Ocean Engineering 216 (2020) 107994

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Kernel-based support vector regression for nonparametric modeling of ship maneuvering motion

Zihao Wang [a,b], Haitong Xu [b], Li Xia [a], Zaojian Zou [a,c,*], C. Guedes Soares [b]

[a] School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China
[b] Centre for Marine Technology and Ocean Engineering (CENTEC), Instituto Superior Técnico, Universidade de Lisboa, Lisboa, Portugal
[c] State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai, China

ARTICLE INFO

Keywords:
Nonparametric modeling
Support vector regression
System identification
Ship dynamics
Kernel method

ABSTRACT

A nonparametric identification method based on ν('nu')-support vector regression (ν-SVR) is proposed to establish robust models of ship maneuvering motion in an easy-to-operate way. Assisted by the kernel trick, the nonlinear model learns implicitly in high-dimensional feature space without a priori model structure. The ν-SVR controls the sparsity automatically, resulting in high efficiency. To improve the practicality, a parameter tuning scheme combining the hold-out validation and the simulation of dynamic processes is designed to avoid overfitting. Taking the KVLCC2 ship as the study object, the experimental data from the SIMMAN database are used to evaluate the method. The selection and pre-processing of training data are discussed. The identified model shows good generalization performance in the prediction of multiple maneuvers not involved in the training set, verifying the effectiveness of the method.

1. Introduction

With the development of autonomous ship technology, the modeling technique for ship maneuvering motion has attracted wide attention. A reliable mathematical model has a highly practical value for providing accurate motion predictions or designing a control system. Due to the change in the ship's operating conditions such as payload and water depth, the dynamic characteristic of a ship is not set in stone. Therefore, it is desirable to develop a modeling method to establish or update the models in an easy-to-operate way.

System identification, as a method for modeling, provides a possible solution. For the problem of ship maneuverability, this method builds the dynamic models based on the measured data of free-running model tests or full-scale trials. Compared with other modeling methods such as captive model tests or computational fluid dynamics (CFD), the system identification method has the advantages of high efficiency and low cost. It is considered as the only viable method for online modeling. In addition, the data-driven feature makes it available for both ship model and full-scale ship.

To conduct the system identification for ship maneuvering motions, a straightforward approach is to assume a parametric model. The hydrodynamic forces and moments are expressed as a Taylor expansion of the state variables or as a modular model (Yoon and Rhee, 2003; Sutulo and Guedes Soares, 2014; Luo et al., 2016; Xu et al., 2018). In this way, the identification problem is converted to a multiple linear regression problem that can be solved by many standard regression techniques: least square (Holzhuter, 1989), extended Kalman filter (Abkowitz, 1980), ridge regression (Yoon and Rhee, 2003), support vector regression (Luo and Zou, 2009), to name but a few. In order to establish a proper dynamic model, a key point is to select an appropriate model structure. Nevertheless, there has been no authoritative or unified method to determine the model structure (Sutulo and Guedes Soares, 2011). It is a trade-off between model capability and complexity, which often need to be iteratively adjusted according to the quality of the prediction or the experience of the modeler. Moreover, different types of ships often correspond to different model structures, and changes in operating conditions may also cause the original model structure no longer applicable. Therefore, determining the model structure is always a challenging problem in practical application for parametric modeling.

An alternative approach is nonparametric modeling, also known as black-box modeling or surrogate modeling. The basic idea is to optimize the model structure according to the training data, without any physical insight. The only prior knowledge is the input and output datasets of the system. This property makes the system identification flexible and easy

* Corresponding author. School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China.
E-mail address: zjzou@sjtu.edu.cn (Z. Zou).

https://doi.org/10.1016/j.oceaneng.2020.107994
Received 17 May 2020; Received in revised form 21 July 2020; Accepted 22 August 2020
Available online 19 September 2020
0029-8018/

to implement. For all its advantages, nonparametric modeling technique is something of a double-edged sword. Compared with parametric modeling, nonparametric modeling is more sensitive to the training data. The noise and outliers can affect the selection of the optimal model structure, resulting in overfitting or underfitting. Thus, there are more stringent requirements to the selection of the training data and the setting of the identification algorithms.

Machine learning techniques provide an effective way for nonparametric modeling of dynamic systems. Rather than postulating the candidate models in a finite-dimensional space, machine learning techniques conduct the function approximation possibly in an infinite-dimensional feature space (Pillonetto et al., 2014).

Neural network algorithms have been studied for the system identification of ship maneuvering motions, such as recursive neural networks (Moreira and Guedes Soares, 2003a, Moreira and Guedes Soares, 2003b; 2012; Hess et al., 2006), feed-forward neural networks (Rajesh and Bhattacharyya, 2008; Zhang and Zou, 2013) and long short-term memory (LSTM)-based recurrent neural network (Woo et al., 2018). The identified models have shown acceptable prediction accuracy. Nevertheless, neural networks have some inherent defects, such as local extrema, less solid mathematical foundation and the risk of suffering the curse of dimensionality. Moreover, since minimizing the empirical risk can make the function estimation sensitive to the training data, the implementation of neural networks often runs a risk of overfitting (Schölkopf et al., 2002), which degrades the generalization performance.

Recently, there has been a considerable interest in the so-called kernel-based regularization methods for system identification (Ljung et al., 2020), such as Kernel Ridge Regression (KRR), Gaussian Processes (GP) and Support Vector Regression (SVR). The basic idea is to map the data into a high-dimensional, implicit feature space by replacing the features with a kernel function. Assisted by the kernel trick, the computational cost is not affected by the dimension of the feature space, so that the curse of dimensionality can be avoided (Cristianini and Shawe-Taylor, 2000). Furthermore, regularization has the effect of reducing variance, thereby improving the model generalization.

Some studies that applied kernel methods have shown good results in the identification of marine hydrodynamics. Ariza et al. (2018) employed multi-output GP for the modeling of four degrees-of-freedom (DOF) ship dynamics. The training and validation were carried out by using the simulation data. Bai et al. (2019) applied locally weighted learning to nonparametric modeling of ship maneuvering motions and proposed a modified genetic algorithm to avoid overfitting or underfitting of the model. Moreno-Salinas et al. (2019) modeled ship maneuvering motion with KRR and Kernel Ridge Regression Confidence Machine (KRRCM). The datasets obtained from the $20^\circ/20^\circ$ zigzag tests were used as training data. In Moreno et al. (2019), KRR with a polynomial kernel and symbolic regression were applied with the experimental data taken from the random trajectories.

These recent studies hint that the kernel method can be effective for the nonparametric modeling of ship maneuvering motions. However, one disadvantage of KRR and GP is the lack of sparsity, which brings some problems in implementation when dealing with large datasets (Cristianini and Shawe-Taylor, 2000). In this paper, a framework based on SVR, a well-known kernel method, is investigated for the nonparametric modeling of ship maneuvering motions. The feature of SVR is that it can produce sparse representations, resulting in extremely high efficiency. Besides, the optimization problems are convex due to the Mercer's conditions on the kernels, thus have a global optimal solution.

In the last decade, the SVR algorithm has been mainly applied in the parametric modeling of ship dynamics with a known model structure. The linear kernel was generally used to estimate the hydrodynamic coefficients. Luo and Zou (2009) first applied the least squares support vector regression (LSSVR) to the white-box modeling of ship maneuvering motions. Afterwards, different kinds of SVR including ε-SVR (Zhang and Zou, 2011), ν-SVR (Wang et al., 2019) and the optimal truncated LS-SVR (Xu and Guedes Soares, 2019; Xu et al., 2020) were studied. To improve the accuracy of parameter estimation, the metaheuristics methods were employed to select the parameters in LSSVR (Luo et al., 2016; Zhu et al., 2017, 2019; Xu and Guedes Soares, 2020). As mentioned above, these studies assumed a known model structure. In practice, however, it is still a problem to determine an appropriate model structure. Moreover, SVR with a linear kernel is approximately equivalent to the least squares regression with a weight delay factor, especially LSSVR. From this perspective, the application of SVR in parametric modeling does not take full advantages of kernel methods.

Until recently, few studies have focused on the nonparametric modeling of ship maneuvering motions based on SVR. It has only been investigated under limited conditions and has not yet been systematically evaluated based on the real data from experiments.

Wang et al. (2015) applied LSSVR for black-box modeling of 4-DOF ship maneuvering motion, but the data were generated by simulation without considering the noise disturbance. Luo et al. (2014) employed LSSVR for a catamaran with the dataset from turning circle tests and obtained the control parameters in SVR by cross-validation. However, due to the noise disturbances, the implicit model can only predict the specific turning circle maneuvers. Some studies on the modeling of ship roll motion can be found in literature (Hou et al., 2018; Sclavounos and Ma, 2018). However, the modeling of ship maneuvering motions is more complicated. The high coupling among the multiple DOF motions magnifies the inaccuracies in modeling, resulting in cumulative deviations and even divergence. In order to accurately extract the nonlinear dynamic characteristics from the real data, it is desirable to set reasonable parameter selection strategy and data pre-processing strategy. An in-depth research on the identification framework is still needed to ensure the nonparametric models stable and with a good generalization ability.

This paper presents a novel nonparametric modeling framework based on ν('nu')-SVR and conducts a comprehensive evaluation based on the experimental data. The ν-SVR model learns nonlinear mappings in the kernel-induced feature space while ensuring the sparsity of the solution through adaptive adjustment. A parameter tuning strategy that considers cumulative errors by combining hold-out validation and simulation of free-running tests is designed to ensure the prediction ability. On top of that, the important issues of training data selection and pre-processing are also discussed. To evaluate the applicability of the method, the experimental dataset of KVLCC2 from the SIMMAN database is taken for training and validation. The effect of the proposed parameter tuning method is tested by comparing the results with those of the 10-fold cross-validation. The identified model is assessed by testing the generalization ability, namely, predicting multiple trajectories not included in the training data.

The rest of the paper is organized as follows: Section 2 presents the problem formulation for nonparametric modeling of ship maneuvering motions. Section 3 describes the mathematical basis of the ν-SVR, discusses the selection of the kernel function, and proposes a parameter tuning method for ship maneuvering modeling. In Section 4, a case study of KVLCC2 is carried out. The ideas of training data selection and pre-processing are discussed first; then identification modeling and a systematic evaluation are conducted based on real data from model tests. Finally, Section 5 summarized with conclusions.

2. Problem formulation

An accurate model of the dynamic system is crucial for planning, control, and many other applications (Nguyen-Tuong and Peters, 2011). This paper focuses on the modeling of 3-DOF ship maneuvering motion, i.e., surge motion, sway motion and yaw motion. In the modeling process, nonlinear mapping is learned from the measured data of free-running model tests, which represents the relationship between the input and the state variables. It can be used to predict the next state given the current state and input. A continuous time-invariant nonlinear state-space model is selected to describe the dynamic system:

$$\begin{aligned} \dot{\mathbf{x}}_s(t) &= \mathbf{f}(\mathbf{x}_s(t), \mathbf{u}(t)) \\ \mathbf{z}(t) &= h\mathbf{x}_s(t) + \mathbf{z}(t-1) \end{aligned} \tag{1}$$

where $\mathbf{x}_s(t)$ is the state vector, referring to the speed items. $\mathbf{u}(t)$ is the input vector, referring to the propeller thrust and steering angle. $\mathbf{f}(\cdot)$ is the nonlinear mapping in matrix notation. Output $\mathbf{z}(t)$ represents the displacement and the heading angle, which is formulated by Euler's method with time step h. The state equation, namely the differential equations of the vehicle's speeds, is given as

$$\begin{aligned} \dot{u}(t) &= f_1(u(t), v(t), r(t), \delta(t)) \\ \dot{v}(t) &= f_2(u(t), v(t), r(t), \delta(t)) \\ \dot{r}(t) &= f_3(u(t), v(t), r(t), \delta(t)) \end{aligned} \tag{2}$$

where u, v, r represent the surge speed, sway speed and yaw rate, respectively. δ represents the steering angle. These three state quantities and one input quantity are used as regressors, which are also the input data for system identification. They can all be collected by the on-board sensors. The accelerations are obtained by differentiating the corresponding speeds with respect to time and used as the output data. Eq. (2) is the regression function for the nonparametric modeling problem. The regressors are selected by referring to the structure of Abkowitz model (Abkowitz, 1969). Consequently, the model is expected to be able to accurately predict the ship maneuvering motion around the normal speed, which can meet the prediction requirements for most ship maneuvers.

The ν-SVR with Radial Basis Function (RBF) kernel is used in this study to identify the mapping $f(\cdot)$ in a kernel-induced feature space. With the help of kernel functions, the model selection is reduced to the parameter tuning of ν-SVR. Because of the high coupling effects of different DOFs and the noise disturbance in the real data, the main challenge is how to avoid overfitting and ensure the generalization ability of the model.

3. Identification algorithm

3.1. The ν-support vector regression algorithm

The ν-Support Vector Regression (ν-SVR) algorithm can adaptively control the sparseness property based on the data structure (Schölkopf et al., 2000). It is a modified algorithm of the ε-SVR. The parameter ν helps to adjust the number of support vectors adaptively; thereby avoiding the manual adjustment of the parameter ε. This feature makes the algorithm smarter, more reliable, and easy to apply in practice. In the modeling of ship maneuvering motion, the data from free-running model tests inevitably contain noise from measurement noise or environmental disturbance. The ν-SVR algorithm is more suitable for this issue since it can ensure stability and generalization ability while taking the modeling efficiency into consideration.

Suppose the nonlinear mapping $\mathbf{\Phi}(\cdot)$ from the space χ of regressors $\mathbf{x}$ to a possible infinite-dimensional hypothesis space $\mathscr{H}$ is described as

$$\begin{aligned} &\mathbf{\Phi}: \chi \rightarrow \mathscr{H} \\ &\mathbf{x} = (x_1, \ldots, x_n) \rightarrow \mathbf{\Phi}(\mathbf{x}) = (\varphi_1(\mathbf{x}), \ldots, \varphi_l(\mathbf{x})) \end{aligned} \tag{3}$$

The nonlinear regression function $f(\cdot)$ in Eq. (2) is expressed as the product of the unknown weight vector $\mathbf{w}$ and the mapped regressor $\mathbf{\Phi}(\mathbf{x})$:

$$f(\mathbf{x}) = \mathbf{w}^{\mathrm{T}}\mathbf{\Phi}(\mathbf{x}) + b \tag{4}$$

where b is the bias.

Conventionally, there are two steps for building a nonparametric model: First, select a nonlinear mapping to transform the data from the input space to a feature space $\mathscr{H}$; and then identify the weights in the feature space. The ν-SVR algorithm merges the two steps with the help of the kernel trick, without the need to explicitly compute the mapping. The optimization problem is formulated as

$$\min_{\mathbf{w},b,\varepsilon} \frac{1}{2}\|\mathbf{w}\|^2 + C\cdot\left(\nu\varepsilon + \frac{1}{l}\sum_{i=1}^{l}(\xi_i + \xi_i^*)\right) \tag{5}$$

subject to

$$y_i - \left(\mathbf{w}^{\mathrm{T}}\mathbf{\Phi}(\mathbf{x}_i) + b\right) \le \varepsilon + \xi_i \tag{6–1}$$

$$\left(\mathbf{w}^{\mathrm{T}}\mathbf{\Phi}(\mathbf{x}_i) + b\right) - y_i \le \varepsilon + \xi_i^* \tag{6–2}$$

$$\xi_i^{(*)} \ge 0, \quad \varepsilon \ge 0 \tag{6–3}$$

where ξ_i, ξ_i^* are the slack variables, also known as ε-insensitive training error. Only the data points with residuals outside the threshold will contribute to the regression fitting. The subset of these data is called support vectors (SVs). The cost function Eq. (5) is based on the structural risk minimization principle (Vapnik, 2006). The regularization constant C controls the trade-off between the model complexity and the empirical risk, which helps to avoid overfitting. If C is too large, the optimization will only minimize the empirical risk, regardless of the model complexity. In the cost function, the parameter ν controls the width of the ε-insensitive zone, which adjusts ε and predetermines the relative number of training samples as support vectors.

To construct the kernel-induced feature space, the optimization problem Eqs. (5) and (6) are transformed into a dual representation. The primal Lagrange formulation is constructed as

$$L(w,b,\alpha^{(*)},\beta,\xi^{(*)},\varepsilon,\eta^{(*)}) = \frac{1}{2}\|\mathbf{w}\|^2 + C\nu\varepsilon + \frac{C}{l}\sum_{i=1}^{l}\left(\xi_i + \xi_i^*\right) - \beta\varepsilon - \sum_{i=1}^{l}\left(\eta_i\xi_i + \eta_i^*\xi_i^*\right) - \sum_{i=1}^{l}\alpha_i\left(\xi_i + y_i - \mathbf{w}^{\mathrm{T}}\mathbf{\Phi}(\mathbf{x}_i) - b + \varepsilon\right) - \sum_{i=1}^{l}\alpha_i^*\left(\xi_i^* + \mathbf{w}^{\mathrm{T}}\mathbf{\Phi}(\mathbf{x}_i) + b - y_i + \varepsilon\right) \tag{7}$$

where $\alpha_i^{(*)}, \beta, \eta_i^{(*)}$ are the Lagrange multipliers. The corresponding dual is deduced based on the saddle point conditions:

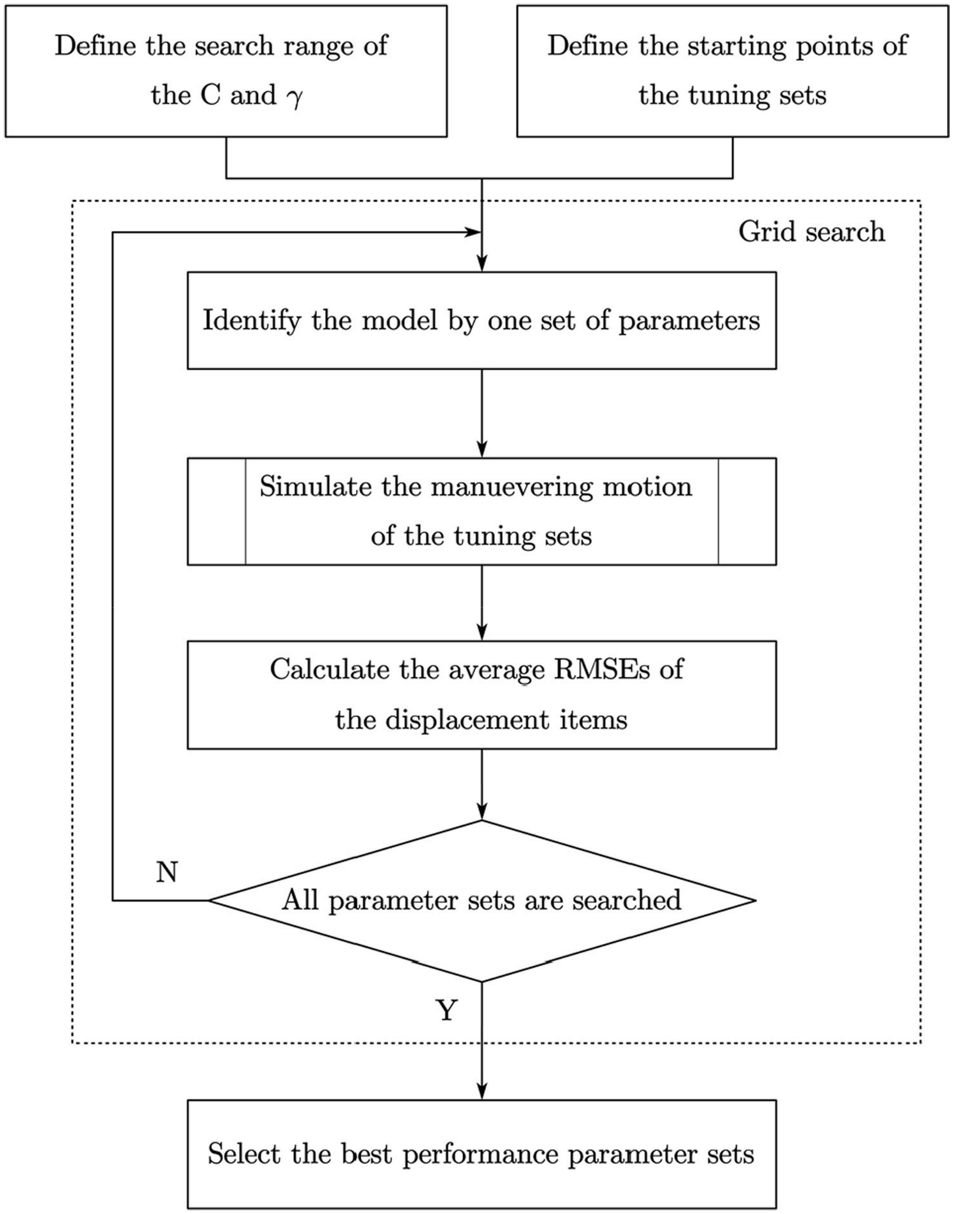


Fig. 1. Flowchart of the dynamic simulation based tuning strategy for parameters C and.γ

$$\begin{aligned}
&\frac{\partial L}{\partial w} = 0 \rightarrow w = \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)\varphi(x_i) \\
&\frac{\partial L}{\partial b} = 0 \rightarrow \sum_{i=1}^{l}(\alpha_i^* - \alpha_i) = 0 \\
&\frac{\partial L}{\partial \xi_i^{(*)}} = 0 \rightarrow C{\cdot}\nu - \sum_{i=1}^{l}(\alpha_i^* - \alpha_i) - \beta = 0 \\
&\frac{\partial L}{\partial \varepsilon} = 0 \rightarrow \frac{C}{l} - \alpha_i^{(*)} - \eta_i^{(*)} = 0
\end{aligned} \tag{8}$$

Substituting Eq. (8) into Eq. (7) and utilizing a kernel function $K(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{\Phi}(\mathbf{x}_i)^{\mathrm{T}}\mathbf{\Phi}(\mathbf{x}_j)$ to replace the dot product of $\mathbf{\Phi}(\mathbf{x})$, the dual representation of the optimization problem is deduced as

$$\min_{\alpha_i} \frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i^* - \alpha_i)\left(\alpha_j^* - \alpha_j\right)K(\mathbf{x}_i, \mathbf{x}_j) - \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)y_i \tag{9}$$

subject to

$$\sum_{i=1}^{l}(\alpha_i^* - \alpha_i) = 0 \tag{10–1}$$

$$\alpha_i^{(*)} \in \left[0, \frac{C}{l}\right] \tag{10–2}$$

$$\sum_{i=1}^{l}(\alpha_i^* + \alpha_i) \leq C{\cdot}\nu \tag{10–3}$$

If the kernel fulfills the Mercer conditions, the optimization problem is convex and hence has a global optimal solution. According to Eq. (8), the weight vector $\mathbf{w}$ is a linear combination of the mapped support vectors $\mathbf{\Phi}(\mathbf{x})$. Consequently, the regression function Eq. (4) can be rewritten as follows:

$$f(\mathbf{x}) = \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)K\left(\mathbf{x}_i, \mathbf{x}_j\right) + b \tag{11}$$

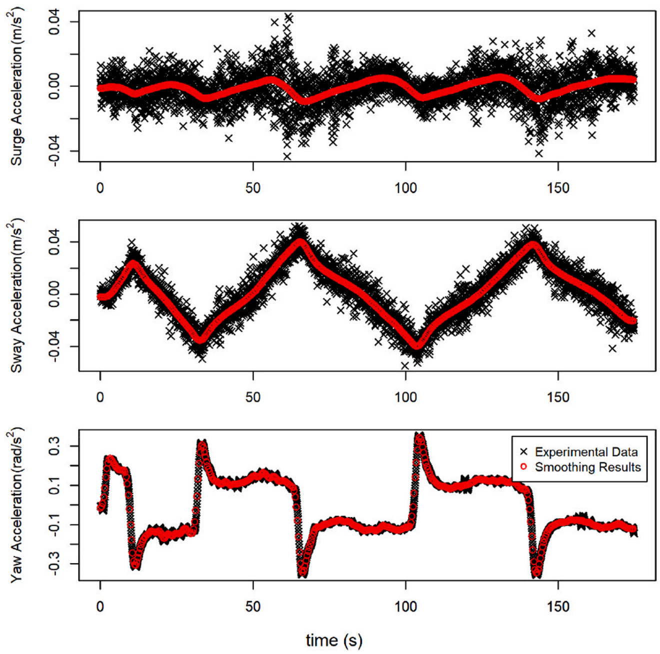


Fig. 2. Smoothing results of the 10°/5° zigzag test data.

The feature vectors are represented by the kernel function implicitly. In this way, the inner product can be efficiently computed in a way independent of the dimension of the feature spaces, thereby avoiding the curse of dimensionality. The nonlinear regression function of ship maneuvering motion corresponds to Eq. (11). The nonparametric modeling is transformed to the selection of the kernel function and its parameters.

3.2. Kernel selection

There are many choices for the kernel function fulfilling the Mercer conditions (Schölkopf et al., 2002). No matter which kernel is chosen, the method for selecting the support vectors does not change. Referring to the model structure based on physical mechanism, the nonlinear mapping can correspond to the function of the hydrodynamic forces and moments expressed in Taylor expansion, where the inertia terms are taken as constants. The RBF kernel (also called Gaussian kernel) and the inhomogeneous polynomial kernel are good choices for this case, expressed as follows:

$$K\left(\mathbf{x}_i, \mathbf{x}_j\right) = \exp\left(-\gamma\left|\left|\mathbf{x}_i - \mathbf{x}_j\right|\right|^2\right) \left(\text{RBF kernel}\right) \tag{12}$$

$$K\left(\mathbf{x}_i, \mathbf{x}_j\right) = \left(\gamma \mathbf{x}_i^{\mathrm{T}} \mathbf{x}_j + r\right)^d \ (d - \text{degree inhomogeneous polynomial kernel}) \tag{13}$$

where γ is the width parameter, and r controls the relative weightings between different degree features and the strength of the constant feature.

The RBF kernel can transform the data space into an infinite-dimensional feature space by using the Taylor expansion. Taking $\gamma = 1$ as example, the RBF kernel is expressed as

The width parameter γ in the RBF kernel plays an important role in controlling the model complexity and needs to be carefully tuned. The polynomial kernel provides control of the complexity through three parameters, i.e., the degree d and the parameters (γ, r). It has an intuitive correspondence with the widely used high-order polynomial of the ship kinematics. Nevertheless, choosing a good combination of the three parameters is not an easy task. In contrast, the RBF kernel has only one parameter, which means that it requires less tuning computation. In addition, the ability to extend the feature space to infinite dimensions makes it more powerful in nonlinear modeling. Considering the applicability of the method, the RBF kernel is selected in this paper to express the nonlinear mapping.

3.3. Parameter tuning

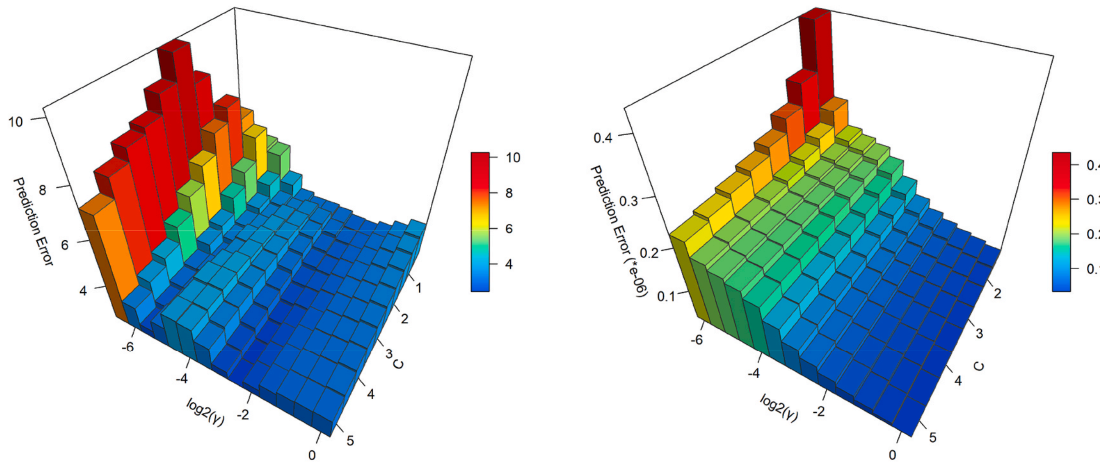
The nonlinear mappings from the regressor space to the output space are closely related to the selection of the parameters. The tuning step helps to find the appropriate parameters for modeling and prevents overfitting. The ν-SVR with RBF kernel has three parameters to be determined in advance, namely γ, C and ν.

A dynamic simulation based tuning strategy is proposed, which mainly focuses on the selection of C and γ that have a significant effect on the model complexity. The selection is conducted by a method combining the hold-out validation and the simulation of free-running tests. At first, the parameter ν is set manually considering the sample size and the level of noise. This is based on the modeler's experience or the prior knowledge on the measurement noise. The value of ν is in the range of $[0, 1]$, which is a lower bound on the relative number of SVs. The smaller the ν, the fewer the support vectors and the faster the computation will be. On the other hand, too few support vectors may result in a low precision for approximating the original data. Therefore, the choice of ν should be made by comprehensively considering accuracy and computing efficiency. More details can refer to Schölkopf et al. (2000).

Fig. 1 shows the flowchart of the tuning strategy for parameters C and γ. First, the search range of C and γ is decided. Theoretically, a reasonable value of C is in the range of output values (Mattera and Haykin, 1999). However, this selection is quite sensitive to the possible outliers. Cherkassky and Ma (2004) advocated to select the value of C based on the prescription $\max(|\bar{y} + 3\sigma_y|, |\bar{y} - 3\sigma_y|)$, where $\bar{y}$ and σ_y are the mean and the standard deviation of the output values. Considering the uncertainties in practice, this paper sets a search range of C on the basis of the empirical value $|\bar{y} \pm 3\sigma_y|$. The search range is defined as $(0, \max(|\bar{y} + 5\sigma_y|, |\bar{y} - 5\sigma_y|]$. Since the scaling of outputs will affect the optimal value of C, the variables in the training set are standardized and centralized beforehand to zero mean and unit variance. The parameter γ plays an important role for the black-box model complexity. Its value is chosen in the range of $[0, 1]$. A too large γ will cause overfitting, making the model lack generalization ability and highly sensitive to the noise. On the other hand, if γ is too small, the mapping will behave almost linearly and lose the nonlinear characteristics.

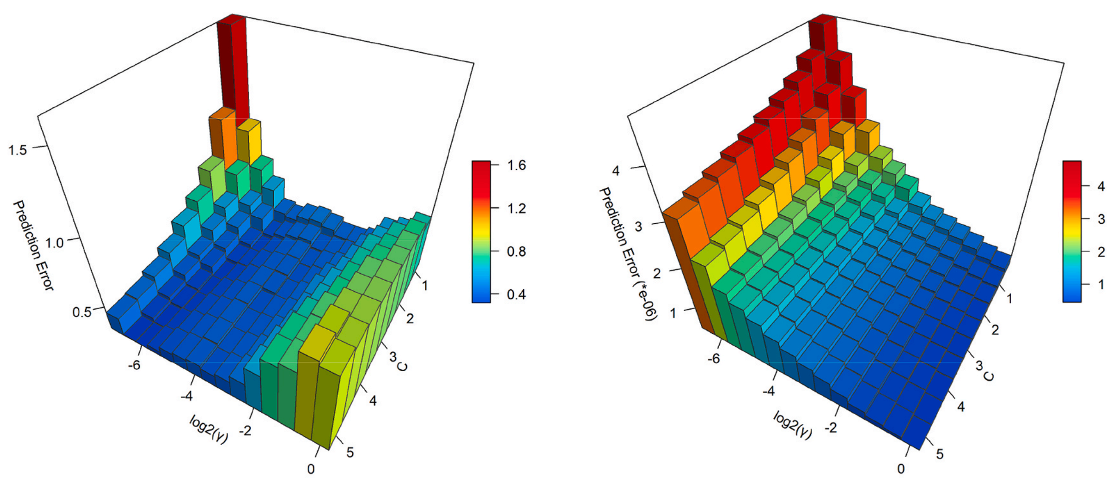
A hold-out strategy combined with the simulation of free-running tests is used to evaluate the performance of the parameters. A subset of the available data is held out for the tuning purpose, which means it is independent of the training set. The grid search is carried out to determine parameters C and γ in limited ranges. In the hold-out dataset, k starting points are selected. Given the initial values of the state variables and the steering angle at subsequent time steps, the simulations of free-

$$K\left(\mathbf{x}_i, \mathbf{x}_j\right) = \exp\left(-\gamma\left|\left|\mathbf{x}_i - \mathbf{x}_j\right|\right|^2\right) = \exp\left[-(x_i)^2\right] \cdot \left(\sum_{k=0}^{\infty} \frac{2^k (x_i)^k (x_j)^k}{k!}\right) \cdot \exp\left[-(x_j)^2\right] \tag{14}$$



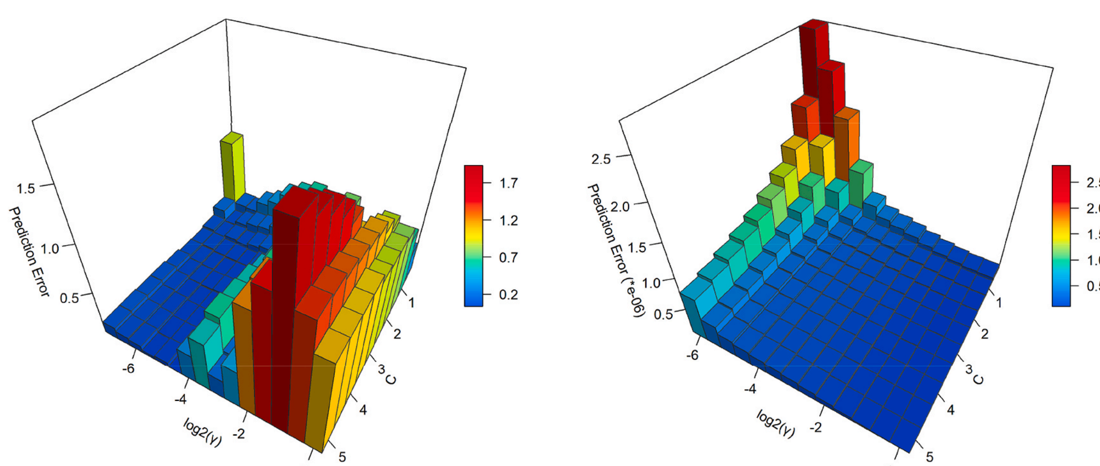
(a) Proposed tuning strategy (b) 10-fold cross-validation

Fig. 3. Parameter tuning results for surge motion.



(a) Proposed tuning strategy (b) 10-fold cross-validation

Fig. 4. Parameter tuning results for sway motion.



(a) Proposed tuning strategy (b) 10-fold cross-validation

Fig. 5. Parameter tuning results for yaw motion.

Table 1
Selection of the ν-SVR parameters.

	Surge motion	Sway motion	Yaw motion
C	5.0	5.0	4.5
γ	0.176	0.0312	0.0442
ν	0.5	0.5	0.5

running tests are conducted with the trained model. The average root-mean-square errors (RMSEs) of the displacement are applied for judging the accuracy of the model. The number of starting points k can be selected with reference to the size of the hold-out data and the expected computation time. Due to the iterative prediction process in the simulation of free-running tests, there will be a cumulative deviation different from the one-step prediction. The selection of multiple starting points in the hold-out dataset and the selection of displacements as evaluation quantities are intended to amplify the weight of the cumulative deviation so as to better search for the minimum generalization error.

4. Modeling of ship maneuvering motion with experimental data

To evaluate the proposed identification method, nonparametric modeling is performed for a KVLCC2 tanker. The experimental dataset from SIMMAN workshop is used. In the following, the selection and pre-processing of training data are introduced first. Then, a 3-DOF maneuvering model is established. The parameter tuning results are compared with those by the widely used cross-validation strategy.

4.1. Selection and pre-processing of training data

Nonparametric modeling requires almost no prior information, neither the model structure nor the inertia constants. The content of training data determines which dynamic characteristics are excited, and the information provided allows the identification of a reliable model. For a nonlinear dynamic system, a high-quality excitation signal of training data should cover the range of the input space and relevant frequencies.

To include more dynamic information, the datasets of 10°/5° and 30°/5° zigzag tests are selected as the training set, which are from the model tests carried out by the Hamburg Ship Model Basin (HSVA). Besides, the dataset of 20°/5° zigzag test from HSVA is selected as the hold-out set for tuning parameters. In other words, around 70% of the data is used for training and 30% of the data is used as a hold-out set. The hold-out set contains different dynamic characteristics from the training set. This makes the prediction of the hold-out data represent the generalization performance to some extent. With this setting, the data almost cover the input space of the steering angle. As the authors' previous paper (Wang et al., 2019) indicated, this setting allows more nonlinear dynamics to be excited, hence makes the identified model robust and stable. If possible, it is also recommended to select the training data based on m-level pseudo-random sequence by experimental design (Wang et al., 2020).

The data from model tests have been filtered by a low-pass filter. To further reduce the fluctuation of the acceleration values, the data are preprocessed by the cubic smoothing spline and linear interpolation with a time step of 0.15s. As an example, Fig. 2 shows the smoothing results of the 10°/5° zigzag test. The acceleration trends are much clearer after pre-processing, which contributes to extracting the dynamic characteristics.

To avoid that the characteristics of larger range of values dominate the characteristics of smaller range of values, the input and output variables are standardized and centralized. Each characteristic is scaled to zero mean and unit variance, expressed as

$$y_i' = \frac{y_i - \overline{y}}{\sigma(y)} \tag{15}$$

where y_i is the original variable and y_i' is the scaled variable; $\overline{y}$ is the mean value of the variable and $\sigma(y)$ is the variance of the variable y.

4.2. Modeling by kernel-based ν-SVR

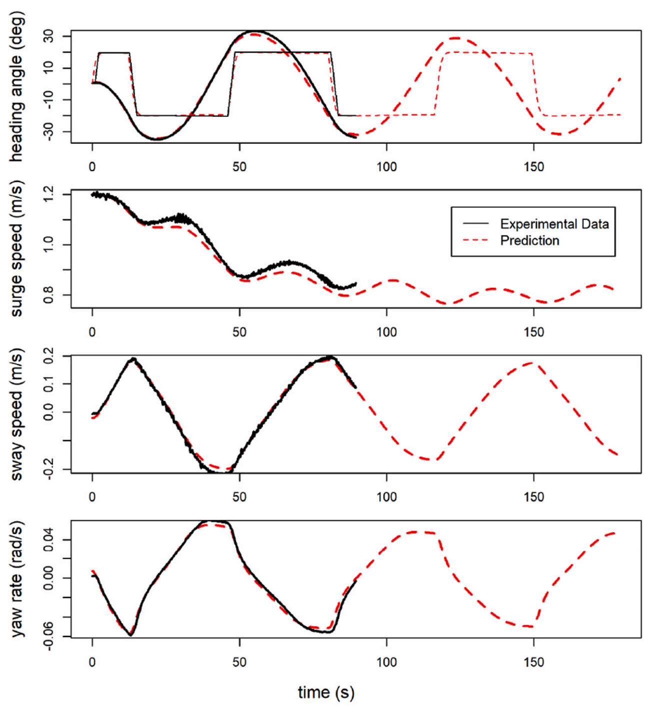
The RBF kernel-based ν-SVR is used for modeling based on the pre-processed data. Considering the computational efficiency and long-term prediction requirements, the parameter ν is set as 0.5, which is also a robust compromise solution in practice (Mattera and Haykin, 1999). On this basis, the value of ν can be adjusted as needed for higher training speed or better accuracy of approximation. Then, the parameters C and γ are selected by the proposed dynamic simulation based tuning method. To evaluate the tuning results, a comparative study is conducted between the proposed tuning method and the 10-fold cross-validation strategy. The k-fold cross-validation is a widely used tuning strategy in machine learning field. It randomly divides the available data into k subsets. At each stage, one subset remains for validation while the other $k-1$ subsets are used for training. The procedure is repeated k times; each time, a different subset is treated as the validation set. Eventually, the average mean squared error measures the prediction capability of the model. The main difference from the proposed method is that the k-fold cross-validation considers only one-step prediction error and does

Table 2
Prediction accuracy assessed by RMSE using HSVA model test data
(All maneuvers started from starboard side).

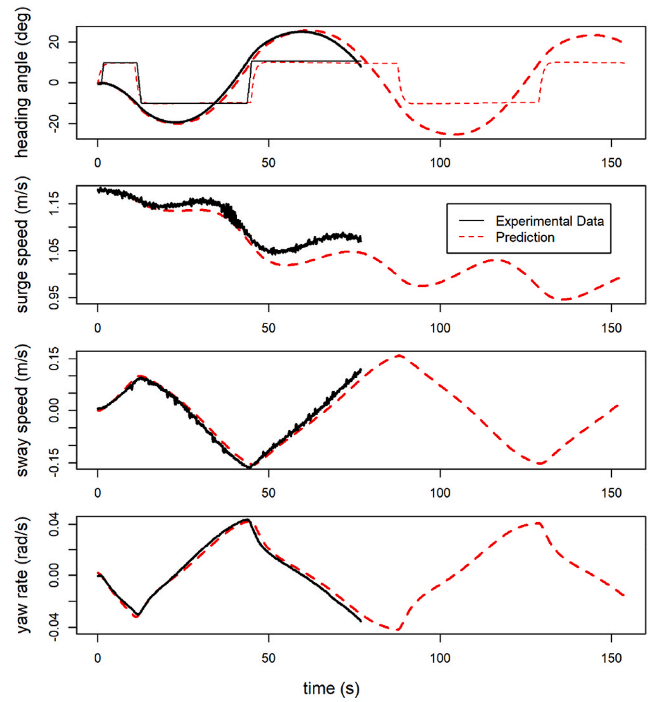
	10°/5° zigzag	10°/10° zigzag	15°/5° zigzag	20°/5° zigzag	20°/10° zigzag	25°/5° zigzag	30°/5° zigzag	35°/5° Zigzag
Surge speed	0.0157	0.0107	0.0115	0.0141	0.0132	0.0145	0.0264	0.0326
Sway speed	0.0366	0.00758	0.0376	0.0515	0.0398	0.0575	0.0897	0.103
Yaw rate	0.00996	0.00258	0.0119	0.0178	0.0133	0.0210	0.0342	0.0417

Table 3
Prediction accuracy assessed by RMSE using MARIN model test data.

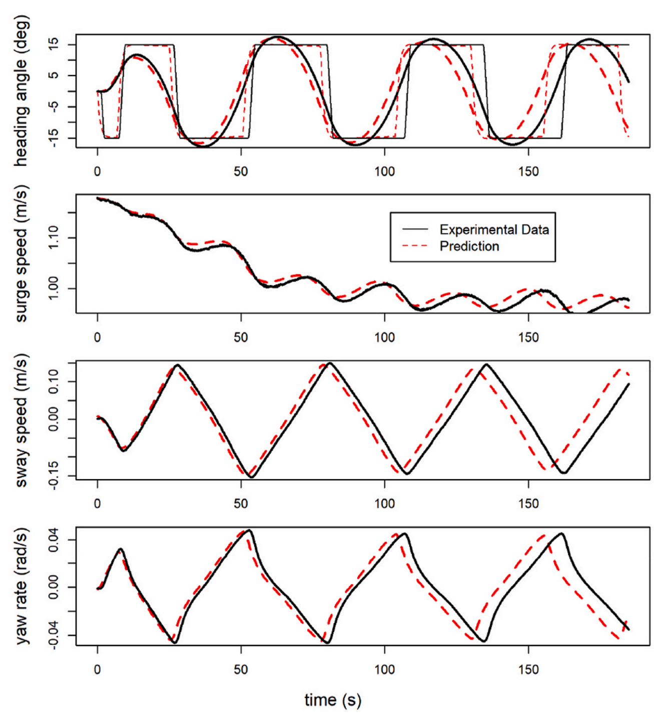
	20°/20° zigzag (starboard)	10°/10° zigzag (starboard)	35° turning (starboard)	20°/20° zigzag (port)	10°/10° zigzag (port)	35° turning (port)
Surge speed	0.0157	0.0107	0.0115	0.0141	0.0132	0.0145
Sway speed	0.0366	0.00758	0.0376	0.0515	0.0398	0.0575
Yaw rate	0.00996	0.00258	0.0119	0.0178	0.0133	0.0210



(a) 20°/20° zigzag maneuver



(b) 10°/10° zigzag maneuver



(c) 15°/5° zigzag maneuver

Fig. 6. Predictions of zigzag maneuvers.

not consider the cumulative bias in the prediction of ship maneuvering motion.

Figs. 3–5 illustrate the results of parameter tuning using the two methods. It is shown that there is a large difference in the tuning results by the two methods. Theoretically, the value of γ can be used to evaluate overfitting, since larger γ corresponds to more complex models. The 10-fold cross-validation tends to choose the largest γ, that is, $\gamma = 1$. This phenomenon implies overfitting, which will make the model lack generalization ability and highly sensitive to the noise. On the other hand, the proposed tuning method suggests the more reasonable solutions with relatively small value of γ. Although the proposed method requires more computational time due to the simulation of free-running tests, approximately 1.5 times that of cross-validation, the comparison of tuning results indicates that the proposed tuning method is more

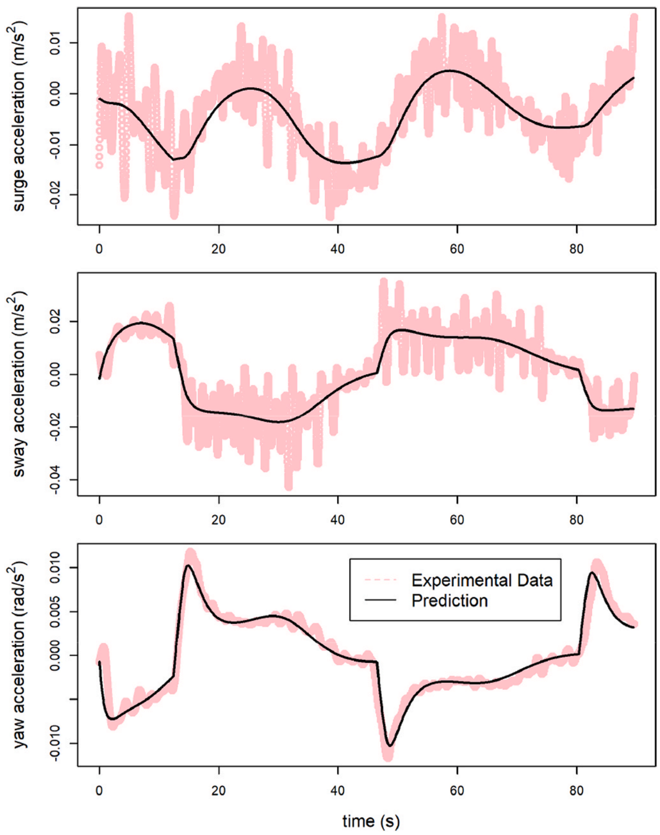


Fig. 7. Predictions of accelerations in 20°/20° zigzag maneuver.

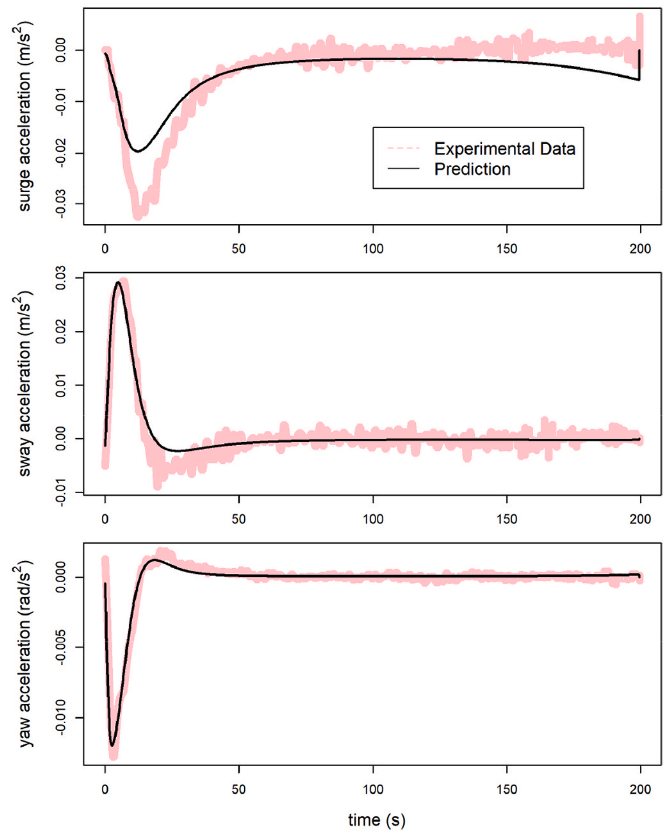


Fig. 9. Predictions of accelerations in 35° turning circle maneuver.

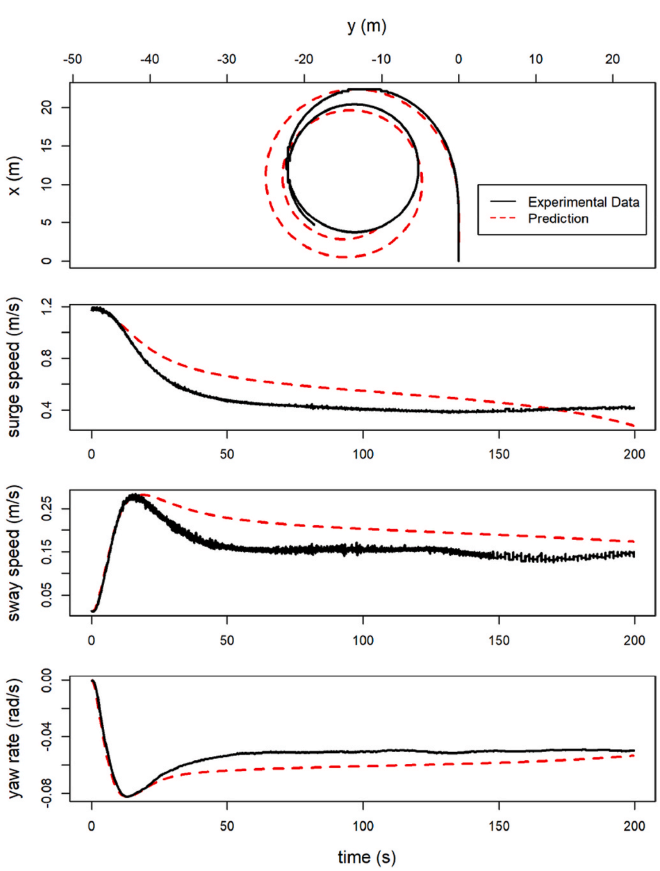


Fig. 8. Predictions of 35° turning circle maneuver.

suitable for the modeling of ship maneuvering motion than the widely used cross-validation.

The results of parameter tuning illustrate that the kernel parameter γ has a significant impact on the tuning results. By contrast, the regularization parameter C is to adjust the generalization performance based on the selected kernel function. There may be several groups of C and γ with an acceptable RMSE value. The model complexity corresponding to the parameter values combined with the modeling knowledge in the field can help the modeler to fine-tune the parameters and improve the quality of the model. The final decision can be made by comparing the results of motion prediction between the training set and test set. If there is not much difference and the fitting is all good, the set of parameters is an appropriate choice. Table 1 lists the final parameter settings for the case studied.

With the preprocessed training data and the adjusted parameters, the nonparametric models for 3-DOF maneuvering motions are established by the RBF kernel-based ν-SVR, according to Eq. (2). The modeling was implemented in RStudio, which is an integrated development environment for the R programming language. It took less than 5s, using an Intel Core i7-6820HQ CPU with 3.3 GHz.

4.3. Validation of the generalization ability

The generalization performance of the identified model is mainly evaluated by predicting the maneuvers not involved in the training data. The validation set includes 10°/10°, 20°/20°, 15°/5°, 20°/10°, 25°/5°, 35°/5° zigzag tests and 35° turning circle test. The datasets from model tests conducted by HSVA and Maritime Research Institute Netherlands (MARIN) are used for comparison. These data were collected from the free-running tests of the KVLCC2 model, which was at a starting speed of 1.18 m/s in deep and calm water. Giving the initial ship's speeds, prediction of ship maneuvering motion is performed by using the identified model to simulate the free-running tests. The rudder steering rate is

Table 4
Comparison of the prediction of 20°/20° zigzag maneuver for KVLCC2.

	System identification by ν-SVR	Free sailing (MARIN)	Empirical modular model with cross flow drag and slender body (MARIN)	PMM analysis of INSEAN data (FORCE)	Analysis of CMT tests (Hiroshima Univ.)	RANS used as virtual PMM (HSVA)
1st ψ_{OS} (deg)	15.0	14.7	15.5	10.6	10.0	15.6
2nd ψ_{OS} (deg)	11.0	12.9	17.2	11.7	9.5	17.6
3rd ψ_{OS} (deg)	12.2	13.5	16.0	10.8	10.3	19.0
1st t_{OS} (s)	67.0	62.8	58.5	54.0	48.5	65.0
2nd t_{OS} (s)	55.0	64.0	58.0	56.0	46.5	76.0
3rd t_{OS} (s)	63.0	58.0	54.5	62.0	48.5	81.0

(ψ_{OS} denotes the overshoot angle and t_{OS} denotes the time to check yaw.).

Table 5
Comparison of the prediction of 35° turning circle maneuver for KVLCC2.

	System identification by ν-SVR	Free sailing (MARIN)	Empirical modular model with cross flow drag and slender body (MARIN)	PMM analysis of INSEAN data (FORCE)	Analysis of CMT tests (Hiroshima Univ.)	RANS used as virtual PMM (HSVA)
T_{90} (s)	165.7	165.0	161.5	232.0	188.5	149.5
T_{180} (s)	325.1	345.0	287.0	500.0	374.0	292.5
T_{360} (s)	666.4	758.0	525.0	1136.0	744.5	798.5
D_{stc}/L_{pp}	2.72	2.46	0.41	4.00	2.19	2.35
AD/L_{pp}	3.10	2.98	2.95	3.84	3.30	3.13
TD/L_{pp}	3.48	3.09	2.59	4.72	3.42	2.95

(T_n denotes the time to reach the yaw angle of n degree. D_{stc} denotes the turning diameter. AD and TD are abbreviations for 'Advance' and 'Tactical diameter'. L_{pp} is the ship length).

15.68°/s on the model scale, corresponding to 2.32°/s on the full scale.

Overall, the results presented below show that good long-term predictions can be achieved for all maneuvers, indicating that the identified nonlinear model can well represent the dynamic characteristics of the ship. The prediction accuracy of the speeds evaluated by RMSE is given in Table 2 and Table 3. As can be seen, the prediction accuracy of these maneuvering motions is balanced: there is no situation where the prediction of the training set is significantly more accurate than that of the validation set. This result implies that the model is generalized well and there is no significant overfitting.

Fig. 6 shows the prediction results of heading angle and speeds for 20°/20°, 10°/10° and 15°/5° zigzag tests. Similar prediction results are obtained for other zigzag maneuvers. As can be seen, high accuracy is achieved in all motion predictions of zigzag maneuvers. The cumulative deviation is small, indicating that the model of each DOF motion has high accuracy. As a representative example, Fig. 7 illustrates the prediction of accelerations in 20°/20° zigzag test. The outputs of the nonlinear regression model fit well and there is no sign of overfitting. Fig. 8 shows the prediction results of 35° turning circle maneuver. The long-term trajectory of the turning circle fits well, but there is a deviation in the predicted speed, especially in surge motion. From the acceleration fitting results in Fig. 9, the surge accelerations are close to but not zero during the steady turning motion, and this small deviation in acceleration amplifies the bias in speeds and displacements as the prediction time increase. This implies that the model does not learn sufficiently about the dynamics of steady-state motion. One main reason could be the difference between the flow fields in the training data and the validation data. From a hydrodynamic point of view, the 35° turning circle maneuver is a large drift angle motion, resulting in a large lateral hydrodynamic force on the hull. When the steady-state motion is reached, the frictional resistance and pressure resistance have changed considerably, which reduces the surge velocity. In contrast, the 10°/5° and 30°/5° zigzag tests have a high steering frequency and a relatively small drift angle. They have stronger unsteady characteristics than the turning circle maneuvers. This result reveals that a high-quality excitation signal should not only cover the range of the input space, but also the relevant frequencies. Nevertheless, the changing trends of accelerations are correct, and the performance of trajectory prediction is acceptable.

Tables 4 and 5 present the comparison results with other modeling methods, including captive model tests (PMM, CMT), RANS computation and the empirical one with modular model of cross-flow drag based on slender body theory. The relevant data for comparison are taken from SIMMAN 2008 workshop. The results indicate that the proposed identification method is sufficiently accurate. It should be noted that in this case the training data are extracted from the HSVA's model tests, while the comparison data for validation contains the results from the MARIN's model tests. In fact, bias may exist in the experimental results from different institutions because of the differences in test sites and ship models. Therefore, this might also be one of the reasons for the discrepancies in these predictions.

It can be found that the selection of the sampling interval is a trade-off between the model capability and computational cost. A larger sampling interval means fewer data points and lower computational cost, but at the same time, the cumulative deviation may be larger when predicting the maneuvers with high steering frequency. Assisted by the kernel trick and the sparsity of ν-SVR, the proposed method of nonparametric modeling has high efficiency. In the case studied, the sampling interval is set to 0.15 s, resulting in a modeling time fewer than 5 s and obtaining a model with great generalization ability. When the sampling interval is set to 0.4 s, the modeling time will be less than 1 s, which can satisfy the requirement of online modeling. This fast-built model has similar accuracy for the predictions of 20°/20° and 10°/10° zigzag maneuvers compared with the model in the case studied, but for the prediction of 30°/5° and 35°/5° zigzag maneuvers, the fast-built model has a larger cumulative deviation. Therefore, the selection of sampling intervals is mainly based on modeling purposes: if high-precision long-term trajectory prediction is desired, relatively small sampling intervals should be chosen; while for quick and short-term predictions, large sampling intervals should be chosen to improve the modeling and prediction efficiency.

In summary, the results demonstrate that the RBF kernel-based ν-SVR has a powerful capability for nonlinear modeling, and the identified model has a good generalization performance. The key issue is to find the right kernel parameters. With the proposed parameter tuning scheme, the kernel parameter corresponding to the minimum generalization error is found. Afterwards, the nonlinear mapping can be learned appropriately. The training data also play an important role for parameter tuning and generalization ability. Since the prior knowledge of nonparametric modeling is limited, the modeling process will face great uncertainty and the risk of overfitting when the identified model is expected to be used for predictions in an extrapolation way, such as training with the data of 20°/20° zigzag maneuver while predicting 10°/10° zigzag maneuver or 35° turning circle maneuver. In this situation, the difficulty of adjusting parameters also increases. When the stability of the model is threatened, the applicability of the system identification method is reduced. By contrast, if the training data cover the input space and relevant steering frequency as much as possible, identification is approximately performed as an interpolation processing with respect to the steering angle input. In this way, the modeling uncertainty and risk of overfitting can be effectively reduced, hence improving the practicality. Through the analysis of experimental data, the generalization validation demonstrates that the data pre-processing and the parameter tuning strategy are effective. A robust model is established in an efficient and practical way.

5. Conclusions

A system identification method based on ν-SVR is proposed for modeling of ship maneuvering motion. Taking the KVLCC2 tanker as the study object, the modeling method is systematically evaluated based on the real data from model tests. From the case study, the following advantages and features of the modeling method can be summarized: Firstly, the kernel function makes prior information on the model structure unnecessary, avoids the curse of dimensionality, and provides powerful nonlinear modeling capability. Secondly, ν-SVR automatically adjusts the amount of support vectors to ensure sparsity, thus achieving high computation efficiency. This feature also enhances the applicability, as it reduces the difficulty of parameter tuning and makes the method robust to the varying noise level. Thirdly, the principle of structural risk minimization allows a balance between approximation accuracy and model complexity, which helps to avoid overfitting. Finally, the proposed dynamic simulation based tuning method gives prominence to the evaluation of generalization error and cumulative deviation, thereby ensuring the quality of the model by selecting appropriate parameters. By comparing with the widely used 10-fold cross-validation, the proposed method of parameter tuning is proven to be more suitable for the modeling of ship maneuvering motion. The prediction results of multiple maneuvers validate the effectiveness and robustness of the identified model.

The kernel-based ν-SVR method has good migration and adaptation ability because of its less dependence on prior information, fast computation, and low cost. It can be applied to other types of marine vehicles or other ship dynamics to establish the dynamic models in an easy-to-implement way. This can also provide supports to the model-based planning or control system design thus to improve the autonomous ship technique. In the future study, the effects of factors such as the configuration of training data and the selection of regressors on nonparametric modeling will be studied to further improve the quality of the identified model.

CRediT author statement

Zihao Wang: Conceptualization, Methodology, Simulation, Analysis, Visualization, Writing - Original Draft.

Haitong Xu: Methodology, Simulation, Analysis, Writing - Original Draft

Li Xia: Methodology, Data processing, Analysis, Writing - Original Draft.

Zaojian Zou: Methodology, Supervision, Writing - Review and Editing, Funding acquisition, Project administration.

C Guedes Soares: Methodology, Supervision, Writing - Review & Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work is financially supported by the National Natural Science Foundations of China [Grant number 51979165]. The authors are grateful to the Hamburg Ship Model Basin (HSVA) and Maritime Research Institute Netherlands (MARIN) for providing the experimental data.

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