PROBABILITY

T SIVA PARVATHI - FWC22089

13.4.5 [1] Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(a) number greater than 4

(b) six appears on at least one die

Solution: Given that a die tossed two times,

Consider each trial results in success or failure. Let X_i where $i = 1, 2$ be the random variables representing the outcome for each die toss.

Variable	Values	Description
n	2	Number of tosses of a die
p_1	$\frac{1}{3}$	Probability of getting number >4
q_1	$1 - p_1$	Probability of not getting number >4
p_2	$\frac{1}{6}$	probability of getting six on a die
q_2	$1 - p_2$	probability of not getting six on a die
X_1	{1,2,3,4,5,6}	possible outcomes of 1st toss of a die
X_2	{1,2,3,4,5,6}	possible outcomes of 2nd toss of a die

Table 2: Variable Description

p and q are the probability of success and failure respectively.

$$p_1 = \frac{1}{3} \tag{13.4.2.1}$$

$$q_1 = 1 - p_1 = \frac{2}{3} \tag{13.4.2.2}$$

Similarly,

$$p_2 = \frac{1}{6} \tag{13.4.2.3}$$

$$q_2 = 1 - p_2 = \frac{5}{6} \tag{13.4.2.4}$$

(a) number greater than 4

In n Bernoulli trials with k success and $(n-k)$ failures, the probablity of k success in n- Bernoulli trials can be given as

$$\Pr\left(X_i = k\right) = \begin{cases} {}^nC_k p^k q^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{13.4.1.5}$$

[1]Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

where, $n = 2$

$$X = X_1 + X_2 \tag{13.4.1.6}$$

$$p_X(k) = {}^nC_k {p_1}^k {q_1}^{n-k}, 0 \le k \le 2, n = 2 \tag{13.4.1.7}$$

Probability distribution of getting number greater than 4 is,

$$p_X(k) = \begin{cases} \frac{4}{9}, & k = 0 \\ \frac{4}{9}, & k = 1 \\ \frac{1}{9}, & k = 2 \end{cases} \tag{13.4.1.8}$$

(b) six appears on at least one die

In n Bernoulli trials with k success and $(n-k)$ failures, the probablity of k success in n- Bernoulli trials can be given as

$$\Pr(X_i = k) = \begin{cases} {}^nC_k p^k q^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \tag{13.4.2.9}$$

where, $n = 2$

$$X = X_1 + X_2 \tag{13.4.2.10}$$

$$p_X(k) = {}^nC_k {p_2}^k {q_2}^{n-k}, 0 \le k \le 2, n = 2 \tag{13.4.2.11}$$

Probability distribution of getting six on atleast one die is,

$$p_X(k) = \begin{cases} \frac{25}{36}, & k = 0 \\ \frac{10}{36}, & k = 1 \\ \frac{1}{36}, & k = 2 \end{cases} \tag{13.4.2.12}$$