1. Implement the Householder QR algorithm on a thin rectangular matrix $A \in \mathbb{R}^{m \times n}$ (where $m \geq n$). As part of this, implement a routine for computing Qb and $Q^T b$ where Q is the unitary matrix obtained using Householder QR algorithm and $b \in \mathbb{R}^{m \times 1}$ is any vector. Perform the detailed complexity analysis for obtaining the QR decomposition. Recall that in the case of Householder QR, we never explicitly construct the matrix Q; We only store the reflectors of appropriate size.

2. Using the above implemenation, write a program to compute the least square solution x_{LS} such that $\|Ax - b\|_2$ is minimized, where $A \in \mathbb{R}^{m \times n}$ with $m \geq n$ and $b \in \mathbb{R}^{m \times n}$.

3. Using the above implemenation, write a program to compute the least norm solution x_{LN} such that $Ax = b$ and $\|x\|_2$ is minimized, where $A \in \mathbb{R}^{m \times n}$ with $m \leq n$ and $b \in \mathbb{R}^{m \times n}$.

4. Problem 1 from Chapter 11 of Trefethen and Bau.

   Let $A \in \mathbb{R}^{m \times n}$, where $m \geq n$. The pseudo inverse of A is defined as $A^{+} = \left(A^T A\right)^{-1} A^T$. If $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$, where A_1 is a square matrix, prove that
   $$\left\|A^{+}\right\|_2 \leq \left\|A_1^{-1}\right\|_2$$

5. Problem 2 from Chapter 11 of Trefethen and Bau.

6. Problem 3 from Chapter 11 of Trefethen and Bau.