed standard deviation for new scheme = 24.0624 (calculated from excel)

So Xbar critical = 5000 + 1.645(24.0624/sqrt(30)) = 5000 + 1.645(4.3931) = 5000 + 7.2266 = 5007.2266

Therefore **Xbar critical = 5007.2266**

This means that New scheme can be continued if the average output is atleast £5007.22



Zs = -1121.75 is less than Zcritical.
Z falls in the acceptance region. Hence we do not have sufficient evidence to reject the null hypothesis.
So alternate hypothesis can not be accepted. So we can say that with 95% confidence level average output will be less than 5000. Hence the new scheme can not be continued. It should be abandoned as Titan is not able to break even.

P-Value Approach:

Xbar = 72.033
Z = -1121.75

Power of test can be computed using excel. Passing the z value as input.

Power of test = 0 (computed using excel)
That implies 1 – beta = 0
Hence beta = 1

p-value =1 and alpha = 0.05
Hence p-value(1) is greater than alpha(0.05).
So by this way also null hypotheis is accepted which means that average output will be less than 5000 and new scheme should be abandoned as Titan will not be able to break even.

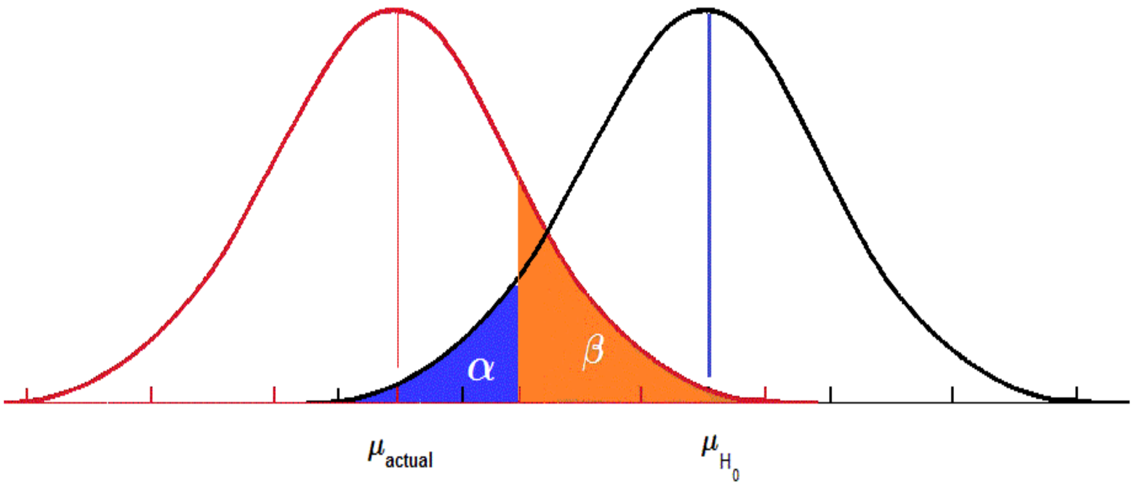
Therefore the required numbers are as follows:
P(Type 1 Error) = Alpha = 0.05 = 5%
Confidence = 1 – Alpha = 1 – 0.05 = 0.95 = 95%
P(Type 2 Error) = Beta = 1 = 100%
Power of test = 1 – Beta = 0

Titan should continue with the new scheme if the average monthly output is atleast £5007.22
As the average output is just £72.033 which is very less than the required, Titan should abandon the new scheme with a confidence level of 95%.

Beta value is 1 which indicates that the probability of type 2 error is 1. That means type 2 error is certain. As type2 error indicates a missed opportunity to take corrective action, it is clearly obvious that Titan is missing an opportunity to improve sales to a very high numbers. However this indicates the probability of accepting null hypothesis when it is false.

(e) Are Type 1 and Type 2 errors the same in this case? Should they be equal? Why or why not? If they are to be equated, suggest a way to do so. (Hint: would a change in sample size work?)"

Alpha is the probability of Type 1 error and Beta is the probability of Type 2 error.



Alpha and Beta can be represented as the areas shown in graph above.

The graph is drawn with two curve where one curve represents null hypothesis and another curve represents alternate hypothesis. So when alpha increases beta decreases. It is not practically possible to make both alpha and beta excatly equal. However the difference can be reduced to maximum extent. Alpha will be constant as we calculate the require statistics assuming some significance level. For a significance level alpha is the same and it will not change. Beta changes as per the variations in other parameters of the Z value computation formula.

If Alpha and Beta are to be equated both should have same value. Alpha is constant as it is determined by the significance level we choose. In this case we have chosen 5% significance. So Alpha = 0.05 If Beta has to be equated with Alpha then other parameters in the Z statistic equation should change as Alpha is constant here and Beta varies.

Let us assume Beta = Alpha So Beta = Alpha = 0.05

When Alpha is 0.05
1 – Alpha =0.95

When we assume that Beta =0.05 (equal to Alpha)
1 – Beta will be 0.95

Z critical can be computed using Excel when Power of test is 0.95
Z critical will be -1.645

So, substituting the values in the equation

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$$

Since we consider μ1 = μ2, the above equation will change as given below

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$$

We assume that the sample size is equal in both samples
Therefore n = n1 = n2

Calculating n from above formula:

n = (Square(Sigma1) + Square(Sigma2))/Square((X1 bar – X2 bar) – (µ1 – µ2)Z)
=(418.447 + 578.999)/ Square(-4/-1.645)
=999.446/Square(-2.4316) = 999.446/5.9126 = 169.0366

n = Sample Size = 169

Therefore, it is recommended to increase the sample size(n) to 169 in order to equate Alpha and Beta where both values will be equal to 0.05

If we want to check this, we can substitute the values in Z Statistic equation. And place n = 169
Z = -4 / Sqrt(((20.456 * 20.456) + (24.0624 * 24.0624))/169)
=-4/(31.5821/13)= -4/2.423 = -1.646 which is approximately -1.645(Z critical)
So above calculation verifies that, in order to equate Alpha and Beta to 0.05 and Z to -1.645, we need to increase sample size to 169.

It is even better if we can increase the sample size to a higher value as accuracy of the results increase with high values of sample size. Beta decreases when sample size increases.