

1. Consider the following integral:

$$I_n = \int_0^1 x^{2n} \sin(\pi x) dx$$

- (a) Obtain a recurrence for I_n in terms of I_{n-1} . (HINT: Integration by parts)
 - (b) Evaluate I_0 by hand
 - (c) Use the recurrence to obtain I_n for $n \in \{1, 2, \dots, 20\}$ in python.
 - (d) Obtain the integral using the builtin **scipy.integrate.quad** command in python and compare the results to above.
 - (e) Explain your observation. We will see later in this course how to evaluate integrals to high accuracy.
2. Recall the coffee-cooling problem discussed in class. In this assignment, let's assume that the cooling rate r is not a constant but decays with time as $r(t) = \exp(-\alpha t^2)$. Take the initial temperature of the coffee-cup to be 85°C and the surrounding temperature to be 25°C . Explore the solution for values $\alpha \in \{0, 1, 2, 3$ per sq. minute. Note that $\alpha = 0$ should give us the solution for constant decay rate.
3. Find the most accurate formula for the first derivative of the function f at x_i utilising known values of f at x_{i-1}, x_i, x_{i+1} and x_{i+2} . The points are uniformly spaced. Give the leading error term and state the order of the method.
4. A general Padé