$A = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4$

$B = b_0 + b_1 r + b_2 r^2 + b_3 r^3 + b_4 r^4$

$C = AB$

$$\boxed{C = c_0 + c_1 r + c_2 r^2 + c_3 r^3 + c_4 r^4 + c_5 r^5 + c_6 r^6 + c_7 r^7 + c_8 r^8 + c_9 r^9}$$

$a_0' = a_0 + a_1 r, \quad a_1' = a_2 r + a_3 r^2 + a_4 r^3$

$b_0' = b_0 + b_1 r, \quad b_1' = b_2 r + b_3 r^2 + b_4 r^3$

$A = a_0' + r a_1', \quad B = b_0' + r b_1'$

$AB = c_0' + c_1' r + c_2' r^2$

$c_0' = a_0' b_0'$

$c_2' = a_1' b_1'$

$c_1' = (a_0' + a_1')(b_0' + b_1') - c_0' - c_2'$

$c_0' = (a_0 + a_1 r)(b_0 + b_1 r)$

$= c_{00}' + c_{01}' r + c_{02}' r^2$

$$\Rightarrow \boxed{\begin{aligned} c_{00}' &= a_0 b_0 \\ c_{02}' &= a_1 b_1 \\ c_{01}' &= (a_0 + a_1)(b_0 + b_1) - c_{00}' - c_{02}' \end{aligned}} \rightarrow \boxed{1}$$

$\rightarrow$ 1st set of eqns.

$$\therefore \boxed{c_0' = c_{00}' + c_{01}' r + c_{02}' r^2 + 0r^3 \ldots + 0r^9} \rightarrow \boxed{1a}$$

$$C_2' = r^2(a_2+a_3r+a_4r^2)(b_2+b_3r+b_4r^2)$$

$$\Rightarrow \frac{C_2'}{r^2} = (a_2+a_3r+a_4r^2)(b_2+b_3r+b_4r^2)$$

$$A_0^2 = a_2 \; ; \; A_1^2 = a_3+a_4r$$

$$B_0^2 = b_2 \; ; \; B_1^2 = b_3+b_4r$$

$$\frac{C_2'}{r^2} = (A_0^2 + rA_1^2)(B_0^2 + rB_1^2)$$

$$\frac{C_2'}{r^2} = C_{20}^2 + C_{21}^2 r + C_{22}^2 r^2$$

$$\boxed{C_{20}^2 = A_0^2 B_0^2 = a_2b_2} \text{ — } \boxed{2a}$$

$$C_{22}^2 = A_1^2 B_1^2 = (a_3+a_4r)(b_3+b_4r).$$

$$C_{22}^2 = C_{220}^2 + C_{221}^2 r + C_{222}^2 r^2$$

$$\boxed{\begin{aligned} C_{220}^2 &= a_3b_3 \\ C_{222}^2 &= a_4b_4 \\ C_{221}^2 &= (a_3+a_4)(b_3+b_4) - C_{220}^2 - C_{222}^2 \end{aligned}} \text{ — } \boxed{2b}$$

$$\boxed{C_{22}^2 = C_{220}^2 + C_{221}^2 r + C_{222}^2 r^2} \text{ — } \boxed{2c}$$

$$C_{20}^2 + C_{22}^2 = (C_{20}^2 + C_{220}^2) + C_{221}^2 r + C_{222}^2 r^2$$

$$C_{21}^2 = (A_0^2 + A_1^2)(B_0^2 + B_1^2) - C_{20}^2 - C_{22}^2$$

$$= \underbrace{(a_2+a_3+a_4r)(b_2+b_3+b_4r)}_{D} - C_{20}^2 - C_{22}^2$$

$$\boxed{\begin{aligned} D &= D_0 + D_1 r + D_2 r^2 \\ D_0 &= (a_2+a_3)(b_2+b_3) \\ D_2 &= a_4b_4 \\ D_1 &= (a_2+a_3+a_4)(b_2+b_3+b_4) - D_0 - D_2 \end{aligned}} \text{ — } \boxed{2D}$$

$$C^2_{21} = D_0 + D_1 r + D_2 r^2$$
$$- (C^2_{20} + C^2_{220}) - C^2_{221} r - C^2_{222} r^2$$

$$\boxed{C^2_{21} = (D_0 - C^2_{20} - C^2_{220}) + (D_1 - C^2_{221}) r + (D_2 - C^2_{222}) r^2}$$

$$\boxed{(2D1)}$$

from (2a), (2b), (2c), (2D), (2D1)

we have.

$$\frac{C^1_2}{r^2} = C^2_{20} + (D_0 - C^2_{20} - C^2_{220}) r$$
$$+ (D_1 - C^2_{221}) r^2 + (D_2 - C^2_{222}) r^3$$
$$+ C^2_{220} r^2 + C^2_{221} r^3 + C^2_{222} r^4$$

$$\Rightarrow \boxed{\frac{C^1_2}{r^2} = C^2_{20} + (D_0 - C^2_{20} - C^2_{220}) r + (D_1 - C^2_{221} + C^2_{220}) r^2 + (D_2 - C^2_{222} + C^2_{221}) r^3 + C^2_{222} r^4} \quad \boxed{3a}$$

$$\Rightarrow \boxed{C^1_2 = C^2_{20} r^2 + (D_0 - C^2_{20} - C^2_{220}) r^3 + (D_1 - C^2_{221} + C^2_{220}) r^4 + (D_2 - C^2_{222} + C^2_{221}) r^5 + C^2_{222} r^6} \quad \boxed{3}$$

C_1':

$$C_1' = (a_0' + a_1')(b_0' + b_1') - C_0' - C_2'$$

from (2a) & (3)

$$C_0' + C_2' = C_{00}' + C_{01}' r + C_{02}' r^2 + C_{20}^2 r^2 + (D_0 - C_{20}^2 - C_{220}^2) r^3 + (D_1 - C_{221}^2 + C_{220}^2) r^4 + (D_2 - C_{222}^2 + C_{221}^2) r^5 + C_{222}^2 r^6$$

$$\therefore \boxed{C_0' + C_2' = C_{00}' + C_{01}' r + (C_{02}' + C_{20}^2) r^2 + (D_0 - C_{20}^2 - C_{220}^2) r^3 + (D_1 - C_{221}^2 + C_{220}^2) r^4 + (D_2 - C_{222}^2 + C_{221}^2) r^5 + C_{222}^2 r^6} \rightarrow \boxed{4a}$$

C_1

$$E = (a_0' + a_1')(b_0' + b_1') \Rightarrow C_1' = E - C_0' - C_2'$$

$$= (a_0 + a_1 r + a_2 r + a_3 r^2 + a_4 r^3)(b_0 + b_1 r + b_2 r + b_3 r^2 + b_4 r^3)$$

$$= (a_0 + (a_1 + a_2) r + a_3 r^2 + a_4 r^3)(b_0 + (b_1 + b_2) r + b_3 r^2 + b_4 r^3)$$

Let $F_0 = a_0 + (a_1 + a_2) r \qquad F_1 = a_3 r + a_4 r^2$

$G_0 = b_0 + (b_1 + b_2) r \qquad G_1 = b_3 r + b_4 r^2$

$$\therefore E = (F_0 + F_1 r)(G_0 + G_1 r)$$

$$E = E_0 + E_1 r + E_2 r^2$$

$$\left.\begin{aligned} E_0 &= F_0 G_0 \\ E_2 &= F_1 G_1 \end{aligned}\right\} \Rightarrow E_1 = (F_0 + F_1)(G_0 + G_1) - E_0 - E_2$$

$$G_0 = F_0 G_0 = (a_0 + (a_1 + a_2)r)\,(b_0 + (b_1 + b_2)r)$$

$$E_0 = E_{00} + E_{01} r + E_{02} r^2$$

$$\boxed{\begin{aligned} E_{00} &= a_0 b_0 \\ E_{02} &= (a_1 + a_2)(b_1 + b_2) \\ E_{01} &= (a_0 + a_1 + a_2)(b_0 + b_1 + b_2) - E_{00} - E_{02} \end{aligned}} \to \boxed{5a}$$

$$\boxed{E_0 = E_{00} + E_{01} r + E_{02} r^2} \to \boxed{5a}$$

$$E_2 = F_1 G_1 = (a_3 r + a_4 r^2)\,(b_3 r + b_4 r^2)$$

$$= r^2 (a_3 + a_4 r)(b_3 + b_4 r)$$

$$\Rightarrow \frac{E_2}{r^2} = (a_3 + a_4 r)(b_3 + b_4 r)$$

$$\frac{E_2}{r^2} = E_{20} + E_{21} r + E_{22} r^2$$

$$E_{20} = a_3 b_3$$

$$E_{22} = a_4 b_4$$

$$E_{21} = (a_3 + a_4)(b_3 + b_4) - E_{20} - E_{22}$$

$$\boxed{\frac{E_2}{r^2} = E_{20} + E_{21} r + E_{22} r^2} \to 5b_1$$

$$\Rightarrow \boxed{E_2 = E_{20} r^2 + E_{21} r^3 + E_{22} r^4} \to 5b$$

$E_0 + E_2 = E_{00} + E_{01} r + E_{02} r^2$

$E_{20} r^2 + E_{21} r^3 + E_{22} r^4$

$\boxed{E_0 + E_2 = E_{00} + E_{01} r + (E_{02} + E_{20}) r^2 + E_{21} r^3 + E_{22} r^4}$ (5C)

Let $H = (F_0 + F_1)(G_0 + G_1)$

$= (a_0 + (a_1 + a_2) r + a_3 r + a_4 r^2)(b_0 + (b_1 + b_2) r + b_3 r + b_4 r^2)$

$= \underbrace{(a_0 + (a_1 + a_2 + a_3) r + a_4 r^2)}_{I} \underbrace{(b_0 + (b_1 + b_2 + b_3) r + b_4 r^2)}_{J}$

$I_0 = a_0$; $I_1 = (a_1 + a_2 + a_3) + a_4 r$

$J_0 = b_0$; $J_1 = (b_1 + b_2 + b_3) + b_4 r$

$\therefore I = I_0 + I_1 r$; $J = J_0 + J_1 r$

$H = IJ$

$= H_0 + H_1 r + H_2 r^2$

$H_0 = I_0 J_0 = a_0 b_0$

$H_2 = I_1 J_1$

$H_1 = (I_0 + I_1)(J_0 + J_1) - H_0 - H_2$

$H_2 = I_1 J_1 = ((a_1 + a_2 + a_3) + a_4 r)((b_1 + b_2 + b_3) + b_4 r)$

$H_2 = H_{20} + H_{21} r + H_{22} r^2$

$H_0 = a_0 b_0$

$H_{20} = (a_1 + a_2 + a_3)(b_1 + b_2 + b_3)$

$H_{22} = a_4 b_4$

$H_{21} = (a_1 + a_2 + a_3 + a_4)(b_1 + b_2 + b_3 + b_4) - H_{20} - H_{22}$

$\boxed{6.}$

$H_0 = a_0 b_0$

$\boxed{H_2 = H_{20} + H_{21} r + H_{22} r^2}$ (6a1)

$H_0 + H_2 = (H_0 + H_{20}) + H_{21} r + H_{22} r^2$

$$H_1 = \Big(\underbrace{(a_0 + a_1 + a_2 + a_3) + a_4 r}_{K} \Big) \Big(\underbrace{(b_0 + b_1 + b_2 + b_3) + b_4 r}_{L} \Big)$$

$K = K_0 + K_1 r$

$L = L_0 + L_1 r.$

$K_0 = a_0 + a_1 + a_2 + a_3$

$K_1 = a_4$

$L_0 = b_0 + b_1 + b_2 + b_3$

$L_1 = b_4.$

$H_1 = H_{10} + H_{11} r + H_{12} r^2$

$H_{10} = K_0 L_0$

$H_{12} = a_4 b_4$

$H_{11} = (K_0 + K_1)(L_0 + L_1) - H_{10} - H_{12}$

$\boxed{7a}$

$$\boxed{H_1 = H_{10} + H_{11} r + H_{12} r^2} \rightarrow \boxed{7}$$

$$H = H_0 + H_{10} r + H_{11} r^2 + H_{12} r^3$$
$$H_{20} r^2 + H_{21} r^3 + H_{22} r^4$$

$$\therefore H = H_0 + H_{10} r + (H_{11} + H_{20}) r^2 + (H_{12} + H_{21}) r^3 + H_{22} r^4$$

$$E_1 = H - H_0 - H_2$$

$$= H_0 + H_{10} r + (H_{11} + H_{20}) r^2 + (H_{12} + H_{21}) r^3 + H_{22} r^4$$
$$- H_0 \cancel{+} H_{20} - H_{21} r - H_{22} r^2$$

$$\therefore E_1 = -H_{20} + (H_{10} - H_{21}) r + (H_{11} + H_{20} - H_{22}) r^2$$
$$+ (H_{12} + H_{21}) r^3 + H_{22} r^4$$

$$E_0 = E_{00} + E_{01} r + E_{02} r^2$$
$$E_2 = E_{20} r^2 + E_{21} r^3 + E_{22} r^4$$
$$E = E_0 + E_1 r + E_2 r^2$$
$$E = E_{00} + E_{01} r + E_{02} r^2 - H_{20} r + (H_{10} - H_{21}) r^2 + (H_{11} + H_{20} - H_{22}) r^3$$
$$+ (H_{12} + H_{21}) r^4 + (H_{22}) r^5 + E_{20} r^4 + E_{21} r^5 + E_{22} r^6$$

$$\therefore E = E_{00} + (E_{01} - H_{20}) r + (E_{02} + H_{10} - H_{21}) r^2 + (H_{11} + H_{20} - H_{22}) r^3$$
$$+ (H_{12} + H_{21} + E_{20}) r^4 + (H_{22} + E_{21}) r^5 + E_{22} r^6$$

$$E = E_{00} + (E_{01} - H_{20})\, r + (E_{02} + H_{10} - H_{21})\, r^2 + (H_{11} + H_{20} - H_{22})\, r^3 + (H_{12} + H_{21} + E_{20})\, r^4 + (H_{22} + E_{21})\, r^5 + E_{22}\, r^6$$

$$C_0^1 = C_{00}^1 + C_{01}^1 r + C_{02}^1 r^2$$

$$C_2^1 = C_{20}^2 r^2 + (D_0 - C_{20}^2 - C_{220}^2)\, r^3 + (D_1 - C_{221}^2 + C_{220}^2)\, r^4 + (D_2 - C_{222}^2 + C_{221}^2) + C_{222}^2 r^6$$

$$C_1^1 = E - C_0^1 - C_2^1$$

$$C_1^1 = E_{00} + (E_{01} - H_{20})\, r + (E_{02} + H_{10} - H_{21})\, r^2 + (H_{11} + H_{20} - H_{22})\, r^3 + (H_{12} + H_{21} + E_{20})\, r^4 + (H_{22} + E_{21})\, r^5 + E_{22}\, r^6 - C_{00}^1 - C_{01}^1 r - (C_{02}^1 + C_{20}^2)\, r^2 - (D_0 - C_{20}^2 - C_{220}^2)\, r^3 - (D_1 - C_{221}^2 + C_{220}^2)\, r^4 - (D_2 - C_{222}^2 + C_{221}^2)\, r^5 - C_{222}^2$$

$\therefore \cancel{C = C_0^1 + C_1^1 r + C_2^1 r^2}$

$\therefore$

$$C_1^1 = (E_{00} - C_{00}^1) + (E_{01} - H_{20} - C_{01}^1) r + (E_{02} + H_{10} - H_{21} - C_{02}^1 - C_{02}^2) r^2$$
$$+ (H_{11} + H_{20} - H_{22} - D_0 + C_{20}^2 + C_{220}^2) r^3$$
$$+ (H_{12} + H_{21} + E_{20} - D_1 + C_{221}^2 - C_{220}^2) r^4$$
$$+ (H_{22} + E_{21} - D_2 + C_{222}^2 - C_{221}^2) r^5 + (E_{22} - C_{222}^2) r^6$$

$$C = C_0^1 + C_1^1 r + C_2^1 r^2$$

$$C = C_{00}^1 + C_{01}^1 r + C_{02}^1 r^2 +$$
$$+ (E_{00} - C_{00}^1) r + (E_{01} - H_{20} - C_{01}^1) r^2 + (E_{02} + H_{10} - H_{21} - C_{02}^1 - C_{02}^2) r^3 +$$
$$+ (H_{11} + H_{20} - H_{22} - D_0 + C_{20}^2 + C_{220}^2) r^4 + (H_{12} + H_{21} + E_{20} - D_1 + C_{221}^2 - C_{220}^2) r^5$$
$$+ (H_{22} + E_{21} - D_2 + C_{222}^2 - C_{221}^2) r^6 + (E_{22} - C_{222}^2) r^7$$
$$+ C_{20}^2 r^4 + (D_0 - C_{20}^2 - C_{220}^2) r^5 + (D_1 - C_{221}^2 + C_{220}^2) r^6$$
$$+ (D_2 - C_{222}^2 + C_{221}^2) r^7 + C_{222}^2 r^8$$

Add term wise and Carrying the over flow to next term, then we will get

$$C = C_0 + C_1 r + C_2 r^2 + C_3 r^3 + C_4 r^4 + C_5 r^5 + C_6 r^6 + C_7 r^7 + C_8 r^8 + C_9 r^9$$