July 27, 2025

1 Assignment 2

1. Define the Wronskian \$ w(y_1, y_2) \$ of any two continuous functions \$ y_1 \$ and \$ y_2 \$ defined in an interval \$ (a, b) $\subset$ $\mathbb{R}$\$.*Showthat*\$$w(y\_1, y\_2) = 0$\$*if*\$*y\_1*\$*and*\$*y\_2*\$*arelinearlydependent.If*\$*y\_1*\$*and*\$*y\_2*\$*areanytwosolutionsofasecondorderlinearhomog* $\mathbb{R}$\$, *then*\$$w(y_1, y_2)$\$*iseitheridenticallyzeroornon* $-$ *zeroatanypointoftheinterval.*

2. If \$ y_1 \$ and \$ y_2 \$ are two linearly independent solutions of a second order linear homogeneous ODE, then prove that \$ y = c_1 y_1 + c_2 y_2 \$, where \$ c_1 \$ and \$ c_2 \$ are constants, is a general solution.

3. Find the general solution of the following second order equations using the given known solution y_1

- \$ x^2 d^2y $\frac{}{dx^2+x\frac{dy}{dx}-y=0 \quad \text{where } y\_1(x)=x\$\$x2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0 \quad \text{where } y\_1(x)=x\hat{}2\$}$

- \$ (x - 1) d^2y $\frac{}{dx^2-x\frac{dy}{dx}+y=0 \quad \text{where } y\_1(x)=x\$\$x\frac{d^2y}{dx^2}-(2x+1)\frac{dy}{dx}+(x+1)y=0 \quad \text{where } y\_1(x)=\hat{e}x\$}$

4. Find the general solution of each of the following equations (\$ D^n $\equiv \frac{d^n}{dx^n}$\$)

- \$ (D^4 - 81)y = 0 \$
- \$ (D^3 - 4D^2 + 5D - 2)y = 0 \$
- \$ (D^4 + 2D^3 + 3D^2 + 2D + 1)y = 0 \$
- \$ (D^2 - 3D - 6)y = 3 $\sin 2x$\$\$$(D\hat{}2 + 1)y = 2\cos x$\$
- \$ (D^2 - 3D + 2)y = (4x + 5)e^{3x} \$

5. Solve using the method of variation of parameters (\$ D^n $\equiv \frac{d^n}{dx^n}$\$)

- \$ (D^2 + 1)y = $\csc x$\$\$$(D\hat{}2 - D - 6)y = \hat{e}\{-x\}$\$
- \$ (D^2 + a^2)y = $\tan(ax)$\$\$$x\hat{}2y'' - 4xy' + 6y = 21x\hat{}\{-4\}$\$
- \$ x^2 y'' - 2x y' + 2y = x^3 $\cos x$\$

6. Solve using the method of undetermined coefficients (\$ D^n $\equiv \frac{d^n}{dx^n}$\$)

- \$ (D^3 - 2D^2 - 5D + 6)y = 18e^x \$
- \$ (D^2 + 25)y = $50\cos 5x + 30\sin 5x$\$\$$(D\hat{}3 - 2D\hat{}2 + 4D - 8)y = 8(x\hat{}2 + \cos 2x)$\$

7. Show that the general solution of \$ L(y) = f_1(x) + f_2(x) \$, where \$ L(y) \$ is a linear differential operator, is \$ y = y} + y^* \$, where \$ y} \$ is the general solution of \$ L(y) = 0 \$ and \$ y^* \$ is the sum of any particular solutions of \$ L(y) = f_1(x) \$ and \$ L(y) = f_2(x) \$.

8. Solve the following using Frobenius method

- $9x(1-x)y" - 12y' + 4y = 0$
- $2x^2y" + xy' - (x^2+1)y = 0$
- $x^2y" + 6xy' + (6+x^2)y = 0$