

CONIC ASSIGNMENT

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IITH - Future Wireless Communication(FWC22089)

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Substitute eq(4) in eq(2)

- 1
- 1 $(2\mathbf{h} - \mathbf{F})^T \mathbf{V}(2\mathbf{h} - \mathbf{F}) + 2\mathbf{u}^T(2\mathbf{h} - \mathbf{F}) + f = 0$
(5)
- 2
- 2 By solving we get the locus parabola equation is,

1 Problem

Q.The locus of the mid-point of the lines segment joining the focus to a moving point on the parabola $y^2=4ax$ is another parabola with directrix

$$\mathbf{x}^T \mathbf{V}_1 \mathbf{x} + 2\mathbf{u}_1^T \mathbf{x} + f_1 = 0 \quad (6)$$

where,

$$\mathbf{V}_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{u}_1 = \begin{pmatrix} -a \\ 0 \end{pmatrix} \text{ and } f_1 = a^2$$

The directrix of a conic is given by,

$$\mathbf{n}^T \mathbf{x} = c \quad (7)$$

2 Solution

The standard conic equation is,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1) \text{ The directrices of eq(6) is given by,}$$

where,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{u} = \begin{pmatrix} -2a \\ 0 \end{pmatrix} \text{ and } f = 0$$

let moving point on the parabola be 'q', then the equation is,

$$\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f = 0 \quad (2)$$

the mid-point when line segment joining the focus to moving point on the parabola be 'h'.

$$\mathbf{h} = \frac{\mathbf{q} + \mathbf{F}}{2} \quad (3)$$

$$\mathbf{q} = 2\mathbf{h} - \mathbf{F} \quad (4)$$

$$\mathbf{n} = \sqrt{\lambda_2} \mathbf{p}_1 \quad (8)$$

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2\mathbf{u}^T \mathbf{n}}, \quad e = 1 \quad (9)$$

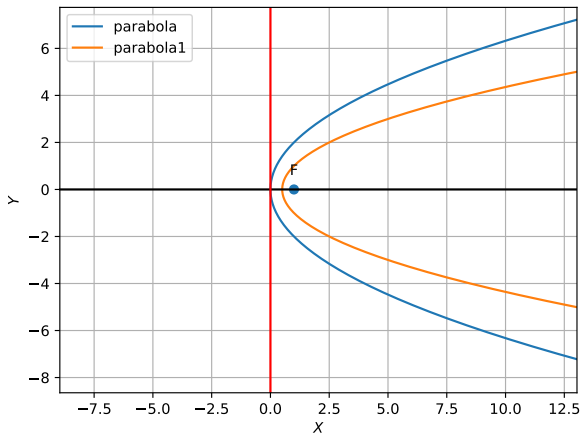
where the eigen values and eigen vectors are,

$$\lambda_1 = 0, \quad \lambda_2 = 1 \quad (10)$$

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (11)$$

substituting values in eq(8) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (12)$$



substituting values in eq(9),

$$c = \frac{a^2 - a^2}{2(-a \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

$$c = 0 \quad (13)$$

put n and c in directrix equation we get,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (14)$$

Therefore, the locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 - 4ax = 0$ is another parabola $y^2 - 2ax + a^2 = 0$ with directrix $x = 0$.

3 Plot

4 Software

We can get the parallel equation of given equation and the plot of two equations by executing the following code:

https://github.com/sivaparvathi-tungala/fwc_module_1/tree/main/conic