



Conic Assignment

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I. PROBLEM

Q.The locus of the mid-point of the lines segment joining the focus to a moving point on the parabola $y^2=4ax$ is another parabola with directrix

II. SOLUTION

The standard conic equation is,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

where, $\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\mathbf{u} = \begin{pmatrix} -2a \\ 0 \end{pmatrix}$ and $f = 0$

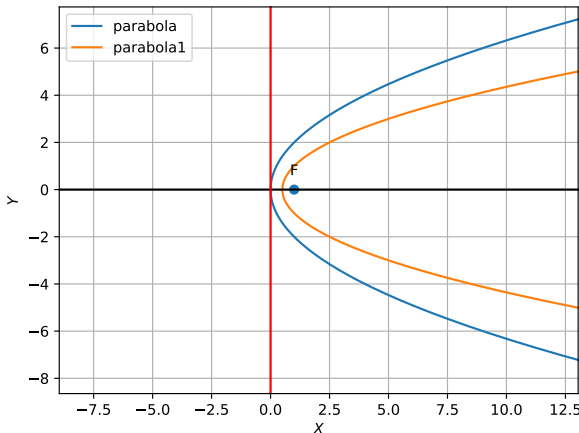


Fig. 1. Labeling locus parabola with directrix for a given parabola

For the standard conic,

$$\mathbf{u} = \frac{\eta}{2} \mathbf{e}_1, e = 1 \quad (2)$$

where, $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ equating values of \mathbf{u} ,

$$\begin{aligned} \frac{\eta}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} &= \begin{pmatrix} -2a \\ 0 \end{pmatrix} \\ \begin{pmatrix} \frac{\eta}{2} \\ 0 \end{pmatrix} &= \begin{pmatrix} -2a \\ 0 \end{pmatrix} \\ \frac{\eta}{2} \mathbf{e}_1 &= -2a \\ \eta &= -4a \end{aligned}$$

III. FIGURE

Foci of the standard parabola is given by,

$$\mathbf{F} = \frac{-\eta}{4\lambda_2} \mathbf{e}_1, e = 1 \quad (3)$$

(1) By substituting η value we get,

$$\begin{aligned} \mathbf{F} &= -\frac{-4a}{4} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \mathbf{F} &= \begin{pmatrix} a \\ 0 \end{pmatrix} \end{aligned}$$

let moving point on the parabola be 'q', then the equation is,

$$\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f = 0 \quad (4)$$

the mid-point when line segment joining the focus to moving point on the parabola be 'h'.

$$\mathbf{h} = \frac{\mathbf{q} + \mathbf{F}}{2} \quad (5)$$

$$\mathbf{q} = 2\mathbf{h} - \mathbf{F} \quad (6)$$

Substitute (6) in (4)

$$(2\mathbf{h} - \mathbf{F})^T \mathbf{V} (2\mathbf{h} - \mathbf{F}) + 2\mathbf{u}^T (2\mathbf{h} - \mathbf{F}) + f = 0 \quad (7)$$

$$4\mathbf{h}^\top \mathbf{V}\mathbf{h} - \mathbf{F}^\top \mathbf{V}\mathbf{F} - 2\mathbf{h}^\top \mathbf{V}\mathbf{F} - 2\mathbf{F}^\top \mathbf{V}\mathbf{h} + 4\mathbf{u}^\top \mathbf{h} - 2\mathbf{u}^\top \mathbf{F} + f = 0$$

By solving we get the new parabola equation,

$$y^2 - 2ax + a^2 = 0 \quad (8)$$

from above the locus parabola equation can be written as,

$$\mathbf{h}^\top \mathbf{V}_1 \mathbf{h} + 2\mathbf{u}_1^\top \mathbf{h} + f_1 = 0 \quad (9)$$

where, $\mathbf{V}_1 = \mathbf{V}$

$\mathbf{u}_1 = \frac{\mathbf{u}}{2}$ and $f_1 = f + d$

The new parabola equation in terms of old parabola equation is,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + \mathbf{u}^\top \mathbf{x} + (f + d) = 0$$

The directrix of a conic is given by,

$$\mathbf{n}^\top \mathbf{x} = c \quad (10)$$

The directrices of (9) is given by,

$$\mathbf{n} = \sqrt{\lambda_2} \mathbf{p}_1 \quad (11)$$

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2\mathbf{u}^\top \mathbf{n}}, e = 1 \quad (12)$$

where the eigen values and eigen vectors are,

$$\lambda_1 = 0, \lambda_2 = 1 \quad (13)$$

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (14)$$

substituting values in (11) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (15)$$

substituting values in (12),

$$c = \frac{a^2 - a^2}{2(-a \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix}} \quad (16)$$

$$c = 0$$

put n and c in directrix equation we get,

$$(1 \ 0) \mathbf{x} = 0 \quad (17)$$

Therefore, the locus of the mid-point of the lines segment joining the focus to a moving point on the parabola $y^2 - 4ax = 0$ is another parabola $y^2 - 2ax + a^2 = 0$ with directrix $x = 0$.

IV. SOFTWARE

https://github.com/sivaparvathi-tungala/fwc_module_1/tree/main/conic