1. Consider the matrix $A_n = aI_n + bK_n + cK_n'$, where I_n is the identity matrix of size $n \times n$, K_n is the matrix with only ones in its super diagonal, i.e., $K_n(i, i+1) = 1$ and the rest are all zeros. Recall that the matrix with $a = 2$, $b = c = -1$ is encountered in second order central finite difference discretization of the second derivative.

   - Compute the determinant of the matrix A_n by deriving an appropriate recurrence relation.
   - Obtain the eigen values of A_n
   - Comment on the singular values of A_n when $b = c$.
   - Obtain the condition number of A_n, when $b = c$. Comment on how the condition number scales as n becomes large.

2. Recall that in class, we suggested that the singular value decomposition of a matrix $A \in \mathbb{R}^{n \times n}$ can be obtained by performing the eigenvalue decomposition of the matrix

$$\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$$

   However, the above can be sped up a bit if we proceed in two phases as follows.

   - **Phase 1**:
     - Show that A can be expressed as $A = U\tilde{A}V^T$, where $\tilde{A}$ is a bidiagonal matrix, i.e., only the entires on the diagonal and super-diagonal of B are non-zero.
     - Explain how to obtain the matrix $\tilde{A}$ and the associated cost of obtaining $\tilde{A}$.
     - Show that the singular value decomposition of A follows immediately once the singular value decomposition of $\tilde{A}$ is obtained.

     Phase 2: This phase is to obtain the SVD of $\tilde{A}$.
     - To find the SVD of $\tilde{A}$ we proceed as we discussed in class. Construct the matrix

$$B = \begin{bmatrix} 0 & \tilde{A} \\ \tilde{A}^T & 0 \end{bmatrix}$$

       Find a permutation matrix P such that $C = PBP^T$ is a tridiagonal matrix.
     - Discuss how to obtain the singular value decomposition of B from the eigenvalue decomposition of C.
     - What is the computational cost of Phase 2?