Problem Set 6; Solutions Part 1

April 26, 2019

1. Sample mean is 168.83. Sample variance is 37.68. Sample standard deviation is 6.14

2. $$\mathbb{E}\left(\hat{\Theta}+W\right)=\mathbb{E}\left(\hat{\Theta}\right)+\mathbb{E}\left(W\right)=\mathbb{E}\left(\hat{\Theta}\right)=\Theta$$
$$\mathbb{E}\left(\frac{\Theta_1-b}{a}\right)=\frac{\mathbb{E}\left(\Theta_1-b\right)}{a}=\frac{\mathbb{E}\left(\Theta_1\right)-b}{a}=\Theta$$

3. $F_{\hat{\Theta}}(x)=\mathbb{P}\left(\hat{\Theta}\le x\right)=\mathbb{P}\left(X_1\le x\right)\mathbb{P}\left(X_2\le x\right)\mathbb{P}\left(X_3\le x\right)\dots\mathbb{P}\left(X_n\le x\right)=\left(\frac{x}{\Theta}\right)^n$ From this find the pdf, the expected value and variance. The estimator is consistent if
$$\lim_{n\to\infty}\mathbb{E}\left(\hat{\Theta}\right)=\Theta$$

4. $$\mathcal{L}\left(p;X_1,X_2,\dots,X_n\right)=p_X\left(X_1\mid p\right)p_X\left(X_2\mid p\right)\dots p_X\left(X_n\mid p\right)=\prod_{k=1}^{n}\left((1-p)^{X_k}p\right)$$
Now find p that maximizes the above function.

5. We have $\mathcal{L}\left(\Theta;X_1,X_2,\dots,X_n\right)=f_X\left(X_1\mid\Theta\right)f_X\left(X_2\mid\Theta\right)\dots f_X\left(X_n\mid\Theta\right)=\left(\frac{1}{\Theta}\right)^n$. Maximize the above the subject to the constraint $\Theta\ge X_i$ for all i. This gives us $\hat{\Theta}=\max_{i=1}^n\{X_i\}$.

6. We have $\frac{(n-1)\,s^2}{\sigma^2}\sim\chi^2_{n-1}$. We have
$$\mathbb{P}\left(s^2\le k\right)=\mathbb{P}\left((n-1)s^2/\sigma^2\le(n-1)k/\sigma^2\right)=\mathbb{P}\left((n-1)s^2/\sigma^2\le 3k/3^2\right)=0.05$$
From the table, we obtain $k/3=0.35185$.

7. We have the expectation of a χ^2_{n-1} random variable to be $n-1$. $(n-1)s^2/\sigma^2\sim\chi^2_{n-1}$.

8. $$C=2.09302$$

9. $$\mathbb{P}\left(-z_{0.05}\le\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\le z_{0.05}\right)=0.9$$
$$\mathbb{P}\left(\bar{X}-z_{0.05}\sigma/\sqrt{n}\le\mu\le\bar{X}+z_{0.05}\sigma/\sqrt{n}\right)=0.9$$
where $z_{0.05}=1.645$.

10. $$\mathbb{P}\left(\bar{X}-z_{0.025}\sigma/\sqrt{n}\le\mu\le\bar{X}+z_{0.025}\sigma/\sqrt{n}\right)=0.95$$
where $z_{0.025}=1.96$.