Lorentz_ESN

January 2, 2020

1 Predicting Lorentz force using Echo State Neural Network

1.0.1 Importing Required Libraries

[1]:
```
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
import ESN
import pandas as pd
import warnings
warnings.filterwarnings('ignore')
```

1.0.2 Set seed for random weights generator

[2]:
```
def set_seed(seed=None):
    """Making the seed (for random values) variable if None"""

    # Set the seed
    if seed is None:
        import time
        seed = int((time.time()*10**6) % 4294967295)
    try:
        np.random.seed(seed)
    except Exception as e:
        print( "!!! WARNING !!!: Seed was not set correctly.")
        print( "!!! Seed that we tried to use: "+str(seed))
        print( "!!! Error message: "+str(e))
        seed = None
    print( "Seed used for random values:", seed)
    return seed
```

[3]:
```
## Set a particular seed for the random generator (for example seed = 42), or␣
↪use a "random" one (seed = None)
# NB: reservoir performances should be averaged accross at least 30 random␣
↪instances (with the same set of parameters)
seed = 42 #None #42
```

[4]:
```
set_seed(seed) #random.seed(seed)
```

```
Seed used for random values: 42
```

[4]:
```
42
```

[5]:
```
initLen = 100
trainLen = initLen + 1300
testLen = 1400
```

[6]:
```
df =  pd.read_excel(r'C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\Input\Lorentz_Data\Lorentz␣
↪data testing and training.xlsx', index = False)
```

[7]:
```
df.head()
```

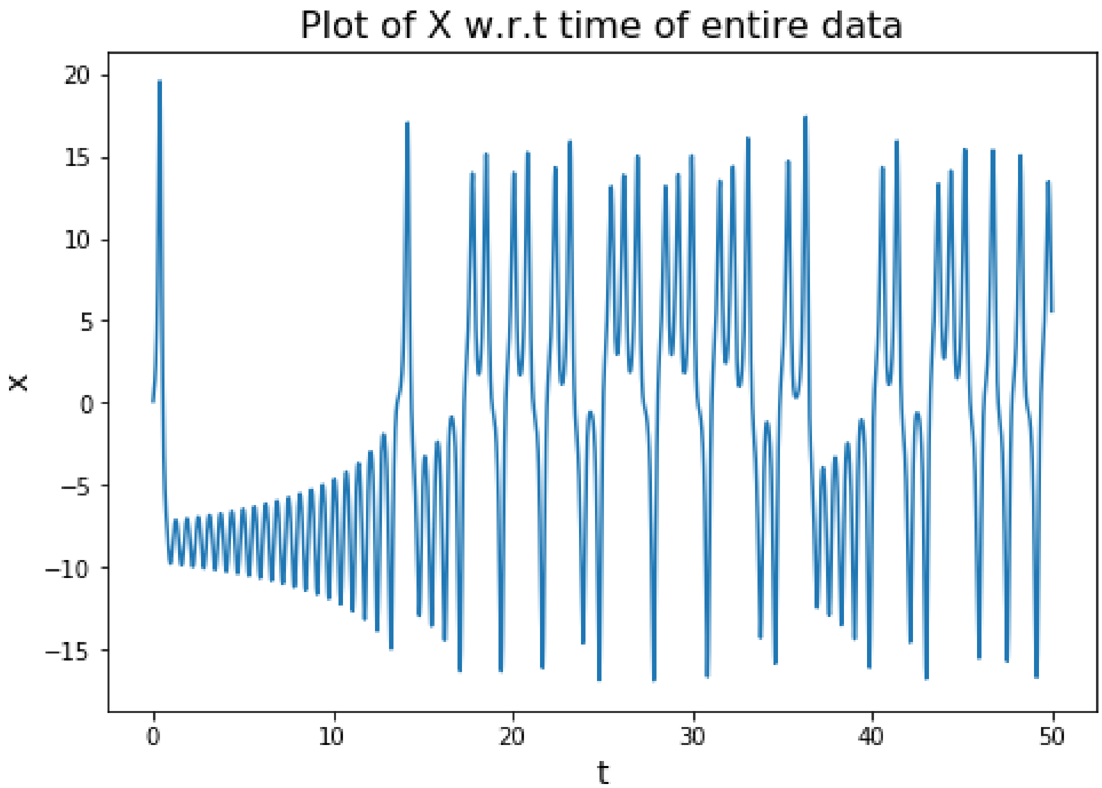
[7]:

	t	x	y	z
0	0.000000	0.100000	1.200000	1.200000
1	0.000457	0.105014	1.200707	1.198596
2	0.000913	0.110008	1.201474	1.197196
3	0.001370	0.114984	1.202302	1.195800
4	0.001827	0.119940	1.203191	1.194409

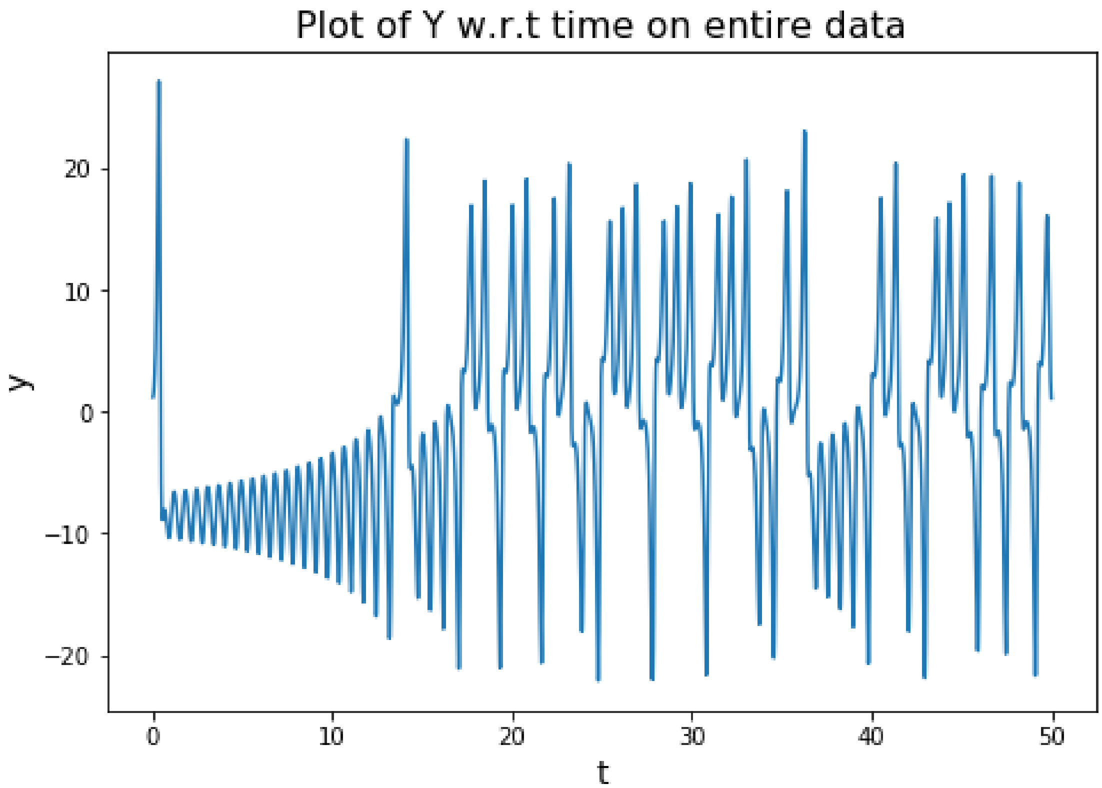
2 EDA

[8]:
```
import os
if not os.path.exists(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data"):
    os.mkdir(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data")
```

[9]:
```
from matplotlib import rcParams
rcParams.update({'figure.autolayout': True})
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(df['t'],df['x'] )
plt.title('Plot of X w.r.t time of entire data',  fontsize=16)
plt.xlabel('t', fontsize = 14)
plt.ylabel('x', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\X_with_Time.
↪png", bbox_inches = "tight")
plt.show()
```

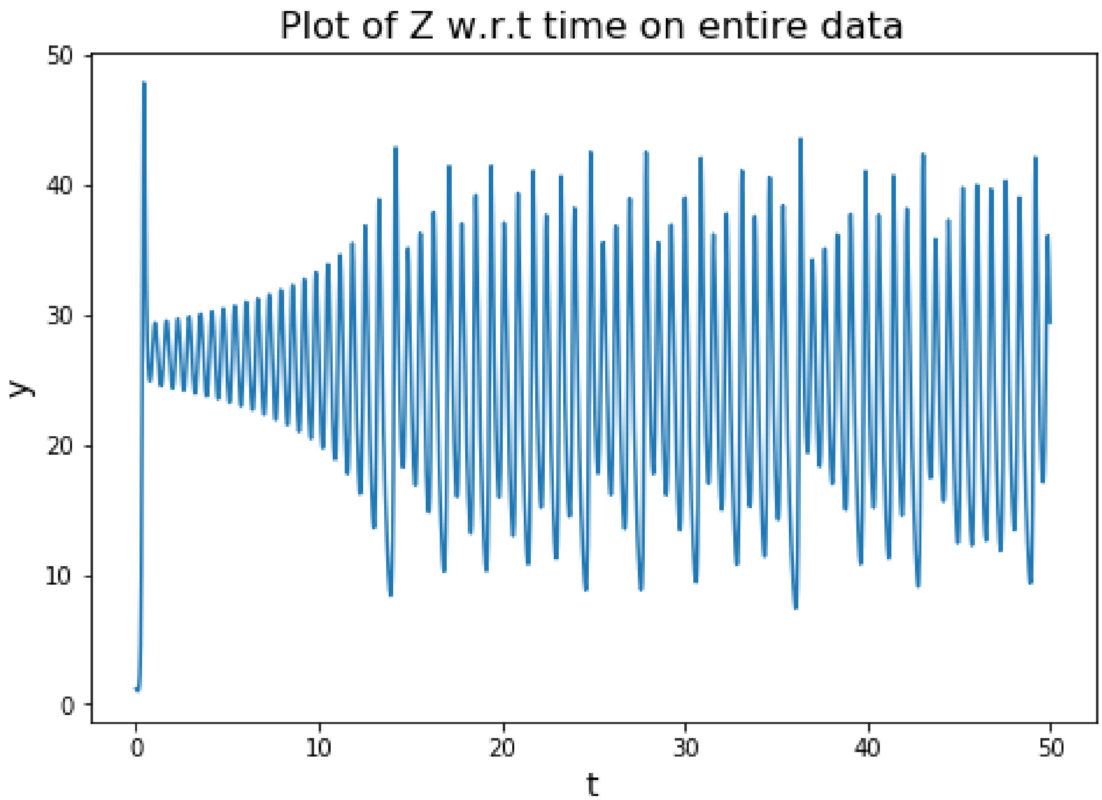


[10]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(df['t'],df['y'] )
plt.title('Plot of Y w.r.t time on entire data',  fontsize=16)
plt.xlabel('t', fontsize = 14)
plt.ylabel('y', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Y_with_Time.
↪png", bbox_inches = "tight")
plt.show()
```

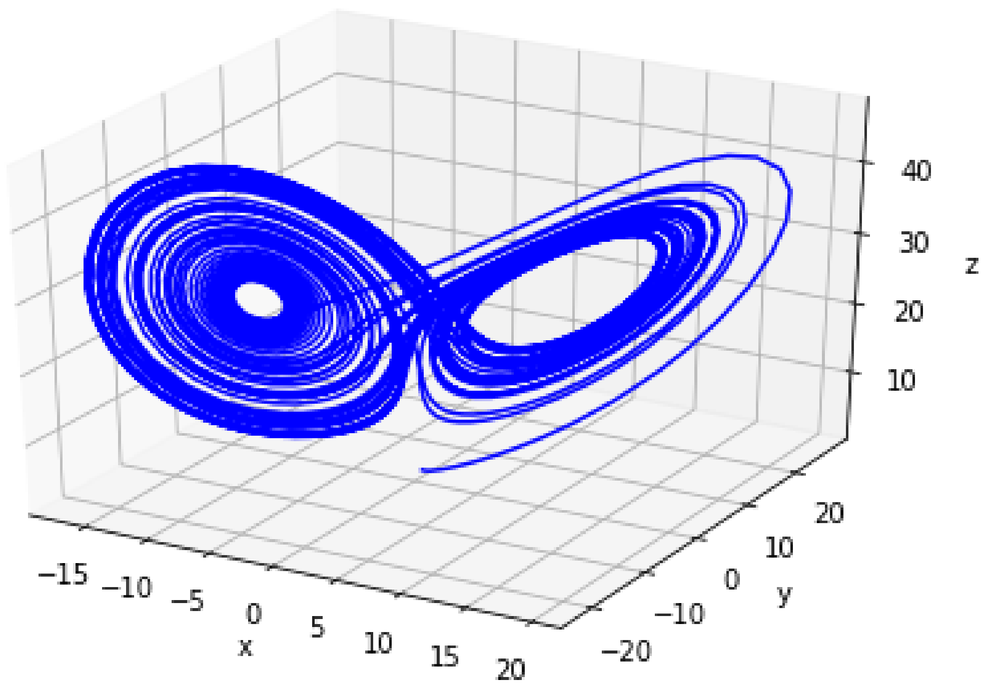


[11]:

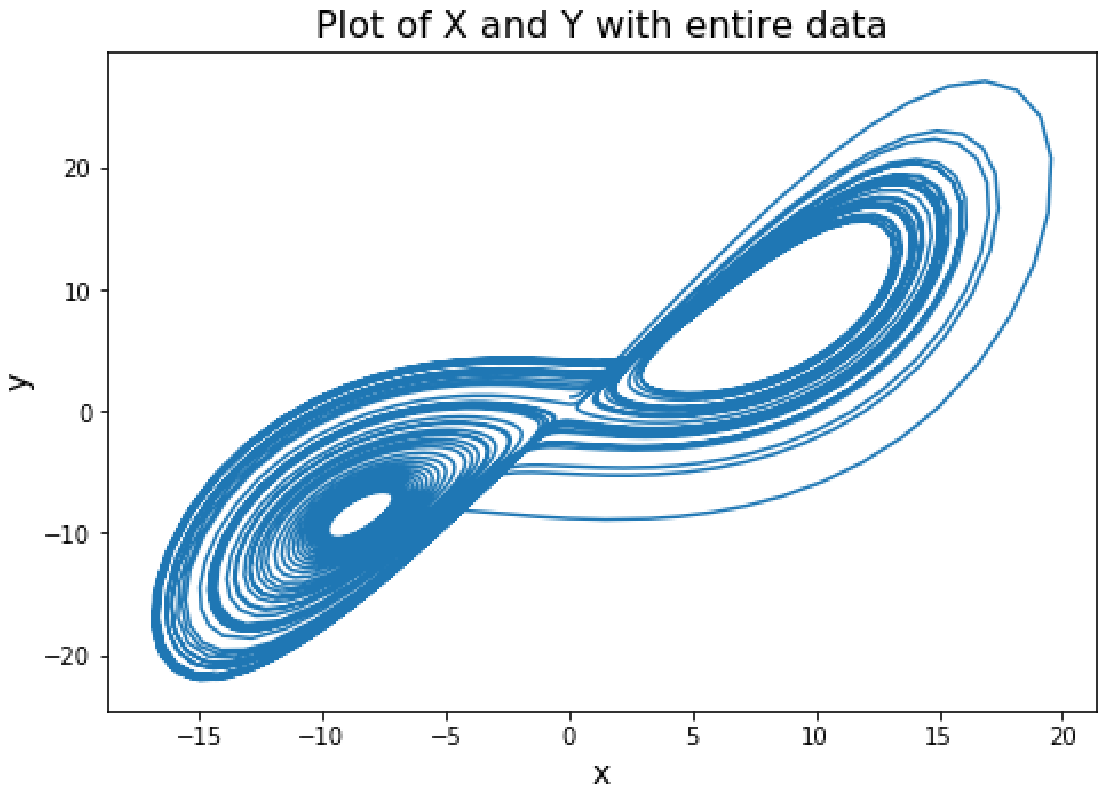
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(df['t'],df['z'] )
plt.title('Plot of Z w.r.t time on entire data',  fontsize=16)
plt.xlabel('t', fontsize = 14)
plt.ylabel('y', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Z_with_Time.
↪png", bbox_inches = "tight")
plt.show()
```



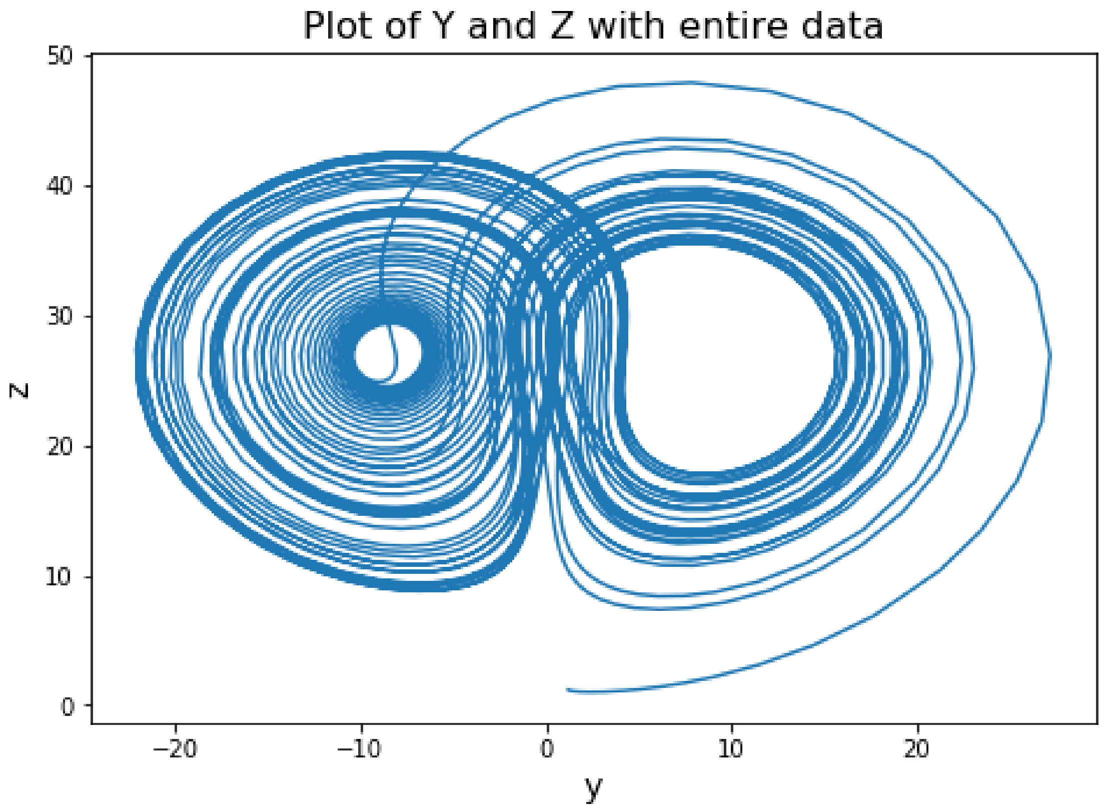
[12]:
```
from mpl_toolkits import mplot3d
fig = plt.figure()
ax = plt.axes(projection="3d")

ax.plot3D(df.x, df.y, df.z, 'blue')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\3D_Plot_X_Y_Z_ent
↪png", bbox_inches = "tight")
plt.show()
```



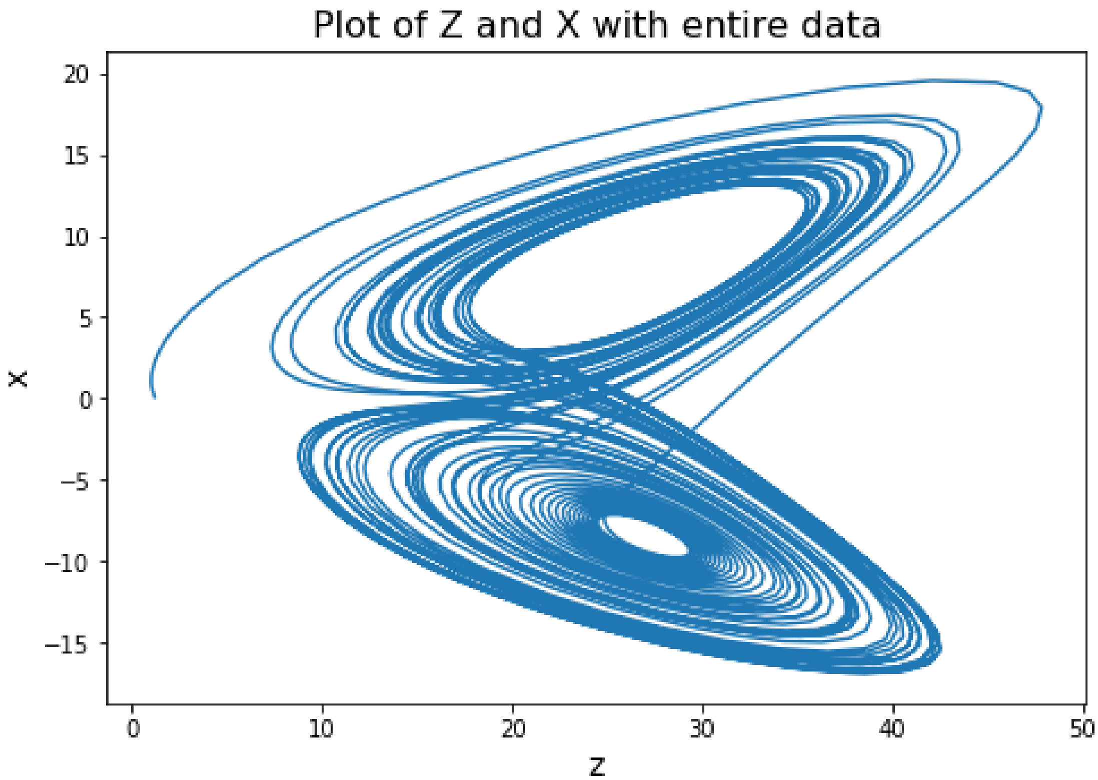
[13]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(df['x'],df['y'] )
plt.title('Plot of X and Y with entire data',  fontsize=16)
plt.xlabel('x', fontsize = 14)
plt.ylabel('y', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
 ↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\X_Y_on_Entire_da
 ↪png", bbox_inches = "tight")
plt.show()
```



[14]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(df['y'],df['z'] )
plt.title('Plot of Y and Z with entire data',  fontsize=16)
plt.xlabel('y', fontsize = 14)
plt.ylabel('z', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Y_Z_on␣
↪Entire_data.png", bbox_inches = "tight")
plt.show()
```



[15]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(df['z'],df['x'] )
plt.title('Plot of Z and X with entire data',  fontsize=16)
plt.xlabel('z', fontsize = 14)
plt.ylabel('x', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Z_X_on␣
↪Entire_data.png", bbox_inches = "tight")
plt.show()
```



2.0.1 Split data for training and testing and creating teaches to train ESN on Input data

[16]:
```
data_in = df[['x','y','z']]
data_T  =df['t']
```

[17]:
```
data_in = np.array(data_in)
data_t = np.array(data_T)
```

[18]:
```
train_in = np.array(data_in[0:trainLen])
train_out = np.array(data_in[0+10:trainLen+10])
test_in = np.array(data_in[trainLen:trainLen+testLen])
test_out = np.array(data_in[trainLen+10:trainLen+testLen+10])
```

[19]:
```
train_in_t = np.array(data_T[0:trainLen])
train_out_t = np.array(data_T[0+10:trainLen])
test_in_t = np.array(data_T[trainLen:trainLen+testLen])
test_out_t = np.array(data_T[trainLen+10:trainLen+testLen+10])
```

[20]:
```
len(test_in_t)
```

[20]: 1400

2.0.2 Modify Parameters to tune ESN for better fit

[21]:
```
n_reservoir = 500 # number of recurrent units
leak_rate = 0.3 # leaking rate (=1/time_constant_of_neurons)
spectral_radius = 1.4 # Scaling of recurrent matrix
input_scaling = 1. # Scaling of input matrix
proba_non_zero_connec_W = 0.2 # Sparsity of recurrent matrix: Perceptage of␣
↪non-zero connections in W matrix
proba_non_zero_connec_Win = 1. # Sparsity of input matrix
proba_non_zero_connec_Wfb = 1. # Sparsity of feedback matrix
regularization_coef =  0.01 #None # regularization coefficient, if None,␣
↪pseudo-inverse is use instead of ridge regression
```

[22]:
```
n_inputs = 3
input_bias = True # add a constant input to 1
n_outputs = 3
```

[23]:
```
N = n_reservoir#100
dim_inp = n_inputs #26
```

2.0.3 Generating weights for input and hidden layers

[24]:
```
### Generating random weight matrices with custom method
W = np.random.rand(N,N) - 0.5
if input_bias:
    Win = np.random.rand(N,dim_inp+1) - 0.5
else:
    Win = np.random.rand(N,dim_inp) - 0.5
Wfb = np.random.rand(N,n_outputs) - 0.5
```

[25]:
```
## delete the fraction of connections given the sparsity (i.e. proba of␣
↪non-zero connections):
mask = np.random.rand(N,N) # create a mask Uniform[0;1]
W[mask > proba_non_zero_connec_W] = 0 # set to zero some connections given by␣
↪the mask
mask = np.random.rand(N,Win.shape[1])
Win[mask > proba_non_zero_connec_Win] = 0
# mask = np.random.rand(N,Wfb.shape[1])
# Wfb[mask > proba_non_zero_connec_Wfb] = 0
```

[26]:
```
## SCALING of matrices
# scaling of input matrix
Win = Win * input_scaling
# scaling of recurrent matrix
```

```
# compute the spectral radius of these weights:
print( 'Computing spectral radius...')
original_spectral_radius = np.max(np.abs(np.linalg.eigvals(W)))
#TODO: check if this operation is quicker: max(abs(linalg.eig(W)[0])) #from␣
↪scipy import linalg
print( "default spectral radius before scaling:", original_spectral_radius)
# rescale them to reach the requested spectral radius:
W = W * (spectral_radius / original_spectral_radius)
print( "spectral radius after scaling", np.max(np.abs(np.linalg.eigvals(W))))
```

```
Computing spectral radius…
default spectral radius before scaling: 2.97582723496286
spectral radius after scaling 1.3999999999999715
```

2.1 Input data dimensions

```
[27]: print('Dimensions of Training data: ', train_in.shape[1])
print('Dimensions of Testing data: ', test_in.shape[1])
```

```
Dimensions of Training data:  3
Dimensions of Testing data:  3
```

2.1.1 Pass Parameters to ESN

```
[28]: reservoir = ESN.ESN(lr=leak_rate, W=W, Win=Win, input_bias=input_bias,␣
↪ridge=regularization_coef, Wfb=None, fbfunc=None)
```

2.2 Input data to reservoir model

```
[29]: internal_trained = reservoir.train(inputs=[train_in], teachers=[train_out],␣
↪wash_nr_time_step=initLen, verbose=False)
output_pred, internal_pred = reservoir.run(inputs=[test_in,], reset_state=False)
errorLen = len(test_out[:]) #testLen #2000
```

2.3 Dimensions of the output data

```
[30]: print('Shape of Output data Dimensions: ', output_pred[0].shape[1])
```

```
Shape of Output data Dimensions:  3
```

2.3.1 Create dataframe for predicted values and test values

```
[31]: import pandas as pd
df_pred = pd.DataFrame(output_pred[0])
```

```
[32]: output_pred[0].shape
```

[32]: (1400, 3)

[33]:
```
test_out = pd.DataFrame(test_out)
```

2.3.2 MSE for X

[34]:
```
## printing errors made on test set
# mse = sum( np.square( test_out[:] - output_pred[0] ) ) / errorLen
# print( 'MSE = ' + str( mse ))
mse_x = np.mean((test_out[0][:] - df_pred[0])**2) # Mean Squared Error: see␣
↪https://en.wikipedia.org/wiki/Mean_squared_error
rmse_x = np.sqrt(mse_x) # Root Mean Squared Error: see https://en.wikipedia.org/
↪wiki/Root-mean-square_deviation for more info
nmrse_mean_x = abs(rmse_x / np.mean(test_out[0][:])) # Normalised RMSE (based␣
↪on mean)
nmrse_maxmin_x = rmse_x / abs(np.max(test_out[0][:]) - np.min(test_out[0][:]))␣
↪# Normalised RMSE (based on max - min)
```

[35]:
```
print("\n**********  MSE and RMSE for Predictions on X  **********")
print("Errors computed over %d time steps" % (errorLen))
print("\nMean Squared error (MSE) for x : \t\t%.4e " % (mse_x) )
print("Root Mean Squared error (RMSE) for x : \t\t%.4e\n " % rmse_x )
print("Normalized RMSE (based on mean) for x : \t%.4e " % (nmrse_mean_x) )
print("Normalized RMSE (based on max - min) for x : \t%.4e " % (nmrse_maxmin_x)␣
↪)
print("*********************************************************\n")
```

```
**********  MSE and RMSE for Predictions on X  **********
Errors computed over 1400 time steps

Mean Squared error (MSE) for x :                1.4905e-01
Root Mean Squared error (RMSE) for x :          3.8606e-01

Normalized RMSE (based on mean) for x :         8.4609e-01
Normalized RMSE (based on max - min) for x :    1.1267e-02
*********************************************************
```

2.3.3 MSE for Y

[36]:
```
## printing errors made on test set
# mse = sum( np.square( test_out[:] - output_pred[0] ) ) / errorLen
# print( 'MSE = ' + str( mse ))
mse_y = np.mean((test_out[1][:] - df_pred[1])**2) # Mean Squared Error: see␣
↪https://en.wikipedia.org/wiki/Mean_squared_error
```

```
rmse_y = np.sqrt(mse_x) # Root Mean Squared Error: see https://en.wikipedia.org/
↪wiki/Root-mean-square_deviation for more info
nmrse_mean_y = abs(rmse_y / np.mean(test_out[1][:])) # Normalised RMSE (based␣
↪on mean)
nmrse_maxmin_y = rmse_y / abs(np.max(test_out[1][:]) - np.min(test_out[1][:]))␣
↪# Normalised RMSE (based on max - min)
```

[37]:
```
print("\n**********  MSE and RMSE for Predictions on Y  **********")
print("Errors computed over %d time steps" % (errorLen))
print("\nMean Squared error (MSE) for Y : \t\t%.4e " % (mse_y) )
print("Root Mean Squared error (RMSE) for Y : \t\t%.4e\n " % rmse_y )
print("Normalized RMSE (based on mean) for Y : \t%.4e " % (nmrse_mean_y) )
print("Normalized RMSE (based on max - min) for Y : \t%.4e " % (nmrse_maxmin_y)␣
↪)
print("**********************************************************\n")
```

```
**********  MSE and RMSE for Predictions on Y  **********
Errors computed over 1400 time steps

Mean Squared error (MSE) for Y :                4.9937e-01
Root Mean Squared error (RMSE) for Y :          3.8606e-01

Normalized RMSE (based on mean) for Y :         8.5515e-01
Normalized RMSE (based on max - min) for Y :    8.5652e-03
**********************************************************
```

2.3.4 MSE for Z

[38]:
```
## printing errors made on test set
# mse = sum( np.square( test_out[:] - output_pred[0] ) ) / errorLen
# print( 'MSE = ' + str( mse ))
mse_y = np.mean((test_out[2][:] - df_pred[1])**2) # Mean Squared Error: see␣
↪https://en.wikipedia.org/wiki/Mean_squared_error
rmse_y = np.sqrt(mse_x) # Root Mean Squared Error: see https://en.wikipedia.org/
↪wiki/Root-mean-square_deviation for more info
nmrse_mean_y = abs(rmse_y / np.mean(test_out[2][:])) # Normalised RMSE (based␣
↪on mean)
nmrse_maxmin_y = rmse_y / abs(np.max(test_out[2][:]) - np.min(test_out[2][:]))␣
↪# Normalised RMSE (based on max - min)
```

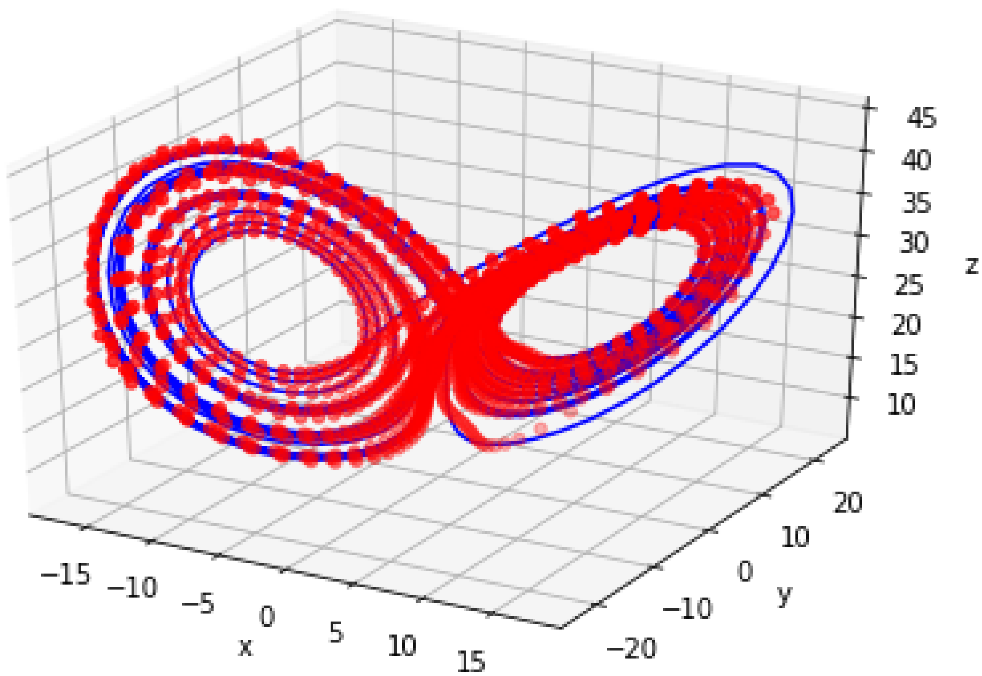
[39]:
```
print("\n**********  MSE and RMSE for Predictions on Z  **********")
print("Errors computed over %d time steps" % (errorLen))
print("\nMean Squared error (MSE) for Z : \t\t%.4e " % (mse_y) )
print("Root Mean Squared error (RMSE) for Z : \t\t%.4e\n " % rmse_y )
print("Normalized RMSE (based on mean) for Z : \t%.4e " % (nmrse_mean_y) )
```

```
print("Normalized RMSE (based on max - min) for Z : \t%.4e " % (nmrse_maxmin_y)␣
 ↪)
print("*******************************************************\n")
```

```
**********  MSE and RMSE for Predictions on Z  **********
Errors computed over 1400 time steps

Mean Squared error (MSE) for Z :                7.2045e+02
Root Mean Squared error (RMSE) for Z :          3.8606e-01

Normalized RMSE (based on mean) for Z :         1.5890e-02
Normalized RMSE (based on max - min) for Z :    1.0675e-02
*********************************************************
```

3 3D Plot with predicted and actural values

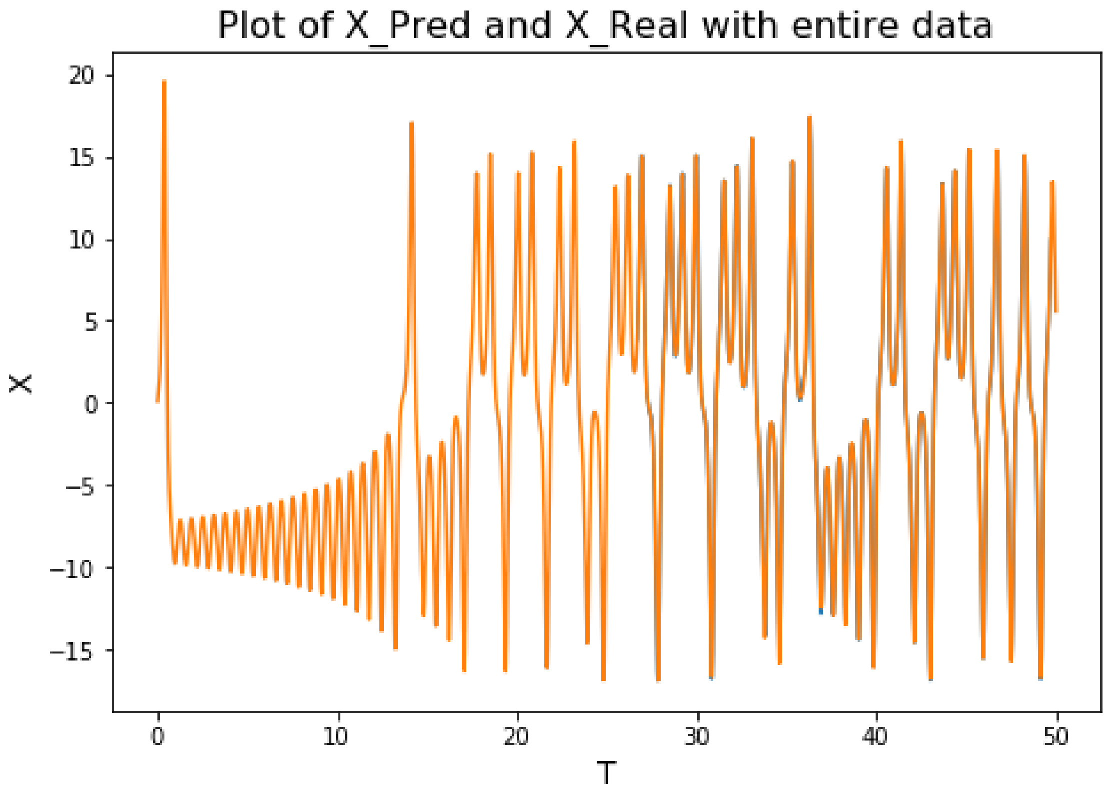
[40]:
```
from mpl_toolkits import mplot3d
fig = plt.figure()
ax = plt.axes(projection="3d")
ax.plot3D(test_out[0], test_out[1], test_out[2], 'blue')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.scatter3D(df_pred[0], df_pred[1], df_pred[2], c='r')
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
 ↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\3D_Plot_X_Y_Z_en
 ↪png", bbox_inches = "tight")
plt.show()
```



[41]:
```
len(df_pred[0])
```

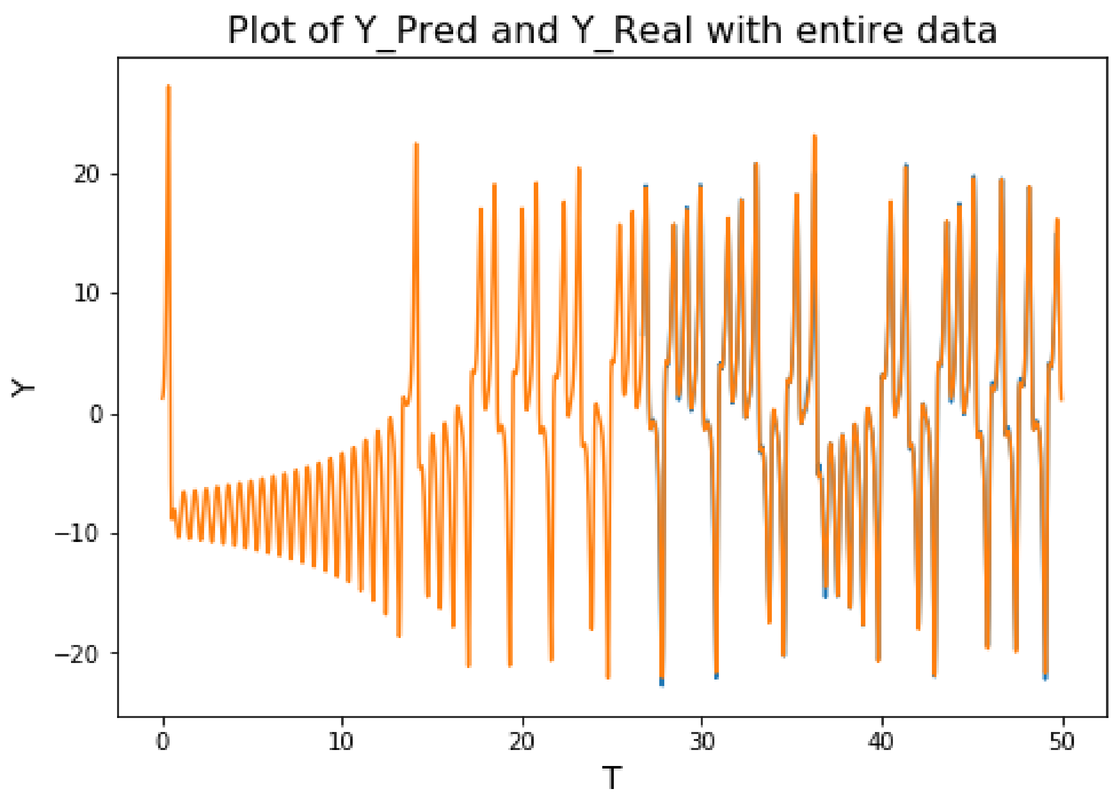
[41]: 1400

[42]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_pred[0])
plt.title('Plot of X_Pred and X_Real with entire data',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('X', fontsize = 14)
ax.plot(df['t'],df['x'])
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\X_Pred_X_Real_on
↪Entire_data.png", bbox_inches = "tight")
plt.show()
```



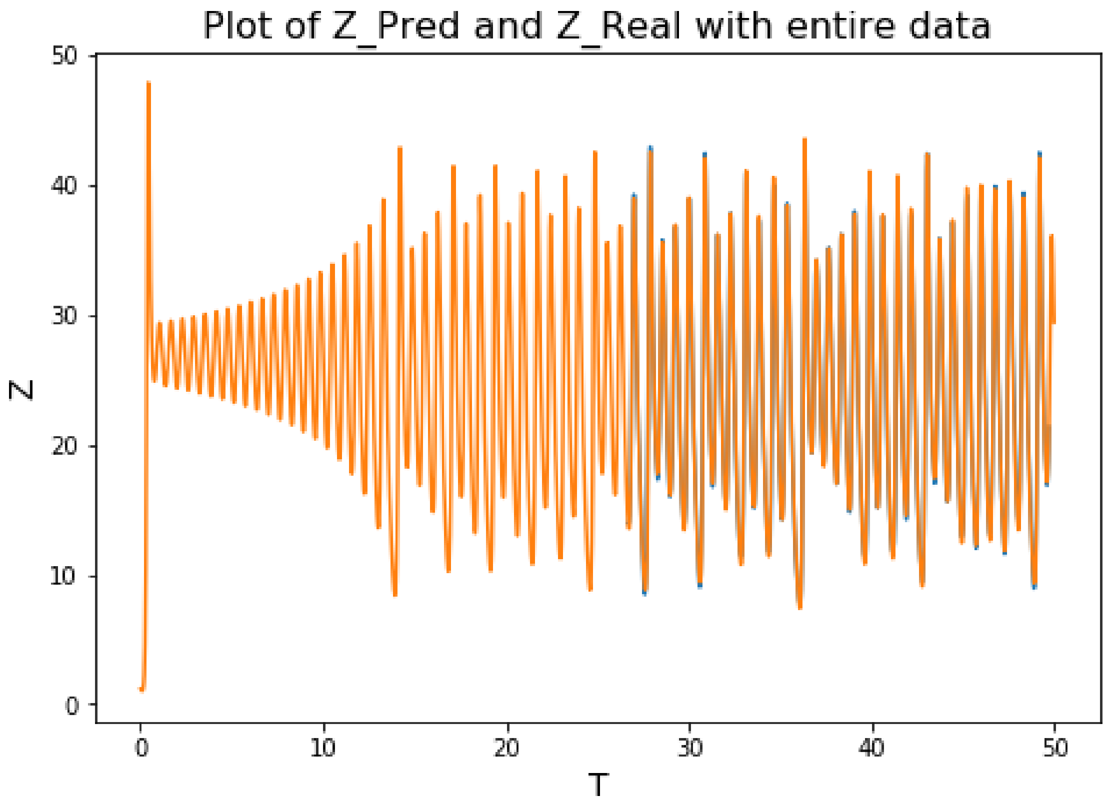
[43]:

```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_pred[1])
plt.title('Plot of Y_Pred and Y_Real with entire data',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('Y', fontsize = 14)
ax.plot(df['t'],df['y'])
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
 ↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Y_Pred_Y_Real_on
 ↪Entire_data.png", bbox_inches = "tight")
plt.show()
```

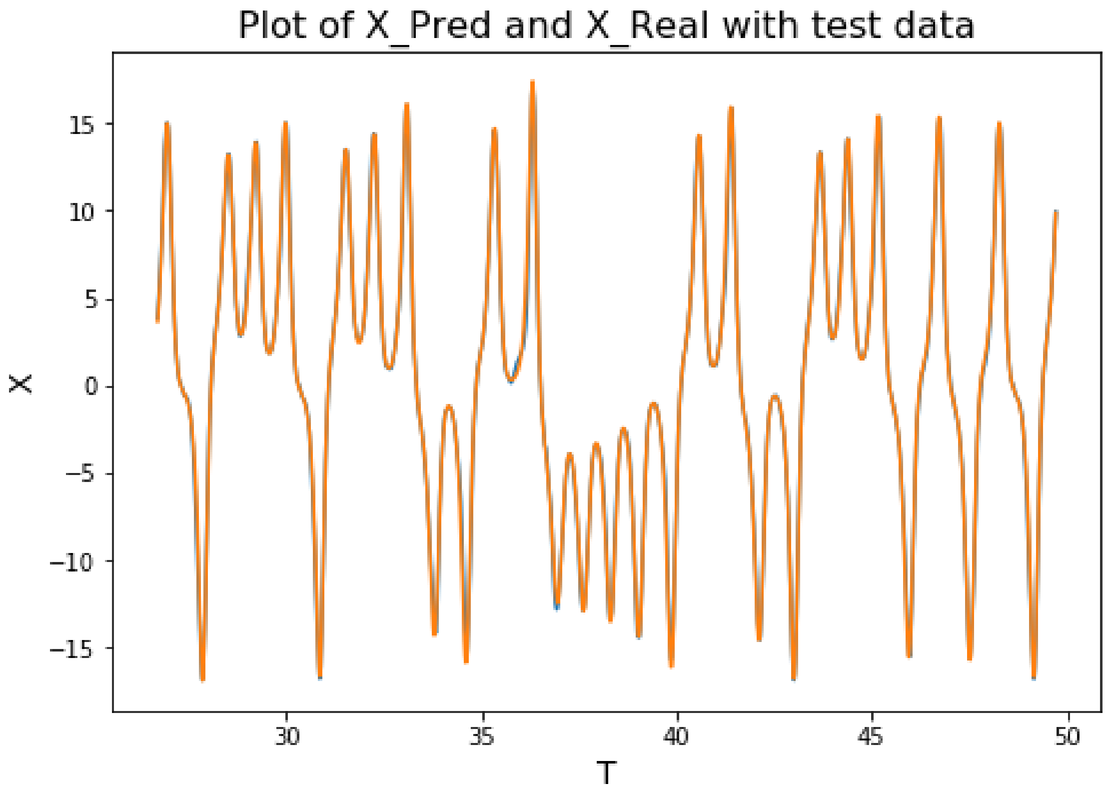


[44]:

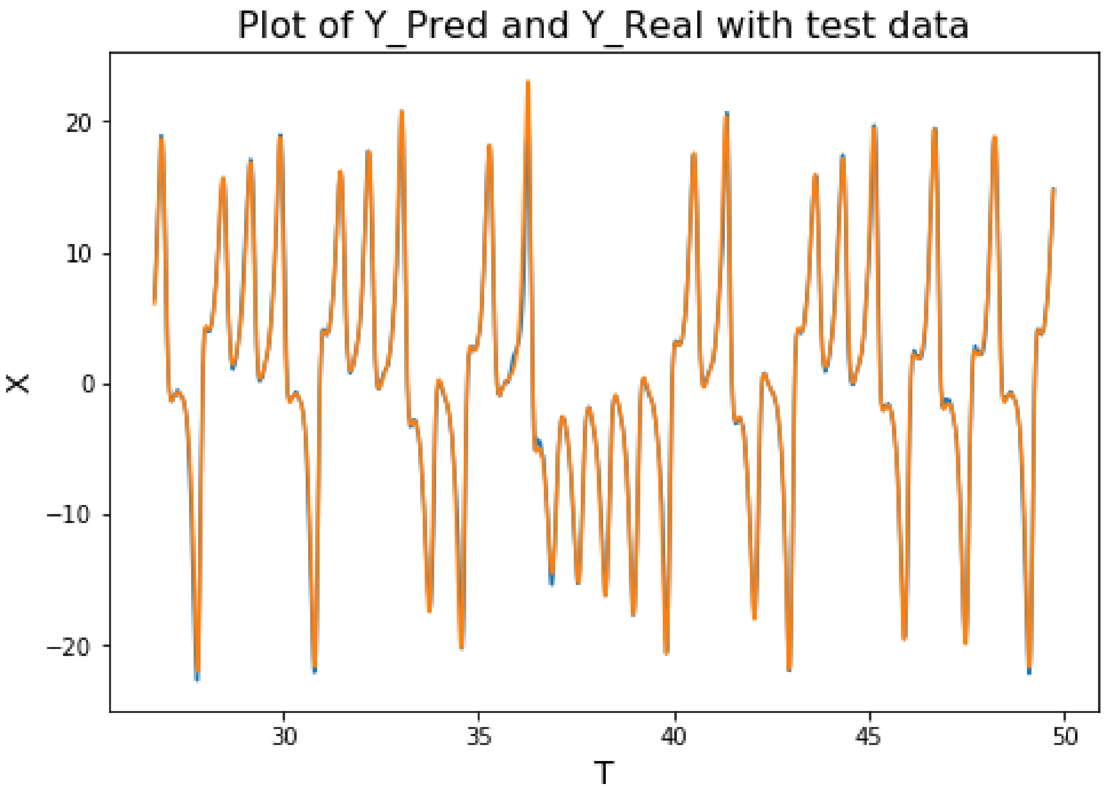
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_pred[2])
plt.title('Plot of Z_Pred and Z_Real with entire data',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('Z', fontsize = 14)
ax.plot(df['t'],df['z'])
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
 ↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Z_Pred_Z_Real_on
 ↪Entire_data.png", bbox_inches = "tight")
plt.show()
```



[45]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_pred[0])
plt.title('Plot of X_Pred and X_Real with test data',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('X', fontsize = 14)
ax.plot(test_out_t,test_out[0])
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
 ↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\X_pred_vs_X_Real
 ↪png", bbox_inches = "tight")
plt.show()
```

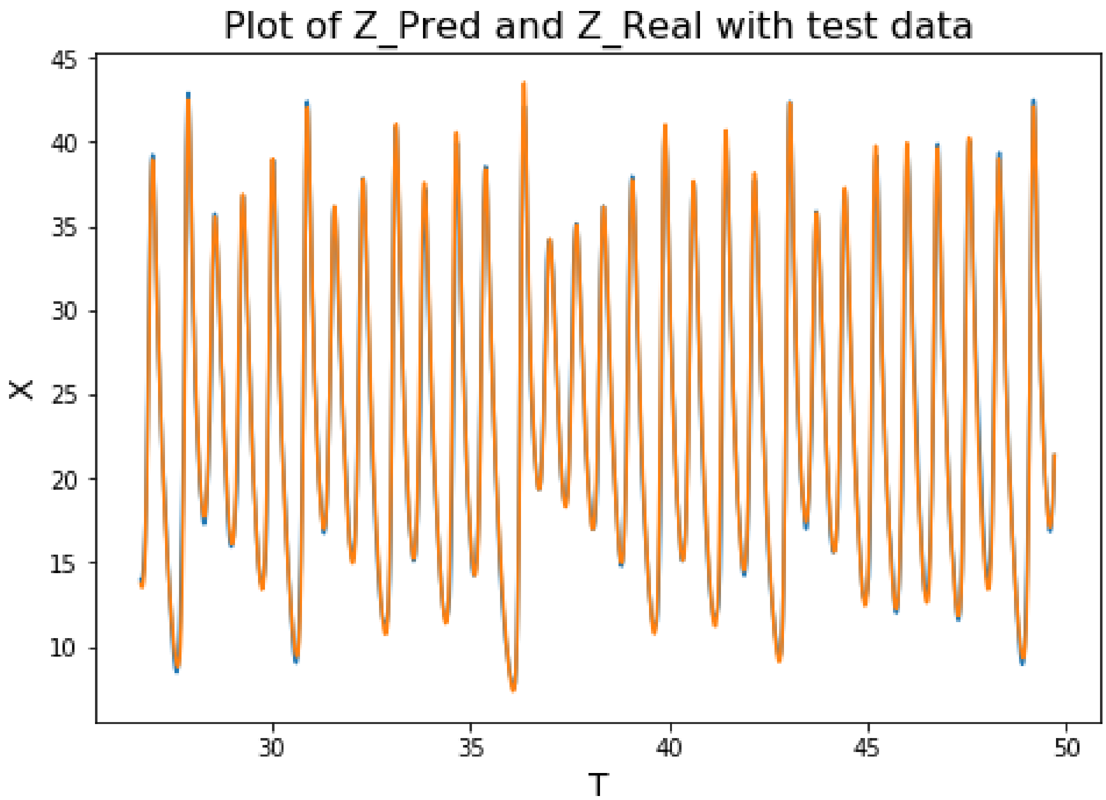



```
[46]: fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_pred[1




                          ])
plt.title('Plot of Y_Pred and Y_Real with test data',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('X', fontsize = 14)
ax.plot(test_out_t,test_out[1])
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
 ↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Y_pred_vs_Y_Real
 ↪png", bbox_inches = "tight")
plt.show()
```



[47]:

```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_pred[2])
plt.title('Plot of Z_Pred and Z_Real with test data',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('X', fontsize = 14)
ax.plot(test_out_t,test_out[2])
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
 ↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Z_pred_vs_Z_Real
 ↪png", bbox_inches = "tight")
plt.show()
```



4 Plotting Local Error from predicted and actual values

[48]:
```
df_local_error = pd.DataFrame()
```

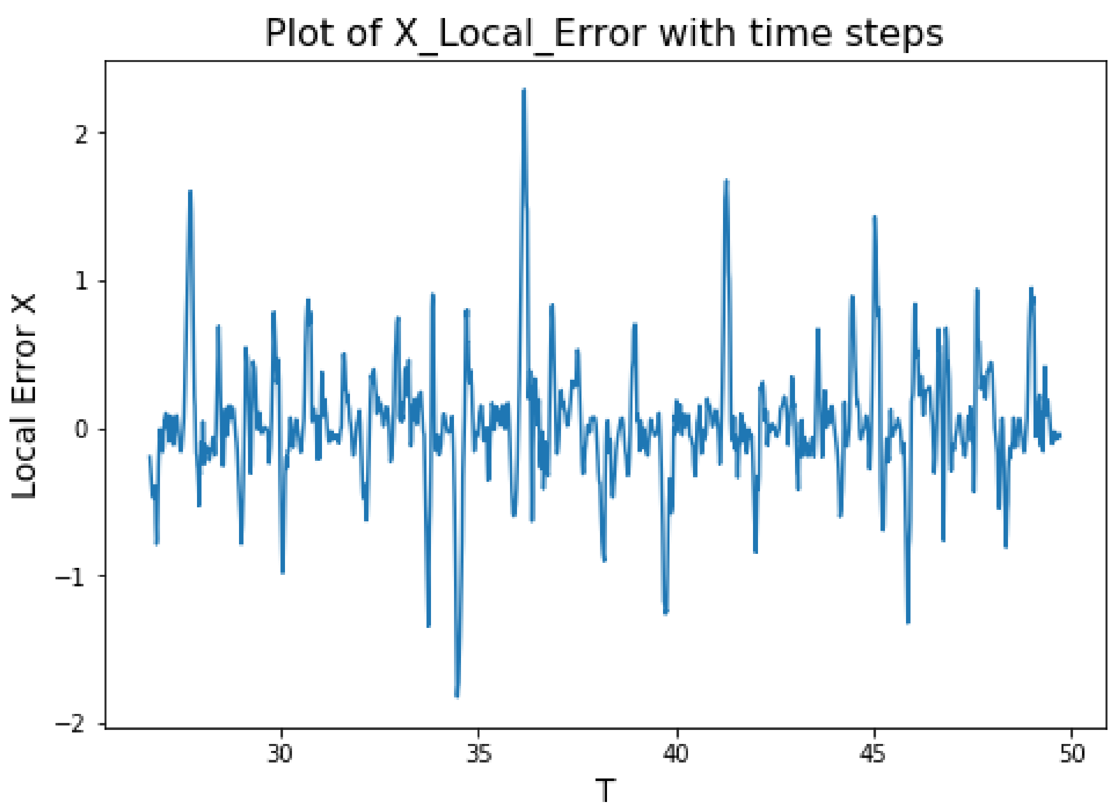
[49]:
```
df_local_error['X_Local_Error'] = test_out[0] - df_pred[0]
df_local_error['Y_Local_Error'] = test_out[1] - df_pred[1]
df_local_error['Z_Local_Error'] = test_out[2] - df_pred[2]
```

[50]:
```
df_local_error.describe()
```

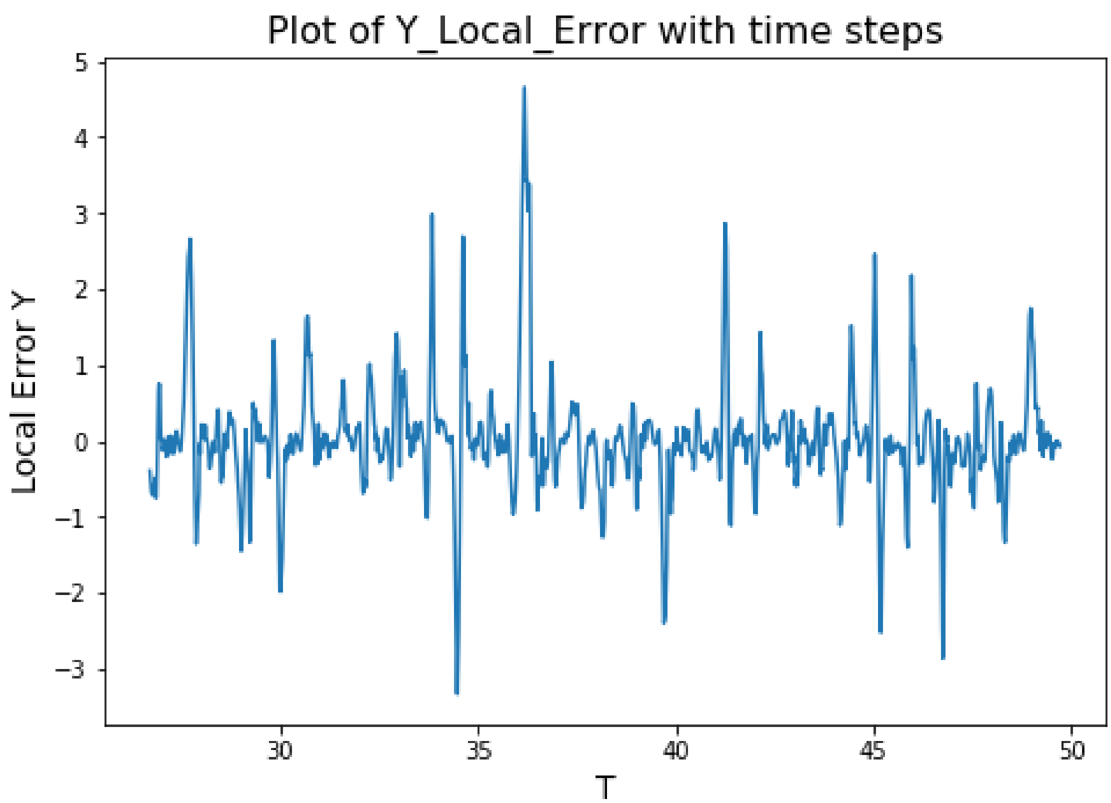
[50]:

	X_Local_Error	Y_Local_Error	Z_Local_Error
count	1400.000000	1400.000000	1400.000000
mean	0.030690	0.026805	0.084786
std	0.384979	0.706405	0.806227
min	-1.826151	-3.332411	-3.343765
25%	-0.119432	-0.205247	-0.294067
50%	0.008579	-0.012049	0.024615
75%	0.146694	0.177234	0.373649
max	2.291610	4.657201	4.877346

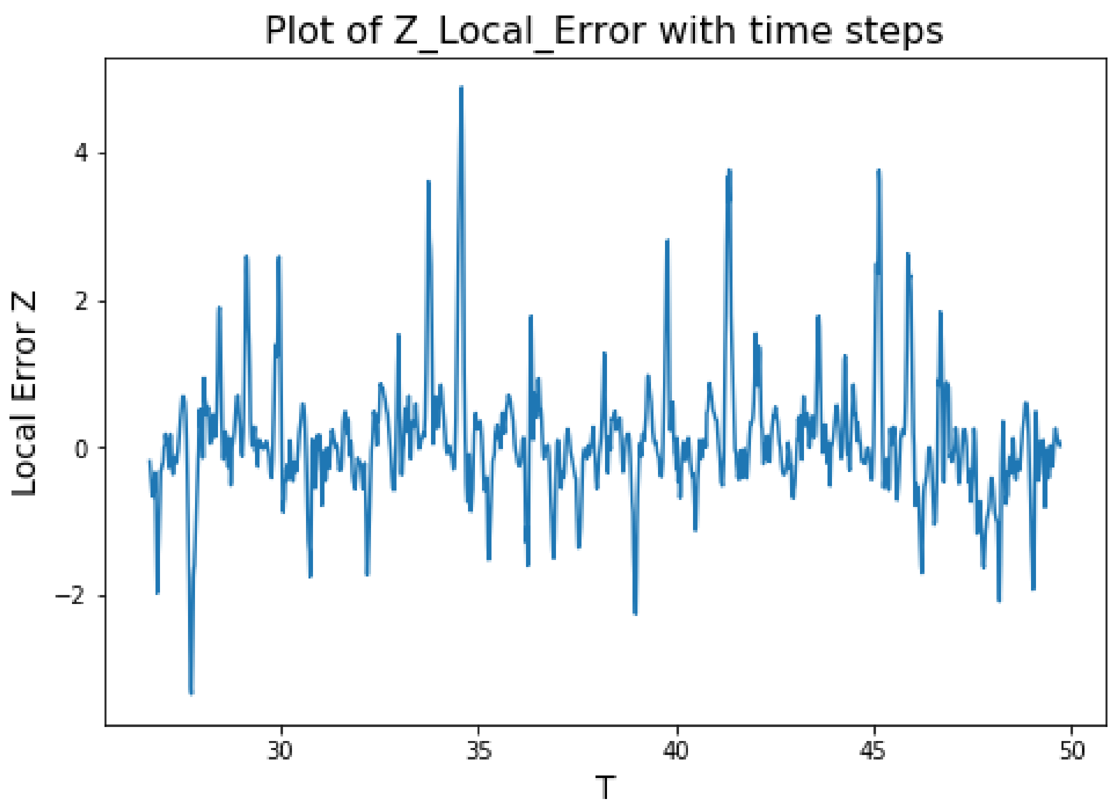
[51]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_local_error['X_Local_Error'] )
plt.title('Plot of X_Local_Error with time steps',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('Local Error X', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Y_Z_on␣
↪Local_Error_X.png", bbox_inches = "tight")
plt.show()
```



[52]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_local_error['Y_Local_Error'] )
plt.title('Plot of Y_Local_Error with time steps',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('Local Error Y', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Y_Z_on␣
↪Local_Error_Y.png", bbox_inches = "tight")
plt.show()
```



[53]:
```
fig = plt.figure()
ax=fig.add_axes([0,0,1,1])
ax.plot(test_out_t,df_local_error['Z_Local_Error'] )
plt.title('Plot of Z_Local_Error with time steps',  fontsize=16)
plt.xlabel('T', fontsize = 14)
plt.ylabel('Local Error Z', fontsize = 14)
plt.savefig(r"C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\images\Lorentz_Data\Y_Z_on␣
↪Local_Error_Z.png", bbox_inches = "tight")
plt.show()
```



[54]:
```
df_pred.columns= ['X_pred', 'Y_pred', 'Z_pred']
```

[55]:
```
df_pred.head()
```

[55]:

	X_pred	Y_pred	Z_pred
0	3.856962	6.491007	13.954546
1	4.541594	7.742547	13.884595
2	5.363696	9.186582	14.056829
3	6.323602	10.806990	14.511188
4	7.415397	12.570400	15.290587

[56]:
```
test_out.columns = ['X_test', 'Y_test', 'Z_test']
```

[57]:
```
df_out = pd.concat([df_pred, test_out], axis = 1)
```

[58]:
```
df_out['Test_T'] = test_out_t
```

[59]:
```
df_out.to_excel(r'C:\Users\INFO-DSK-02\Desktop\Lorentz Multi Dimension␣
↪Prediction-Phase-2\Final_Version\3D_ReservoirComputing\Output\Lorentz\Lorentz_Output.
↪xlsx', index= False)
```