Back Propagation

– classification

Entropy of $P(A)$:

$$H(P) = -E_P[\ln P] \quad \text{---} (1)$$

Expectation

Measure of Uncertainity

$$= -\sum_A P(A) \ln P(A)$$

for $K = 5$,

$P(A)$: 1 0 0 0 0

$H(P) = 0$ {where certain}

$P(A)$: 1/5 1/5 1/5 1/5 1/5

$H(P) = -\ln(1/5) = \ln K$ {highly uncertain}

$$0 \le H(P) \le \ln(K) \quad \text{---} (2)$$

Cross Entropy } $H(P,Q) = -E_P[\ln Q]$

$$= -\sum_A P(A) . \ln Q(A)$$

$\bar{t} \to$ Targered Probability Distribution (desired)

$S_K^o, \bar{y} \to$ Computed P.D from Softmax function

$$\boxed{\tilde{E}_{CE} = -\sum_{K=1}^{K} t_K \ln(y_K)} \quad \text{---} (3)$$

If $\bar{x} \in C_K$, $t_K = 1$

$t_l = 0 \;\forall\, l \ne K$

$$\tilde{E}_{CE} = -\ln y_K \quad \text{---} (4)$$

$$\Delta w(m) = -\eta \frac{\partial \tilde{E}_{CE}}{\partial w}$$

$$\Delta w_{jK} = -\eta \frac{\partial \bar{E}}{\partial w_{jK}} = \eta \frac{\partial}{\partial w_{jK}} (\ln y_K)$$

$$= \eta \frac{\partial}{\partial w_{jK}} (\ln S_K^o)$$

$$= \eta \frac{\partial}{\partial w_{jK}} \left[\ln \frac{e^{a_K^o}}{\sum_{l=1}^{K} e^{a_l^o}} \right]$$

for class K,

$$= \eta \cdot \frac{\partial}{\partial w_{jK}} \left[\ln e^{a_K^o} - \ln \left[\sum_{l=1}^{K} e^{a_l^o} \right] \right]$$

$$= \eta \left[\frac{\partial a_K^o}{\partial w_{jK}} - \frac{e^{a_K^o}}{\sum_{l=1}^{K} e^{a_l^o}} \cdot \frac{\partial a_K^o}{\partial w_{jK}} \right]$$

$$= \eta \left[1 - S_K^o \right] \frac{\partial a_K}{\partial w_{jK}}$$

$$\boxed{\Delta w_{jK} = \eta \underbrace{(1 - S_K^o)}_{\delta_K^o} S_j^h} \quad \text{---} (5)$$

δ_K^o = local gradient

K is the class in which the example belongs to.

Then $l \ne K$, l – is not the class in which $\bar{x} \in C_K$.

$$\Delta w_{jl} = -\eta \frac{\partial \tilde{E}}{\partial w_{jl}}$$

$$= -\eta \cdot \frac{\partial}{\partial w_{jl}} \left[-\ln s_k^o \right]$$

$$= -\eta \cdot \frac{\partial}{\partial w_{jl}} \left[\ln \frac{e^{a_k^o}}{\sum_{l=1}^{K} e^{a_l^o}} \right]$$

$$= -\eta \cdot \frac{\partial}{\partial w_{jl}} \left[\ln e^{a_k^o} - \ln \left(\sum_{l=1}^{K} e^{a_l^o} \right) \right]$$

$$= \eta \left[\frac{-1}{\sum_{l=1}^{K} e^{a_l^o}} \right] \cdot e^{a_l} \cdot \frac{\partial a_l}{\partial w_{jl}}$$

$$\boxed{\Delta w_{jl} = -\eta\, s_l^o s_j^h} \quad \text{------ (6)}$$

$$\Rightarrow \quad \Delta w_{ij}^h = -\eta \cdot \frac{\partial \bar{E}}{\partial w_{ij}^h}$$

$$= -\eta \cdot \frac{\partial}{\partial w_{ij}^h} \left[-\ln y_k \right]$$

$$= \eta \cdot \frac{\partial}{\partial w_{ij}^h} \left[\ln s_k^o \right]$$

$$= \eta \cdot \frac{\partial}{\partial w_{ij}^h} \left[\ln e^{a_k^o} - \ln \sum_{l=1}^{K} e^{a_l^o} \right]$$

$$= \eta \cdot \frac{\partial a_k^o}{\partial w_{ij}^h} - \frac{1}{\sum_{l=1}^{K} a_l^o} \cdot \frac{\partial}{\partial w_{ij}^h} \left[\sum_{l=1}^{K} e^{a_l^o} \right] \quad \text{------ (7)}$$

$$\frac{\partial a_k^o}{\partial w_{ij}^h} = \frac{\partial \left[\sum_j w_{jk} s_j^h \right]}{\partial w_{ij}^h}$$

$$\frac{\partial a_k^o}{\partial w_{ij}^h} = \frac{\partial \left[w_{jk} s_j^h \right]}{\partial w_{ij}^h}$$

$$= w_{jk} \cdot \frac{\partial s_j^h}{\partial w_{ij}^h}$$

$$= w_{jk} \cdot \frac{\partial s_j^h}{\partial a_j^h} \cdot \frac{\partial a_j^h}{\partial w_{ij}^h}$$

$$= w_{jk} \cdot \frac{\partial f^h(a_j^h)}{\partial a_j^h} \cdot s_i$$

(7) $\Rightarrow$

$$\boxed{\Delta w_{ij}^h = \eta \left[w_{jk} - \sum_{m=1}^{K} w_{jm} s_m^o \right] \frac{df^h(a_j^h)}{da_j^h} \cdot s_i}$$