where

Ex 3.7

4) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $D = \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix}$

Augmental matrix $= ([AD]) = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \\ 2 & -1 & 3 & 9 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$\begin{array}{cccc} 2 & -1 & 3 & 9 \\ 2 & 2 & 2 & 12 \\ - & - & - & - \\ \hline 0 & -3 & 1 & -3 \end{array}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 1 & -3 \end{bmatrix}$

$R_3 \to 2R_3 - 3R_2$

$\begin{array}{cccc} 0 & -6 & 2 & -6 \\ 0 & -6 & 0 & -12 \\ - & + & - & + \\ \hline 0 & 0 & 2 & +6 \end{array}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ — (1)

Rank (A) = No. of non zero rows in A = 3

Rank (AD) = No. of non zero rows in AD = 3

Rank (A) = Rank (AD) = 3

Hence the given system of equation is consistent and It has unique solutions.

from (1)

$x + y + z = 6$ — (2)

$-2y = -4 \Rightarrow y = 2$

$2z = 6 \Rightarrow z = 3$

from (2)

$x + z + 3 = 6$

$\Rightarrow x = 6 - 2 - 3 = 1$

$\therefore x = 1, y = 2, z = 3$ is only solution.

---

*** 5) $x + y + z = 1$

$2x + y + z = 2$

$x + 2y + 2z = 1$

$x + y + z$

Sol: Given system of equations can be represented by the matrix equation $AX = D$

where

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Augmental matrix $= ([AD]) =$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow (1)$$

Rank of $A$ = no. of non zero rows in $A = 2$

Rank of $([AD])$ = no. of non zero rows in $([AD]) = 2$

Rank of $A$ = Rank of $([AD]) < 3$

Given system of equations is consistent & it has infinitely many solutions

from (1) $\Rightarrow$

$x + y + z = 1 \longrightarrow (2)$

$-y - z = 0 \longrightarrow (3)$

Let $\boxed{z = K}$. then 'n' is real number

from (3) $\Rightarrow -y - K = 0 \Rightarrow \boxed{y = -K}$

from (2) $\Rightarrow x \cancel{-K} \cancel{+K} = 1$

$\Rightarrow \boxed{x = 1}$

$\therefore x = 1;\ y = -K;\ z = K$ is the solution where 'K' is real number

---

***

(5) $x + y + z = 9$; $2x + 5y + 7z = 52$

$2x + y - z = 0$

Given system of equations can be represented by the matrix equation $AX = D$; where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\ D = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

Augmented matrix $([AD])$ =

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{bmatrix} - (1)$$

Rank of $A$ = no. of non zero rows in $A = 3$

Rank of $([AD])$ = no. of non zero rows in $([AD]) = 3$

Rank of $A$ = Rank of $([AD]) = 3$

Given system of equation is consists & it has unique solution

from (1) $\Rightarrow$

$x + y + z = 9 \to (2)$

$3y + 5z = 34 \to (3)$

$-4z = -20 \to (4)$

(4) $\Rightarrow \boxed{z = 5}$

(3) $\Rightarrow 3y = 34 - 25 = 9$

$\boxed{y = 3}$

(2) $x = 9 - 3 - 5$ $\boxed{x = 1}$

$\therefore x = 1;\ y = 3;\ z = 5$ is only solution.

---

(2) $x - 3y - 8z = -10$

Sol: $3x + y - 4z = 0$

$2x + 5y + 6z = 13$

Given system of equations can be represented by the matrix equation $AX = D$, where

$$A = \begin{bmatrix} 1 & -3 & -8 \\ 3 & 1 & -4 \\ 2 & 5 & 6 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$;\ D = \begin{bmatrix} 10 \\ 0 \\ 13 \end{bmatrix}$$

Augmented matrix $[AD]$

$$= \begin{bmatrix} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{bmatrix}$$

$R_2 \to \frac{R_2}{10}$

$R_3 \to \frac{R_3}{11}$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to ①$$

Rank of (A) = no. of non Zero rows in A = 2

Rank of ([AD]) = no. of non zero rows in ([AD]) = 2

Rank of A = Rank of ([AD]) < 3

Given system of equations is consistant, and has Infinetly and has many solutions.

from ① ⇒

$x-3y-8z=-10$ — ②

$y+2z=3$ — ③

Let us take

$\boxed{z=k}$ where 'k' is real number

from ③ ⇒

$y+2z=3$

$y+2k=3$

$\boxed{y=3-2k}$

from ② ⇒

$x-3(3-2k)-8(k)=-10$

$\Rightarrow x=-10+3(3-2k)+8k$

$\Rightarrow x=-10+9-6k+8k$

$\Rightarrow \boxed{x=2k-1}$

$\therefore x=2k-1;\ y=3-2k$

$z=k$ is the solution where 'k' is real number.

Solution of Homogenous system of linear equations

Consider the following Homogenous linear equations

$a_1x+b_1y+c_1z=0$

$a_2x+b_2y+c_2z=0$

$a_3x+b_3y+c_3z=0$

$x=y=z=0$; is a solution of given equation

[Trivial solution (or) zero solution]

So, a system of Homogenous equations always consistent

*** Note: The system of *** equations $AX=0$; has

(i) the trivial solution only If rank (A) = 3

(ii) An Infinitely no. of solutions If rank (A) < 3

## Exercise 3.8

SAQ

① $2x+3y-z=0$

$x-y-2z=0$

$3x+y+3z=0$

Sol The Given system of equations can be represented by a matrix equation $AX=0$, where,

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

$R_2 \longrightarrow 2R_2 - R_1$

$R_3 \longrightarrow 2R_3 - 3R_1$

$$\sim \begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & -3 \\ 0 & -7 & 9 \end{bmatrix}$$

$R_3 \longrightarrow 5R_3 - 7R_2$

$$\sim \begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & -3 \\ 0 & 0 & 66 \end{bmatrix}$$

Rank of A = no. of non zero rows = 3

∴ solution trival solution.

∴ Given system of equation has trivial solution and the trivial solution is $x=0$; $y=0$, $z=0$

③ $x+y-2z=0$, $2x+y-3z=0$, $5x+4y-9z=0$

Sol The Given system of equations can be represented by a matrix equation $AX=0$, where

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1$

$R_3 \longrightarrow R_3 - 5R_1$

$$\sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{—}(1)$$

Rank of A = No. of non zero rows = 2

Rank of $A < 3$

Given system equation has Infinitely many solutions.

from (1)

$x + y - 2z = 0$ —(2)

$-y + z = 0$ —(3)

let $z = k$; $k$ is any real number

from (3) $-y + k = 0$

$\Rightarrow y = k$

from (2) $x + k - 2k = 0$

$\Rightarrow x = k$

$\therefore x = k, y = k, z = k$ is the solution

$\therefore k$ is any real number.

---

(3) $2x + 5y + 6z = 0$,

$x - 3y - 8z = 0$,

$3x + y - 4z = 0$

Sol Given system of equations can be represented by the matrix equation $AX = 0$ where,

$$A = \begin{bmatrix} 2 & 5 & 6 \\ 1 & -3 & -8 \\ 3 & 1 & -4 \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$

$R_3 \to 2R_3 - 3R_1$

$$\sim \begin{bmatrix} 2 & 5 & 6 \\ 0 & -11 & -22 \\ 0 & -13 & -26 \end{bmatrix}$$

$R_2 \to \frac{R_2}{-11}$; $R_3 \to \frac{R_3}{-13}$

$$\sim \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{—}(1)$$

Rank of A = No. of non-zero rows = 2

Rank of $A < 3$

Given system equation has Infinitely many solutions

from (1)

$2x + 5y + 6z = 0$ —(2)

$y + 2z = 0$ —(3)

Let $z = k$; $k$ is any real number

from (3) $y + 2k$.

$\Rightarrow y = -2k$

from (2) $2x + 5(-2k) + 6k = 0$

$\Rightarrow x = 2k$

$\therefore x = 2k, y = -2k, z = k$, is the solution, $k$ is any real number

---

## METHODS OF SOLVING NON - HOMOGENEOUS SYSTEM

### 1, CRAMER'S RULE

The solution of the system of linear equation

$a_1x + b_1y + c_1z = d_1$,

$a_2x + b_2y + c_2z = d_2$,

$a_3x + b_3y + c_3z = d_3$,

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta},$$

$$z = \frac{\Delta_3}{\Delta}; \quad \Delta \neq 0$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix};$$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix};$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix};$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

---

### (2) MATRIX INVERSION METHOD

A system of linear equations is given by, $AX = B$

If the coefficient matrix A is non singular then the system has a solution given by $\boxed{X = A^{-1}B}$

3, GAUSS - JORDAN METHOD

Let the Linear equation be

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix}$$

If the augmented matrix can be reduced to the form

$$\begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \beta \\ 0 & 0 & 1 & \gamma \end{bmatrix}$$

then $x = \alpha, y = \beta, z = \gamma$