SRI VENKATESWARA COLLEGE OF ENGINEERING
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Question bank

Name of the Branch/Course	Common to All Branches
Subject	**Linear Algebra & Calculus**
Subject Code	**MA23ABS101**
Year & Sem	I&I

10 Marks Questions

	Unit – I
1	Define Rank of Matrix? Find rank of a matrix $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by using Echelon form.
2	Find rank of a matrix A = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ by using Normal form
3	Show that the system of equations $x+2y+z=3, 2x+3y+2z=5, 3x-5y+5z=2, 3x+9y-z=4.$ are Consistent and solve them.
4	Investigate for what values of λ and μ the simultaneous equations $x+y+z=6;\ x+2y+3z=10; x+2y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) An infinite number of solutions
5	Solve the equations by using Gauss elimination method 2x+y+z=10, 3x+2y+3z=18, x+4y+9z=16
6	Find the inverse of the matrix A= $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ by using Gauss-Jordan method.
7	Solve by Jacobi iteration method, the equations $5x-y+z=10, 2x+4y=12, x+y+5z=-1$ Start with the solutions (2,3,0)
8	Solve by Jacobi iteration metod,the equations 20x+y-2z=17,3x+20y-z=-18,2x-3y+20z=25
9	Solve the equations $10x+y+z=12$, $2x+10y+z=13$, $2x+2y+10z=14$ by using Gauss- Seidel iteration method
10	Determine whether the following equations will have a non-trivial solution, if so solve them $4x+2y+z+3w=0, 6x+3y+4z+7w=0, 2x+y+w=0.$
	Unit-II
1	Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$
2	Find the Eigen values and the corresponding Eigen vectors of the matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

3	Verify Cayley-Hamilton theorem for the matrix A = $\begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ and hence Find A^{-1} and A^4
4	Verify Cayley-Hamilton theorem for the matrix A = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence Find the value of the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.
5	Using Cayley-Hamilton theorem for the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ Find A^{-1} and A^4
6	Diagonalize of the Matrix A= $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$
7	Diagonalize the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and hence find A^4.
8	Use the orthogonal transformation, reduce the given quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy + 2xz - 2yz$ into canonical form and also find the rank, index and signature of the quadratic form
9	Use the orthogonal transformation, reduce the given quadratic form $6x^2 + 3y^2 + 3z^2 - 2yz + 4zx - 4xy$ into canonical form and also find the rank, index and signature of the quadratic form.
10	Reduce the real quadratic form $3x_1^2 - 3x_2^2 - 5x_3^2 - 2x_1x_2 - 6x_2x_3 - 6x_3x_1$ to canonical form.
	Unit – III
1	Verify Rolle's theorem for $f(x) = (x-a)^m (x-b)^n$ where m and n are positive integers *in* $[a,b]$
2	Verify Rolle's theorem for the function $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b]$, $a. > 0$, $b > 0$.
3	Verify Rolle's theorem for the function $\frac{\sin x}{e^x}$ in $(0,\pi)$.
4	Verify Lagrange's mean value theorem $f(x) = x(x-1)(x-2)$, $a = 0$ *and* $b = \frac{1}{2}$
5	Show that for 0<a<b<1 $\frac{1}{1+b^2} < \frac{\tan^{-1} b - \tan^{-1} a}{b-a} < \frac{1}{1+a^2}$ using Lagrange's mean value theorem and hence deduce that (i) $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$ (ii) $\frac{5\pi+4}{20} < \tan^{-1}2 < \frac{\pi+2}{4}$
6	Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagranges mean value theorem
7	Verify cauchy's mean value theorem for the following functions (i) $f(x) = e^x$, $g(x) = e^{-x}$ in $[a, b]$ (ii) $f(x) = \log x$, $g(x) = g'(x)$ in $[1, e]$
8	Verify Taylor's theorem for $f(x) = (1-x)^{\frac{5}{2}}$ with Lagrange's form of remainder up to two terms in $[0,1]$
9	Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \ldots\ldots$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \ldots\ldots$
10	Obtain the Maclaurin's series expansion of the following functions $(i)\ e^x$ $(ii) \cos x$ $(iii) \sin x$

	Unit – IV
1	If U = log($x^3+y^3+z^3-3xyz$), prove that $(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z})^2 U = \frac{-9}{(x+y+z)^2}$
2	If $u = f(r,s,t)$ where $r = \frac{x}{y}$, $s = \frac{y}{z}$ and $t = \frac{z}{x}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
3	If x + y + z = u, y +z = uv, z = u v w , then Show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$
4	If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$ and find $\frac{\partial(r,\theta,\phi)}{\partial(x,y,z)}$
5	If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$ *find* $\frac{\partial(u,v)}{\partial(x,y)}$. Show that u and v are functionally dependent and find the relation between them.
6	(a) Examine for maximum and minimum values of sinx + siny + sin(x + y). (b))Find a point on the plane 3x+2y+z-12 = 0 which is nearest to the origin
7	Investigate the maxima and minima, if any of the function $f(x,y) = x^3y^2(1-x-y)$.
8	A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction
9	Find the maximum value of $x^2+y^2+z^2$ subject to the condition $xyz = a^3$
10	Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$
	Unit – V
1	Evaluate the following double integrals. (a) $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$ (b) $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dxdy$ (c) $\int_0^1\int_x^{\sqrt{x}} x^2y^2(x+y)dydx$
2	Evaluate the following triple integrals a) $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\ dx\ dy\ dz$ b) $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dz\,dy\,dx$
3	Evaluate $\int_1^e\int_1^{\log y}\int_1^{e^x} \log z\ dz\,dy\,dx$
4	Evaluate $\iint(x^2+y^2)dxdy$ in the positive quadrant for which x + y ≤ 1
5	a) Evaluate $\iint r^3\,dr\,d\theta$ over the area included between the circles r =2sinθ and r =4sinθ b) Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$
6	Change the order of integration and evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$.
7	Change the order of integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} x\ y\ dx\ dy$
8	By changing the order of integration , evaluate $\int_0^1\int_0^{\sqrt{1-x^2}} y^2\ dx\ dy$
9	Evaluate the triple integral $\iiint xy^2z\ dx\,dy\,dz$ taken through the +ve octant of sphere $x^2+y^2+z^2 = a^2$
10	Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$

2 Marks Questions

Unit-I

1	Define rank of a matrix and give an example.
2	Obtain the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 2 & 2 \end{bmatrix}$
3	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 8. \\ 2 & 4 & 0 & 6 \end{bmatrix}$
4	State Cauchy–Binet formulae and give an example.
5	Explain the working rule for finding the Inverse of a matrix by Gauss-Jordan method
6	Define system of linear equations with suitable example.

Unit-II

1	Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.
2	Find the sum and product of the eigen values of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$
3	Find the eigen vector of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ corresponding to $\lambda = 15$.
4	Explain the working rule of diagonalization form.
5	State Cayley Hamilton theorem.
6	Write the quadratic forms of the matrices A = $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 4 & 5 \\ -1 & 5 & 2 \end{bmatrix}$ B= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Unit-III

1	State Rolle's Theorem
2	Verify whether Rolle's Theorem can be applied to the following function in the given interval f(x) = tanx in [0,π].
3	State Lagrange's Mean value theorem
4	Verify the result of Cauchy's Mean Value theorem for the functions sinx and cosx in [a, b]
5	Obtain the Taylor's series expansion of e^x about x=-1.
6	If f(0)=0, $f^{1}(0)= 1$, $f^{11}(0)= 1$, $f^{111}(0)=1$, then write the Maclaurin's series expansion of f(x).

Unit-IV	
1	Find the first and second order partial derivatives of $ax^2 + 2hxy + by^2$
2	If u = f (x-y, y-z, z-x), then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
3	If $f(x,y) = e^x \cos y$, Find f_x at $(1, \frac{\pi}{4})$
4	If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$
5	If x=u(1+v), y= v(1+u), then find Jacobian.
6	If f(x,y) =xy+(x-y),then find the stationary points

Unit-V	
1	Evaluate $\int_0^2 \int_0^x e^{x+y}\, dy\, dx$
2	Evaluate $\int_0^\infty \int_0^{\pi/2} e^{-r^2} r\, d\theta dr$
3	Evaluate $\int_0^{\pi/2} \int_0^\infty \frac{r}{(r^2+a^2)^2}\, dr d\theta$
4	Evaluate the triple integral $\int_0^1 \int_y^1 \int_0^{1-x} x\, dzdxdy$
5	Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$
6	Evaluate $\int_0^1 \int_1^2 \int_2^3 xyzdxdydz$

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