LINE ASSIGNMENT

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IITH - Future Wireless Communication(FWC22089)

Contents

1 Problem

Q.Straight lines 3x+4y=5 and 4x-3y=15 intersect at point A. Points B and C are choosen on these two lines such that AB=AC. Determine the possible equations of the line BC through the point (1,2).

2 Solution

we know that vector equation of the lines intersecting at point A,

$$\mathbf{n}_1^\top \mathbf{A} = c_1 \tag{1}$$
$$\mathbf{n}_2^\top \mathbf{A} = c_2 \tag{2}$$

The vector equation of the line1 and line2 are

$$\mathbf{n}_1^\top \mathbf{B} = c_1 \tag{3}$$
$$\mathbf{n}_2^\top \mathbf{C} = c_2 \tag{4}$$

Symbol	Co-ordinates
n1	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
n2	$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$
R	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
p	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

from equation (1) and (2),

$$\mathbf{A} = \begin{pmatrix} \mathbf{n}_1^\top \\ \mathbf{n}_2^\top \end{pmatrix}^{-1} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
$$\mathbf{A} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Points B and C are choosen on these two lines such that AB = AC i.e.,

$$\|\mathbf{A}\text{-}\mathbf{B}\| = \|\mathbf{A}\text{-}\mathbf{C}\| \tag{5}$$

Consider,

$$\mathbf{B} = \mathbf{A} + K_1 \mathbf{m}_1 \tag{6}$$
$$\mathbf{C} = \mathbf{A} + K_2 \mathbf{m}_2 \tag{7}$$

when a line passing through a point the vector equation is,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{p}) = 0 \tag{8}$$
$$where, \mathbf{n} = \mathbf{R}(\mathbf{B} - \mathbf{C}) \tag{9}$$

Substituting B and C in eq(5),

$$|K_1|\|\mathbf{m}_1\| = |K_2|\|\mathbf{m}_2\|$$

$$if\ \|\mathbf{m}_1\| = \|\mathbf{m}_2\| = 1 \tag{10}$$
$$then, K_1 = K_2 (or) K_1 = -K_1 \tag{11}$$
$$\tag{12}$$

To satisfy equation(10),

$$\mathbf{m}_1 = \frac{\mathbf{R}\mathbf{n}_1}{\|\mathbf{n}_1\|} \tag{13}$$
$$\mathbf{m}_2 = \frac{\mathbf{R}\mathbf{n}_2}{\|\mathbf{n}_2\|} \tag{14}$$

let K_1 be λ then ,
Case(i) :

$$\mathbf{B} = \mathbf{A} + \lambda \mathbf{m}_1 \tag{15}$$
$$\mathbf{C} = \mathbf{A} + \lambda \mathbf{m}_2 \tag{16}$$

Substitute B and C in eq(9),

$$\mathbf{n} = \lambda \mathbf{R}^2 (\frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} - \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|}) \tag{17}$$

$$\mathbf{n} = -\lambda (\frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} - \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|}) \tag{18}$$

As eq(8) is satisfied by B, substitute n and B in (8)

$$-\lambda (\frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} - \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|})^\top (\mathbf{A} - \mathbf{p} + \lambda \mathbf{m}_1) = 0 \tag{19}$$

$$\lambda = \frac{(\frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} - \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|})^\top (\mathbf{A} - \mathbf{p})}{\frac{\mathbf{n}_2^\top \mathbf{m}_1}{\|\mathbf{n}_2\|}} \tag{20}$$

$$\lambda = \frac{(\|\mathbf{n}_2\| \mathbf{n}_1 - \|\mathbf{n}_1\| \mathbf{n}_2)^\top (\mathbf{A} - \mathbf{p})}{\mathbf{n}_2^\top \mathbf{n}_1 \mathbf{R}} \tag{21}$$

Similarly, Case(ii) :

$$\mathbf{B} = \mathbf{A} + \lambda \mathbf{m}_1 \tag{22}$$

$$\mathbf{C} = \mathbf{A} - \lambda \mathbf{m}_2 \tag{23}$$

Substitute B and C in eq(9),

$$\mathbf{n} = -\lambda (\frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} + \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|}) \tag{24}$$

$$\lambda = -\frac{(\|\mathbf{n}_2\| \mathbf{n}_1 - \|\mathbf{n}_1\| \mathbf{n}_2)^\top (\mathbf{A} - \mathbf{p})}{\mathbf{n}_2^\top \mathbf{n}_1 \mathbf{R}} \tag{25}$$

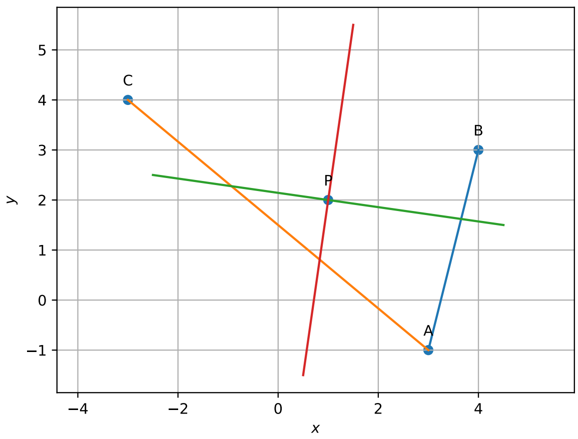
By sustituting in eq(19) and (23),

$$\lambda = \begin{pmatrix} 230 \\ 110 \end{pmatrix} \tag{26}$$

from Case(i), by substituting the values

$$\mathbf{B} = \begin{pmatrix} -181 \\ 137 \end{pmatrix} \; and \;\; \mathbf{C} = \begin{pmatrix} 141 \\ 183 \end{pmatrix} \tag{27}$$

$$\mathbf{n} = \begin{pmatrix} 46 \\ 183 \end{pmatrix} \tag{28}$$



from Case(ii), by substituting the values

$$\mathbf{B} = \begin{pmatrix} -85 \\ 65 \end{pmatrix} \; and \;\; \mathbf{C} = \begin{pmatrix} 63 \\ -89 \end{pmatrix} \tag{29}$$

$$\mathbf{n} = \begin{pmatrix} -154 \\ -22 \end{pmatrix} \tag{30}$$

Therefore,the possible equations passing through the point(1,2) are 7y-x-13=0 and 7x+y-9=0.

3 Plot

4 Software

We can get the parallel equation of given equation and the plot of two equtions by executing the following code:

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https://github.com/sivaparvathi−tungala
    /fwc_module_1/tree/main/line
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