Assignment Sheet 1

MA2020 Differential Equations (July - November 2012)

1. Solve the following first order differential equations

(a) $x\frac{dy}{dx} + y = x^3y^6$

(b) $xy^2\frac{dy}{dx} + y^3 = x\cos(x)$

(c) $x\frac{dy}{dx} + y = y^2\log(x)$

(d) $(x^2y^3 + xy)\frac{dy}{dx} = 1$

(e) $\frac{dy}{dx} + (2x\tan^{-1}y - x^3)(1 + y^2) = 0$

(f) $(x^2 + y)dx + (y^3 + x)dy = 0$

(g) $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$

(h) $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$

(i) $(x\,dx + y\,dy)(x^2 + y^2) = y\,dx - x\,dy$

(j) $y\cos(x)dx + 2\sin(x)dy = 0$

(k) $x\,dy + y\,dx + 3x^3y^4dy = 0$

(l) $(1 + xy)y\,dx + (1 - xy)x\,dy = 0$

(m) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

(n) $(xy - 1)dx + (x^2 - xy)dy = 0$

2.

Find the curve passing through the point (1, 0) and having at each of its points the slope $-\frac{x}{y}$

3. Solve the following Initial Value Problems (IVP)

(a) $\frac{dy}{dx} = 2y + e^{2x}, \quad y(0) = 3$

(b) $\frac{dy}{dx} = 3y + 2e^{3x}, \quad y(0) = 2$

(c) $\frac{dy}{dx} = y\tan(x) + \sec(x), \quad y(0) = -1$

(d) $\frac{dy}{dx} = \frac{2}{x}y + x, \quad y(1) = 2$

4.

Define the Wronskian $w(y_1, y_2)$ of any two continuous functions y_1 and y_2 defined in an interval $(a, b) \subset \mathbb{R}$. Show that $w(y_1, y_2) = 0$ if y_1 and y_2 are linearly dependent.

5.

If y_1 and y_2 are any two solutions of a second order linear homogeneous ordinary differential equation defined in an interval $(a, b) \subset \mathbb{R}$, then $w(y_1, y_2)$ is either identically zero or non-zero at any point of the interval.

6.

If y_1 and y_2 are two linearly independent solutions of a second order linear homogeneous ODE, then prove that $y = c_1y_1 + c_2y_2$, where c_1 and c_2 are constants, is a general solution.

7. Find the general solution of the following second order equations using the given known solution y_1

(a) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ where $y_1(x) = x$

(b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$ where $y_1(x) = x^2$

(c) $(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ where $y_1(x) = x$

(d) $x\frac{d^2y}{dx^2} - (2x+1)\frac{dy}{dx} + (x+1)y = 0$ where $y_1(x) = e^x$

8. Find the general solution of each of the following equations ($D^n \equiv \frac{d^n}{dx^n}$)

(a) $(D^4 - 81)y = 0$

(b) $(D^3 - 4D^2 + 5D - 2)y = 0$

(c) $(D^4 - 7D^3 + 18D^2 - 20D + 8)y = 0$

(d) $(D^4 + 2D^3 + 3D^2 + 2D + 1)y = 0$

(e) $(D^2 - 3D - 6)y = 3\sin 2x$

(f) $(D^2 + 1)y = 2\cos x$

(g) $(D^2 - 3D + 2)y = (4x + 5)e^{3x}$

(h) $(D^2 - 1)y = 3e^{2x}\cos 2x$

(i) $(D^2 - 2D + 3)y = 3e^{-x}\cos x$

9. Solve using the method of variation of parameters $(D^n \equiv \frac{d^n}{dx^n})$

(a) $(D^2 + 1)y = \csc x$

(b) $(D^2 - D - 6)y = e^{-x}$

(c) $(D^2 + a^2)y = \tan ax$

(d) $(D^2 + a^2)y = \sec 2x$

(e) $x^2y'' - 4xy' + 6y = 21x^{-4}$

(f) $4x^2y'' + 8xy' - 3y = 7x^2 - 15x^3$

(g) $x^2y'' - 2xy' + 2y = x^3\cos x$

(h) $xy'' - y' = (3 + x)x^2e^x$

10. Solve using the method of undetermined coefficients $(D^n \equiv \frac{d^n}{dx^n})$

(a) $(D^3 - 2D^2 - 5D + 6)y = 18e^x$

(b) $(D^2 + 25)y = 50\cos 5x + 30\sin 5x$

(c) $(D^3 - 2D^2 + 4D - 8)y = 8(x^2 + \cos 2x)$

(d) $(D^3 + 3D^2 - 4)y = 12e^{-2x} + 9e^x$

11.

Show that the general solution of $L(y) = f_1(x) + f_2(x)$, where $L(y)$ is a linear differential operator, is $y = \bar{y} + y^*$, where $\bar{y}$ is the general solution of $L(y) = 0$ and y^* is the sum of any particular solutions of $L(y) = f_1(x)$ and $L(y) = f_2(x)$.