भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Conic Assignment

T.Siva Parvathi(FWC22089)

CONTENTS

I. PROBLEM

Q.The locus of the mid-point of the lines segment joining the focus to a moving point on the parabola y^2=4ax is another parabola with directrix

II. SOLUTION

The standard conic equation is,

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{1}$$

where,
$\mathbf{V}=\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$
$\mathbf{u}=\begin{pmatrix}-2a\\0\end{pmatrix}$ and $f=0$

For the standard conic,

$$\mathbf{u}=\frac{\eta}{2}e_1, e=1 \tag{2}$$

where, $e_1=\begin{pmatrix}1\\0\end{pmatrix}$ equating values of u,

$$\tfrac{\eta}{2}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-2a\\0\end{pmatrix}$$
$$\begin{pmatrix}\frac{\eta}{2}\\0\end{pmatrix}=\begin{pmatrix}-2a\\0\end{pmatrix}$$
$$\tfrac{\eta}{2}e_1=-2a$$
$$\eta=-4a$$

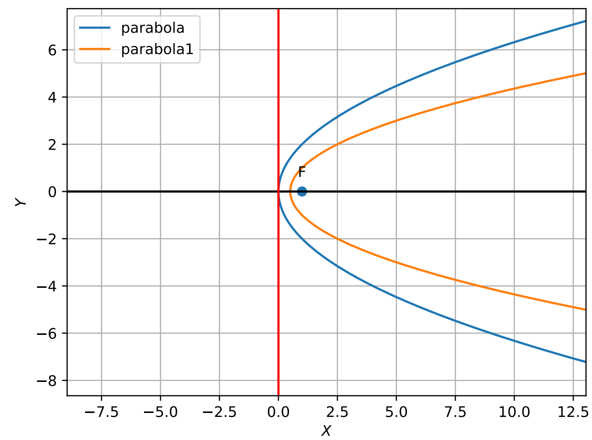

Fig. 1. Labeling locus parabola with directrix for a given parabola

III. FIGURE

Foci of the standard parabola is given by,

$$\mathbf{F}=\frac{-\eta}{4\lambda_2}e_1, e=1 \tag{3}$$

By substituting η value we get,

$$\mathbf{F}=-\tfrac{-4a}{4}\begin{pmatrix}1\\0\end{pmatrix}$$
$$\mathbf{F}=\begin{pmatrix}a\\0\end{pmatrix}$$

let moving point on the parabola be 'q',then the equation is,

$$\mathbf{q}^\top\mathbf{V}\mathbf{q}+2\mathbf{u}^\top\mathbf{q}+f=0 \tag{4}$$

the mid-point when line segment joining the focus to moving point on the parabola be 'h'.

$$\mathbf{h}=\frac{\mathbf{q}+\mathbf{F}}{2} \tag{5}$$

$$\mathbf{q}=2\mathbf{h}-\mathbf{F} \tag{6}$$

Substitute (6) in (4)

$$(2\mathbf{h}-\mathbf{F})^\top \mathbf{V}(2\mathbf{h}-\mathbf{F})+2\mathbf{u}^\top(2\mathbf{h}-\mathbf{F})+f=0 \tag{7}$$

(2**h**-$\begin{pmatrix} a \\ 0 \end{pmatrix}$)$^\top$ $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ (2**h**-$\begin{pmatrix} a \\ 0 \end{pmatrix}$) $+\ 2\begin{pmatrix} -2a & 0 \end{pmatrix}$ (2**h**-$\begin{pmatrix} a \\ 0 \end{pmatrix}$) $+0=0$

By solving we get the new parabola equation,

$$y^2-2ax+a^2=0 \tag{8}$$

from above the locus parabola equation can be written as,

$$\mathbf{x}^\top \mathbf{V}_1\mathbf{x}+2\mathbf{u}_1^\top\mathbf{x}+f_1=0 \tag{9}$$

where,
$\mathbf{V}_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u}_1 = \begin{pmatrix} -a \\ 0 \end{pmatrix}$ and $f_1=a^2$
The new parabola equation in terms of old parabola equation is,

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+\mathbf{u}^\top\mathbf{x}+(f+d)=0$$

The directrix of a conic is given by,

$$\mathbf{n}^\top\mathbf{x}=c \tag{10}$$

The directrices of (9) is given by,

$$\mathbf{n}=\sqrt{\lambda_2}\mathbf{p}_1 \tag{11}$$

$$c=\frac{\|\mathbf{u}\|^2-\lambda_2 f}{2\mathbf{u}^\top\mathbf{n}}, e=1 \tag{12}$$

where the eigen values and eigen vectors are,

$$\lambda_1=0,\lambda_2=1 \tag{13}$$

$$\mathbf{p}_1=\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{p}_2=\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{14}$$

substituting values in (11) we get,

$$\mathbf{n}=\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{15}$$

substituting values in (12),
$c=\frac{a^2-a^2}{2\begin{pmatrix} -a & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$

$$c=0 \tag{16}$$

put n and c in directrix equation we get,

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x}=0 \tag{17}$$

Therefore,the locus of the mid-point of the lines segment joining the focus to a moving point on the parabola y^2-$4ax = 0$ is another parabola y^2-$2ax$+$a^2=0$ with directrix $x=0$.

IV. SOFTWARE

https://github.com/sivaparvathi−tungala/fwc_module_1/tree/main/conic