Assign1

July 27, 2025

1 Assignment 1

1. Consider the following integral:

$$I_n = \int_0^1 x^{2n} \sin(\pi x) dx$$

   - Obtain a recurrence for I_n in terms of I_{n-1}. (HINT: Integration by parts)
   - Evaluate I_0 by hand
   - Use the recurrence to obtain I_n for $n \in \{1, 2, \ldots, 20\}$ in python.
   - Obtain the integral using the builtin scipy.integrate.quad command in python and compare the results to above.
   - Explain your observation. We will see later in this course how to evaluate integrals to high accuracy.

2. Solve the following differential equations:

- $xy' + y = x^3y^6$
- $xy^2y' + y^3 = x\cos(x)$
- $xy' + y = y^2\cos(x)$
- $(x^2y^3 + xy)y' = 1$
- $y' + (2x\arctan(y) - x^3)(1 + y^2) = 0$
- $(x^2 + y)dx + (y^3 + x)dy = 0$
- $(1 + \exp(x/y))dx + \exp(x/y)(1 - x/y)dy = 0$
- $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$
- $(x\,dx + y\,dy)(x^2 + y^2) = y\,dx - x\,dy$
- $y\cos(x)dx + 2\sin(x)dy = 0$
- $x\,dy + y\,dx + 3x^3y^4dy = 0$
- $(1 + xy)y\,dx + (1 - xy)x\,dy = 0$
- $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$
- $(xy - 1)dx + (x^2 - xy)dy = 0$

3. Find the curve passing through the point (1, 0) and having at each of its points the slope $-\frac{x}{y}$

4. Solve the following initial value problem (IVP):

- $\frac{dy}{dx} = 2y + e^{2x}, \quad y(0) = 3$
- $\frac{dy}{dx} = 3y + 2e^{3x}, \quad y(0) = 2$
- $\frac{dy}{dx} = y\tan(x) + \sec(x), \quad y(0) = -1$
- $\frac{dy}{dx} = \frac{2}{x}y + x, \quad y(1) = 2$