PROBABILITY

T SIVA PARVATHI - FWC22089

13.4.5 [1] Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(a) number greater than 4

(b) six appears on at least one die

Solution: Given that a die tossed two times,

Variable	Values	Description
n	2	Number of tosses of a die
X_1	{1,2,3,4,5,6}	1st toss outcomes of a die
X_2	{1,2,3,4,5,6}	2st toss outcomes of a die
p_1	$\frac{1}{3}$	$Pr(X > 4)$
q_1	$1 - p_1$	$Pr(X \leq 4)$
p_2	$\frac{1}{6}$	$Pr(X = 6)$
q_2	$1 - p_2$	$Pr(X \neq 6)$

Table 2: Variable Description

(a) number greater than 4

Consider each trial results in success or failure. Let $X_i \in \{1, 2, 3, 4, 5, 6\}$ where $i = 1, 2$ be the random variables representing the outcome for each die toss.

p_1 and q_1 are the probability of success and failure respectively.

$$p_1 = \frac{1}{3} \tag{13.4.1.1}$$

$$q_1 = 1 - p_1 = \frac{2}{3} \tag{13.4.1.2}$$

In n Bernoulli trials with k success and $(n-k)$ failures, the probablity of k success in n-Bernoulli trials can be given as,

$$p_{X_i}(k) = \begin{cases} {}^{n}C_k {p_1}^k {q_1}^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \tag{13.4.1.3}$$

where, $n = 2$

X_1 and X_2 are independent events, so the desired outcome is

$$X = X_1 + X_2 \tag{13.4.1.4}$$

[1] Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

Probability distribution of getting number greater than 4 is,

$$p_X(k) = \begin{cases} \frac{4}{9}, & k = 0 \\ \frac{4}{9}, & k = 1 \\ \frac{1}{9}, & k = 2 \end{cases} \tag{13.4.1.5}$$

where,

k=0, not getting a favourable outcome on either die

k=1, getting a favourable outcome on one die

k=2, getting a favourable outcome on both die

(b) six appears on at least one die

p_2 and q_2 are the probability of success and failure respectively.

$$p_2 = \frac{1}{6} \tag{13.4.2.6}$$

$$q_2 = 1 - p_2 = \frac{5}{6} \tag{13.4.2.7}$$

In n Bernoulli trials with k success and $(n-k)$ failures, the probablity of k success in n-Bernoulli trials can be given as,

$$p_{X_i}(k) = \begin{cases} {}^nC_k {p_2}^k {q_2}^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \tag{13.4.2.8}$$

where, $n = 2$

X_1 and X_2 are independent events, so the desired outcome is

$$X = X_1 + X_2 \tag{13.4.2.9}$$

Probability distribution of getting six on atleast one die is,

$$p_X(k) = \begin{cases} \frac{25}{36}, & k = 0 \\ \frac{10}{36}, & k = 1 \\ \frac{1}{36}, & k = 2 \end{cases} \tag{13.4.2.10}$$