2.1) $\min\limits_{x_1,x_2 \in \mathbb{R}} \quad (x_1-4)^2+7(x_2-4)^2+4x_2$ (1)

$$f(x_1,x_2) = x_1^2+16-8x_1+7(x_2^2+16-8x_2)+4x_2$$
$$= x_1^2+7x_2^2-8x_1-52x_2+128$$

$$\frac{\partial f(x_1,x_2)}{\partial x_1} = 2x_1-8=0 \Rightarrow x_1 = 8/2 = \underline{4}$$

$$\frac{\partial f(x_1,x_2)}{\partial x_2} = 14x_2-52=0 \Rightarrow x_2 = 52/14 = \underline{3.7}$$

optimal value $= (x_1-4)^2+7(x_2-4)^2+4x_2$
$$= (4-4)^2+7(3.7-4)^2+4(3.7)$$
$$= 0.63+14.8 = \underline{15.43}$$

2.2) $\min\limits_{x_1,x_2,x_3 \in \mathbb{R}} \quad x_1^3+(x_2-x_3)^2+x_3^3+2$

$$\Rightarrow x_1^3+x_2^2+x_3^2-2x_2x_3+x_3^3+2$$

$$\frac{\partial f}{\partial x_1} = 3x_1^2 = 0 \Rightarrow x_1 = 0$$

$$\frac{\partial f}{\partial x_2} = 2x_2-2x_3=0 \Rightarrow \cancel{2}x_2 = \cancel{2}x_3 \Rightarrow x_2 = x_3$$

$$\frac{\partial f}{\partial x_3} = 2x_3-2x_2+3x_3^2 = 0$$

substitute $x_2 = x_3$; $\quad \cancel{2x_3}-\cancel{2x_3}+3x_3^2=0$
$$x_3 = 0$$

i.e optimal variable values: $x_1=0$; $x_2=0$; $x_3=0$.

optimal value $= x_1^3+(x_2-x_3)^2+x_3^3+2$
$$= 0+(0-0)^2+0+2 = \underline{2}$$

Hessian matrix (H) $= \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \partial^2 f/\partial x_2 \cdot \partial x_3 \\ \frac{\partial^2 f}{\partial x_3 \cdot \partial x_1} & \frac{\partial^2 f}{\partial x_3 \cdot \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} 6x_1 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -2 & 2+6x_3 \end{bmatrix}$

$$|\lambda I - H| = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 6x_1 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -2 & 2+6x_3 \end{bmatrix} = \begin{bmatrix} \lambda-6x_1 & 0 & 0 \\ 0 & \lambda-2 & 2 \\ 0 & 2 & \lambda-(2+6x_3) \end{bmatrix}$$

2

$$\Rightarrow (\lambda-6x_1)\left[(\lambda-2)\left[\lambda-(2+6x_3)\right]-4\right] \geq 0$$

$$\Rightarrow (\lambda-6x_1)\left[\lambda(\lambda-2-6x_3)-2(\lambda-2-6x_3)-4\right] \geq 0$$

$$\Rightarrow (\lambda-6x_1)\left[\lambda^2-2\lambda-6x_3\lambda-2\lambda+\cancel{4}-12x_3-\cancel{4}\right] \geq 0$$

$$\Rightarrow (\lambda-6x_1)\left(\lambda^2-\lambda(4+6x_3)-12x_3\right) \geq 0$$

$$\boxed{\begin{array}{l} ax^2+bx+c=0 \\ a=1 \\ b=-(4+6x_3) \\ c=-12x_3 \end{array}}$$

$\lambda-6x_1 \geq 0$

$\boxed{\lambda \geq 0}$

$$\lambda^2-\lambda(4+6x_3)-12x_3 \geq 0$$

$$\text{roots} = \frac{-(-(4+6x_3)) \pm \sqrt{(-(4+6x_3))^2-4(1)(-12x_3)}}{2(1)}$$

$$= \frac{4+6x_3 \pm \sqrt{(-4-6x_3)^2+48x_3}}{2} \geq 0$$

$$= \frac{(4+6x_3)+\sqrt{(-4-6x_3)^2+48x_3}}{2} \geq 0$$

$$= (4+6x_3)^2 \geq (4+6x_3)^2-48x_3$$

is this condiation is true only for

$\boxed{x_3 \geq 0}$

so, For make it this problem to

convex $\boxed{x_3 \geq 0 \;\&\; \lambda \geq 0}$.

$$= \frac{(4+6x_3)-\sqrt{(-4-6x_3)^2+48x_3}}{2} \geq 0$$

$$= (4+6x_3)^2 \geq (-4-6x_3)^2+48x_3$$

$$(4+6x_3)^2 \geq -(4+6x_3)^2+48x_3$$

2.3) $\min\limits_{x_1, x_2 \in \mathbb{R}} (x_1-2)^2 + 3x_2$ s.t. $-x_1 - x_2 \le -4$

③

$-x_1 - x_2 + 4 \le 0$

$L(x, y, \lambda) = (x_1-2)^2 + 3x_2 + \lambda(-x_1 - x_2 + 4)$

$= x_1^2 + 4 - 4x_1 + 3x_2 - \lambda x_1 - \lambda x_2 + 4\lambda$

$\frac{\partial L}{\partial x_1} = 2x_1 - 4 - \lambda = 0 \Rightarrow x_1 = \frac{4+3}{2} = \frac{7}{2} = \underline{\underline{3.5}}$

$\frac{\partial L}{\partial x_2} = 3 - \lambda = 0 \Rightarrow \underline{\underline{\lambda = 3}}$

$\frac{\partial L}{\partial \lambda} = -x_1 - x_2 + 4 \Rightarrow x_2 = 4 - x_1 = 4 - 3.5 = \underline{\underline{0.5}}$

optimal value $= (x_1-2)^2 + 3x_2$

$= (3.5-2)^2 + 3(0.5) = 2.25 + 1.5 = \underline{\underline{3.75}}$

In log barrier: for $t=1 \Rightarrow$ optimal value $= 5.84$; $x_1 = 3.5$; $x_2 = 0.83$

$t=10 \Rightarrow$ optimal value $= 4.19$; $x_1 = 3.5$; $x_2 = 0.53$

$t=100 \Rightarrow$ optimal value $= 3.81$; $x_1 = 3.5$; $x_2 = 0.50$

$t=1000 \Rightarrow$ optimal value $= 3.75$; $x_1 = 3.5$; $x_2 = 0.50$

$t=10000 \Rightarrow$ optimal value $= 3.75$; $x_1 = 3.5$; $x_2 = 0.5$

$t=100000 \Rightarrow$ optimal value $= 3.75$; $x_1 = 3.5$; $x_2 = 0.5$

3.1) Maximum limit = 5,00,000 lit/day $\Rightarrow$ $x_1 + x_2 = 5,00,000$ lit/day

4

let, x_1 = water from stream $\leq 1,00,000$ lit/day

x_2 = water from reservoir $\Rightarrow$ $x_2 = 5,00,000 - 1,00,000 = 4,00,000$ lit/day

pollutants limit ≤ 100 PPM

From table, PPM for reservoir = 50

PPM for stream = 250

x_1 = stream
x_2 = Reservoir

$$\frac{x_1(250)}{5,00,000} + \frac{x_2(50)}{5,00,000} \leq 100$$

$$\Rightarrow \frac{(1,00,000)(250)}{5,00,000} + \frac{4,00,000(50)}{5,00,000} \leq 100$$

$\boxed{90 \text{ PPM}}$ $50 + 40 \leq 100$ $\Rightarrow$ PPM satisfied by x_1 & x_2 values.

water cost per reservoir = $\frac{100}{1000}$ = 0.1 € per lit

stream = $\frac{50}{1000}$ = 0.05 € per lit

$$\boxed{\min f(x) = (0.1)x_2 + (0.05)x_1}$$

s.t, $x_1 + x_2 = 500000$,

$x_1 \leq 100000$,

$$\frac{(x_1)250}{500000} + \frac{(x_2)50}{500000} \leq 100.$$

optimal value = $(0.1)4,00,000 + (0.05)1,00,000$

= 40,000 + 5000 = $\underline{\underline{45000}}$ €

3.2) Let, 'p' be the % of blend 1 $\Rightarrow 0.10 \le p \le 0.25$

'q' be the % of blend 2 $\Rightarrow 0.05 \le q \le 0.20$

'r' be the % of blend 3 $\Rightarrow r \ge 0.30$

's' be the % of blend 4 $\Rightarrow s \ge 0$

(5)

Final % of rose w.r.to blenders must be at most 35%.

i.e $\boxed{(0.30)p+(0.20)q+(0.40)r+(0.20)s \le 0.35}$

Final % of orange w.r.to blenders must be at most 50%.

i.e $\boxed{(0.35)p+(0.60)q+(0.35)r+(0.40)s \le 0.50}$

Final % of lily w.r.to blenders must be at least 19%.

i.e $\boxed{(0.20)p+(0.15)q+(0.05)r+(0.30)s \ge 0.50}$

Final % of thymus w.r.to blenders must be between 8% to 13%.

i.e $\boxed{0.80 \le (0.15)p+(0.05)q+(0.20)r+(0.10)s \le 0.13.}$

~~[illegible]~~ By adding mixes of 4 blenders for get one perfume below condiation make it sense. i.e $\boxed{p+q+r+s=1}$

For get optimum cost, $\underset{f(x)}{\min} = (55)p+(65)q+(35)r+(85)s$

So, For solving by using CVXPY considering $f(x)$ is optimal solution and remaining all condiations are constraints.

By taking $p+q+r+s=1$ to constraint $\Rightarrow$ optimal value $= \underline{\underline{63}}$

with out taking $p+q+r+s=1$ to constraint $\Rightarrow$ optimal value $= \underline{\underline{61}}$

3.3) Total amount to spend = 200M € has to

i.e. $x_{rural} + x_{urban} \le 200M€$

(6)

let $x_1 = x_{rural}$; $x_2 = x_{urban}$

$$\max_{f(x)} = B_{rural} + B_{urban} - x_{rural} - x_{urban}$$

$B_{rural} = 7000 \log(1 + x_{rural})$

$B_{urban} = 5000 \log(1 + x_{urban})$

i.e. $$\max_{f(x)} = (7000.\log(1+x_1)) + (5000.\log(1+x_2)) - x_1 - x_2$$

$$\text{s.t. } x_1 + x_2 \le 200M$$

so, By solving with CVXPY, optimal value $= 55348$

$x_1 = 116.83$; $x_2 = 83.16$

∴ Benefit from spending x_{rural};

$B_{rural} = 7000.\log(1+116.83) = 7000(2.07) = \underline{14,490}$

∴ Benefit from spending x_{urban};

$B_{urban} = 5000.\log(1+83.16) = 5000(1.92) = \underline{9,625}$

$\max_{f(x)} = 14,490 + 9,625 - 116.83 - 83.16 = \underline{23,915.01}$

By giving 200, I am getting these values. But, I think we should give 200M = 200000000 in constraint. By using 200M ; am getting error in my code. :(