1. Extend the argument discussed in the class to prove that the Gauss-Seidel method converges for strictly row diagonally dominant matrices.
2. Let $A, B \in \mathbb{R}^{n \times n}$ such that A and $A - B - B^T$ are symmetric positive definite. Prove that the spectral radius of $(A - B)^{-1} B$ is strictly less than 1.
3. Use the above to prove that if A is symmetric positive definite, then the Gauss-Seidel method converges.
4. For a matrix A, let $\tilde{A}$ be defined as follows:
$$\tilde{A}_{ij} = \begin{cases} A_{ij} & \text{if } i = j \\ -A_{ij} & \text{if } i \neq j \end{cases}$$
Prove that if A and $\tilde{A}$ are symmetric positive definite, then the Jacobi method converges.
5. Consider the tridiagonal linear system
$$Ax = b$$
where $A \in \mathbb{R}^{n \times n}$, $A_{kk} = 2$, $A_{k,k+1} = A_{k+1,k} = -1$ and $b_k = \dfrac{k}{n^3}$.
   - Implement Jacobi, Gauss-Seidel, Steepest Descent and Conjugate Gradient for solving this linear system. Try with $n \in \{10, 20, 50, 100, 200, 500, 1000, 2000, 5000\}$ and for different tolerance $10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}$. The tolerance is used to bound the relative error in the residual, i.e., $\dfrac{\|b - Ax\|}{\|b\|}$.
   - Plot the number of steps for each method as a function of n for different tolerances.
   - Arrive at a rough scaling of the number of steps for each method in terms of n and tolerance.