1. We will now analyze the distribution of growth factor due to complete and partial pivoting emperically as follows.
   - Generate a set of 1 million random matrices A of size 100×100, where each entry in A is a normal random variable with mean zero and variance 1.
   - Compute the growth factor of A due to partial and complete pivoting on the above 1 million sample.
   - Plot the histograms.
   - Obtain the mean and standard deviation of the growth factor due to partial and complete pivoting.
   - What were the maximum and minimum growth factors you obtained?
   - How do these maximum and minimum compare with the upper bound derived in the last assignment?
   - Comment on the distribution of these growth factors.

2. Recall that while discussing Cholesky decomposition, we did not worry about pivoting and stability and we will see why. The growth factor for Cholesky decomposition on a matrix A is defined as

$$\rho(A) = \frac{\|L\|_{\max} \left\|L^T\right\|_{\max}}{\|A\|_{\max}}$$

   (identical to the way growth factor is defined for partial and complete pivoted LU. Note that the maximum entry in L is always bounded by 1 in LU decomposition, whereas in Cholesky it is not the case). Show that for Cholesky the growth factor never exceeds 1. Hence, conclude that there is no need for pivoting in Cholesky decomposition.

3. Recall that multiplying two real matrices of size $n \times n$ costs $\mathcal{O}(n^3)$. A matrix is said to be a Toeplitz matrix if $A_{ij} = A_{i+k,j+k}$, i.e., the entries on each diagonal are identical.

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \cdots & a_{n-1} \\ a_{-1} & a_0 & a_1 & a_2 & \cdots & a_{n-2} \\ a_{-2} & a_{-1} & a_0 & a_1 & \cdots & a_{n-3} \\ a_{-3} & a_{-2} & a_{-1} & a_0 & \cdots & a_{n-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{-(n-1)} & a_{-(n-2)} & a_{-(n-3)} & a_{-(n-4)} & \cdots & a_0 \end{bmatrix}$$

   Note that Toeplitz matrices are highly structured. It is sufficient to store $2n-1$ entries (one entry for each diagonal). Devise an algorithm to compute the product of two Toeplitz matrices at a cost of $\mathcal{O}(n^2)$. Obtain the exact flop count.