Dynamic Mode Decomposition in Finance

21DUM123 - Financial Time Series Analysis

Team 5

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Abstract

Because the stock market is so unpredictable, stock price prediction is a difficult task. We propose a price prediction method based on Dynamic Mode Decomposition, assuming the stock market is a dynamic system. DMD is a data-driven, equation-free spatiotemporal technique that decomposes a system into modes that have specified temporal behavior. These modes aid in determining how the system evolves and predicting the system's future state. These modes have been utilized to make predictions about the stock market. We used time series data from companies that are listed on the National Stock Exchange. The time granularity was quite fine. We selected a few companies from various industries listed on the National Stock Exchange and used minute-by-minute stock prices to forecast their price in the next minutes. The findings of the price prediction were compared to actual stock prices. The variation of anticipated price from actual price for each company was calculated using Mean Absolute Percentage Error. Each company's price projection was made in three different ways. In the first, we used a sample of companies from the same industry to forecast future prices. For the latter, we used a sample of organizations from various industries to make predictions. The sample and prediction window sizes were both fixed in the first and second methods. In the third method, companies from all sectors were randomly selected. The sample window was kept constant, but predictions were made until a threshold error was reached. Prediction accuracy was found to be higher when samples were gathered from all sectors rather than just one. For specific instances of sampling, predictions might be produced for longer periods when the sampling window was fixed.

Dynamic Mode Decomposition in Finance

Dynamic mode decomposition (DMD) is a dimensionality reduction (transformation of data from a high-dimensional space into a low-dimensional space so that the low-dimensional representation retains some meaningful properties of the original data) algorithm developed by Peter Schmid in 2008.The data-based techniques have been rapidly developed over the past two decades and widely used in numerous industrial sectors nowadays. The core of data-based methods is to take full advantage of the enormous amounts of available data, aiming to acquire the useful information within. Compared with the well-developed model-based approaches, data-based techniques provide efficient alternative solutions. Data-based modal decomposition is frequently employed to investigate complex dynamical systems. When dynamical operators are too sophisticated to analyze directly because of nonlinearity and high dimensionality, then in such cases data-based techniques are often more practical. Such methods numerically analyze empirical data produced by experiments or simulations, yielding modes containing dynamically significant structures.

Brief History about DMD

Introduced in the fluid mechanic's community, Dynamic Mode Decomposition (DMD) traces its origins to Bernard Koopman in 1931 and be a particular case of Koopman theory. Initially, DMD was used to analyze the time evolution of fluid flows. In recent years DMD has quickly gained its popularity and has emerged as a significant tool for analyzing dynamics of nonlinear systems.

When we see DMD in the last few years alone, it has made immense progress in both theory and application. When we look into the theory of DMD, it has seen innovations around compressive architectures, multi-resolution analysis, and de-noising algorithms. DMD has not only made its progress in fluid dynamics, but also it has been applied to new domains such as neuroscience, epidemiology, robotics, and the current application of video processing and computer vision.

In the latter part, there will be a brief discussion on applications of DMD on different domains to understand the potential of DMD. The dynamic mode decomposition (DMD) is an equation-free, data-driven matrix decomposition method.

This method can provide accurate reconstructions of coherent spatiotemporal structures arising in nonlinear dynamical systems or short-time future estimates of such systems. The DMD method offers a regression technique for the least square fitting of snapshots to a linear dynamical system. Dynamic mode decomposition approximates the modes of the Koopman operator, which is a linear, infinite-dimensional operator that represents nonlinear, infinite-dimensional dynamics without linearization and is the adjoint of the Perron-Frobenius operator.

The method can be viewed as computing eigenvalues and eigenvectors of a linear model that approximates the underlying dynamics, even if those dynamics are nonlinear. Unlike Proper Orthogonal Decomposition (POD) and balanced POD, this decomposition yields growth rates and frequencies associated with each mode, which can be found from the magnitude and phase of each 2 corresponding eigenvalue.

If the data generated by a linear dynamical operator, then the decomposition recovers the leading eigenvalues and eigenvectors of that operator. If the data generated is periodic, then the decomposition is equivalent to a temporal discrete Fourier transform (DFT). Describing a nonlinear system by a superposition of modes whose dynamics are governed by eigenvalues may seem dubious at first place. After all, one needs a linear operator to talk about eigenvalues. However, we can see in (Tu, Rowley, Luchtenburg, Brunton, & Kutz, 2013) that DMD is closely related to a spectral analysis of the Koopman operator.

The Koopman operator is a linear but infinite-dimensional operator whose modes and eigenvalues capture the evolution of observables describing any (even nonlinear) dynamical system. The use of its spectral decomposition for data-based modal decomposition and model reduction was proposed first in (Mezi´c, 2005). DMD analysis can be a numerical approximation to Koopman spectral analysis, and it is in this sense that DMD applies to nonlinear systems.

In fact, the terms DMD mode and Koopman mode are often used interchangeably in this literature.

Theory

- Dynamic mode decomposition (DMD) is a mathematical method that was developed in order to understand, control, or simulate inherently complex, nonlinear systems without necessarily knowing fully, or partially, the underlying governing equations that drive the system.
- Experimental (or simulated) data, collected in snapshots through time, can be processed with DMD in order to mimic, control, or analyze the current state and/or dimensionality of a system, or even to predict future states of, or find coherent structures within, that system.
- The power of DMD is found in exploiting the intrinsic low-dimensionality of a complicated system, which may not be obvious a priori, and then rendering that system in a more computationally and theoretically tractable (low-dimensional) form.
- DMD has been traditionally used in the fluid mechanics, atmospheric science, and nonlinear waves communities as a popular method for data-base learning.

Pre-Requisites

ARIMA (Autoregressive integrated moving average) & MAPE (Mean absolute percentage error)

ARIMA

To understand ARIMA, it's helpful to examine the name. The "AR" stands for autoregression, which refers to the model that shows a changing variable that regresses on its own prior or lagged values. In other words, it predicts future values based on past values.

The "I" stands for integrated, which means it observes the difference between static data values and previous values. The goal is to achieve stationary data that is not subject to seasonality. That means the statistical properties of the data series, such as mean, variance and autocorrelation, are constant over time. Data scientists use an Augmented Dickey-Fuller (ADF) test to determine whether the data is stationary.Finally, "MA" represents the moving average, which is the dependency between an observed value and a residual error from a moving average model applied to previous observations.

An ARIMA model has three component functions: AR (p), the number of lag observations or autoregressive terms in the model; I (d), the difference in the nonseasonal observations; and MA (q), the size of the moving average window. An ARIMA model order is depicted as (p,d,q) with values for the order or number of times the function occurs in running the model. Values of zero are acceptable.

The ARIMA model uses differenced data to make the data stationary, which means there's a consistency of the data over time. This function removes the effect of trends or seasonality, such as market or economic data.

Seasonality occurs when data exhibits predictable, repeating patterns. It is critical to control for seasonality because it could impact the accuracy of the results. ARIMA models can be built using seasonal and nonseasonal formats. A seasonal model must take into account the number of events in each season in addition to the autoregressive, differencing and average terms for each season.

ARIMA models can be built in an array of software tools, including Python. Before deciding on an ARIMA model, the data scientist must confirm that the process in question fits the model. If the data is an appropriate fit for the ARIMA model, the data scientist builds the model and trains it on a dataset before inputting live data to develop and plot a forecast.

MAPE

The mean absolute percentage error (MAPE) is a measure of how accurate a forecast system is. It measures this accuracy as a percentage, and can be calculated as the average absolute percent error for each time period minus actual values divided by actual values.

$$\mathrm{M} = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|$$

Where:

- n is the number of fitted points,
- At is the actual value,
- Ft is the forecast value.
- Σ is summation notation (the absolute value is summed for every forecasted point in time).

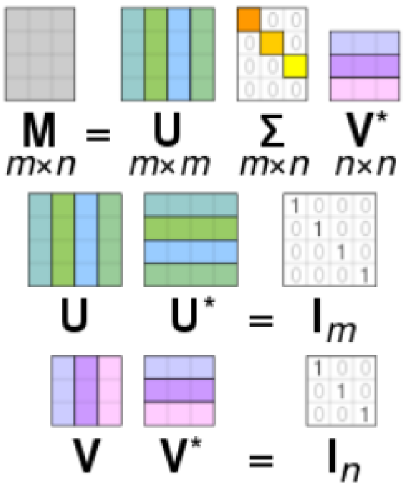
The mean absolute percentage error (MAPE) is the most common measure used to forecast error, and works best if there are no extremes to the data (and no zeros).

SVD

In linear algebra, the singular value decomposition (SVD) is a factorization of a real or complex matrix that generalizes the eigen decomposition of a square normal matrix to any matrix via an extension of the polar decomposition.

Specifically, the singular value decomposition of an mxn real or complex matrix M is a factorization of the form UΣV*, where U is an mxm real or complex unitary matrix, Σis an mxn rectangular diagonal matrix with non-negative real numbers on the diagonal, and V is an V real or complex unitary matrix. If M is real, U and V^T are real orthogonal matrices. The diagonal entries of $\sigma_i = \Sigma_{ii}$ of $\mathbf{\Sigma}$ are known as the singular values of M. The number of non-zero singular values is equal to the rank of M. The columns of U and the columns of V are called the **left-singular vectors** and **right-singular vectors** of M, respectively.

$$A_{mxn} = U_{mxm}\,\Sigma_{mxn}\,V^{*}_{nxn}$$



Algorithm

At a particular instant,the data snapshot of the system in the matrix:

$$X_1 = \begin{bmatrix} | & | & | & & | \\ x_1 & x_2 & x_3 & \cdots & x_m \\ | & | & | & & | \end{bmatrix}$$

The delayed version of the above matrix is:

$$X_2 = \begin{bmatrix} | & | & | & & | \\ x_2 & x_3 & x_4 & \cdots & x_{m+1} \\ | & | & | & & | \end{bmatrix}$$

DMD finds a matrix A such a way that,

$X_2 = AX_1$

i.e it maps:

$$\begin{matrix} | & & | & & | & & & & | & & | \\ x_1 & \rightarrow & x_2 & \rightarrow & x_3 & \rightarrow & \cdots & & x_m & \rightarrow & x_{m+1} \\ | & & | & & | & & & & | & & | \end{matrix}$$

To peform DMD for a given data,we will be following these steps:

Step 1:

SVD of X_1 gives,

$X_1 = U \sum V^*$

Step 2:

$A = X_2 X_1^{\dagger}$

$A = X_2 V \sum^{-1} U^*$

Now we introduce a low rank matrix which is equivalent of A.

$\tilde{A} = U^* A U = U^* X_2 \sum^{-1} U^* U$

$= U^* X_2 V \sum^{-1}$

Step 3:

$\tilde{A} W = W \Lambda$

(Here W is the eigen vectors and Λ is diagonal matrix of eigen values)

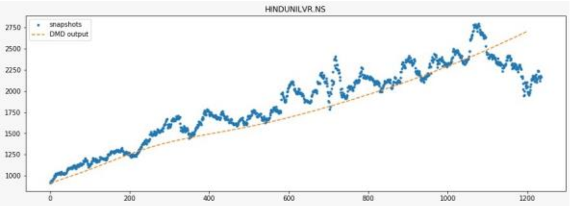
Step 4:

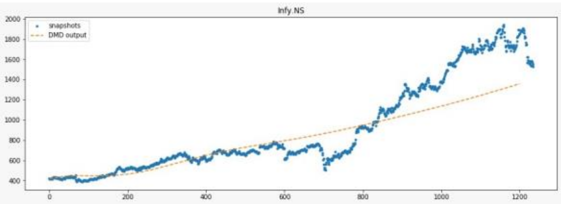
$\phi = X_2 V \sum^{-1} W$

$$X = \sum_{i=1}^{r} \phi_i e^{\omega_i t} b_i$$

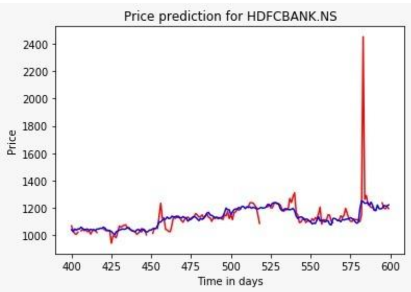
Results:

We approached our problem in three different ways DMD algorithm. In the first method we used, the snap consisted of the companies sampled from same sector. I second method the snapshots consisted of the companies sampled across different sectors. In both these approaches, the size of the sampling window and prediction window was kept same. The sampling window size chosen for our experiment was 6. On comparison with the actual values, it was observed that the predicted result using second approach was more accurate. In the third method sampling window size was kept fixed at 20, but prediction was made until it crossed a threshold error of 3%.

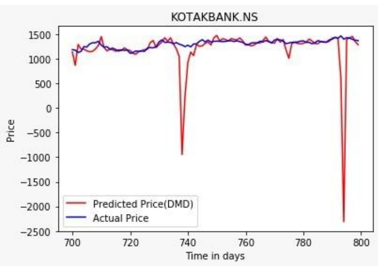




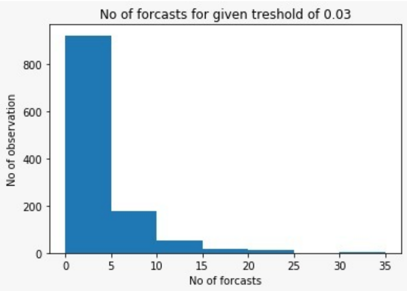
In the above figures we have fitted DMD model to snapshots consisting of 12 companies across different sectors.



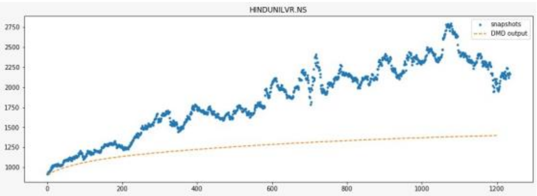
In the above figures we have fitted DMD model to snapshots consisting of 12 companies across different sectors with window size of 20 and plotted prediction for company HDFC BANK.

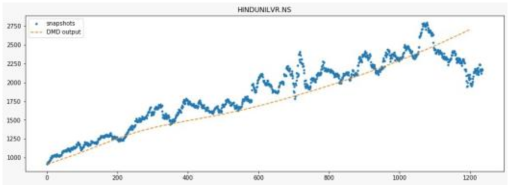


In the above figures we have fitted DMD model to snapshots consisting of 5 companies across banking sector with window size of 20 and plotted prediction for company KOTAK MAHINDRA BANK.

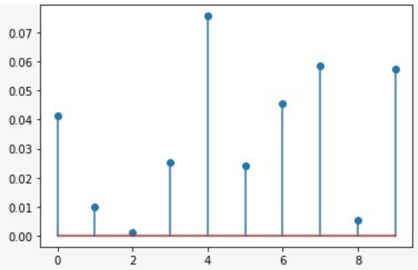


The above figure represents the number of continuous predictions made by DMD until it crosses the threshold error of 3%.





In the left figure 3, snapshots of three companies were used to fit the DMD model. In the right figure snapshots of 12 companies was used to fit the DMD model. When more snapshots of more companies is used the line of best fit is more precise.



The figure represents MAPE value for banking sector.

	DIFFERENT SECTORS	SAME SECTORS
DMD	0.014	0.005
ARIMA	0.034	0.01

Conclusion:

In this work, a powerful decomposition technique known as Dynamic mode decomposition was presented. The concept of dimensionality reduction is advantageous to build models based on dynamics of the data if there exists a subspace or low-dimensional structure in the high-dimensional data. DMD plays a significant role in dimensionality reduction provided there exist a subspace in the big data. Therefore DMD can reconstruct the original data with fewer dimensions. When compared to other modal decomposition techniques, DMD not only provides modes but also helps in predicting the system dynamics(how modes evolve). Also, it's an equation-free technique which can be applied to complex nonlinear systems. DMD has certain disadvantages as discussed in example cases, despite its limitation DMD is a powerful tool in analysing and predicting dynamical systems. As it finds application in various domains such as computer vision, neuroscience, biology, finance, etc, it would be great for the data scientists from different backgrounds to have good knowledge of DMD to explore its true potential.

References:

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2. https://www.mastersindatascience.org/learning/what-is-arima-modeling/