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Modified multiscale entropy for short-term time series analysis

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HIGHLIGHTS

- A Modified Multiscale Entropy (MMSE) algorithm is proposed in this study.
- The MMSE is an algorithm for measuring the complexity.
- Compared with MSE, MMSE is more precise and reliable for short-term time series.

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ABSTRACT

Multiscale entropy (MSE) is a prevalent algorithm used to measure the complexity of a time series. Because the coarse-graining procedure reduces the length of a time series, the conventional MSE algorithm applied to a short-term time series may yield an imprecise estimation of entropy or induce undefined entropy. To overcome this obstacle, the modified multiscale entropy (MMSE) was developed. The coarse-graining procedure was replaced with a moving-average procedure and a time delay was incorporated for constructing template vectors in calculating sample entropy. For conducting short-term time series analysis, this study shows that the MMSE algorithm is more reliable than the conventional MSE.

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1. Introduction

In the past few decades, many studies have attempted to develop entropy-estimation algorithms, such as Shannon entropy [1], Kolmogorov entropy [2], spectral entropy [3], SVD entropy [4], wavelet entropy [5], permutation entropy [6], approximate entropy [7], and sample entropy (SampEn) [8], to quantify the complexity of various time series. However, the results of most algorithms for estimating entropy are not always associated with complexity. A good example of this is that the SampEn of white noise is higher than that of $1/f$ noise, a case against Fogedby's research [9]. Costa [10] proposed multiscale entropy (MSE) to calculate SampEn over a range of scales to represent the complexity of a time series. By using the MSE method, the result is consistent with the result of Fogedby. The MSE method has been applied to quantify the complexity of many physiological signals [11,12] and vibration signals [13]. The works demonstrate the effectiveness of the MSE algorithm for the analysis of the complex time series.

Although previous studies have shown the successful use of MSE, its reliability remains questionable for short-term time series analysis. The MSE method incorporates two procedures: (1) the representation of the dynamics of a system on different time scales is derived by conducting a coarse-graining procedure; and (2) the regularities of the coarse-grained

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time series are quantified by applying SampEn with unity delay. The coarse-graining procedure shortens the length of time series, and SampEn may yield an imprecise estimation of entropy or induce undefined entropy when the time series is too short. Therefore, the conventional MSE method cannot provide a reliable analysis for short-term time series. In this study, we proposed a modified MSE (MMSE) algorithm to overcome the obstacles in the short-term time series analysis. The effectiveness of the MMSE algorithm is evaluated through two synthetic noise signals and a real vibration data set provided by Case Western Reserve University (CWRU) [14].

2. Methods

2.1. Sample entropy

Let $\mathbf{x} = \{x_1\ x_2\ \cdots\ x_N\}$ represent a time series of length N. The SampEn algorithm proposed by Richman [8] and modified by Govindan [15] can be written as follows:

(1) Construct the ith template vector with dimension m by using the following equation:

$$\mathbf{x}_i^m(\delta) = \left\{x_i \quad x_{i+\delta} \quad \cdots \quad x_{i+(m-1)\delta}\right\}, \quad 1 \le i \le N - m\delta \tag{1}$$

where the parameter δ represents the time delay between the successive components of the template vector [15].

(2) Calculate the Euclidean distance d_{ij}^m for each pair of template vectors $\left(\mathbf{x}_i^m, \mathbf{x}_j^m\right)$ by using infinity norm:

$$d_{ij}^m(\delta) = \left\|\mathbf{x}_i^m(\delta) - \mathbf{x}_j^m(\delta)\right\|_\infty, \quad 1 \le i, j \le N - m\delta,\ j > i + \delta. \tag{2}$$

We call $\left(\mathbf{x}_i^m, \mathbf{x}_j^m\right)$ a matched vector pair when $d_{ij}^m(\delta) \le r$ holds, where r is a pre-defined tolerance threshold.

(3) Let $n(m, \delta, r)$ represent the total number of matched vector pairs of dimension m. Increase $n(m, \delta, r)$ by one when $d_{ij}^m(\delta) \le r$ holds.

(4) Repeat Steps 1–3 for the template vectors reconstructed in the $m + 1$ dimension. Calculate the total number of matched vector pairs of dimension $m + 1$, $n(m + 1, \delta, r)$.

(5) The SampEn is then calculated by using the following equation:

$$\mathrm{SampEn}(\mathbf{x}, m, \delta, r) = -\ln \frac{n(m + 1, \delta, r)}{n(m, \delta, r)}. \tag{3}$$

Let $N_t(m + 1, \delta)$ represent the total number of template vectors of length $m + 1$. To ensure that the template vectors of length $m + 1$, $\mathbf{x}_i^{m+1}(\delta)$, are all defined, the following equation should hold:

$$N_t(m + 1, \delta) = N - m\delta. \tag{4}$$

In the SampEn algorithm, $N_t(m, \delta)$ must be equal to $N_t(m + 1, \delta)$; that is,

$$N_t(m, \delta) = N_t(m + 1, \delta) = N - m\delta \equiv N_\delta. \tag{5}$$

To exclude the self matches [8] and avoid the temporal correlation effect [15], the constraint $j > i+\delta$ in Eq. (2) is imposed in the SampEn algorithm. Therefore, the total number of paired template vectors, N_c, which are determined in Steps 2–3, is given as

$$N_c = \frac{N_\delta \times (N_\delta - 1)}{2} - \left((\delta - 1) \times \left(N_\delta - \frac{\delta}{2}\right)\right). \tag{6}$$

If δ is set to be unity, Eq. (6) can be simplified to

$$N_c|_{\delta=1} = \frac{(N - m - 1) \times (N - m)}{2}. \tag{7}$$

Eq. (7) is consistent with the formula obtained by Richman [8].

When the length of time series, N, is too small, the total number of paired template vectors, N_c, is also small. Therefore, the following two outcomes may occur: (1) $n(m, \delta, r) = 0$, which indicates that no regularity of the time series has been detected; and (2) $n(m + 1, \delta, r) = 0$, which corresponds to a conditional probability of zero. These two conditions result in an undefined SampEn [8]. In most cases, the SampEn yields an imprecise estimation when N_c is not adequately large. Generally, to obtain a reasonable SampEn, N is suggested to be in the range of 10^m–30^m [16].

2.2. Multiscale entropy

This algorithm was developed by Costa [10] to quantify the complexity of time series for a range of scales. Two procedures of the MSE algorithm are briefly described as follow:

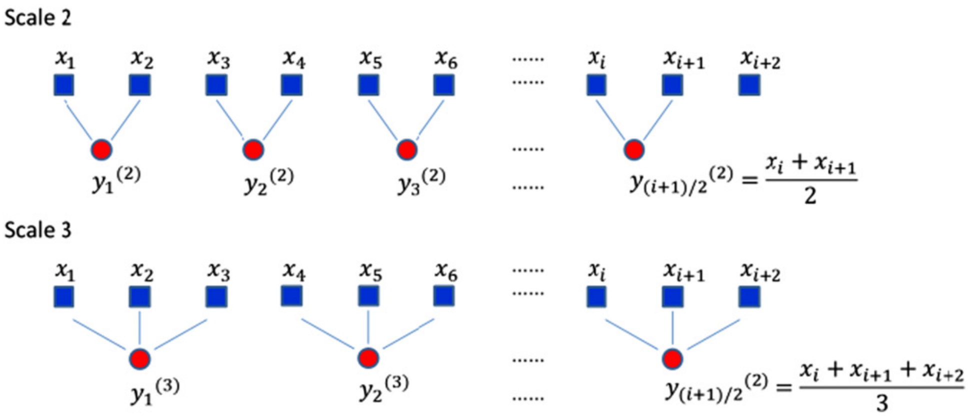

Fig. 1. Schematic illustration of the coarse-grained procedure.
Source: Modified from Ref. [10].

(1) To obtain the coarse-grained time series at a scale factor of τ, the original time series $\mathbf{x}$ is divided into disjointed windows of length τ, and the data points are averaged inside each window. In other words, the coarse-grained time series at a scale factor of τ, $\mathbf{y}^\tau$, can be constructed according to the following equation [10]:

$$y_j^\tau = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} x_i, \quad 1 \le j \le \left\lfloor \frac{N}{\tau} \right\rfloor, \tag{8}$$

where $\lfloor\cdot\rfloor$ denotes the floor function.
(2) SampEn with unity delay is calculated for each coarse-grained time series and then plotted as the function of the scale factor τ.

In the conventional MSE method, the tolerance threshold, r, of SampEn is set at a certain percentage of the standard deviation of the original time series and remains constant for all scales [17]. SampEn with unity delay ($\delta = 1$) is used to calculate the entropies of the coarse-grained time series for all scales.

Fig. 1 illustrates the coarse-graining procedure. The length of the coarse-grained series at a scale factor of τ is reduced to $\lfloor N/\tau \rfloor$. Therefore, the number of template vectors used to calculate SampEn at a scale factor of τ is given by

$$N_\delta(\tau)|_{\delta=1} = \lfloor N/\tau \rfloor - m \equiv N_\tau. \tag{9}$$

If $N_{c,\tau}$ represents the total number of paired template vectors that is checked at a scale factor of τ, then the following equation holds:

$$N_{c,\tau} = \frac{(N_\tau - 1) \times (N_\tau)}{2}. \tag{10}$$

From the perspective of signal processing, the coarse-graining procedure in MSE consists of two parts: (1) averaging the data within a window of length τ to reduce the high frequency component; and (2) the averaged data is downsampled by a factor of τ [15]. The downsampling procedure reduces the length of the coarse-grained time series substantially and may induce an imprecise, even undefined SampEn at large time scales. In the following subsection, we propose a modified MSE (MMSE) algorithm to overcome this obstacle.

2.3. Modified multiscale entropy

The proposed MMSE method incorporates two procedures: (1) the representation of the dynamics of a system on different time scales is derived by conducting a moving-averaging procedure; and (2) the regularities of the moving-averaged time series at a scale factor of τ are quantified by applying SampEn with a time delay τ.

1. Let $\mathbf{z}^\tau$ represent the moving-averaged time series at a scale factor of τ, constructed according to the following equation:

$$z_j^\tau = \frac{1}{\tau} \sum_{i=j}^{j+\tau-1} x_i, \quad 1 \le j \le N - \tau + 1. \tag{11}$$

2. SampEn with time delay τ is calculated for the moving-averaged time series, $\mathbf{z}^\tau$. This SampEn is defined as the modified MSE (MMSE) value of the time series under analysis at a scale factor of τ. That is,

$$\text{MMSE}(\mathbf{x}, m, \tau, r) = \text{SampEn}(\mathbf{z}^\tau, m, \delta = \tau, r). \tag{12}$$

Table 1
Means of SampEn values of noise signals at a scale of factor 20 ($\tau = 20$) obtained using MSE and MMSE with various data lengths.

	Method	Data length							
		500	1000	1500	2000	2500	5000	10 000	30 000
White noise	MSE	1.126	1.027	1.015	1.021	1.018	1.013	1.011	1.008
	MMSE	1.060	1.029	1.020	1.019	1.018	1.012	1.011	1.009
$1/f$ noise	MSE	[a]	[a]	[a]	1.931	1.912	1.843	1.798	1.750
	MMSE	2.113	1.960	1.926	1.890	1.875	1.838	1.802	1.752

[a] Represents undefined entropy.

The length of the moving-averaged time series at a scalar of τ is given by

$$\tilde{N}(\tau) = N - \tau + 1. \tag{13}$$

According to Eqs. (4)–(5), in the MMSE method, the number of template vectors used to calculate SampEn with time delay τ is given by

$$\tilde{N}_\delta(\tau) = N - \tau + 1 - m\tau = N - (m+1)\tau + 1 \equiv \tilde{N}_\tau. \tag{14}$$

The total number of paired template vectors that must be determined in SampEn is given as:

$$\tilde{N}_{c,\tau} = \frac{\tilde{N}_\tau \times \left(\tilde{N}_\tau - 1\right)}{2} - \left((\tau - 1) \times \left(\tilde{N}_\tau - \frac{\tau}{2}\right)\right). \tag{15}$$

According to Eqs. (9) and (14), the template vectors used in the MMSE algorithm are greater than those used in the MSE algorithm. Large numbers of template vectors used in SampEn can avoid obtaining an undefined entropy value and perform a more precise estimation. In case of $N = 1000$, $m = 2$, and $\tau = 20$, there are 941 template vectors used in the MMSE algorithm, but only 48 template vectors used in the MSE algorithm. In this case, according to Eqs. (10) and (15), the total number of paired template vectors used in MMSE is 376.4 times than that used in MSE. Therefore, for short-term time series analysis, the proposed MMSE algorithm can provide a more reliable analysis than the MSE algorithm. However, the computational cost of the MMSE is more than that of MSE since more paired template vectors are used in the MMSE method.

3. Experiments

This section presents two synthetic noise signals, white noise and $1/f$ noise, as well as a real vibration data set, which were applied to evaluate the effectiveness of the proposed MMSE method.

3.1. White noise and $1/f$ noise

According to Costa, the SampEn curve of white noise decreases monotonically as the time scale factor increases, whereas the SampEn curve of $1/f$ noise remains constant for all time scales [10]. Fig. 2 shows the SampEn curves of white and $1/f$ noises with 1000 data points obtained using both the MSE and MMSE methods. Fig. 2(a) shows how the SampEn curve of white noise obtained through MMSE decreases monotonically, whereas the SampEn curve obtained through MSE fluctuates. Regarding $1/f$ noise, Fig. 2(b) shows how the fluctuation of the SampEn curve obtained through MMSE is smaller than that obtained through MSE. Furthermore, the SampEn values of $1/f$ noise calculated using MMSE remain nearly constant for all time scales. These numerical simulations indicate that, compared with the conventional MSE method, the proposed MMSE method can detect more effectively the behaviors of white noise and $1/f$ noise and provide a more accurate estimation.

To investigate the statistical behaviors of the MSE and MMSE methods further, we applied MSE and MMSE analyses of white and $1/f$ noises. There were 200 independent noise samples used in each simulation, and each noise sample contained 500 data points. Figs. 3 and 4 present the simulation results. The error bar at each scale indicates the standard deviation (SD) of an entropy value that was calculated from 200 independent noise signals. For white noise (see Fig. 3), the means of the entropy values obtained using MSE and MMSE are nearly equal, but the SD of MMSE is less than that of MSE. This result indicates that the MMSE and MSE methods are nearly equivalent statistically; however, the MMSE method can provide a more accurate estimate than the MSE method can. For $1/f$ noise (see Fig. 4), the MSE method induced undefined entropies when the time scale was larger than 5, whereas the MMSE method was applicable for all time scales.

To investigate the effect of data lengths on the MSE and MMSE methods, the SampEns of white noise and $1/f$ noise at a time scale of 20 ($\tau = 20$) were calculated with several data lengths ($N = 500$, 1000, 1500, 2000, 2500, 5000, 10 000, 20 000, and 30 000). In each case, there were 200 independent noise samples used in simulations. The means and standard deviations (SDs) of the simulations are summarized in Tables 1 and 2, respectively.

Table 1 shows that, for white noise, the means of SampEns obtained using MSE and MMSE are nearly the same when the data length is larger than 2000. The SampEns converge to a theoretical value as the length of data increases. Suppose that the theoretical SampEn can be obtained by applying MSE/MMSE on white noise with 30 000 data points. When $N = 500$, the

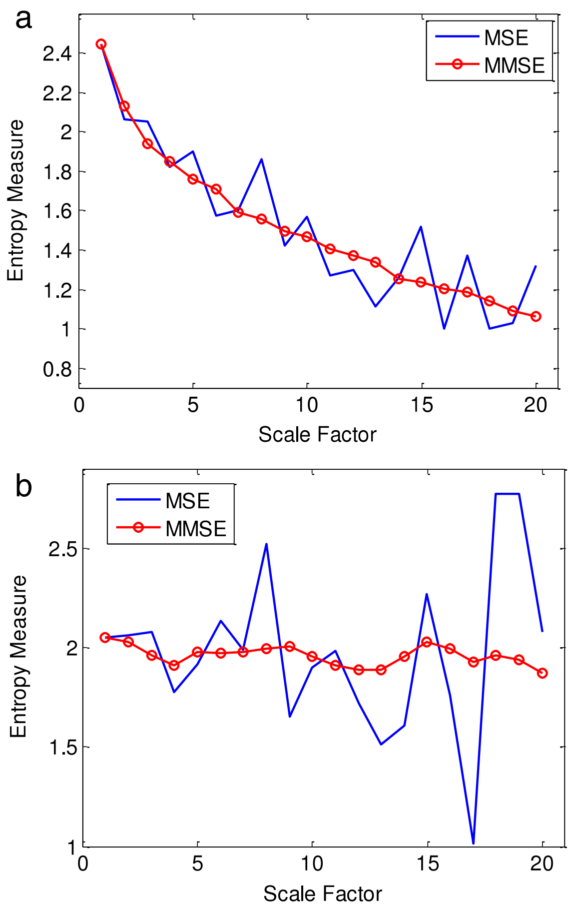

Fig. 2. MSE and MMSE analyses of (a) white noise and (b) $1/f$ noise.

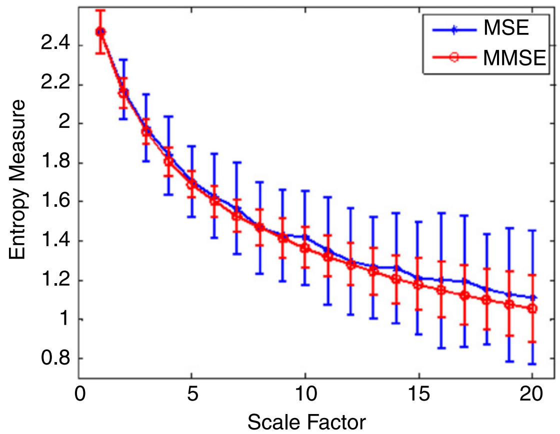

Fig. 3. MSE and MMSE analyses of 200 simulated white noises with 500 data points.

corresponding estimation errors of SampEn for the MSE and MMSE methods are 11.7% and 5%. In the case of $1/f$ noise, the means of SampEns obtained using these two methods are nearly equal when the data length is larger than 5000. However, the MSE method induces undefined entropy, whereas the MMSE method still provides reliable estimation when the data length is smaller than 1500.

Table 2 shows that, for both white noise and $1/f$ noise, the SDs of SampEns obtained using the MMSE method are all smaller than those obtained using the MSE method. To achieve the same SD when analyzing white noise, the data length used in MMSE was half of that used in MSE. For example, to achieve the performance level with SD $= 0.097$, the data lengths

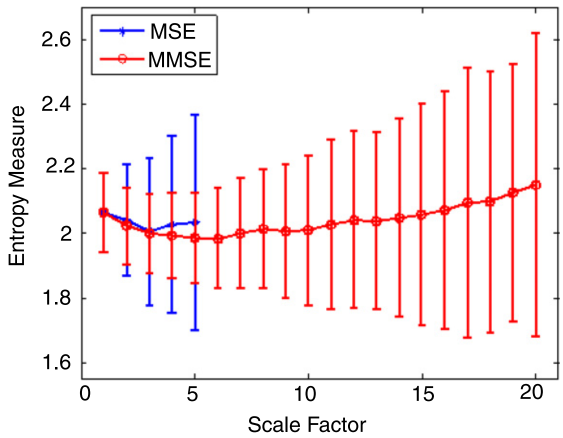

Fig. 4. MSE and MMSE analyses of 200 simulated $1/f$ noises with 500 data points.

Table 2
Standard deviations of SampEn values of noise signals at a scale of factor 20 ($\tau = 20$) obtained using MSE and MMSE with various data lengths.

	Method	Data length							
		500	1000	1500	2000	2500	5000	10 000	30 000
White noise	MSE	0.365	0.165	0.118	0.097	0.083	0.056	0.037	0.021
	MMSE	0.155	0.096	0.077	0.061	0.054	0.038	0.026	0.015
$1/f$ noise	MSE	[a]	[a]	[a]	0.315	0.252	0.141	0.086	0.068
	MMSE	0.399	0.197	0.144	0.122	0.114	0.080	0.068	0.065

[a] Represents undefined entropy.

used in MMSE and MSE were 1000 and 2000, respectively. To achieve the same SD when analyzing $1/f$ noise, the data length used in MMSE is nearly one third of that used in MSE. For example, to achieve the performance level with SD $= 0.144$, the data lengths used in MMSE and MSE were 1500 and 5000, respectively.

Based on the data listed in Tables 1 and 2, we conclude that the proposed MMSE method can avoid obtaining undefined entropy and provide a more precise estimation of entropy compared with the MSE method when analyzing a short-term time series.

3.2. Real vibration data

To validate the utility of the MMSE algorithm for real data, experimental analysis on bearing faults was conducted. The bearing fault data used in this paper were obtained from the Case Western Reserve University (CWRU) Bearing Data Center [14]. The time-domain vibration signals of bearings were collected from the normal case, the ball fault case, the inner race fault case, and the outer race fault case at the 6 o'clock position. Bearing conditions of the experiments included normal states, ball faults, inner race faults, and outer race faults located at the 3 o'clock, 6 o'clock, and 12 o'clock positions, which were respectively at 0°, 270°, and 90° on the front section diagram of the bearing. The sampling frequency of the vibration signal was 48 kHz, and the shaft rotating speeds of the motor were 1730, 1750, and 1772 rpm. The bearings were seeded with single point faults by using electro-discharge machining. The fault diameters used in the experiments were 7 and 14 mils.

In the following experiments, the vibration signals were divided into several non-overlapping segments with a specified data length of $N = 1000$. Each non-overlapping segment was regarded as one sample; that is, a sample was a time series containing 1000 data points collected from different time intervals. There were approximately 485 samples for each bearing condition. We then calculated the MSE and MMSE values up to scale 20 for each sample. Therefore, the dimension of the samples in the feature space is 20. Fig. 5 shows partially measured vibration signals under different bearing conditions. Fig. 6 shows the means and SDs of the corresponding features under various bearing conditions extracted using the MSE and MMSE methods. For every bearing condition, the means of the features' values obtained using MMSE are extremely close to those obtained using MSE, whereas fewer SDs of the features' values are achieved using the MMSE method. This is consistent with the analysis results of white and $1/f$ noises.

When the MSE and MMSE methods were used as feature extractors for bearing fault classification, the features extracted by MMSE were expected to have a higher distinguishability. To demonstrate this fact, a multi-class support vector machine (SVM) [18] was used as a classifier to distinguish bearing conditions. The SVM algorithm used in these experiments was C-SVC, and the regularization parameter C was set at 100. A radial basis function (RBF) was chosen as the kernel function of the C-SVC algorithm. The kernel parameter γ of RBF was set to be equal to the reciprocal of the number of features (i.e., $\gamma = 1/20$). In each experiment, 50% of the samples for each bearing condition were randomly selected for training the SVM

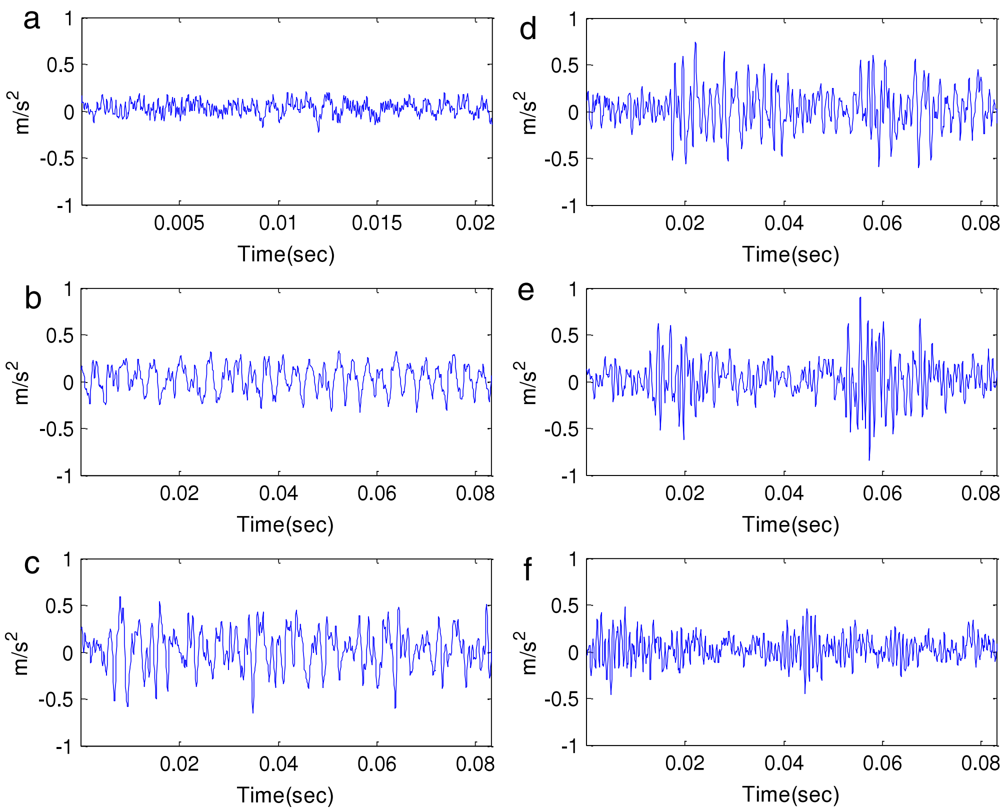

Fig. 5. Bearing vibration data in the time domain (1730 rpm, 7 mils): (a) normal state; (b) ball fault; (c) inner race fault; (d) outer race fault (3 o'clock position); (e) outer race fault (6 o'clock position); (f) outer race fault (12 o'clock position).

Table 3
Accuracy of prediction results using MSE and MMSE ($N = 1000$).

Rotating speed (rpm)	1730		1750		1772	
Fault diameter (mils)	7	14	7	14	7	14
MSE	87.17%	92.44%	94.19%	91.51%	93.23%	88.80%
MMSE	**97.64**%	**99.19**%	**98.84**%	**99.59**%	**99.00**%	**98.58**%

model. The remaining 50% of the samples were then used for a classification test. The average accuracy of prediction for each experiment was obtained by repeating these procedures 200 times. Table 3 lists the results.

Table 3 indicates that, compared with MSE, MMSE as a feature extractor can increase the accuracy of prediction for all cases. When the rotating speed is 1750 rpm and the fault diameter is 7 mils, the accuracy of prediction is 94.19% for MSE, whereas it became 98.84% for MMSE. The accuracy of prediction is improved by approximately 4.65%. When the rotating speed is 1730 rpm and the fault diameter is 7 mils, the accuracy of prediction is improved by approximately 10.47%. For all cases, the accuracies of prediction are larger than 97.64% when MMSE is used. The experimental results show that the MMSE is an effective feature extractor for detecting bearing faults.

4. Conclusion

MSE is an effective method for measuring the complexity of time series, such as white noise, $1/f$ noise, and the vibration signals of the bearings in our experiments. However, MSE may yield an imprecise estimation of entropy or induce undefined entropy when the time series is too short. This paper presents the MMSE algorithm to overcome this obstacle. Compared with MSE, MMSE can estimate entropy more precisely and avoid inducing undefined entropy for analyzing short-term time series. In applying bearing fault detection, MMSE can extract the features exhibiting high distinguishability. However, for long-term time series, the improvement in precision for entropy estimation is not significant, and the computational cost of the MMSE is more than that of the MSE. Therefore, the MMSE method is not suitable for direct application in analyzing long-term time series. To make a trade-off between precision and computational cost in working with long-term time series, one can use MSE to perform small-scale analysis of the time series, and use MMSE to conduct a large-scale analysis.

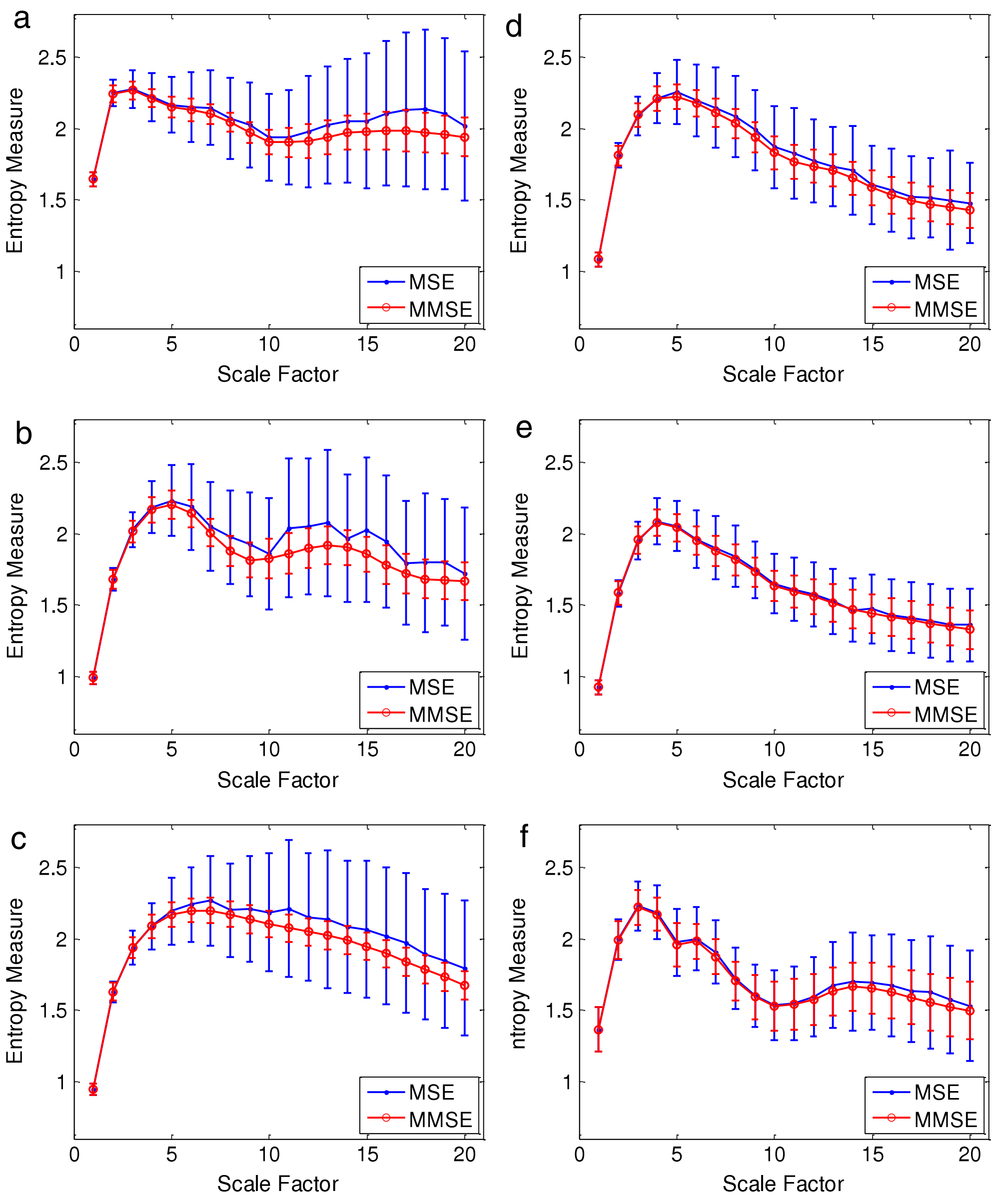

Fig. 6. MSE and MMSE results of bearing vibration data (1730 rpm, 7 mils): (a) normal state; (b) ball fault; (c) inner race fault; (d) outer race fault (3 o'clock position); (e) outer race fault (6 o'clock position); and (f) outer race fault (12 o'clock position).

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Appendix. The Matlab code for the modified multiscale entropy algorithm

```
function E = MMSE(data, scale)
r = 0.15*std(data);
for i = 1:scale
        buf = movingaverage(data, i);
        E(i) = SampEn2(buf,r,i);
end
% moving average procedure. See Eq. (9)
% data: input signal; s: scale numbers ;
```

```
function data = movingaverage(data,s)
   N = length(data);
for i = 1:N − s + 1
        data(i) = mean(data(i:i + s − 1));
end
%function to calculate sample entropy with delay.
function entropy = SampEn(data,r,delay)
N = length(data);
Nn = 0;
Nd = 0;
for i = 1:N − 3*delay
        for j = i + delay:1:N − 2*delay
            if abs(data(i)-data(j))<r && abs(data(i + delay)-data(j + delay))<r
                Nn = Nn + 1;
                if abs(data(i + 2*delay)-data(j + 2*delay))<r
                    Nd = Nd + 1;
                end
            end
        end
end
entropy = −log(Nd/Nn);
```

References

[1] C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27 (1948) 379–423.
[2] A.N. Kolmogorov, Entropy per unit time as a metric invariant of automorphism, Dokl. Russ. Acad. Sci. 124 (1959) 754–755.
[3] G.E. Powell, I.C. Percival, Spectral entropy method for distinguishing regular and irregular motion of Hamiltonian-systems, J. Phys. A: Math. Gen. 12 (1979) 2053–2071.
[4] S.J. Roberts, W. Penny, I. Rezek, Temporal and spatial complexity measures for electroencephalogram based brain–computer interfacing, Med. Biol. Eng. Comput. 37 (1999) 93–98.
[5] O.A. Rosso, S. Blanco, J. Yordanova, V. Kolev, A. Figliola, M. Schurmann, E. Basar, Wavelet entropy: a new tool for analysis of short duration brain electrical signals, J. Neurosci. Methods 105 (2001) 65–75.
[6] C. Bandt, B. Pompe, Permutation entropy: a natural complexity measure for time series, Phys. Rev. Lett. 88 (2002).
[7] S. Pincus, Approximate entropy (Apen) as a complexity measure, Chaos 5 (1995) 110–117.
[8] J.S. Richman, J.R. Moorman, Physiological time-series analysis using approximate entropy and sample entropy, Am. J. Physiol. Heart Circ. Physiol. 278 (2000) H2039–H2049.
[9] H.C. Fogedby, On the phase–space approach to complexity, J. Stat. Phys. 69 (1992) 411–425.
[10] M. Costa, A.L. Goldberger, C.K. Peng, Multiscale entropy analysis of complex physiologic time series, Phys. Rev. Lett. 89 (2002) 068102.
[11] R.A. Thuraisingham, G.A. Gottwald, On multiscale entropy analysis for physiological data, Physica A 366 (2006) 323–332.
[12] M. Costa, C.K. Peng, A.L. Goldberger, J.M. Hausdorff, Multiscale entropy analysis of human gait dynamics, Physica A 330 (2003) 53–60.
[13] S.D. Wu, C.W. Wu, S.G. Lin, C.C. Wang, K.Y. Lee, Time series analysis using composite multiscale entropy, Entropy 15 (2013) 1069–1084.
[14] Case western reserve university bearing data center website, in, http://csegroups.case.edu/bearingdatacenter/pages/download-data-file.
[15] R.B. Govindan, J.D. Wilson, H. Eswaran, C.L. Lowery, H. Preissl, Revisiting sample entropy analysis, Physica A 376 (2007) 158–164.
[16] Q. Liu, Q. Wei, S.Z. Fan, C.W. Lu, T.Y. Lin, M.F. Abbod, J.S. Shieh, Adaptive computation of multiscale entropy and its application in EEG signals for monitoring depth of anesthesia during surgery, Entropy 14 (2012) 978–992.
[17] M. Costa, A.L. Goldberger, C.K. Peng, Multiscale entropy analysis of biological signals, Phys. Rev. E 71 (2005) 021906.
[18] C.C. Chang, C.J. Lin, LIBSVM: a library for support vector machines, ACM Trans. Intell. Syst. Technol. (TIST) 2 (2011) 27.