1. We will now look at another way to obtain the QR factorization. This is called as the method of *Given's rotation.* Consider the 2×2 orthogonal matrices
$$F = \begin{bmatrix} -c & s \\ s & c \end{bmatrix}, J = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$
where $s = \sin(\theta)$ and $c = \cos(\theta)$ for some θ.
   - Compute the determinant of F and G.
   - Describe the action of F and G on a vector in two dimensions.
   - Drawing inspiration from matrix G, describe an algorithm analogous to the Householder reflection to obtain the QR factorization of a matrix A.
   - Show that the algorithm above is called the *Given's rotation* algorithm. Prove that the flop count for Given's rotation is 50% more than that for Householder reflection.
2. Consider the matrix we looked at in the class
$$A = \begin{bmatrix} 1 & 1 & 1 \\ \delta & \delta & 0 \\ \delta & 0 & \delta \end{bmatrix}$$
where δ is such that $\delta^2 < \epsilon_m$, where ϵ_m is the machine precision on a double precision machine. **Work out by hand** the QR decomposition using the classical Gram Schmidt, modified Gram Schmidt, Huseholder reflection and Given's rotation algorithm. Obtain the following error bounds:
$$\frac{\|A - QR\|_F}{\|A\|_F}, \text{ and } \left\|Q^T Q - I_{3\times 3}\right\|_F$$
Comment on the stability of the four algorithms.