1. Compute the first two iterates of steepest descent and Newton method for the Rosenbrock function:

$$f(x) = (x-1)^2 + 100\left(y - x^2\right)^2$$

starting at $(0, -1)$.

2. Consider a convex function $f : \mathbb{R} \mapsto \mathbb{R}$. Recall that f is convex if $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$. Also, recall that if $f(x) \in C^2$, then an equivalent condition is that $f''(x) \geq 0$.

   - Prove that for any n

   $$f\left(a_1 x_1 + a_2 x_2 + \cdots + a_n x_n\right) \leq a_1 f(x_1) + a_2 f(x_2) + \cdots + a_n f(x_n)$$

   where $a_1 + a_2 + \cdots + a_n = 1$, where $a_i \geq 0$.
   - Prove that $-\log(x)$ is convex
   - Hence or otherwise conclude the Arithmetic Mean-Geoemtric Mean inequality, i.e., if x_i's are non-negative real numbers, then

   $$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n}$$

3. Prove that in any triangle ABC, we have

$$\sin(A) + \sin(B) + \sin(C) \leq \frac{3\sqrt{3}}{2}$$

HINT: Consider an appropriate convex function.

4. Find the first two intervals using the bisection method to compute the minimum of the function

$$f(x) = x^2 \cos(x)$$

on the interval $[\pi/2, 3\pi/2]$.

5. Find the monic quadratic polynomial, $q(x)$, such that

$$\int_{-1}^{1} |q(x)|\, dx$$

is minimized.

6. Show that among all rectangles with a given perimeter, the square is the one with the maximum area.