

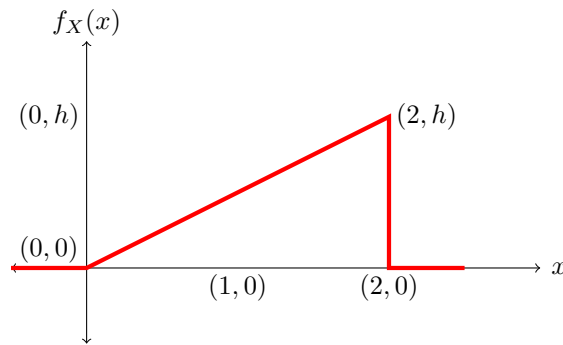
# MA2040: Probability, Statistics and Stochastic Processes

## Problem Set-III

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1. If  $X_1, X_2, \dots, X_n$  are independent random variables having the same probability density function  $f_X(x)$ , what is the probability density function for the random variable  $Y = \min\{X_1, X_2, \dots, X_n\}$ ?
2. A random variable  $X$  has a probability density function as shown below.



- (a) Determine  $h$
  - (b) Determine the cumulative distribution function
  - (c) Compute the mean
  - (d) Compute the variance
  - (e) Determine the probability that  $X \in (1, 2)$ .
3. The median  $m$  of a probability density function is defined as the value of  $m$  such that

$$\int_{-\infty}^m f(x)dx = \int_m^{\infty} f(x)dx = 1/2$$

Essentially, the median splits the distribution into two equal halves. Prove that the median is the best predictor if one wants to minimize the expected value of the absolute error, i.e.,  $\mathbb{E}(|X - c|)$  is minimized when  $c$  is the median of the underlying distribution.

4. Let  $X$  be a random variable, whose pdf is given by

$$f_X(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ xe^{-x^2/2} & \text{if } x > 0 \end{cases}$$

Find the pdf for the random variable  $Y = X^2$ .

5. Let  $X$  be a uniform random variable on the interval  $[0, 1]$ . Consider the random variable  $Y = g(X)$ , where

$$g(x) = \begin{cases} 1 & \text{if } x \leq 1/3 \\ 2 & \text{else} \end{cases}$$

Find the probability mass function of  $Y$  and compute its expected value.

6. Show the expected value of a random variable  $X$  can also be obtained as

$$\mathbb{E}(X) = \int_0^\infty \mathbb{P}(X > x) dx - \int_0^\infty \mathbb{P}(X < -x) dx$$

7. Let the random variables  $X$  and  $Y$  have a joint PDF, which is uniform over the triangles with vertices  $(0, 0)$ ,  $(0, 1)$  and  $(1, 0)$ .

- (a) Find the joint PDF of  $X$  and  $Y$ .
- (b) Find the marginal PDFs.
- (c) Find the conditional PDFs.

8. Chennai's temperature is modeled as a normal random variable with a mean temperature of  $34^\circ\text{C}$  and a standard deviation of  $5^\circ\text{C}$ . What is the probability that the temperature at a randomly chosen time will exceed  $45^\circ\text{C}$ ?

9. A surface is ruled with parallel lines, which are at a distance  $d$  from each other. Suppose that we throw a needle of length  $l$  on the surface at random. What is the probability that the needle will intersect one of the lines? (NOTE: You will need to treat the case  $d < l$  and  $d > l$  separately.)

10. Consider two continuous random variables  $Y$  and  $Z$  and a random variable  $X$  that is equal to  $Y$  with a probability  $p$  and equals  $Z$  with a probability  $1 - p$ . Obtain the pdf of  $X$  in terms of the pdf's of  $Y$  and  $Z$ .