Conic Assignment

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CONTENTS

I. PROBLEM

Q.The locus of the mid-point of the lines segment joining the focus to a moving point on the parabola y^2=4ax is another parabola with directrix

II. SOLUTION

The standard conic equation is,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where,
$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u} = \begin{pmatrix} -2a \\ 0 \end{pmatrix}$ and $f = 0$
let moving point on the parabola be 'q',then the equation is,

$$\mathbf{q}^\top \mathbf{V}\mathbf{q} + 2\mathbf{u}^\top \mathbf{q} + f = 0 \tag{2}$$

the mid-point when line segment joining the focus to moving point on the parabola be 'h'.

$$\mathbf{h} = \frac{\mathbf{q}+\mathbf{F}}{2} \tag{3}$$

$$\mathbf{q} = 2\mathbf{h} - \mathbf{F} \tag{4}$$

Substitute (4) in (2)

$$(2\mathbf{h}-\mathbf{F})^\top \mathbf{V}(2\mathbf{h}-\mathbf{F}) + 2\mathbf{u}^\top(2\mathbf{h}-\mathbf{F}) + f = 0 \tag{5}$$

By solving we get the locus parabola equation is,

$$\mathbf{x}^\top \mathbf{V}_1\mathbf{x} + 2\mathbf{u}_1^\top \mathbf{x} + f_1 = 0 \tag{6}$$

where,
$\mathbf{V}_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u}_1 = \begin{pmatrix} -a \\ 0 \end{pmatrix}$ and $f_1 = a^2$
The new parabola equation in terms of old parabola equation is,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + \mathbf{u}^\top \mathbf{x} + (f+d) = 0$$

The directrix of a conic is given by,

$$\mathbf{n}^\top \mathbf{x} = c \tag{7}$$

The directrices of (6) is given by,

$$\mathbf{n} = \sqrt{\lambda_2}\mathbf{p}_1 \tag{8}$$

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2\mathbf{u}^\top \mathbf{n}}, e = 1 \tag{9}$$

where the eigen values and eigen vectors are,

$$\lambda_1 = 0, \lambda_2 = 1 \tag{10}$$

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{11}$$

substituting values in (10) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{12}$$

substituting values in (12),
$c = \frac{a^2 - a^2}{2\begin{pmatrix} -a & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$

$$c = 0 \tag{13}$$

put n and c in directrix equation we get,

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 0 \tag{14}$$

Therefore,the locus of the mid-point of the lines segment joining the focus to a moving point on the parabola y^2-$4ax = 0$ is another parabola y^2-$2ax$+$a^2 = 0$ with directrix $x = 0$.

III. FIGURE

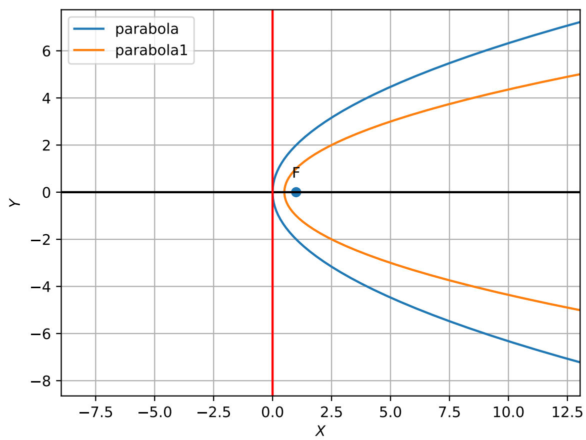

Fig. 1. labeling

IV. SOFTWARE

https://github.com/sivaparvathi−tungala/fwc_module_1/tree/main/conic