ZF Assignment - on Control System of a Car

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Contents

1 System Modelling

system has been modelled based on the given governing equation. it has been tested for positive, negative Fig.1 and alternate positive and negative Fig.2 steering control for one second with $0.01s$ time interval.

2 Yaw Rate Control ($\dot{\psi}$)

- Control Methodology : **Deep Q Networks**
- Agent initial state $(y, \dot{y}, \psi, \dot{\psi})$: $(0, 0, 0, 0)$
- Episode Termination criteria : **MPCE** simulation for 400 time steps in 0.01s interval.
- Reward Function : **Based on the Difference between $\dot{\psi}_{actual}$ and $\dot{\psi}_{desire}$**

$$\mathcal{R}(\dot{\psi}) = -abs(\dot{\psi}_{Target} - \dot{\psi}_{Actual}) \times 100 \tag{1}$$

DQN Hyper Parameters	
Shape of the NN	4-64-64-5
Activation function	Sigmoid
Step size parameter	0.98
Target Update Interval	10 episodes
Optimizer	Adam's Method
Learning Rate	0.001
β_1,β_2	0.9,0.999
Batch size	128
Exploration threshold	0.2
Memory Buffer Size	3000

Comments of Yaw Rate Control: DQN with high number of action space seems to have a better performance. Eqn.1 represents the reward function.

3 Comments on Increasing the stability

System ensures the stability. The hand written notes are added in the end...!

4 Path Following

The path following problems mostly depend on the spatial position of the agent. Even though the car model has the four variable state space, x directional position can be found by appropriate trigonometrical relations. For example case, initial position of the car is taken as (0,0,0,0).

State Space : $(y, \dot{y}, \psi, \dot{\psi})$

$$\Delta x = V_x \times \sin \dot{\psi} \times dt \tag{2}$$

$$\Delta y = V_x \times \cos \dot{\psi} \times dt \tag{3}$$

From the Eqn.2 and Eqn.3, spatial position of the car can be calculated.Δx and Δy are the increment with respect to time step. With this (x,y) position, corresponding $\dot{\psi}$ will be calculated. Two approaches has been performed for the curvature tracking problem.

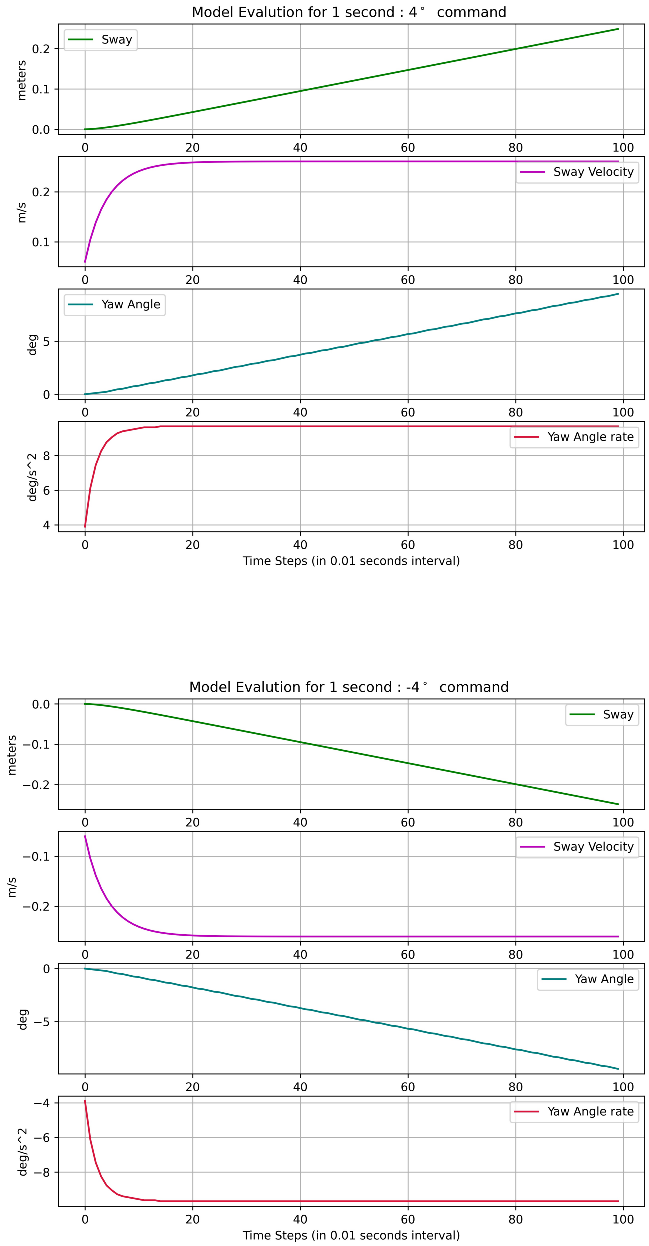

Figure 1: Model Testing for Positive and Negative Steering Command

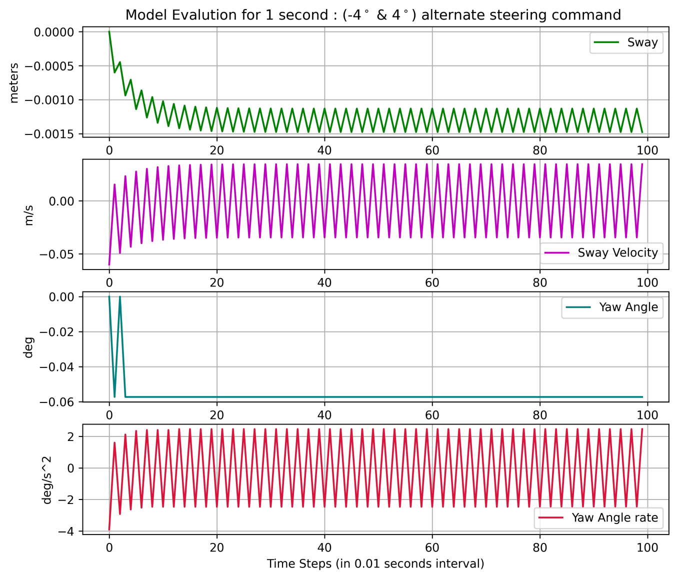

Figure 2: Model Test for alternate positive and negative steering command

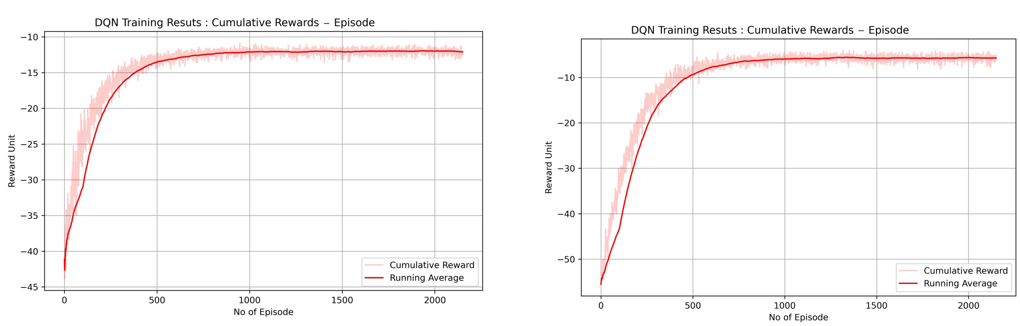

Figure 3: Reward Plots for 3 and 5 input cases

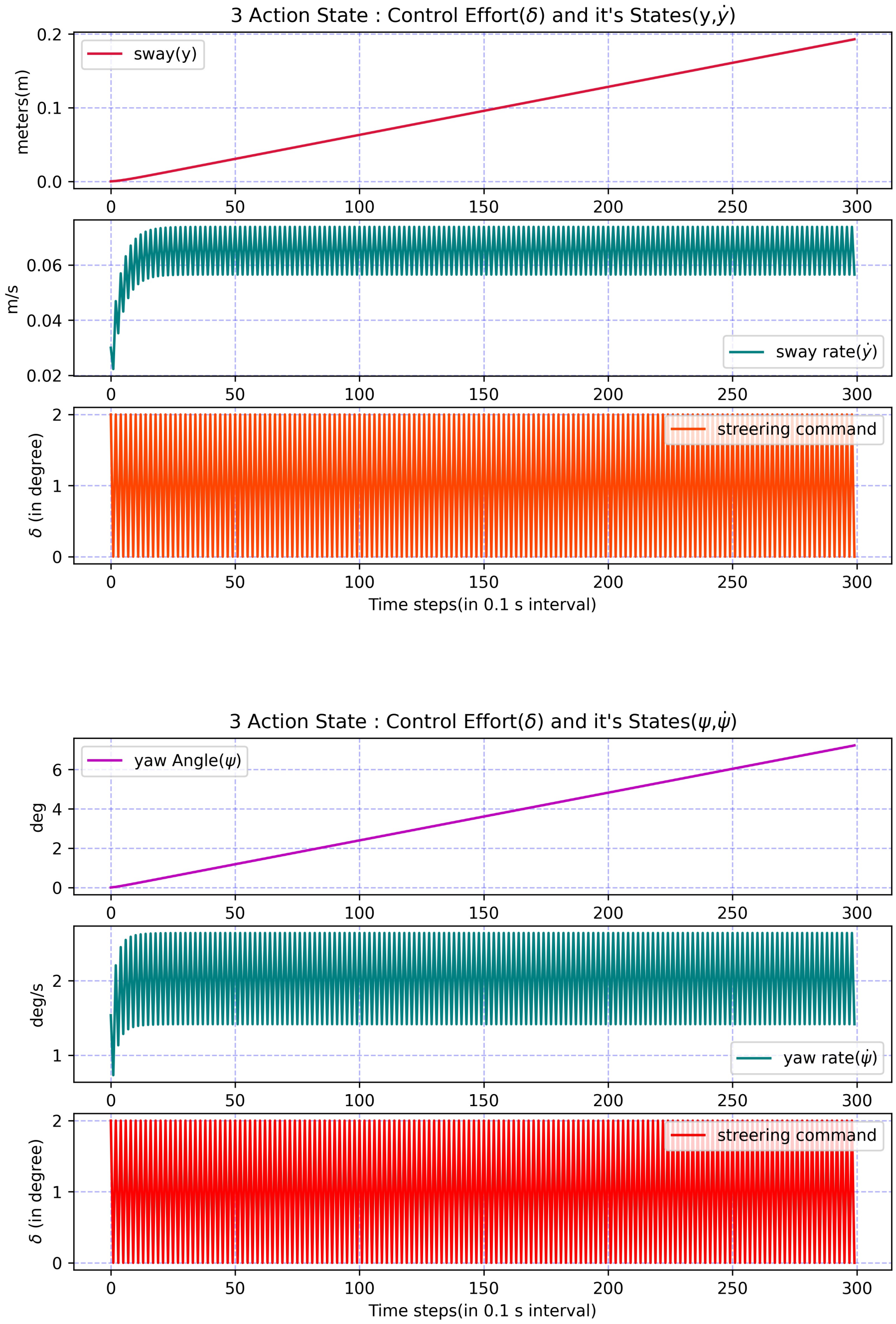

Figure 4: Yaw Rate Control for 3 action method (0.01s time step)

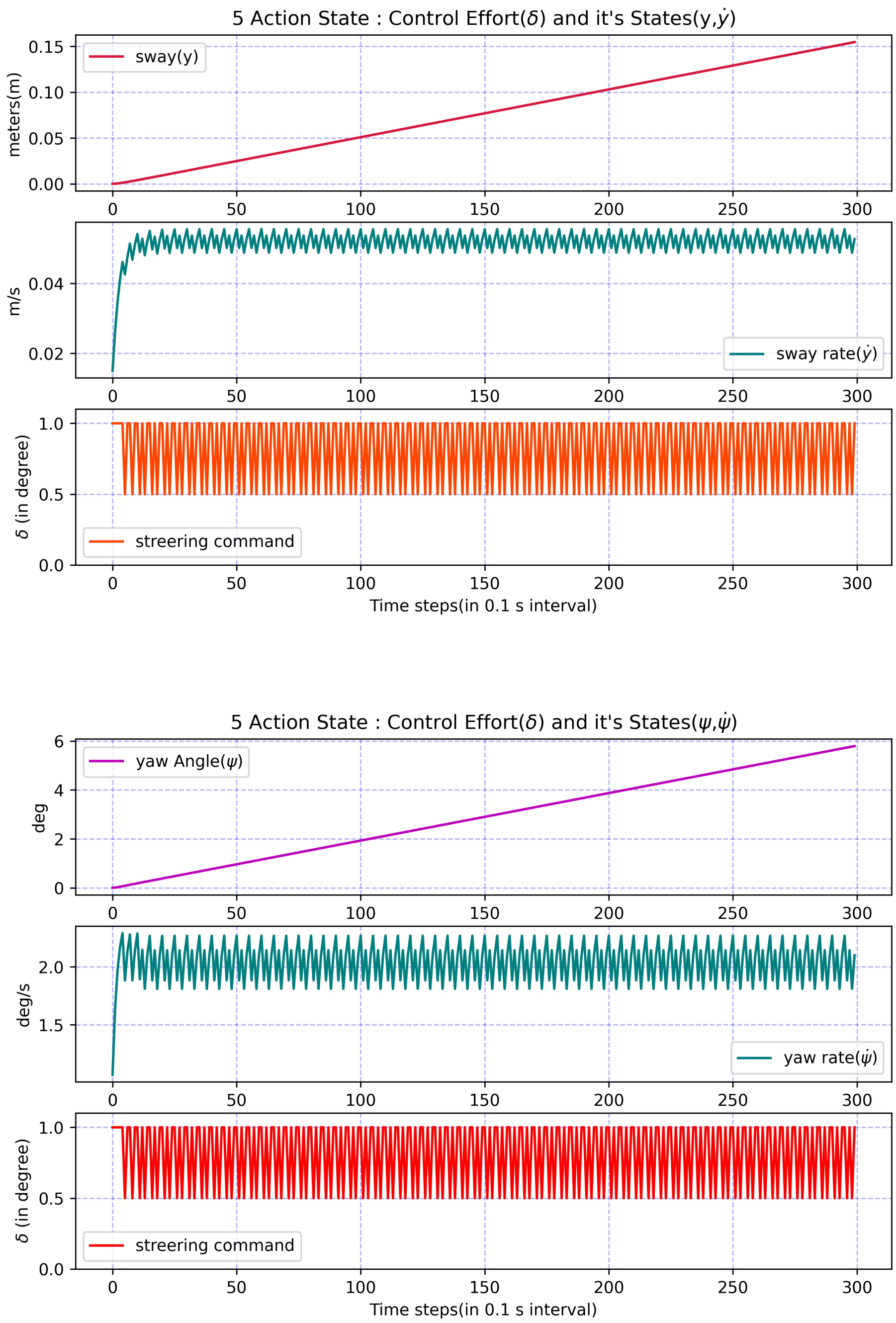

Figure 5: Yaw Rate Control for 5 action method (0.01s time step)

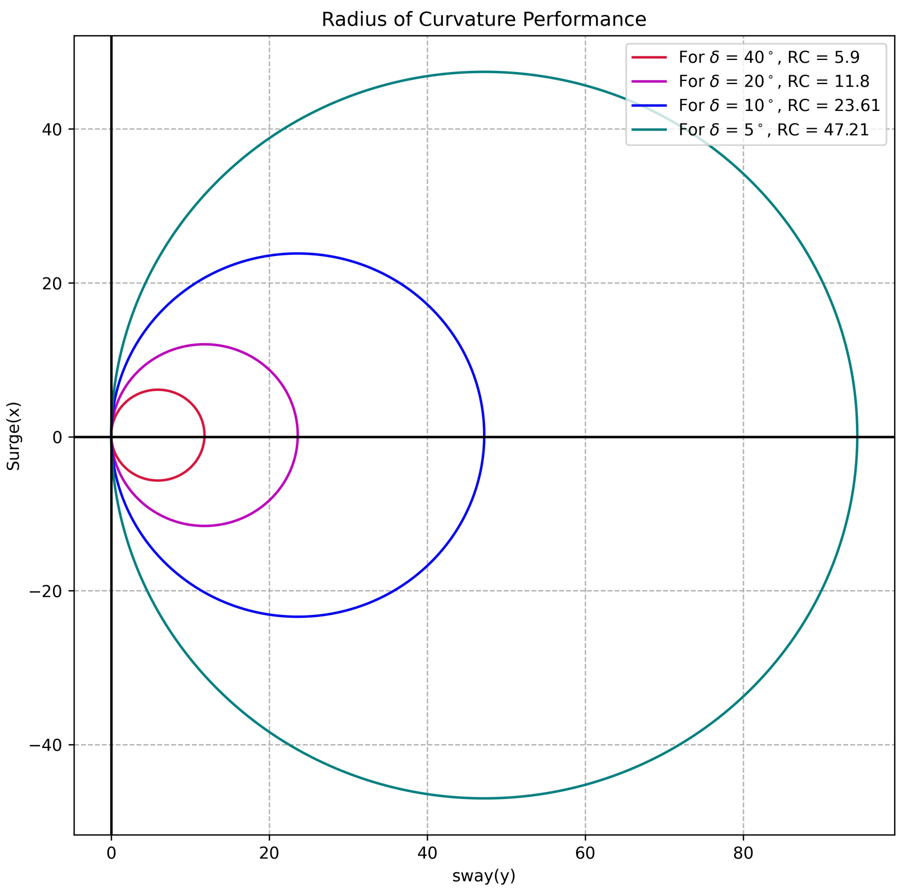

Figure 6: Radius of Curvature for different steering command

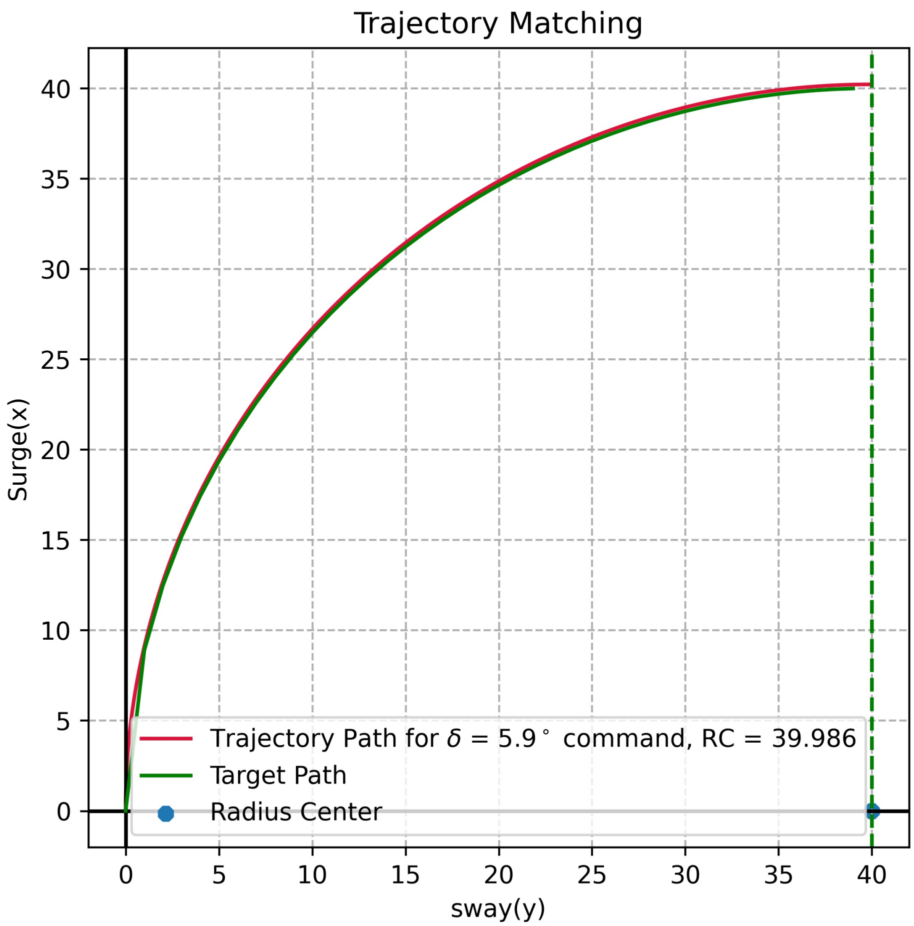

Figure 7: Trajectory matching of Radius of curvature by Interpolation method

4.1 Interpolation Based Control

The Model is evaluated as shown in Fig.6 for different steering command. The observation has been noted that for the change in steering command, radius of curvature is linearly varied. From the studied relation, the required radius of curvature (40 m) has been interpolated. The steering command of 5.9° will make the 40 m radius of curvature path as shown in Fig.7.

$$\delta' = \frac{(10-5)}{(47.212-23.606)} \tag{4}$$

$$\delta(R_{curvature}) = \delta' \times R_{curvature} \tag{5}$$

since the steering command and radius of curvature make the linear relation, the slope of the relation is found by Eqn.4 and corresponding steering action relation can be determined from the Eqn.5.

4.2 DQN Based Path Tracking

I used 7 state action for path tracking varying from (-20,-5,-1,0,1,5,20). Moderate result has been attained as shown in Fig.8. Eqn.1 is used as reward function.

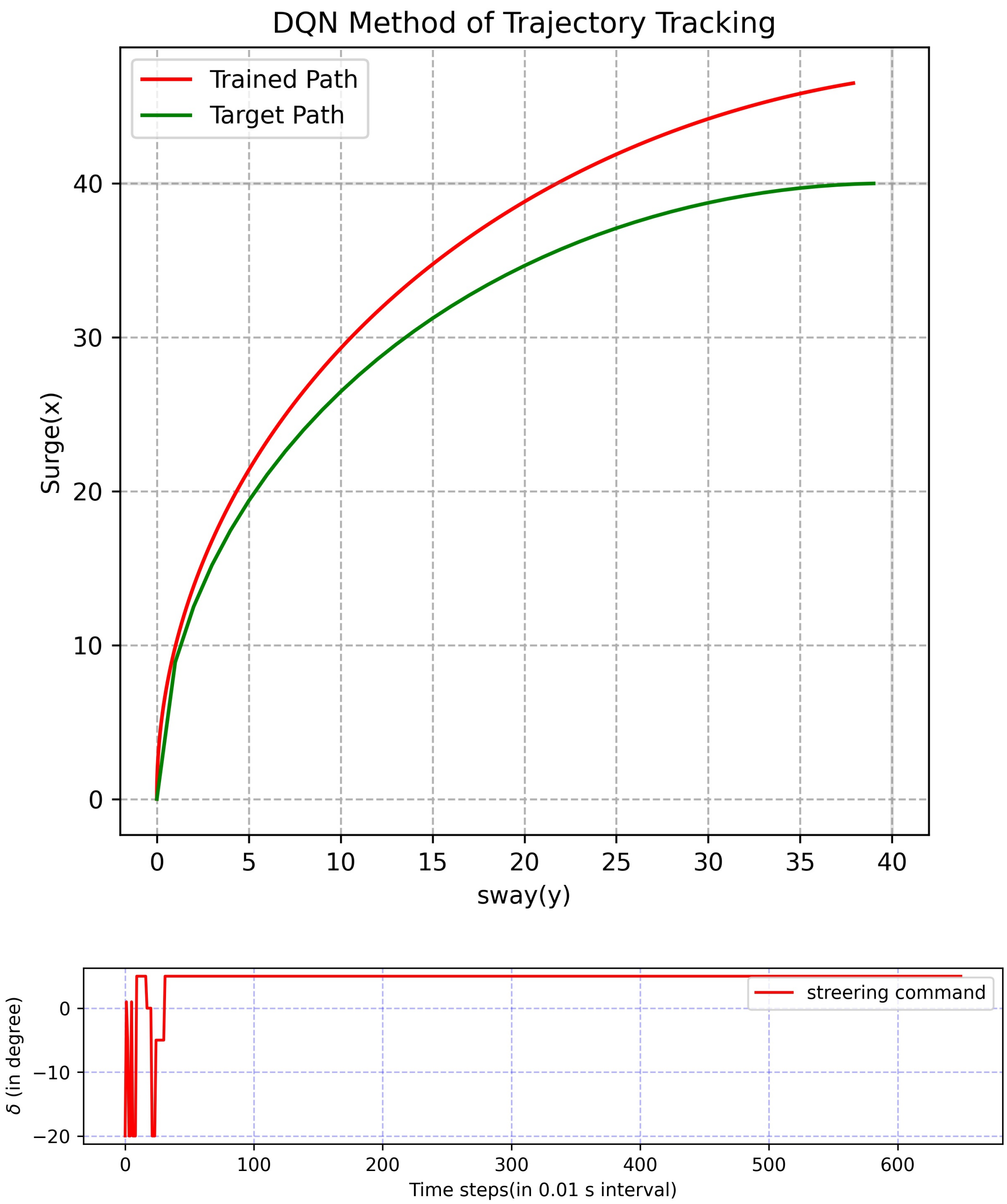

Figure 8: DQN method for Path following and it's rudder command

Lyapunov Stability Analysis

Given State Matrix :

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{2C_{\alpha f} + 2C_{\alpha r}}{mV_x} & 0 & -V_x - \dfrac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{2l_fC_{\alpha f} - 2l_rC_{\alpha r}}{I_zV_x} & 0 & -\dfrac{2l_f^2C_{\alpha f} + 2l_r^2C_{\alpha r}}{I_zV_x} \end{bmatrix}$$

here. $\dot{y} = \frac{dy}{dt}$ & $\dot{\psi} = \frac{d\psi}{dt}$ and 2^{nd} & 4^{th} row of the state matrix.

$$\ddot{y} = A_{22}\dot{y} + A_{24}\dot{\psi}$$

$$\ddot{\psi} = A_{42}\dot{y} + A_{44}\dot{\psi} \quad \text{- - - - (1)}$$

further, $x_1 = \dot{y}$ & $x_2 = \dot{\psi}$, equ(1) becomes,

$$\dot{x}_1 = A_{22}x_1 + A_{24}x_2$$

$$\dot{x}_2 = A_{42}x_1 + A_{44}x_2 \quad \text{- - - - (2)}$$

from the given informations,

$$\dot{x}_1 = -19.44x_1 - 5.52x_2$$

$$\dot{x}_2 = 2.52x_1 - 43.76x_2$$

Assume $V(x) = x_1^2 + x_2^2$

$$\dot{V}(x) = 2x_1 \cdot \dot{x}_1 + 2x_2 \cdot \dot{x}_2$$

$$= x_1[-19.44x_1 - 5.52x_2] + x_2[2.52x_1 - 43.76x_2]$$

$$= -19.44x_1^2 - 5.52x_1x_2 + 2.52x_1x_2 - 43.76x_2^2$$

$\boxed{\dot{V}(x) < 0}$ $\boxed{\dot{V}(x) = -19.44x_1^2 - 3.0x_1x_2 - 43.76x_2^2}$ ---> (3)

* From the eqn (3), we can conclude that the system has a set constraints for non-decreasing stability. [as it should satisfies $\dot{V}(x) < 0$].

* Approriate $V(x)$ function may yeild stability asymptotically.