# Modeling the US National Debt

A Stochastic Approach

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# Summary Sheet


When the United States spends more money than it makes in taxes, it supplements its income by taking out debt. For the past two decades, the United States has run a deficit, meaning that its expenditures are greater than its income. This is not entirely unnatural, however in recent years, multiple wars, economic recessions, and a global pandemic, have caused the U.S. debt to grow quite sharply.

We used the White House Historical tables that tracks annual government expenditures and incomes, and narrowed down our data to the years 2000 to 2020. We also used the pre-existing categories used in the historical tables for income and expenditure. Within our model, we based the decrease or increase of the deficit on 3 factors:

1. Spending policy
2. Tax policy
3. Economic catastrophes

Within our calculations for the deficit, we split up our model into expenditures and income. Using the categories for expenditures and income in the historical tables, we calculated the average growth rate over the years 2000-2020 and formed a normal distribution of growth rates for each category.

After establishing the demographic stochasticity for categories within expenditure and income for our model, we incorporated the environmental stochasticity. For expenditures, we used data since 1964 to find the probability of a catastrophe that affects government spending significantly. For income, we found the probability of a tax increase and decrease through legislative action.

Using our recursive model with demographic and environmental stochasticity, we used the existing expenditures and income from 2020 to simulate the deficit over the years 2021-2031. In our model, we observed that the deficit increased greatly, much faster than the predicted deficit from the White House Budget tables.

We then formulated 2 different plans with expenditure and tax policies aimed at reducing the deficit. In one plan, we modeled what would happen to the deficit if we cut back on expenditures, reducing both the National Defense budget and Human Resources budget growth rate. In the next plan, we modeled the deficit after implementing higher tax policies, with a focus on high Individual Income taxes. Looking at the total deficit result from both plans, we estimate that national debt will continue to grow exponentially unless changes in expenditure or tax policy are made.

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# 1. Introduction

The growing US debt is an issue often ignored or unknown, despite the magnitude of intergovernmental and public debt as well as the potential impacts it poses for the U.S. population. The US debt changes largely according to government policies managing government expenditures and income. This debt has increased in an exponential pattern, with a current outstanding debt of $28,908,663,008,400.59[1]. By creating a model of the change in the amount of debt, a better understanding of the US debt can be achieved and be used by government intel and voters to affect government policy.

### 1.1 Problem Statement

The US' debt's increasing growth poses problems for the future. As the debt to GDP ratio increases to an alarming amount, expected to be 137.2% by the end of 2021, there is increasing pressure to control government expenditures and income in order to decrease the yearly budget deficit [2].

We use data analysis to build a model that **predicts the growth of U.S. debt**. In our model, we implement the following financial factors: **government expenditures, government income (primarily from taxes), and random economic catastrophes**. We then use this model to enact different scenarios with varying policies that correspond to differing political parties in office and amount of catastrophes.

#### 1.1.2 Data Sources

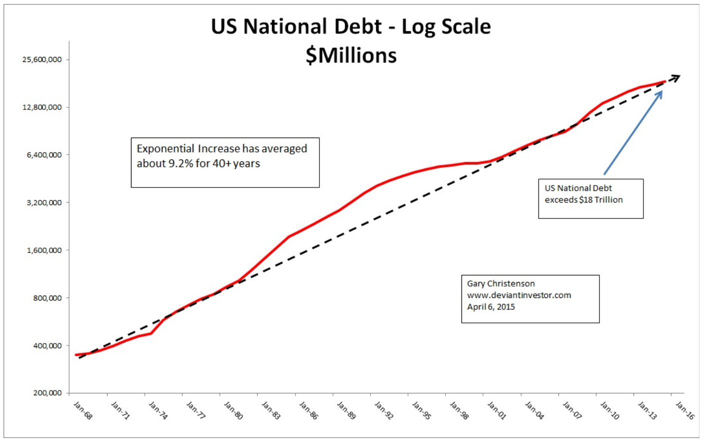
Our model is informed by the datasets from the U.S. Treasury Fiscal Data website, which records debt to the penny as well as budget deficits from Monthly Treasury Statements (MTS).

#### 1.1.3 Existing Models

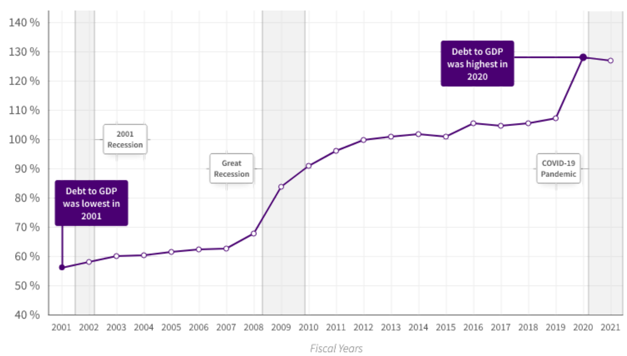
Models of the US debt come in many variations, including raw fiscal data and as a ratio to other indicators of the economy such as GDP.

The graph of raw outstanding US debt since 1950 can be easily compared to an exponential model, and the model of debt on a log scale shows a close relationship with the exponential line of best fit[3].

Many models graph the outstanding debt as a percentage of GDP as it gives context and significance to the debt[2]. These charts also show a pattern of general growth in percentage of GDP, and other models include the corresponding president in office to show the effects of policy[4].


US National Debt - Log Scale
$Millions
25,600,000
12,800,000
6,400,000
3,200,000
1,600,000
800,000
400,000
200,000
Exponential Increase has averaged about 9.2% for 40+ years
US National Debt exceeds $18 Trillion
Gary Christenson
www.deviantinvestor.com
April 6, 2015
Jan-68
Jan-71
Jan-74
Jan-77
Jan-80
Jan-83
Jan-86
Jan-89
Jan-92
Jan-95
Jan-98
Jan-01
Jan-04
Jan-07
Jan-10
Jan-13
Jan-16

[3]


140 %
130 %
120 %
110 %
100 %
90 %
80 %
70 %
60 %
50 %
40 %
Debt to GDP was highest in 2020
2001 Recession
Great Recession
COVID-19 Pandemic
Debt to GDP was lowest in 2001
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
Fiscal Years

[2]


GROWTH & FORECAST OF U.S. NATIONAL DEBT IN RELATION TO GDP
NATIONAL DEBT IN RELATION TO GDP
110%
109%
108%
107%
106%
105%
104%
103%
103.41%
105.41%
105.05%
105.20%
107.11%
108.14%
107.83%
107.94%
108.24%
108.79%
109.62%
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
YEAR
Source: https://www.statista.com/statistics/269960/national-debt-in-the-us-in-relation-to-gross-domestic-product-gdp/

[4]

### 1.2 Background

The US debt is a calculation of how much the US government owes its creditors. This debt decreases or increases based on how much the US takes in income, mostly through taxes, and how much the US spends. US expenditures include government sectors, projects, and social welfare. The total amount of money from income minus expenditures is the yearly calculated surplus or deficit that contributes to debt. [5]

#### 1.2.1 US debt holders

US debt holders come in two categories: public holders and intragovernmental holders. Debt is accumulated as the US borrows money from individuals, organizations, or government sectors that purchase Treasury securities. Public debt includes debt that the US government owes to individuals or organizations who purchase Treasury Securities. The top owners of public debt include US investors, the Federal Reserve, Japan, and China. Intragovernmental debt is debt that one sector of the government owes to another sector. The most significant intragovernmental debt holders are Social Security, Department of Defense, and the Civil Service Retirement and Disability Fund[6]. As of 11/26/2021, the approximate calculated public debt is 22.6 trillion and the intragovernmental debt is 6.3 trillion[1].

#### 1.2.2 Impact of Debt

While the raw calculations of debt do not offer much insight into the effects of this debt, the context around this debt conveys that the US debt has potential consequences. The debt to GDP directly compares the debt to an indicator of US economic performance, and the graph displays the debt as a growing percentage of the GDP. In 2012, the debt to GDP ratio met 100%, meaning US debt matched the value of all goods and services in the US economy[2]. As the debt to GDP ratio increases, it becomes more difficult for the US to pay its interest. A worst case scenario is if the US could become unable to pay off it's interest, or defaults on debt. A default on debt would cause massive economic effects, as markets and investors lose faith in the US and the dollar falls in value. These and other effects could trigger a recession and seriously harm the ability of American companies to operate[7].

### 1.3 Nomenclature

$E(n)$ - Annual Government Expenditure

ND - National Defense
HR - Human Resources
PR - Physical Resources
NI - Net Interest
OF - Other Functions

$I(n)$ - Annual Government Income

IIT - Individual Income Tax
CIT - Corporate Income Tax
SSM - Social Security Medicare
EXT - Excise Tax
MISC - Miscellaneous Revenue Sources

$D(n)$ - Annual Government Deficit

$R_x$ - Catastrophe, synonymous with an stochastic event
$P$ - Annual rate of change, equal to $1 + r$
$\bar{r}$ - mean increase
$YoY$- Year over Year
σ - Standard Deviation

**1.4 Assumptions**

- The different sections of public debt and intragovernmental debt do not affect the change in government debt, and can subsequently be grouped together into one category.
- The expenditure and income growth rates from 2000-2020 best reflect the predictions for the years 2021-2031. We are assuming that the 2021 data we found is not yet complete since the year is not over. Thus we believe 2020 is the best starting point for our model, as it is the last complete year of data.
- The occurrences of economic catastrophes that affect government expenditures can be modeled by a binomial distribution
- The amount of money the U.S. government borrows will not change drastically, other than the scenarios of debt growth through revenue, expenditure, and catastrophe.
- The US will not go through a period of sustained hyperinflation
- A singular catastrophe will only have an effect on one year. Because our model is recursive, if a catastrophe occurs during one year, the spending will continue on the trend it was on current trend
- The current spending will not decrease significantly even after covid
- Catastrophes (war or economic recession) are independent events from each other and from year to year.
- Spending increase or decrease in one category does not affect any other categories
- We did not account for possibility of recovery from a catastrophe, we simply assumed that if spending increased suddenly, the following years would exhibit the following trend
- The nature of catastrophes has shifted in the past two decades, and future events would be equally bad should they occur. The past 20 years worth of catastrophic events have been extremely bad, there is still a possibility that it will not be as bad in the future.

- Environmental spending stochasticity only affects Individual Income Tax and Corporate Income Tax. The other three revenue categories aren't affected by drastic changes in legislative policy.
- Looking back at the last twenty-one years, there have been four individual income tax cuts, and no tax raises. Thus, we model an environmental stochasticity with an effect to increase taxes by 3.25% taken as an average of the four tax cuts. We take the probability of this event occurring as $\frac{4}{21}$ or 19%.
- Looking back at the last twenty years, there has been a corporate tax increase by 15% and a corporate tax decrease by 14%. Therefore, we model two environmentally stochastic events: tax increase by 15% with a 5% chance of occurring, and a tax decrease by 14% with a 5% chance of occurring.
- A corporate tax increase and individual income tax cut are independent events from each other and from year to year.

**1.5 Our Model**

In order to simulate different effects from external factors on the change of U.S. debt, we have decided to find the impact of tax policies, expenditure policies, and economic catastrophes on the US debt. Throughout the last two decades, the nature of the national debt has drastically changed, with the debt to GDP ratio increasing rapidly and economic phenomena such as the dot com boom and housing recession affecting the stability of the US economy. Because of this, we will be basing our model on the tax and expenditure policies and economic catastrophes of the last 20 years

# 2. Model Development

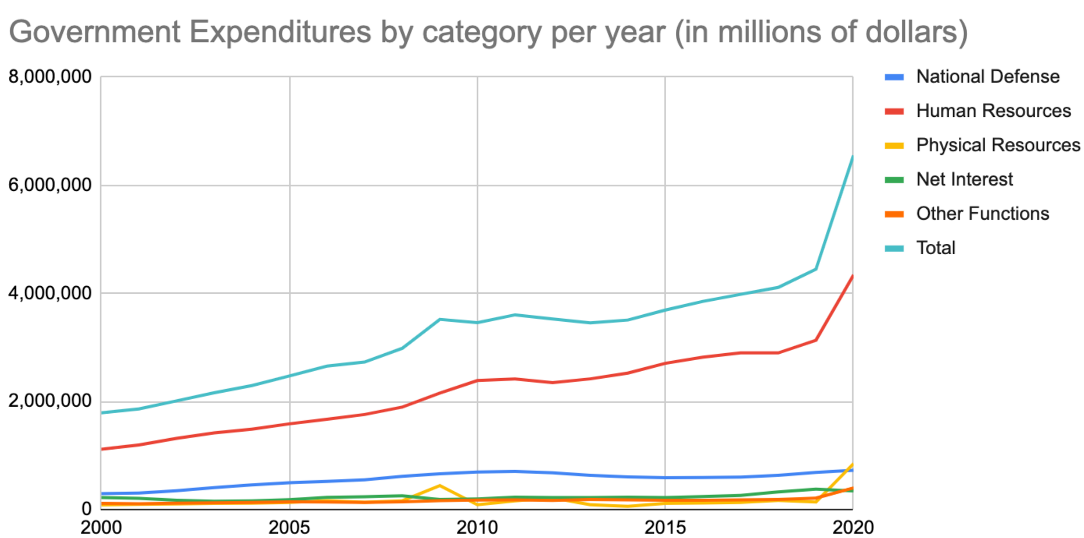
### 2.1 Model Description

We believe that modeling the national debt can best be broken down into 2 main parts: modeling the government's annual expenditures and annual revenue. Once modeling these 2 sections, we were able to model the annual deficit by subtracting the expenditure values from the revenue. We used a recursive model with demographic and environmental stochasticity to simulate both the expenditures and income models.

### 2.2 Methodology

We use income and expenditure data from 2000-2020 as our observed period for normal distributions of income and expenditure growth rates. We decided to start with the year 2021 since 2021 is incomplete, as the first time step in our recursive model. We assume that there will be no random drastic change in the amount of government expenditures and that the only changes are due to the growth rate and a catastrophe. The amount of expenditures and income each are are dependent on the year before it, and we sample the growth rates for each year using the normal distribution from the observed data, assuming that the US will not go through a period of sustained hyperinflation.

For the first part of our model, expenditures, we use a recursive model that depends on the growth rates of 5 different categories in expenditures: National defense, Human Resources, Physical resources, Net interest, and other functions. These rates derived from the average growth rate of each category from 2000-2020. The data for growth rates of expenditure categories from 2000-2020 is then made into a normal distribution.

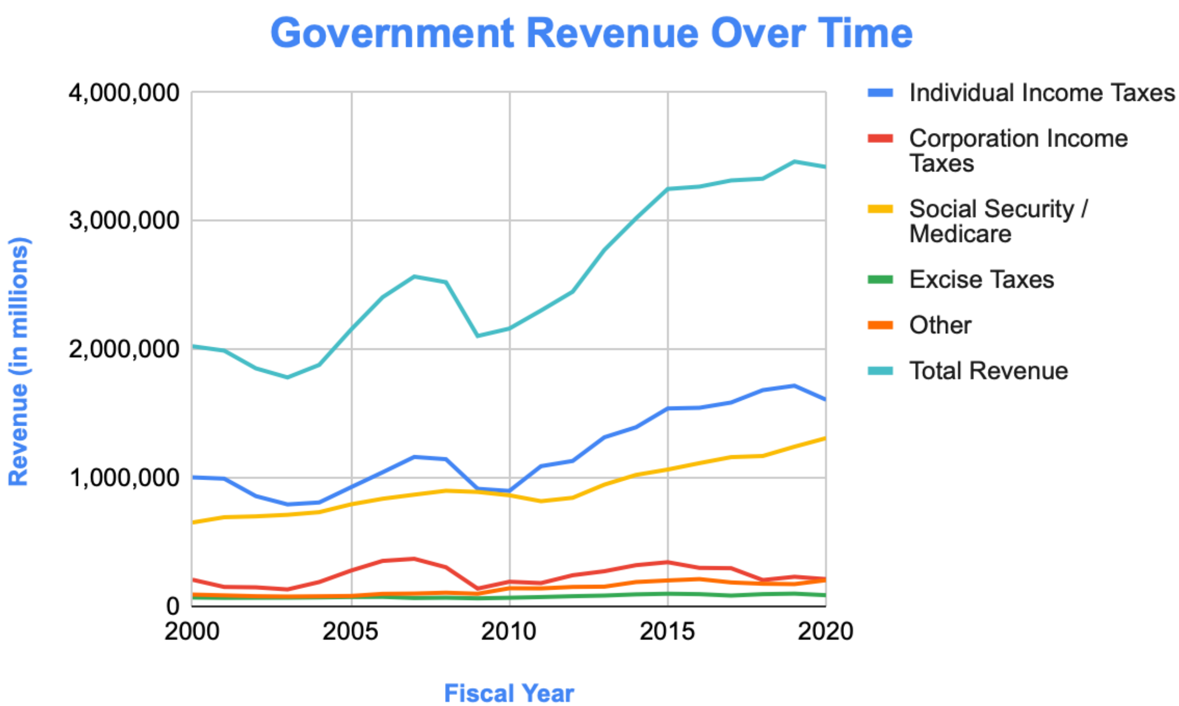


The graph shows the government expenditures by category in millions for each year from 2000-2020. The average rate of change for each category over every 1 year time step is used to make the normal distribution for expenditures.[8]

For the next part of our model, we calculate the annual government income. Modeling government annual revenue is achieved by individually modeling revenue categories and aggregating their results to form an overall revenue projection model.

Government annual revenue can be broken down into five categories: individual income tax, corporate income tax, social security / medicare, excise tax, and miscellaneous income.

Over the past 20 years, the breakdown of annual revenue can be visualized below:




The visualization above shows annual revenue by category in millions from 2000-2020. The average rate of change for each category over every 1 year time step is used to make the normal distribution for revenue modeling forecasting. [8]

## 2.3 Parameter Construction

### 2.3.1 Expenditure Stochasticity

Looking at previous data starting in 1964, we identified economic recessions and wars as relatively unlikely events that could both drastically increase spending in a given year, or catastrophe. Because of this, our model added a stochastic element to our calculation for the yearly rates of change in affected categories.

For both economic recessions and war, we measured the likelihood of these events occurring using data from 1964. However, the nature of government spending has changed so much in the past two decades, we did not feel it was reasonable to use data from before 2001 to predict the effects of such an event. Therefore, we only used data from 2001 onward to predict the change a war or economic recession would have on government expenditures.

Since 1964 (the past 57 years), the United States has entered a war or notable conflict 4 times. Therefore, we estimated that the U.S. enters a conflict every 14.25 years, or has a probability of $0.07$ for entering a conflict during any given year. Based on data from the past 20 years, National Defense spending had an average increase of 9.8% during the years when the U.S. entered a conflict. In our model, when we simulated the $P_{ND}$ value of the recurrence relation, we generated a random number between 0 and 1. If that number was less than 0.07, we ignored the number generated from the normal distribution of yearly rates of change for National Defense spending and set $P_{ND} = 1.098$ for the simulated year.

Similarly, since 1964, the U.S. has entered 7 economic recessions. Therefore, we estimated that the U.S. enters a recession every 8.14 years, or has a probability of 0.122 for entering a recession during any given year. Based on data from the past 20 years, both Human Resources and Physical Resource spending increased an average of 30.8% during the years the U.S. entered an economic recession. In our model, when we simulated the $P_{PR}$ and $P_{HR}$ values of their respective recurrence relations, we generated a random number between 0 and 1. If that number was less than 0.122, then we ignored the number generated from the normal distribution of yearly rates of change for Physical and Human Resources spending and set $P_{PR} = 1.308$ and $P_{HR} = 1.308$ the simulated year.

**2.3.2 Income Stochasticity**

Looking at previous data starting in 2000, we identified legislative tax changes due to economic crisis or new political administrations as relatively unlikely events that could both drastically increase/decrease income in a given year, or catastrophe. Because of this, our model added a stochastic element to our calculation for the yearly rates of change in affected categories.

First, we see this with drastic changes in annual revenue from individual income tax due to legislative policy changes. Looking back at the last 21 years, there have been four individual income tax cuts, and no tax raises. Thus, we model an environmental stochasticity with an effect to decrease taxes by 3.25% taken as an average of the four tax cuts. As tax cuts have happened four times in the last 21 years, we take the probability of this catastrophe occurring as 19%. In our model, when we simulated the $I_{ITC}$ value of the recurrence relation, we generated a random number between 0 and 1. If that number was less than 0.19, we ignored the number generated from the normal distribution of yearly rates of change for individual income tax income and set $I_{ITC} = 1.0325$ for the simulated year.

Second, we see this with drastic changes in annual revenue from corporate tax due to legislative policy changes as well. Looking back at the last twenty years, there has been a corporate tax increase by 15% and a corporate tax decrease by 14%. Therefore, we model two environmentally stochastic events: tax increase by 15% with a 5% probability of occurring, and a tax decrease by 14% with a 5% probability of occurring. In our model, when we simulated the $I_{CTI}$ and $I_{CTC}$ values of their respective recurrence relations, we generated a random number between 0 and 1 for both. If either number was less than 0.05, then we ignored the number generated from the normal distribution of yearly rates of change for corporate tax income and set $I_{CTI} = 1.15$ and $I_{CTC} = 1.14$ the simulated year, respectively.

**2.4 Model Construction**

In order to create the predictions for 2021-2031, we had to base it off of previous data. Our overall model is a recursive model that grows based on the previous year's amount. We followed a 4-step process.

**2.4.1 Deterministic Model**

Step one was a deterministic model. We averaged the growth rates for each of the categories from the years 2000-2020. The rates are based on these averages.

We split the model up into 3 parts: income, spending, and deficit.

Income, as stated before, is the sum of: individual income tax (iit), corporate income tax (cit), social security / medicare (ssm), excise tax (et), and miscellaneous income (mi). Part 1 (income) was modeled as:

$$I(n) = (1 + r_{iit}) * iit(n-1) + (1 + r_{cit}) * cit(n-1) + (1 + r_{ssm}) * ssm(n-1) + (1 + r_{et}) * et(n-1) + (1 + r_{mi}) * mi(n-1)$$

where: $I(n_i)$ is the income for a given year *i*, and *r* is the growth rate.

Expenditures: as stated before, is the sum of: national defense (nd), human resources (hr), physical resources (pr), net interest (ni), and other functions (of). Part 2 (spending) was modeled as:

$$E(n) = (1 + r_{nd}) * nd(n-1) + (1 + r_{hr}) * hr(n-1) + (1 + r_{pr}) * pr(n-1) + (1 + r_{ni}) * ni(n-1) + (1 + r_{of}) * of(n-1)$$

where: $E(n_i)$ is the income for a given year *i*, and *r* is the growth rate.

Deficit is modeled as $D(n) = E(n) - I(n)$.

**2.4.2 Demographic Stochastic Model**

Step two was factoring in demographic stochasticity. Each of the growth rates could be normally approximated, with $\bar{r}$ equaling the average growth rates from the years 2000-2020, and $\sigma_r$ being the standard deviation of the growth rates. The general model is the same as above, with the main difference being all the growth rates *r* can be approximated with a normal distribution (ie: r$\sim Norm(\bar{r}, \sigma_r)$).

**2.4.3 Environmental Stochastic Model**

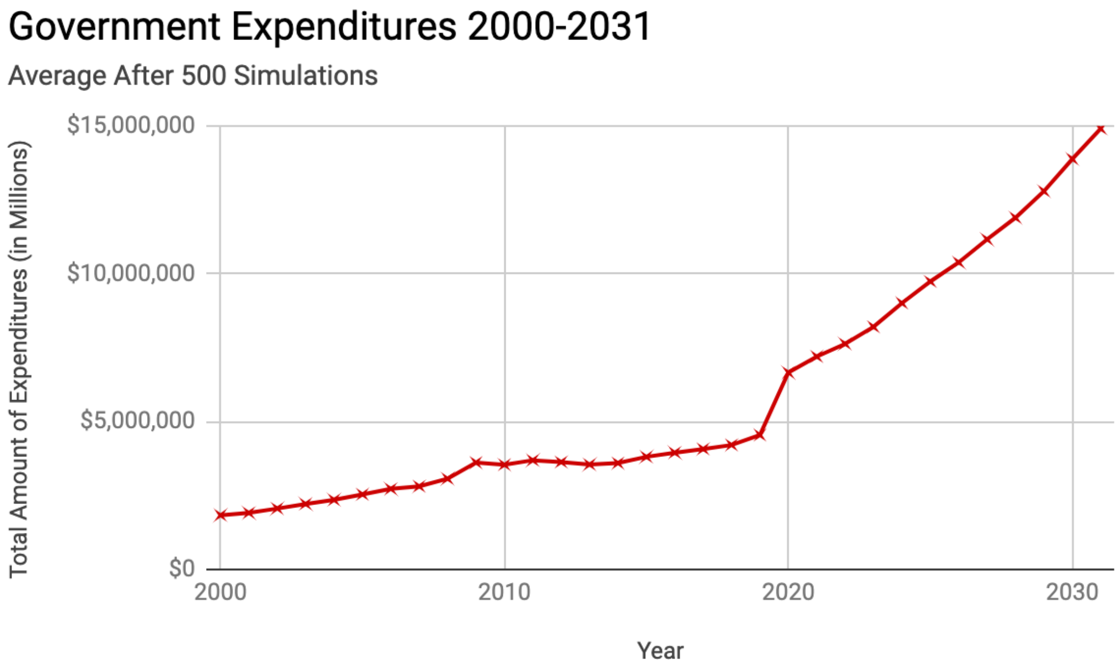
Step three was factoring in environmental stochasticity. In the event of a war, or a human resources catastrophe, or a physical resources catastrophe, there would be a different growth rate for those expenditures. Similarly, if there was a tax cut for individuals or corporations, there would be a different growth rate for that income.

- Mathematically, our model remains very similar. The factors that change are $r_{hr}, r_{pr}, r_{iit}, r_{cit}$, these depend on probabilities, which we will define as $p_{hr}, p_{pr}, p_{iit}, p_{cit}$. We sampled from a uniform distribution, from 0 to 1. If the probability sampled is less than the probability defined previously, we define this as catastrophe. Catastrophe will have its own rate to multiply by.

  For example, in the case of war:
  - If $Unif(0,1) < p_{nd}$ Then $E(n) = (1 + eff_{nd}) * nd(n-1) + \ldots$ (remaining factors). Note how this is **not** $r_{nd}$ but rather a new growth rate that has been previously determined.
  - Else $E(n) = (1 + r_{nd}) * nd(n-1) + \cdots$ (remaining factors), where $r_{hr} = Norm(\bar{r_{hr}}, \sigma_{r_{hr}})$

**2.4.4 Simulations Based on Model**

Step 4 was running 5000 simulations of the model from 2021 to 2031. Then we averaged the values for each year. This accounts for years with lots of catastrophe, no catastrophe, crazy high growth rates, crazy low growth rates etc. The Law of Large Numbers helps us realize 5000 simulations will be closer to the true value of what 2021-2031 values will look like. This graph below shows the average expenditures after 5000 simulations.

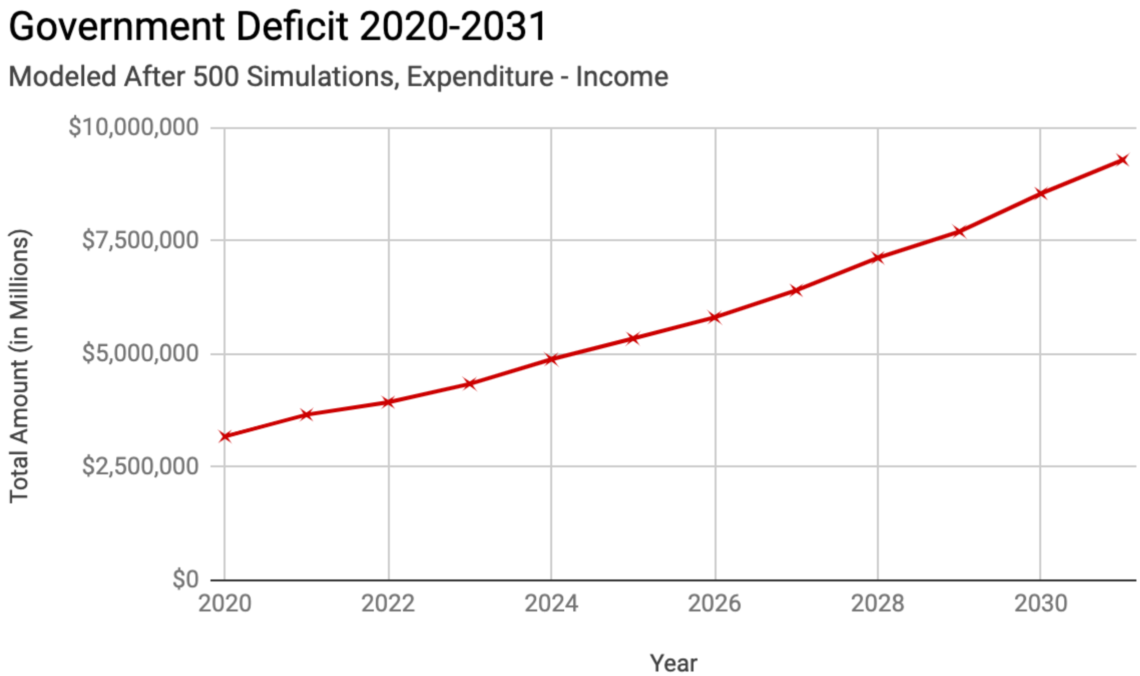

Government Expenditures 2000-2031
Average After 500 Simulations
Total Amount of Expenditures (in Millions)
$15,000,000
$10,000,000
$5,000,000
$0
2000
2010
2020
2030
Year

# 3. Model Results

### 3.1 Outlook for the next decade

Our model is relatively pessimistic in regards to the national deficit outlook until 2031. While the debt is large now, our model demonstrates substantial growth: the 2020 national deficit of $3.18 trillion is a mere fraction of the 2031 national deficit at **$9.29 trillion**. These values were calculated using our models for expenditure and income. We calculated the deficit by taking Expenditure - Income for each respective year. (The 2020 values were our starting values, 2021-2031 values were modeled based on 2000-2020 data).

| | **EXPENDITURE (E)** | **INCOME (I)** | **DEFICIT (E-I)** |
|---:|---:|---:|---:|
| *2020* | *$6,601,157* | *$3,421,162* | *$3,179,995* |
| 2021 | $7,189,382 | $3,528,330 | $3,661,052 |
| 2022 | $7,586,080 | $3,650,920 | $3,935,160 |
| 2023 | $8,078,593 | $3,736,039 | $4,342,553 |
| 2024 | $8,737,606 | $3,851,888 | $4,885,718 |
| 2025 | $9,306,828 | $3,962,233 | $5,344,595 |
| 2026 | $9,887,886 | $4,076,471 | $5,811,415 |
| 2027 | $10,654,948 | $4,245,988 | $6,408,960 |
| 2028 | $11,442,769 | $4,315,432 | $7,127,337 |
| 2029 | $12,104,213 | $4,396,767 | $7,707,446 |
| 2030 | $13,051,836 | $4,503,270 | $8,548,566 |
| 2031 | $13,832,445 | $4,540,721 | **$9,291,724** |




When looking at the graph of the deficit, we can notice how the graph seems to grow at a steady rate for the next decade. More specifically, when running exponential regression on our model visualized above, we retrieve an $R^2$ value of 0.999. Thus, it is fair to say that our model predicts that the deficit will follow exponential growth from 2021 to 2031.

From the modeled data for Expenditure and Income, these were both calculated using the sum of the categories specified earlier. Each category, simulated individually, was summed together to get these final values, and then we subtracted them to retrieve the final column of deficit values annually. The national deficit is added to the national debt every year – thus our model is essentially the annual growth rate of the debt. By summing the deficit values, we see that the national debt will have grown by **$67,064,526** from now to 2031.

### 3.2 Alternative Plans

We decided the best way to solve our problem was to look at different plans on how we decrease the growth of the deficit over the upcoming decade. As a result, we took 2 different approaches: one looking mainly at decreasing expenditures, while the other mainly focuses on how we can increase taxes to make up for the difference. We figured it would not be politically feasible to include a drastic increase in taxes and simultaneous expenditure cuts in the same plan. Thus, we believe it makes the most sense to split the solution into 2 plans: one that mainly focuses on cutting expenditures while the other prompts the government to tax much more.

**3.2.1 Decreasing National Defense, Human Resources Expenditure**

Our first plan of action would take place under a fiscally conservative administration. When looking at our model, we noticed that the 2 largest categories contributing to expenditures were National Defense and Human Resources. While these categories are very important to preserving the health and wellness of US Citizens, we believe that the best way to decrease expenditures would be by looking at these areas.

National Defense, constituting funding that goes to wars, had an original YoY growth-rate mean of 1.047 from 2000 to 2020. We used this value for our normal distribution to sample future YoY growth rates until 2031 for just National Defense. We figured that decreasing national defense spending would be one effective way towards shrinking the deficit. Thus, by decreasing the YoY growth-rate mean to 1.017 (decrease of 3%), we see that the defense spending increases at a much slower rate. While it is still increasing nearly every year (depending on new sampled growth rate), defense spending no longer takes the alarming exponential jump it did in our old model.

A similar approach can be applied to Human Resources. While many of these services are relevant, we believe it would not be out of line to decrease the near-surplus spending we contribute to this area. Human resources had a YoY growth-rate mean of 1.057 up until 2020, and thus we believe it would be best to once again shrink that number to 1.017 (decrease of 4%). As a result, the HR also begins to grow at a much slower rate than in our previous model.

While we were able to simply lower the growth rates for Human Resources and National Defense, we then took into account the after effect of these changes. If spending for National Defense decreases, this means that the country would be less prepared to protect itself, and thus the likelihood of war would increase. A similar notion can be applied to a catastrophe for human resources. As a result, we adjusted the probability of catastrophe for defense and HR to be higher than they were previously. The probability of war was adjusted from 1/14.5 to 1/12, and a human resources catastrophe was adjusted from 1/8.14 to 1/7. These probabilities were estimated.

This new model achieves many key points that we believe are necessary for an effective plan: it substantially lowers the growth rate of the national debt, it does not make drastic changes to spending policy, and it is politically feasible. From our original model, the deficit was projected to be $9,291,724 million in 2031. With this adjusted model, the deficit is now projected to be $6,154,250 million in 2031. We believe that a change of nearly $3,000,000 million is enough to validate these new spending policies.

**3.2.2 Increasing Taxes Across the Board**

In reverse to decreasing spending, we believe that another opportunity to decrease the national deficit would be to increase the revenue the government brings in annually. As specified previously, we broke down government revenue into 5 main categories: Individual Income Tax, Corporate Income Tax, Social Security/Medicare, Excise Taxes, and Miscellaneous Taxes. Out of these categories, the one with the most influence on overall revenue is Individual Income Tax. However, we do not believe that it would be politically feasible to only increase the tax on citizens. Thus we would suggest increasing taxes on all 5 categories – that way no individual category has too much of a drastic increase.

The main category to focus on is Individual Income Tax because of its weight on the overall revenue. From 2000-2020, IIT increased at a rate of 5% on average. We believe that it is essential to take a more liberal approach and substantially increase taxes – specifically on upper class citizens. While our model does not take economic class into account, we believe that an overall increase to 8% would be sufficient towards helping our cause. Thus, the government revenue will see a sustained increase annually.

For the other 4 categories, a similar process was conducted. We increased the CIT tax growth rate from 3.7% to 6.7%, Social Security was raised from 3.6% to 5%, Excise Taxes from 1.4% to 2%, and everything else from 4.8% to 6%. These increases in growth rates influenced the overall increase in revenue in the model. Once again, the same growth rate is not used every year in the model, rather a sample from a distribution modeled after the means and growth rates of YoY changes. We did not alter the standard deviations for our alternative models, just the means per category.

While these changes were made to the growth rates, we once again needed to take into account the possibility for catastrophe. With an increase in taxes, we predicted that economic catastrophe would be slightly more likely. Thus, we increased the probability of IIT catastrophe from 4/21 to 5/21 and the probability of CIT catastrophe from 1/20 to 1/10.

We believe that once again, this model achieves many key points that we believe are necessary for an effective plan: it lowers the growth rate of the national debt, it does not make drastic changes to tax policy, and it is politically feasible. If this plan was implemented, we would see the deficit in 2031 amount to $8,155,572 million – over $1,000,000 million less than our original model predicted. We believe that this change is enough to validate our tax plan for the upcoming decade.

# 4. Model Assessment

**4.1 Sensitivity Analysis**

Because we used multiple different categories each with a recurrence relation, the models for categories with large initial values will have a very large effect on the overall deficit model. This model is the most sensitive to the parameters for the categories with the highest initial values.

Human Resources had the highest initial value of any category by far with $4,339,671 million. We modeled the increase in Human Resource expenditures using a mean $P$ value $P_{HR} = 1.0565$ with a standard deviation $\sigma_{HR} = 0.0391$. Increasing $P_{HR}$ will cause the deficit to increase faster, while decreasing $P_{HR}$ will cause the deficit to increase slower or decrease. Because Human Resources had such a high initial value, the overall debt model is the most sensitive to $P_{HR}$.

In the Income category, Individual Income Tax and Social Security Medicare had similarly high initial values of $1,608,661 million and $1,309,955 million respectively. Individual Income Tax was modeled using a mean $P$ value of $P_{IIT} = 1.05$ with a standard deviation of $\sigma_{IIT} = 0.1018$. Social Security Medicare was modeled using a mean $P$ value of $P_{SSM} = 1.036$ with a standard deviation of $\sigma_{SSM} = 0.2644$. Increasing either of these $P$ values would cause the deficit to increase slower or possibly decrease, while decreasing either of these $R$ values will cause the deficit to increase faster.

**4.2 Strengths and Weaknesses**

The main aspects that make our model strong are that it is flexible and accounts for many events that may introduce stochasticity. The model accounts for the different behavior and stochastic issues regarding both the national expenditures and revenue. All of the sub-categories that make up the expenditures and revenue are also taken into account and controlled for. For example, the model can isolate a singular tax policy and model its effects of the total revenue. Smaller categories that make up less than 10% of either the expenditures or revenue are also modeled for, eliminating the need for a hacking number which would not be as accurate. Accounting for both demographic and environmental stochasticity helps us to not overshoot the projection for each category.

Along with taking these factors into account, our model aims to pinpoint any facet of the overall deficit that may help control this number. The overall deficit is broken up into two categories which are then split into their own respective subcategories. The multivariable model is able to isolate either the spending or income to use it as the independent variable. All of the sub-categories that make up the expenditures and revenue are also taken into account and controlled for. For example, the model can isolate a singular tax policy and model its effects of the total revenue. This makes the model very flexible and easily interpretable as each aspect of the total model is shown in the equation.

While our model has many strengths, it still has some limitations and overall assumptions that are key to note. First, the model assumes that The US will not go through a period of sustained hyperinflation. The hyperinflation is very hard to predict and has not shown a clear trend over the data that we analyzed. It does not factor for the breakdown in sub-categories (i.e. raising taxes is not broken up by tax brackets). Additionally, we used the last 20 years of data with an unusual spike in 2021 due to the Covid-19 Pandemic which skews models slightly. It is possible that the past 20 years saw exceptionally costly events that will not occur again in the next decade. Lastly, it does not factor for White House plans for future spending or taxation as the policy which leads to different deficit projections when compared to White House estimates. Overall, these limitations do not bar us from developing a fairly accurate model for the National debt over the next 10 years.

# 5. Conclusion

Due to the tumultuous nature of the current political climate, the national debt has become more apparent than ever. Monitoring and making projections on this deficit involves countless variables and very few available projections tell the full story. Some fail to account for stochasticity and others are overly optimistic.

While modeling for the national debt, the model can be broken up into two parts: national expenditures and national revenue. Each of these categories have their own behavior and are affected differently by financial crises. Events like the Covid-19 Pandemic cause spikes in the total expenditures. Other events like the election of a new president cause policy changes that affect the federal income. Both the spending and revenue experience spikes and troughs as they are subject to environmental stochasticity. The government has increased its spending over the last 20 years and must find a way to generate the needed income to ensure that the national deficit is not too large.

Policy on factors like  Individual Income tax or Corporate Income tax are especially key as these taxes make up a large portion of the total revenue. The government currently spends trillions of dollars on welfare and social programs which aid Americans. For example, during the pandemic, a stimulus was given out to all working American citizens and yet when this spending must be matched by increasing taxes, the government faces backlash. The U.S. government must find a way to model their income and spending values and experiment to see if they can raise certain taxes and cut down on specific expenditures while maintaining public appeal and a steady national deficit.

### 5.1 Summary of Results

Using the observed year over year growth-rates of change for the categories within expenditures and income, our model shows that the total expenditures in 2031 is $13.83 trillion and our total income is $4.54 trillion. By subtracting the expenditures from income, we find our prediction for the deficit to be $9.29 trillion in 2031. By adding together all the deficit values from 2021 to 2031, we calculate the predicted national debt to be $67 trillion, 40.3 trillion more than the current US national debt.

Our first alternative plan was to decrease expenditures. The two largest categories in expenditures in National Defense and Human Resources. When we decrease the YoY growth-rate of National Defense by 3% and the Human Resources YoY growth-rate by 4%, we see a much smaller growth of expenditures in our model. In addition to changing these growth rates, we also increased the probability for a Public Defense or Human Resources catastrophe.

Using this alternative plan, we observe the predicted deficit in 2031 to be 6.15 trillion, more than 3 trillion less than our original model.

Our second alternative plan was to increase taxes. The biggest contributor to income is Individual Income Taxes, however, greatly increasing taxes in this category is not feasible so we decided to increase taxes in all categories. For Individual Income Tax, we implemented a 3% increase from 5% to 8%. For CIT tax, Social Security, Excise Taxes and Other, we increased the growth rates by 4%, 1.4%, 0.6%, and 1.2% respectively. After altering the probability of catastrophe for individual income tax and CIT, we found that this plan also lowers the growth rate of the national debt. The predicted deficit in 2031 under this plan is $8.16 trillion, more than $1 trillion less than our original plan.

**5.2 Further Discussion**

While both alternative plans successfully decrease the predicted deficit and slow down the growth of the national debt, it is arguable how politically feasible they are. While our model can be used to show the effect of different tax and expenditure policies on the deficit, we can also apply our model in other directions.

One addition to our model could be more specific catastrophe events and effects. While we generalized these catastrophes on expenditures and tax cuts on income, there are more patterns and specific impacts that these catastrophes have. By focusing on the intricacies of catastrophe, we could use our model to show how the national debt would behave through both small and big catastrophes. This could show how stable the national debt is and whether or not policy makers should be more worried about this debt if the model shows volatile behavior.

Another application of our model could be analyzing how plausible a debt ceiling is. A debt ceiling is a limit set by the legislative branch on the public debt, or how much the US can borrow. The US has passed the debt ceiling multiple times, with the latest being in March of 2019, when the US passed it's borrowing limit of $22 trillion. In our model, we see that without any legislative changes, the US debt continues to grow exponentially, with a predicted debt of $67 trillion in 2031. Our original model shows that if a debt ceiling was implemented, the exponential growth of debt would soon overtake it. By using our model, we could find a plausible rate of growth for the debt ceiling as well as testing how different strategies can maintain a debt ceiling.

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