Ocean Engineering 186 (2019) 106111

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

On the application of empiric methods for prediction of ship manoeuvring properties and associated uncertainties

Serge Sutulo, C. Guedes Soares *

Centre for Marine Technology and Ocean Engineering, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001, Lisbon, Portugal

ARTICLE INFO

Keywords:
Ship manoeuvrability
Mathematical models
Empiric methods
Numerical simulation
Comparative study

ABSTRACT

In relation to development of ship manoeuvrability standards, in particular those accounting for adverse weather conditions there is a need for relatively simple but reliable prediction methods. A number of popular published empiric methods for predicting manoeuvring properties of surface displacement ships have been briefly described and analysed. An extension of the Pershitz method to low speed manoeuvres has been implemented. Sixteen empiric methods and hybridizing combinations of hull and rudder sub-models were tested numerically through simulation of standard manoeuvres for the KVLCC2 benchmark ship. The obtained numerical results have demonstrated that very few methods or combinations give reasonably accurate results for this vessel and some simulated responses even look unacceptable. The primary conclusion drawn from the study is that in general application of universal empiric methods can lead to unacceptably large prediction uncertainties and such methods must be used with great care and preferably tuned on prototype ships.

1. Introduction

The problem of predicting manoeuvring performance of ships and other marine craft has always been of considerable theoretical and practical interest. As long as manoeuvring qualities are important for the overall performance of a vessel, it is important for a ship designer to be able to predict them at different stages of the design process similarly to how it has always been the case with the ship resistance and propulsion. However, the difference with the latter is that while the very design process is unthinkable without the ability of predicting the ship design speed and the required engine power, the manoeuvring performance has been treated as secondary and typically was only verified and adjusted on the later design stages when the shape of the hull was fixed and its scaled model was available. Then, the manoeuvring performance was in most cases tested in the free-running mode directly demonstrating compliance with certain manoeuvring criteria, mostly the turning ability and satisfactory degree of the directional stability.

However, even a superficial comparison with the methods of handling the powering issues in course of the design process, reveals that the common approach is multi-step, realized at each design phase and at each phase methods of different and increasing degree of complexity and accuracy are applied.

For instance, at a conceptual design stage, the ultimately simple Admiralty coefficient formula has been very popular during many decades and still can be very useful nowadays (Barras, 2004). On the preliminary design stage, the classic approach is to rely on the methods associated with various model series which have been developed, manufactured and tested during many decades and typically represent certain databases for the resistance and sometimes also for the propeller–hull interaction coefficients accompanied by simple interpolation procedures to account for the form variations (Bertram and Schneekluth, 1998). Within each series, the hull shape is varying within restricted limits and is defined with relatively few parameters. In practice it means that most of these empiric methods can only provide reliable estimates when the hull form fits some tested series well enough which explains why so many various series have been tested. Although some of the empiric methods like the well-known Holtrop method (Betram and Schneekluth, 1998) were based on several model series and are claimed to be universal, if a proposed ship form has some revolutionary elements or unusual particulars, the prediction error may grow considerably and become inacceptable. While in the past this situation often stimulated development and testing of new series, an alternative later appeared to recourse to the Computational Fluid Dynamics (CFD). The CFD codes are already matured enough to provide estimates that, in general, can be viewed as being even more credible than the series database method. It is expected that the CFD modelling can even become sufficient for the

* Corresponding author.
E-mail address: c.guedes.soares@centec.tecnico.ulisboa.pt (C. Guedes Soares).

https://doi.org/10.1016/j.oceaneng.2019.106111
Received 22 September 2018; Received in revised form 23 April 2019; Accepted 5 June 2019
Available online 9 July 2019

final design stage although the dedicated model tests are still considered desirable.

It would be natural to expect a similar hierarchy of approaches when it goes about manoeuvrability. However, in this case hydrodynamics is much more complicated and, in the same time, long-term investments into the experimental studies in the field of manoeuvring of surface ships were much inferior to those allocated for resistance and propulsion. To a large extent it is explained by the fact that the design process has always been much more focused on the contractual speed and propulsion characteristics which are of considerable importance for the commercial or tactical efficiency of the future ship. This situation seems to be slowly changing after first sets of manoeuvring standards were introduced, first, by the Russian Register of Shipping in 1979 (these standards were further integrated into the Rules (RS, 2018)) and somewhat later—by the International Maritime Organization (IMO, 1993, 2002a,b). In the first case an empiric method for predicting manoeuvrability was de facto embedded into the implicit standards and in the second case any approved empiric method was recognized as a legitimate way to demonstrate compliance with the standards.

However, an increased interest in the empiric methods for predicting manoeuvring performance still has not resulted in really adequate investments into their development. Moreover, any attempt to develop a versatile and reliable method of this kind would, apparently, face unsurmountable difficulties even if much more significant resources are provided. The reasons for these difficulties are:

1. While the overall number of scaled models tested for the purpose of estimating manoeuvring performance is considerable, rarely those results were published and very few of them constituted even small systematic series, the number of which is no match to that of the resistance-and-propulsion series.
2. The situation exposed above can hardly be overcome even if the number of available specialized experimental facilities is increased considerably. The reasons for this are much more complex kinematics and hydrodynamics of manoeuvring motion. While in the straight run regime considered in the resistance and propulsion studies the ship speed is the only variable defining parameter when the hull form and the attitude are fixed, at least 3 kinematic parameters are required to specify the manoeuvring motion in the horizontal plane which results in a tremendous increase of the number of required test runs. This is true not only for the hull forces but also for the propeller-hull and also for the rudder–hull and rudder–propeller interaction. The latter two kinds of interaction are almost completely ignored in the propulsion studies but are of primary importance in manoeuvrability.
3. The rudder deflection angle represents an additional parameter further increasing the complexity of the problem.
4. The number of scalar dynamic responses (forces and moments) is also superior to that in the straight run which further complicates the resulting ship mathematical model.

Regarding all the circumstances outlined above it is clear that it would be practically impossible to develop a reliable empiric method for predicting manoeuvring performance of ships which would be valid for a more or less wide range of hull forms and steering arrangement configurations.

However, in spite of these evident deficiencies, some empiric methods continue to be used in practice. Partly, this is driven by certain favourable circumstances. In particular, the accuracy requirements in manoeuvrability are traditionally less strict than in the resistance and propulsion. For instance, if some manoeuvring measure is predicted with an error not exceeding 15%, the result is viewed as quite satisfactory and even good while the tolerance in predicting the design speed must not exceed 5%. This explains why e.g. the scale effect is treated and studied as a factor of major importance in resistance, while in manoeuvrability it is given much less attention and mostly neglected except for the care about the shift of the propulsion point. Also, most of empiric methods predict correctly, at least qualitatively, the influence of certain changes in the hull attitude and form parameters and in the effectiveness of the steering device.

Finally, sometimes obvious successes in application of empiric methods are faced. For instance, prediction of the manoeuvring motion of a shuttle tanker performed by Sutulo and Guedes Soares (2015a,b) with the help of a modified empiric method showed a rather good agreement with the available full-scale results without any special tuning. The latter is especially important as in fact such tuning can be practised relatively easily, is a common practice in development of ship handling simulators and can be applied in some published comparisons involving empiric methods without notice.

A retrospective analysis of the literature on ship mathematical models and empiric methods reveals a definite increase of interest to this topic in connection with the adoption of the IMO manoeuvring standards as long as these standards declared a possibility of using recognized empiric methods during the ship design process for checking compliance with the mentioned standards.

Of special interest are several contributions focused on general analysis and comparison of different prediction methods and approaches. Ogawa and Kasai (1978) have formulated certain principles that must be laid in the foundation of any viable empiric method. In particular, the importance of the modular principle as opposed to the holistic approach was emphasized. Some of the formulated principles, as, for instance, necessity to account for the scale effect still remain unrealized even after 40 years of progress. Barr (1993) discussed a number of general problems related to the simulation of the manoeuvring motion but he also presented results of comparison of mathematical models developed at various hydrodynamic centres for the then principle benchmark ship Esso Osaka. The comparison was mainly performed on two levels: (1) values of the parameters of regression models (all of polynomial or quasi-polynomial type), and (2) dependencies of the hull sway force and yaw moment coefficients on the dimensionless velocities of sway and yaw. The revealed scatter turned out quite significant. However, very few comparisons of practically more relevant kinematical output were present.

Ishiguro et al. (1996) focussed on an early version of the Kijima method and its value is assessed through comparisons with results of full-scale trials and by the sensitivity analysis which demonstrated relative influence of various regression parameters.

Burnay and Ankudinov (2003) analysed simulation results for 2 benchmark ships: Esso Osaka and Golden Princess obtained with several mathematical models used by various hydrodynamic centres. The unique feature of this publication was the idea of averaging the responses provided by various models under the assumption that inevitable biases of various method will be mutually cancelled, at least partly, and the averaged response will be more accurate. This approach is de facto a kind of assimilation method applied to the manoeuvring simulation problem. However, this technique has never gained popularity partly due to its complexity but also because in the cited publication the averaged responses did not show any definite gain in the prediction accuracy.

Independent and blind testing of various methods for manoeuvring performance prediction has been undertaken by two workshops: SIMMAN2008 (Stern and Agdrup, 2009) and SIMMAN2014 (Quadvlieg et al., 2015), (SIMMAN, 2018) where, in particular, empiric methods implemented by different institutions did not permit to work out any conclusive statements regarding all empiric methods in general or their comparative quality and practical value. In particular, the responses submitted by the authors (Sutulo and Guedes Soares, 2018a) were successful in some respects but not acceptable in others. It seems that it makes sense to further continue comparisons in order to accumulate more information about performance and limitations of various methods and to acquire better understanding concerning relative importance of their elements and components. It can be recognized that lately the interest directed towards empiric methods was also re-animated in connection with the development of additional manoeuvring criteria in adverse conditions, in particular within the framework of the project SHOPERA (Papanikolaou et al., 2016) as some suggested standards are based on prediction of manoeuvring qualities of ships with methods of reasonable and even reduced complexity.

All practical empiric methods are indeed of the modular type, which means that the ship hull, the steering device (rudder in most cases) and the propellers are handled separately and the corresponding forces and moments are summed to produce the total force and moment acting upon the ship. Of course, the hydrodynamic interaction between the components is not completely neglected but is accounted for by means of relatively few empiric coefficients.

Such approach makes possible hybridization i.e. a model for the hull forces and moments from one method can be combined with the rudder forces model from another method. Sometimes, such hybridization can be successful but in general this must be performed with great care as some empirical methods were pre-tuned in such a way that they are capable to provide reasonable prediction of the manoeuvring motion in spite of much worse estimation of the hydrodynamic loads on each of the constituting elements.

Apparently, the mathematical models for the rudders or other similar steering devices may suffer considerably from prediction uncertainties. This should not be surprising as because, on one hand, the rudder forces are of primary importance and, on the other hand, the rudder is working inside a complex flow whose parameters are not well determined as they depend on the hull boundary layer and wake, and on the propeller slipstream, and all that in curvilinear motion. The mentioned parameters are not constant but depend on the actual motion of the ship.

It is evident that only a detailed CFD modelling can provide more or less reliable description of the flow and prediction of the loads but, obviously, this does not match the philosophy of empiric methods where only simple and fast approaches are welcome. Hence, the influence of the hull is taken into account through a certain generalization of the wake fraction model traditionally and successfully used in ship propulsion. The peculiarity of its manoeuvrability-oriented extension is that instead of a single and constant wake fraction coefficient, also the flow straightening factors must be considered and some of the parameters may be not constant but dependent on the local sidewash. In practice, it is necessary to consider at least 3–4 parameters which are not well supported by estimation methods or databases.

Influence of the propeller slipstream is accounted for either empirically or on the basis of the actuator disk theory which is used either explicitly or implicitly when that theory is used for creating some approximating structure whose coefficients are to be estimated from some model tests which are inevitably of a rather limited scale.

Possible uncertainties associated with the explicit actuator disk model are caused by such secondary phenomena as the contraction and turbulent expansion of the slipstream, variation of the axial induced velocity along the jet, its deflection under influence of the sidewash, and the influence of its boundaries. The latter is de facto ignored in virtually all practical methods which are based on the so-called load separation method presuming that the forces on the parts of the rudder blade working inside and outside the slipstream are handled separately as if the corresponding parts worked in unbounded streams with different velocities. In the present study, a review of several empiric methods for predicting manoeuvring performance of surface displacement ships is presented and these methods are applied to a popular modern benchmark ship design KVLCC2. The methods are described with varying degree of elaboration as most of them were more than once described in available English-language literature. In those cases the authors preferred to focus on conceptual issues rather than on specific approximations. An exception is made for two methods developed in Russia at the Krylov Ship Research Institute and associated with the names of Robert Pershitz and Alexander Tumashik. The first method since many years ago (its first version appeared in 50s) has been known as the Pershitz method and was developed for predicting the turning ability, first of all, of naval combatants but later was extended to cover a wider range of ship types and configurations. The second method called here the Pershitz–Tumashik method is an extension of the former, permitting to model arbitrary manoeuvres including hard low-speed ones. The extension was not trivial and required a number of additional sets of model tests. Although the method is not familiar to the majority of specialists, it served as basis for the tunable core mathematical model for at least 2 commercial bridge simulators. The authors implemented the both mentioned methods using their last official description in the reference book (Voytkunsky, 1985) but some uncertainties were resolved with the books (Pershitz, 1983) and (Vassilyev, 1989). The description of the both methods given in the present article besides partial replacement of nomenclature is believed to be freed of several discovered printing errors. In general, the possibility of presence of errors and ambiguities in published descriptions is one of major problems when this or that method is implemented from its published description, as mentioned in the Discussion to (Barr, 1993).

Presentation of results of simulation of standard manoeuvres by a number of empiric methods and their combinations is concluded by a brief discussion of observed uncertainties and their possible causes.

A preliminary study of similar nature where the Pershitz model was tested has already been carried out by Sutulo and Guedes Soares (2018b) but some implementation flaws were discovered later and fewer methods were there compared. On the other hand, some historic details related to the development and evolution of the Pershitz method discussed in the cited publication will be omitted here.

2. Ship mathematical model

2.1. General remarks and equations of motion

All ship mathematical models for simulating manoeuvring motion of a surface displacement ship must include equations of the surge, sway and yaw motions and the minimum consistent number of degrees of freedom is 3. However, if a ship is characterized by low transverse hydrostatic stability and/or it is relatively fast, the fourth degree of freedom in roll is added. For instance, the equation of dynamic roll constitutes part of the well-known Inoue model (Inoue et al. 1981a,b) and one of the latest investigations of the effect of coupling with the dynamic roll was undertaken by Fukui et al. (2015). The trim and—to a lesser extent—sinkage are also changing in manoeuvring but these changes are mainly caused by speed changes in course of manoeuvring which is happening rather slowly and there is no need to consider dynamic equations of pitch and heave. In fact, a parametric account for the dynamic trim is quite sufficient and is so implemented in the Pershitz model. Finally, 6DOF models are only necessary when manoeuvring simulation in sea waves is considered.

As every 4DOF manoeuvring model reduces to its 3DOF version when the roll angle is fixed at zero value, all models can be treated as 3DOF and so will be assumed in the present study to facilitate comparisons.

Two standard frames of reference are sufficient for the description of the 3DOF manoeuvring motion: the right-handed Earth-fixed Cartesian frame $O\xi\eta\zeta$ with the origin on the undisturbed free surface and the ζ-axis directed vertically downwards and the body axes $Cxyz$ coinciding with the Earth axes at the initial time moment $t = 0$. The origin C lies in the centerplane of the ship (most often also in the midship plane) and if the body frame is not central, the position of the centre of mass of the ship is characterized by its abscissa x_G.

The standard and absolutely accurate in 3DOF kinematical equations are:

$$\begin{aligned}\dot{\xi}_C &= u\cos\psi - v\sin\psi,\\ \dot{\eta}_C &= u\sin\psi + v\cos\psi,\\ \dot{\psi} &= r,\end{aligned} \tag{1}$$

where $\xi_C(t)$ and $\eta_C(t)$ are the instantaneous advance and transfer of the point C, $\psi(t)$ is the heading angle, $u(t)$, $v(t)$ are the velocities of surge and sway, and $r(t)$ is the angular velocity of yaw.

The most convenient form of dynamic equations valid for all empiric methods is:

$$\begin{aligned}(m+\mu_{11})\dot{u} - mvr - mx_G r^2 &= X_H + X_P + X_R,\\ (m+\mu_{22})\dot{v} + (mx_G + \mu_{26})\dot{r} + mur &= Y_H + Y_R,\\ (mx_G + \mu_{26})\dot{v} + (I_{zz} + \mu_{66})\dot{r} + mx_G ur &= N_H + N_R,\end{aligned} \tag{2}$$

where m is the mass of the ship; μ_{ij} are the added mass coefficients, I_{zz} is the moment of inertia in yaw; X, Y, N are the forces and moment of surge, sway, and yaw respectively and the subscripts H, P and R stand for the hull, propeller and rudder respectively.

The equation (2) are in fact used by all empiric methods although they can be written in different forms (Sutulo and Guedes Soares, 2011). Alternative forms are either equivalent to (2) or are not quite correct. In some methods central body axes are assumed, and in that case $x_G = 0$ but typically the position of the centre of mass is accounted for albeit in a different way, at the right-hand side of the equations. The exception is the Pershitz method where the influence of the position of the centre of mass is completely ignored except for its influence on the dynamic trim at high Froude numbers.

2.2. Hull forces

The hull hydrodynamic forces are represented as

$$X_H = x'(v', r')\frac{\rho}{2}V^2LT, \quad Y_H = Y'(v', r')\frac{\rho}{2}V^2LT, \quad N_H = N'(v', r')\frac{\rho}{2}V^2L^2T, \tag{3}$$

or as

$$X_H = X''(\beta, r'')\frac{\rho}{2}V_G^2LT, \quad Y_H = Y''(\beta, r'')\frac{\rho}{2}V_G^2LT, \quad N_H = N''(\beta, r'')\frac{\rho}{2}V_G^2L^2T, \tag{4}$$

where $X', \ldots, N''$ are the force/moment coefficients, ρ is the water density, $V = \sqrt{u^2+v^2}$ is the ship speed, L is the ship's length, T—its draught, $v' = v/V$, $r' = rL/V$ are the dimensionless velocities of sway and yaw, $\beta \in (-\pi, \pi]$ is the drift angle defined in such a way that $v' = -\sin\beta$; $V_G = \sqrt{V^2 + L^2r^2}$, $r'' = r'/\sqrt{1+r'^2}$.

Most empiric methods apply the formulae (3) which are only valid for moderate manoeuvres but the methods applicable for arbitrary manoeuvres use the generalized formulae (4). Specifics of any particular method is conditioned by the definition of the non-dimensional functions at the right-hand sides of equations (3) and (4).

There is some difference in representation of the surge force coefficient and of the sway force and yaw moment coefficients. For instance, in the Inoue (Inoue et al., 1981a) and Kijima (Kijima and Nakiri, 2003) methods:

$$x' = x'_0(u) + x'_{vr}v'r', \tag{5}$$

where

$$x'_0 = \frac{-2R_T(u)}{\rho V^2 LT}; \quad x'_{vr} = \frac{2C_m\mu_{22}}{\rho L^2 T}, \tag{6}$$

and where $R_T(V)$ is the drag curve of the ship, and C_m is the so-called viscous correction coefficient. Originally, it was recommended to assume $C_m = 0.5 \div 0.75$ but the formula $C_m = 1.5 - 1.66C_B$ with C_B being the hull block coefficient was recommended in (Ishiguro et al., 1996). In the Pershitz model it is assumed $C_m = 1$.

Sway force and yaw moment coefficients in moderate manoeuvring are described by the following quasi-polynomial regression models:

$$\begin{aligned}Y'_H &= Y'_v v' + Y'_r r' + Y'_{vv} v'|v'| + Y'_{vr} v'|r'| + Y'_{rr} r'|r'|,\\ N'_H &= N'_v v' + N'_r r' + N'_{rr} r'|r'| + N'_{vvr} v'^2 r' + N'_{vrr} v' r'^2\end{aligned} \tag{7}$$

—in the Inoue model;

$$\begin{aligned}Y'_H &= Y'_v v' + Y'_r r' + Y'_{vv} v'|v'| + Y'_{vr} v'|r'| + Y'_{vvr} v'^2 r' + Y'_{vrr} v' r'^2,\\ N'_H &= N'_v v' + N'_r r' + N'_{vv} v'|v'| + N'_{rr} r'|r'| + N'_{vvr} v'^2 r' + N'_{vrr} v' r'^2\end{aligned} \tag{8}$$

—in the Kijima and Matsunaga models;

$$\begin{aligned}Y'_H &= Y'_v v' + Y'_{vv} v'|v'|,\\ N'_H &= N'_v v' + N'_r r'\end{aligned} \tag{9}$$

—in the Pershitz model.

Each of the regression models above can be represented in the alternative form where v' is substituted by $(-\beta)$. As the absolute value of the drift angle in moderate manoeuvres is limited by 30°, practically no corrections of the coefficients are required.

The coefficients $Y'_v, \ldots, N'_{vrr}$, which are often traditionally called "hydrodynamic derivatives", depend on various geometric parameters of the ship hull and are typically defined by rather simple formulae or by plots. In the latter case it is always possible to build a suitable analytic approximation as was repeated more than once by various researchers with respect to all "nonlinear" coefficients of the Inoue method. The approximations for some coefficients of the Inoue method developed by the authors and used in this study are given in Appendix A. All the remaining necessary information for estimation of the coefficients in (7) and (8) can be found in the publications: (Inoue et al., 1981b), (Matsunaga, 1993), (Kijima and Nakiri, 2003) and (Kijima, 2003). Unfortunately, in the latter paper used by the authors in (Sutulo and Guedes Soares 2018a,b) one of the coefficients was erroneously printed with the misplaced decimal point. The problem of such kind of misprints is rather common and a cross-check must be performed whenever possible.

In all Japanese methods the "hydrodynamic derivatives" depend on the following main non-dimensional hull form parameters: the aspect ratio of the doubled hull $k_H = 2T/L$, the hull block coefficient C_B and the ratios B/L and T/B. In Kijima's method (Kijima and Nakiri, 2003) two additional parameters were introduced: the aft prismatic coefficient C_{PA} and the aft waterplane area coefficient C_{WA}. Both parameters are defined similarly to the standard prismatic coefficient C_P and the waterplane area coefficient C_W but are based on the aft quarter of the hull.

The coefficients in the Pershitz model can be represented in the following form:

$$\begin{aligned}&Y'_v = -c_1C_L, \quad Y'_{vv} = -c_2C_L, \quad N'_v - (2m_1 + m_2)C_L,\\ &N'_r = \left[-(0.739 + 8.7T/L)\left(1.611C_L^2 - 2.873C_L + 1.33\right)\right]C_L,\end{aligned} \tag{10}$$

where c_1, c_2, m_1, m_2, C_L are the parameters defined in Appendix B, In particular, C_L is the effective centerplane area coefficient which enters as a factor into all the formulae (10) because the effective centerplane area is used as the reference area in the Pershitz method instead of the more common LT.

It is evident that the degree of complexity of the three sets of approximating formulae is different and the Pershitz model looks oversimplified. However, this simple structure is sufficient to capture all qualitative specifics of manoeuvring models. At the same time, at present there is no sufficient evidence of whether this of that degree of complexity of a polynomial or quasi-polynomial model is necessary and sufficient for adequate description of the hull forces.

The 4-quadrant generalized coefficients used in the Pershitz–Tumashik model are defined as follows:

$$\begin{aligned}
X'' &= -0.075C_L \sin\{[\pi - \arcsin(x'_0/0.075)][1-|\beta|/\beta_x]\} \\
&\quad \cdot\left(1-r''^2\right), \\
Y'' &= C_L\left(\frac{1}{2}c_1 \sin 2\beta\cos\beta + c_2\sin\beta|\sin\beta|\right. \\
&\quad \left.+c_3\sin^3 2\beta|\sin 2\beta|\right)\left(1-r''^2\right), \\
N'' &= N''_1 + N''_2 + N''_3, \quad \text{where} \\
N''_1 &= C_L\left(m_1\sin 2\beta + m_2\sin\beta + m_3\sin^3 2\beta\right. \\
&\quad \left.+m_4\sin^3 2\beta|\sin 2\beta|\right)\left(1-r''^2\right), \\
N''_2 &= \frac{C_L L^2}{V^2}N'_{rr}r|r|, \\
N''_3 &= \frac{C_L}{\pi}\left\{N'_r + A_1|\sin\beta| + \frac{1}{2}A_2[1-\cos((2\pi-4.0|\beta|)\cos\beta\right. \\
&\quad \left.+0.1|\sin\beta|)]\right\}\sin\pi r''\left(1-r''^2\right).
\end{aligned} \tag{11}$$

where $\beta_x\ :\ X''(\beta_x)=0$, $N'_{rr}=-0.059c_2$ and the remaining coefficients are given in Appendix.

Another 4-quadrant trigonometric model for hull forces had been developed by the authors as an extension of the Inoue model. That model is asymptotically close to the original Inoue model and is described in detail in Sutulo (1994) and Sutulo and Guedes Soares (2005). In the following exposure this model is designated as Inoue Extended or InoueEx.

2.3. Rudder forces

The rudder surge and sway forces can be expressed either through the rudder lift and drag (Crane et al., 1989) or through the rudder normal force N and its tangential force. The latter approach is preferred in most empiric methods as the tangential force can always be neglected. Then,

$$\begin{aligned}
X_R &= -N(1+a_{Hx})\sin\delta_R, \\
Y_R &= -N\left(1+a_{Hy}\right)\cos\delta_R, \\
N_R &= Y_R x_{RH},
\end{aligned} \tag{12}$$

where $a_{Hx,y}$ are the hull influence coefficients, δ_R is the rudder deflection angle, positive to the starboard and the rudder normal force, $x_{RH}=x_R+a_{Hy}x_H$ is the effective rudder abscissa which can be not identical to its geometric abscissa x_R and $x_H=x'_H L$ is the abscissa of the rudder-induced sway force on the hull;

$$N = C_N^\alpha \frac{\rho V_R^2}{2}A_R\sin\alpha_R, \tag{13}$$

where C_N^α is the effective normal force coefficient gradient, V_R is the effective rudder velocity with respect to water, A_R is the rudder lateral area, $\alpha_R=\delta_R-\beta_R$ is the rudder attack angle, where β_R is the sidewash angle at the rudder.

The normal force gradient in (13) can be and normally is assumed to be identical to the lift gradient which is typically estimated with the Prandtl formula obtained in the lifting line theory or some equivalent one. The thus defined C_N^α is not the true gradient but gives a reasonable approximation which is commented in more detail by Sutulo and Guedes Soares (2011). The sine function in (13) is often replaced by its argument making the equation fully linear.

The variables V_R and β_R depend on the ship kinematical parameters u, v, r but also on the influence of the hull and of the propeller slipstream. In all practical methods the hull influence is described by a small number of constant empiric parameters like the wake fraction and straightening coefficients while the influence of the propeller is typically based on the actuator disk theory with some empiric corrections which vary from one specific method to another.

In the original Inoue model (Inoue et al. 1981a,b) (Hirano, 1980), used in many methods:

$$\begin{aligned}
&a_{Hx}=0;\quad a_{Hy}=0.635C_B-0.153;\quad \beta_R=\kappa_\beta\beta-\kappa_r x'_R r'; \\
&\kappa_r=2\kappa_\beta;\quad \kappa_\beta=\kappa_H\kappa_P;\quad \kappa_H=\min\left\{\frac{1}{2},\ 0.45|\beta-2x'_R r'|\right\}; \\
&\kappa_P=\left[1+\frac{3}{5}\frac{D_P}{h_R}\frac{2-1.4s}{(1-s)^2}\right]^{-\frac{1}{2}},
\end{aligned} \tag{14}$$

where $x'_R=x_R/L$ is the relative rudder abscissa, κ with various subscripts is the rudder inflow straightening factor and its components, $s=1-J/J_0$ is the slip ratio with J being the propeller advance ratio (see below), and J_0 is the zero thrust advance ratio. It must be noted that in the slip ratio definition often the propeller pitch ratio is used instead of J_0, which, however, is not desirable as can result in too small values of the slip ratio; D_P is the propeller diameter, and h_R is the rudder height.

The effective rudder velocity in the same method is defined by the formula:

$$V_R=V(1-w_R)\left[1+(1-0.065\operatorname{sign}\delta_R)\frac{D_P}{h_R}k_{PR}\frac{s[2-s(2-k_{PR})]}{(1-s)^2}\right]^{\frac{1}{2}}, \tag{15}$$

where

$$k_{PR}=\frac{3(1-w_P)}{5(1-w_R)}, \tag{16}$$

and where w_P, w_R are the propeller and rudder wake fraction respectively. Both of them depend on the geometric propeller sidewash angle $\beta_P=\beta-x'_P r'$ as

$$w_{P,R}=w_{P0,R0}\exp\left(-4\beta_P^2\right), \tag{17}$$

where the wake fraction in the straight run for the propeller w_{P0} is estimated using methods developed in the ship propulsion while the corresponding parameter for the rudder is approximately defined as $w_{R0}=0.4$. This recommendation used in the present study looks rather rough and in earlier publications it was recommended to assume $w_{R0}=0.25$ while Ishiguro et al. (1996) suggest also the approximations

$$\frac{1-w_{R0}}{1-w_{P0}}=[-156.2(C_B B/L)+41.6](C_B B/L)-1.76 \tag{18}$$

and

$$w_{R0}=[0.35(1-w_{P0})-0.78](1-w_{P0})+0.6525. \tag{19}$$

The same authors recommend to use instead of $a_{Hx}=0$ the approximation:

$$a_{Hx}=0.28C_B-0.45. \tag{20}$$

The following approximations were obtained by the authors using plots from (Ishiguro et al., 1996):

$$a_{Hy}=(2.909C_B-1.5912)C_B+0.2263, \tag{21}$$

$$a_{Hy}=(3.4257C_B-3.3178)C_B+1.039, \tag{22}$$

$$x'_H=(7.1111C_B-4.8)C_B-1.08478, \tag{23}$$

$$x'_H=-0.46. \tag{24}$$

and

$$\kappa_H=(1.083333C_B-1.581667)C_B+1.002 \tag{25}$$

Regarding the alternatives, it must be noted that while the approximations (18), (21) and (23) were declared as standard, making part of an early version of the Kijima method, the approximations (19), (22), (24) and (25) based on the data published by Yoshimura, Kose and Yumoto in Japanese were proposed by Ishiguro et al. (1996) as an attempt to improve the Kijima method. They all are supposed to be applicable for $C_B \in [0.5,\ 0.85]$.

A somewhat different model for the rudder inflow was suggested by Ogawa et al.(1980). In that model the hull and propeller corrections apply not to the total speed of the ship but to the velocity of surge which results in the formula:

$$u_R = \frac{u(1-w_R)}{s-1}\sqrt{1-2s\left(1-k_{PR}\frac{D_P}{h_R}\right)+s^2\left[1-k_{PR}\frac{D_P}{h_R}(2-k_{PR})\right]}. \tag{26}$$

The total rudder velocity is then restored using the obvious formula

$$V_R = \frac{u_R}{\cos\beta_R} \tag{27}$$

Although the formulae (15) and (26) look rather different, in many cases the simulation results obtained with the original Inoue and Ogawa models are very close at least for constant rotation frequency of the propeller. However, the Ogawa model is preferable when the rotation frequency drops following the diesel engine limiting characteristic. In this case the effect of the ship speed reduction can be overestimated by the Inoue rudder model causing fast increase of the propeller torque which even sometimes may result in the modelled engine's failure.

The rudder-in-the-slipstream model proposed by Söding (1982) is based directly on the actuator disk theory and the main parameter defining the action of the propeller is the loading coefficient $C_{TA} = 8T/(\rho\pi u^2(1-w_P)^2 D_P^2)$. This model was used in this study in an advanced form suitable for any combination of the propeller rotation and for a ship advancing or backing. The model is described in detail in (Sutulo and Guedes Soares 2015a,b) and the original description of the Söding model can also be found in (Söding, 1982) and (Brix, 1993).

The Pershitz model for the rudder forces is much more schematic and looks much less consistent than all models outlined above. It makes sense to give its description in a comparative style:

1. Instead of using (12), the action of the rudder is described directly in terms of the rudder surge and sway forces and yaw moment

$$\begin{aligned} X_R &= -C_{XR}(\alpha_R)\frac{\rho V^2}{2}A_{RE}, \\ Y_R &= -C_{YR}(\alpha_R)\frac{\rho V^2}{2}A_{RE}, \\ N_R &= x_R Y_R, \end{aligned} \tag{28}$$

where $C_{XR},\ C_{YR}$ are the rudder surge and sway force coefficients, A_{RE} is the effective rudder area. It is clear that the forward displacement of the sway force due to rudder induction on the hull is here ignored. It is also clear that instead of the local rudder speed V_R the speed of the ship V is applied.

2. The rudder surge and sway forces are implicitly assumed to be identical to the rudder drag and lift respectively and the latter is defined as:

$$C_{YR} = kC^{\alpha}_{YR0}\alpha_R, \tag{29}$$

where $k = 1.0$ for all-movable (spade) rudders, and $k = 0.9$ for a steering nozzle. The base lift gradient C^{α}_{YR0} is estimated using the Prandtl formula taken in the form:

$$C^{\alpha}_{YR0} = \frac{2\pi C_R k_R}{2+k_R}, \tag{30}$$

where k_R is the aspect ratio of the rudder and the correction factor $C_R = 1$ for all-movable rudders and $C_R = 7/6 - k_R/3$ for horn rudders.

The Pershitz method does not provide any standardized estimation of the rudder surge/drag force advising use of specific experimental data. So, in the present study the formula recommended by Crane et al. (1989) was used:

$$C_{XR}(\alpha_R) = 0.0065 + \frac{C^2_{YR}}{0.9\pi k_R}. \tag{31}$$

3. The rudder attack angle α_R is defined almost in the same way as in the Inoue model by the formula:

$$\alpha_R = \delta_R - \kappa_H\kappa_P(\beta - x'_R r'), \tag{32}$$

where the slipstream straightening factor is defined as

$$\kappa_P = \frac{A_{R0} + A_{RP}\sqrt{1+C_{TA0}}}{A_{R0} + A_{RP}(1+C_{TA0})}, \tag{33}$$

where A_{RP} is the part of the rudder area inside the slipstream under conditions that the rudder is not deflected and the slipstream is horizontal, and $A_{R0} = A_R - A_{RP}$; C_{TA0} is the propeller loading coefficient as defined above but corresponding to the approach straight-run phase. The latter condition is another factor apparently reducing the consistency of the method.

The hull straightening factor κ_H is assumed following the recommendations (Voytkunsky, 1985):

- $\kappa_H = 0.3$ when the gap between the rudder and the stern skeg does not exceed half of the rudder mean chord or, in the case of twin rudders, when each rudder has a nonzero transverse projection on the skeg;
- $\kappa_H = 0.5$ if the rudder(s) are placed at a longitudinal distance from the skeg exceeding half of the mean chord, or for a rudder behind a rudder post, or for a steering nozzle in the centerplane behind the skeg;
- $\kappa_H = 0.7$ for steering nozzles located at the sides while the stern skeg is small or absent;
- $\kappa_H = 0.9$ in the case of a cruiser type stern with a small or no skeg;
- $\kappa_H = 1.0$ in the case of a transom stern with no skeg or with a skeg ending at most at a 17th theoretical section—this configuration is typical for most naval combatants.

4. While the rudder inflow velocity is assumed the same as the ship speed, a correction for the influence of the slipstream is introduced through the effective rudder area defined as:

$$A_{RE} = A_{M0} + A_{MP}(1 + C_{TA0}), \tag{34}$$

where M in the subscripts stands for the movable part of a rudder i.e. the horn area should be excluded. Of course $A_{RE} \equiv A_{M0}$ if the rudder is working outside the slipstream.

In the case of a steering nozzle, the effective lift gradient is estimated using the empiric formula:

$$C^{\alpha}_{YR0} = \left\{\left[1+\frac{1}{4}\left(\sqrt{1+C_{TA0}}+1\right)^2\right](0.55+0.35L_D)+\frac{1}{2}C_{TA0}\right\}\Big/(2k_D), \tag{35}$$

where $k_D = L_D/D_D$ is the aspect ratio of the nozzle with L_D being the nozzle's length (chord) and D_D—the minimum inner diameter of the nozzle.

The effective area of the nozzle is defined as:

$$A_{RE} = L_D D_D. \tag{36}$$

The Pershitz method presumes also a special treatment of the rudder behind the rudder post. However, this configuration is rather rare nowadays and the corresponding description is dropped here although it can be found in Sutulo and Guedes Soares (2016).

A different rudder model was developed as part of the Pershitz–Tumashik method. According to this model

$$X_R = C_{XR}(\alpha_R)\left(\frac{\rho V^2}{2}A_R + cT\right),$$
$$Y_R = C_{YR}(\alpha_R)\left(\frac{\rho V^2}{2}A_R + cT\right), \tag{37}$$
$$N_R = Y_R x_R,$$

where T is the actual propeller thrust, C_{XR} is the rudder drag curve defined for arbitrary attack angles. The rudder lift coefficient is defined by

$$C_{YR} = \begin{cases} C^{\alpha}_{YR}\sin\alpha_R & \text{at} \quad |\alpha_R| \le \alpha_{ss} \\ 1.0 & \text{at} \quad |\alpha_R| > \alpha_{ss} \end{cases}, \tag{38}$$

where α_{ss} is the rudder stall angle and

$$C^{\alpha}_{YR} = C^{\alpha}_{YRB} + \left(C^{\alpha}_{YRB} - C^{\alpha}_{YR0}\right)\kappa_P(2-\kappa_P), \tag{39}$$

where C^{α}_{YRB} is the lift gradient in bollard pull regime which can be estimated as $C^{\alpha}_{YRB} = 0.82$ unless more specific data are available. The parameters C^{α}_{YR0} and κ_P are defined as before but the straightening coefficient in the definition of the attack angle is defined differently:

$$\kappa = \frac{1}{2}[1+\kappa_H - (1-\kappa_H)\cos 2\beta_R][\kappa_P + 0.65\overline{x}_R\kappa_P(1-\kappa_P)], \tag{40}$$

where $\beta_R = \beta - x'_R r'$ is the geometric rudder inflow angle, $\overline{x}_R = (x_{RL} + b_R)/D_P$, x_{RL} is the distance from the propeller to the leading edge of the rudder and b_R is the rudder chord.

Finally, the coefficient c in eq. (37) is defined as

$$c = \min\left\{\frac{4A_{RP}}{\pi D_P^2}, \quad 1.0\right\}. \tag{41}$$

However, analysis of the structure of the Pershitz–Tumashik model for the rudder showed that apparently it was designed exclusively for the low-speed manoeuvring and may be not uniformly valid. That is why, in the computations whose results are given in the next section the Pershitz–Tumashik model for the hull forces was combined with the original Pershitz rudder model.

2.4. Propeller model

In all existing practical empiric mathematical models the sway force and the yaw moment generated by the propeller are neglected which is justified for most merchant ships unless there is a necessity to model the so-called Hovgaard force appearing in active stopping. The propeller surge force is de facto the same as the effective thrust T_E and can be represented as:

$$X_P \equiv T_E = \rho K_T(J)n^2 D_P^4(1-t_P), \tag{42}$$

where K_T is the open-water thrust coefficient depending on the advance ratio $J = u(1-w_P)/(nD_P)$, w_P is the propeller wake fraction, n is the propeller rotation frequency, D_P is the propeller diameter, and t_P is the thrust deduction coefficient.

The equation (42) has the same form as used in ship propulsion and specifics related to the manoeuvring motion are limited to using the surge velocity u instead of the ship speed V and to the wake fraction w_P being dependent on the propeller sidewash angle β_P.

The representation (42) is only applicable for simulating moderate manoeuvres without covering e.g. the stopping manoeuvre. More general 4-quadrant models and also methods for appropriate balancing the propeller and hull surge forces in straight run are discussed by Sutulo and Guedes Soares (2011, 2015a,b).

It must be realized that no one of existing empiric methods for manoeuvrability predictions is specifying some particular approximation for the propeller open-water characteristic $K_T(J)$ which could be considered as standard for the method in concern. This may lead to additional scatter of the results obtained with different implementations of one and the same empiric method. In the present study, one and the same propeller model based on the well-known approximations by Oosterveld and van Oossanen (1975) and adjusted following the procedure described in (Sutulo and Guedes Soares, 2011) was used with all methods.

2.5. Additional elements of ship mathematical models

The full ship mathematical model must also include the equation of the propeller shaft torque accompanied by some model for engine dynamics. However, the torque submodel is dropped in the present study as it was assumed that the propeller rotation frequency remains constant in all simulations performed.

Another secondary submodel is that of the steering gear. Although it is not absolutely necessary, its presence is always desirable as it is the most convenient way to account for the constraints on the rudder angle and the rudder deflection rate. A rather simple but adequate model of the gear is described in (Sutulo and Guedes Soares 2015a,b). However, in most cases application of the ideal gear is also possible provided that the maximum rudder angle and the maximum deflection rate are modelled correctly.

Finally, any model must be complemented by control programmes and laws to simulate standard manoeuvres but this part is more or less obvious and does not create any additional uncertainty as long as implemented correctly.

2.6. On applicability limitations of empiric methods

Of course, one cannot expect from any empiric method that it would be equally applicable for all types of surface ships but, strangely, in no one of the known publications the limitations are formulated explicitly. However, inspection of these publications reveals that implicit information on these limitations can be traced. In particular, all empiric methods contain some plots, nomograms or approximations based on them where the range of input geometric parameters is defined in the natural way (see Appendices A and B). Also, an indirect indication on the limits can be extracted from the publications as long as there particulars of the database ship models or of tested configurations are mentioned. For instance, regarding the Kijima method the database limits (Ishiguro et al., 1996) are from 0.48 to 0.84 for the block coefficient, $L/B \in [4.5,\ 7.0]$ and $B/T \in [2.4,\ 6.3]$ (the same ship in full load and ballast condition is treated as two different configurations). The corresponding limits for the Inoue or Matsunaga methods based on the tested configurations are: $C_B \in [0.5,\ 0.825]$, $L/B \in [5.0,\ 7.15]$, and $B/T \in [2.7,\ 5.8]$. It is also clear from the publications that all MMG methods are focused on single-screw merchant ships with a single rudder working in the slipstream. The rudder is supposed to be all-movable (suspended or “simplex” type) although a horn-rudder correction, which is non-significant for typical horns, can be introduced. On the other hand, the Pershitz method is positioned as fully all-purpose i.e. applicable to any monohull surface displacement ship equipped with any main control device except for azipods which still were not in use when the last version of the method was published.

3. Numerical results

3.1. Ship data

The description and body plan of the benchmarking virtual vessel KVLCC2 can be found in a number of publications, sea e.g. (Quadvlieg

Table 1
Main parameters of KVLCC2 ship.

m, t	320437.6	$k_{H2} = T/L$	0.065	$A_{CO}/(LT)$, %	2.84
L, m	320.0	C_B	0.81	i_{UV}	13.5
L/B	5.52	C_P	0.82	$A_R/(LT)$, %	2.05
B/T	2.79	C_{PA}	0.52	k_R	1.95
$k_H = 2T/L$	0.13	C_{WA}	0.747	D_P/h_R	0.605

et al., 2015), (Yasukawa et al., 2015), (SIMMAN, 2018). The main parameters of the ship are given in Table 1 where emphasis is given on dimensionless parameters used in the tested empiric methods.

It is clear that no one of the geometric parameters of this vessel violates the limits which can be established for all empiric methods considered here.

3.2. Turning manoeuvre

The turning manoeuvre with the 35° helm to starboard was simulated with the initial speed 15.5 knots corresponding to the value 0.142 of the approach Froude number. The same value of the approach speed was also kept for the spiral and zigzag manoeuvres.

The standard numerical measures of the turning manoeuvre are shown in Table 2 where are also added data from 2 published sources obtained with free-running model tests and CFD computations, and the Free Running Model Test data (FRMT) submitted by MARIN to SIMMAN 2014, see (SIMMAN, 2018) and recognized as quite reliable. It makes sense to assume that these values lying within the intervals $D_T/L \in [3.0,\ 3.32]$ for the relative tactical diameter and $A_d/L \in [2.5,\ 3.4]$ for the relative advance are credible enough even more so that they correspond to what could be intuitively expected from a ship with the characteristics as in Table 1. Then, it can be established that only the following combinations predict reasonably well the mentioned parameters: (1) Kijima + Ogawa, (2) Pershitz + Pershitz, (3) Pershitz + Soeding, and (4) Inoue + Soeding. In the third case the advance is somewhat overpredicted and in the last case the discrepancy is somewhat larger. In general, MMG (Japanese) models tend to underpredict the turning ability while the Pershitz family models rather overpredict it.

Table 2
Numerical measures of the 35 deg turning manoeuvre.

Hull model	Rudder model	Relative Advance	Relative Transfer	Relative Tactical diameter
Inoue	Inoue	4.218	2.236	4.886
Inoue	Ogawa	4.263	2.290	5.014
Inoue	Soeding	3.920	1.833	3.750
Matsunaga	Inoue	4.272	2.338	5.110
Inoue Extended	Inoue	4.123	1.977	4.257
Inoue Extended	Soeding	4.106	1.948	4.077
Kijima	Inoue	4.428	1.936	5.108
Kijima	Ogawa	3.497	1.043	3.266
Kijima	Soeding	4.284	1.681	4.267
Pershitz	Pershitz	3.635	1.066	3.231
Pershitz	Inoue	4.347	1.296	3.347
Pershitz	Soeding	3.989	0.873	1.764
Pershitz–Tumashik	Pershitz–Tumashik	2.879	0.616	1.891
Pershitz–Tumashik	Pershitz	2.700	0.562	1.906
Pershitz–Tumashik	Soeding	3.593	0.850	2.145
Yasukawa et al. (2015)		3.0–3.4	no data	3.1–3.25
Stern & Agdrup (2009)		2.5–3.0	no data	3.0–3.3
FRMT: SIMMAN (2018)		3.09	1.37	3.32

Predicted trajectories in the turning manoeuvre are presented in Fig. 1 with the FRMT added to each plot. There inspection confirms good behaviour of the Kijima + Ogawa model while the three remaining combinations listed above exhibit a somewhat exaggerated steady turning rate. The Pershitz group of models tends to produce turning trajectories with self-intersections. As this effect does not depend substantially on the rudder model, it is most likely caused by peculiarities of the hull model. Although there are no proves that this kind of trajectories is impossible, they do not look natural and are only registered in full-scale trials due to effect of wind.

Somewhat unexpected was the effect of moving from the original Inoue to the Ogawa rudder model. While the trajectory alteration is very small when the rudder is combined with the Inoue hull model, this is not the case when the Kijima hull model is used. So far, it was not possible to obtain a definite explanation of this fact but apparently it is connected to possible violation of domains of validity of this or that submodel. For no one of the models such domains are defined explicitly but there are no doubts they exist. In the future, this issue should be subject to a special study.

The time histories for the dimensionless yaw rate $r'(t)$, drift angle $\beta(t)$ and for the current ship speed $V(t)$ related to the same turning manoeuvre are given in Figs. 2–4. On these and some further plots the abbreviation "PershTum" is used instead of "Pershitz–Tumashik". Analysis of time histories for the yaw rate reveals that in most cases these transients do not show any overshoot. The exceptions are: (1) Pershitz + Söding, (2) Pershitz + Pershitz, and (3) Pershitz–Tumashik + Söding. The first of these three cases is also characterized by unrealistically high values of r' reached in steady turn. Further, all the models can be grouped into three rather distinct clusters: (1) combinations underpredicting r' which include Inoue + Inoue/Ogawa, Kijima + Inoue, and Matsunaga + Inoue, (2) combinations apparently overpredicting r': Pershitz + Söding and all combinations based on the Pershitz–Tumashik hull model, and (3) the remaining combinations which look acceptable from this viewpoint. On these plots the FRMT data may show some additional scatter and even some biases conditioned by the quality of primary plots from (SIMMAN, 2018) processed with the screen digitizer Grafula.

Time histories for the drift angle show more cases with overshoot which include all models of the Pershitz group and the Kijima + Ogawa model. Also, an almost insignificant overshoot can be spotted on the Kijima + Ogawa combination. All models under-predicting r' do also under-predict β except for the combination Kijima + Ogawa. The drift angle is definitely overpredicted by all combinations including the Pershitz–Tumashik hull model and by the combination Pershitz + Söding.

The speed reduction curves (Fig. 4) look somewhat less informative with 5 of them i.e. (1) Pershitz + Pershitz, (2) Pershitz-Tumashik + Pershitz, (3) Pershitz + Söding, (4) Pershitz-Tumashik + Söding, and (5) Pershitz-Tumashik + Pershitz-Tumashik showing overshoots and the last 3 of the listed combinations also apparently predict speed drops which look too excessive for constant propeller rotation frequency.

3.3. Spiral manoeuvre

The Dieudonné spiral manoeuvre was simulated for the same model combinations and all results are assembled in Fig. 5. It is clearly seen that the combinations Kijima + Ogawa, Pershitz–Tumashik + Pershitz and Pershitz–Tumashik + Söding show extremely wide hysteresis loops corresponding to highly unstable and even practically uncontrollable ships. These cases are excluded from further analysis.

Of the remaining MMG models (Fig. 6) those based on the Kijima hull model are directionally unstable with a degree of instability which is definitely too high. Inoue- and Matsunaga-based models are directionally stable and the combinations Inoue + Söding and Inoue Extended + Söding also show decent level of turning ability at large helms (see Fig. 7).

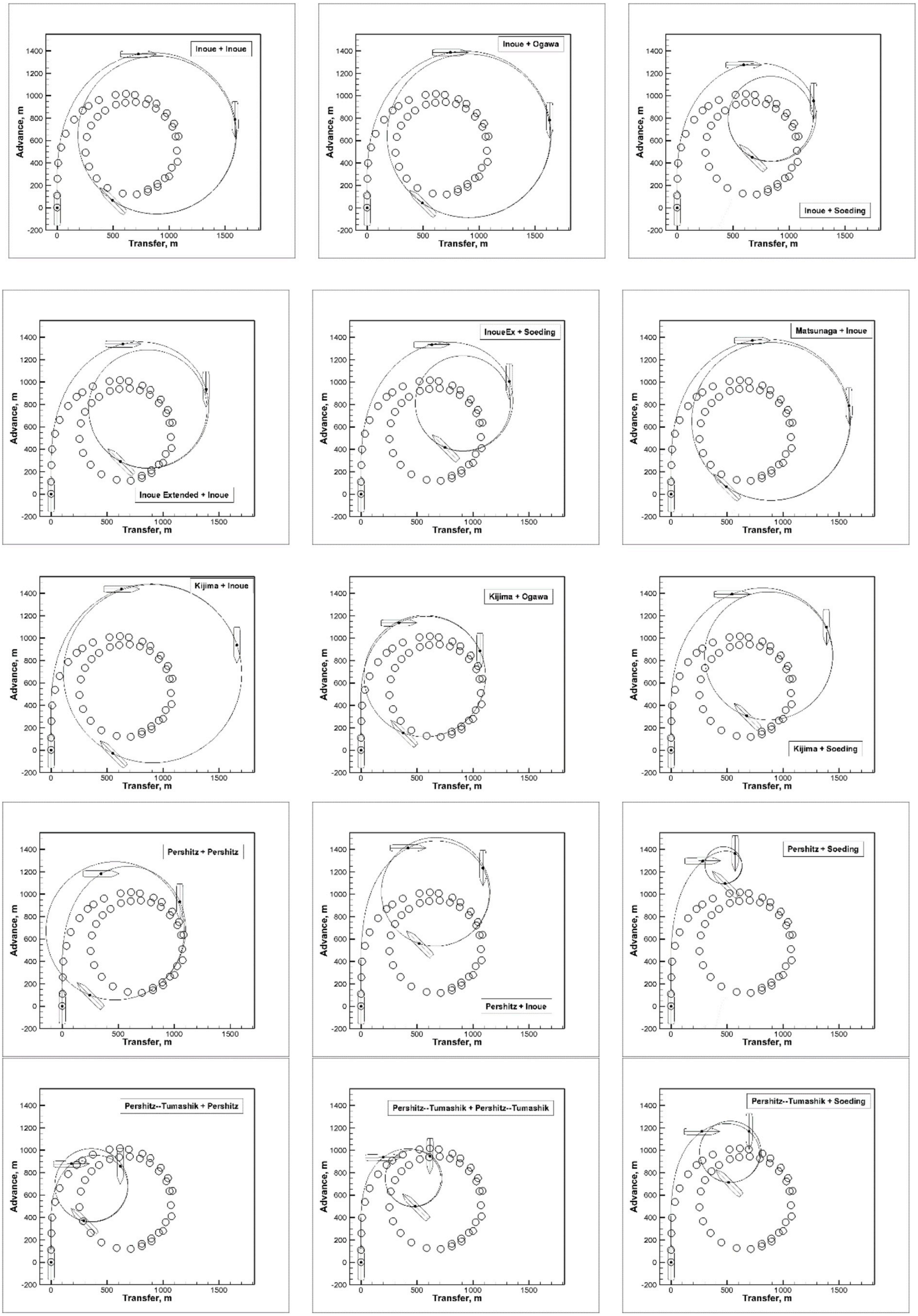

Fig. 1. Ship trajectories in 35 deg helm turn (empty circles correspond to FRMT data).

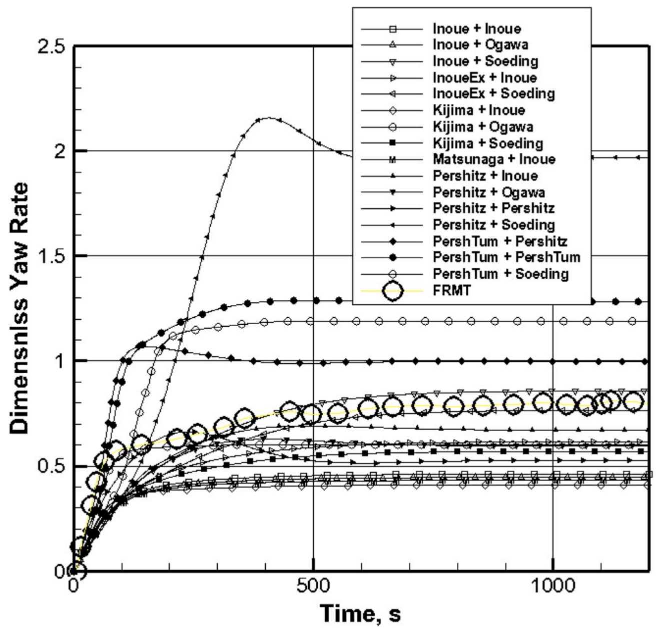

Fig. 2. Time histories for dimensionless yaw rate in 35 deg helm turn.

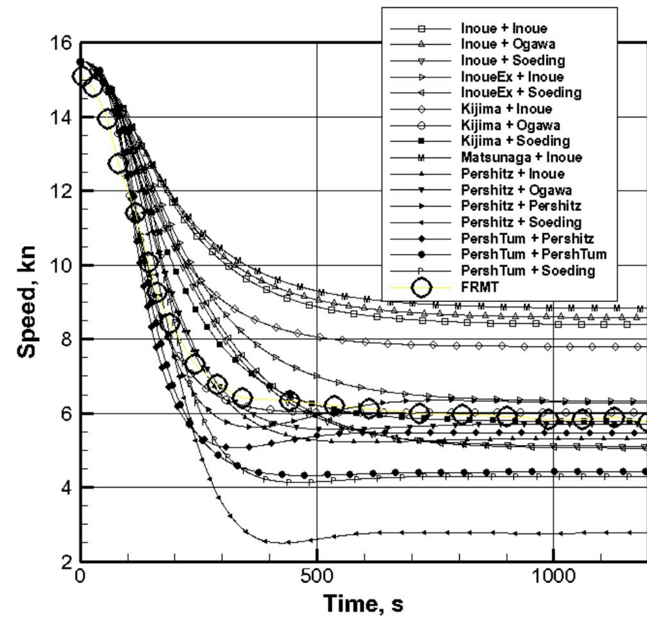

Fig. 4. Time histories for instantaneous speed in 35 deg helm turn.

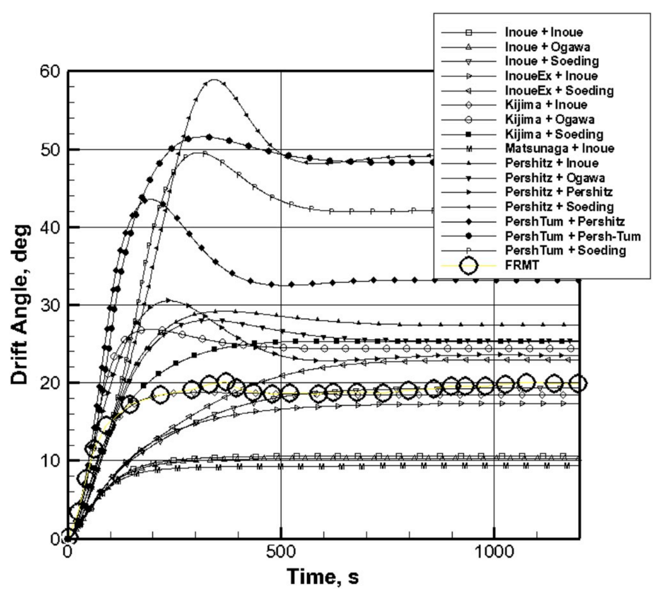

Fig. 3. Time histories for drift angle in 35 deg helm turn.

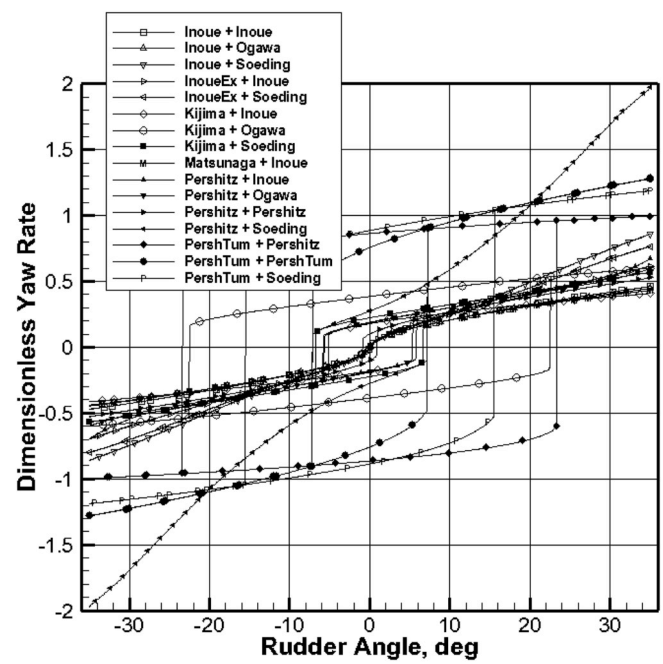

Fig. 5. Spiral curves: all models.

The spirals based on the Pershitz hull model all correspond to an unstable ship but only the combination Pershitz + Pershitz can be viewed as acceptable albeit with insufficient turning ability.

3.4. Zigzag manoeuvres

The standard 10° – 10° and 20° – 20°zigzag manoeuvres were modelled. The typical numerical measures of these zigzags are the first overshoot i.e. the absolute value of the exceedance of the first maximum yaw angle deviation from the second zigzag parameter and the second overshoot defined similarly. The values of the overshoots obtained as results of simulations and independent values from the literature are all assembled in Table 3. The comparative analysis for the 10°– 10° zigzag indicates that only the Kijima hull model based combinations and most of the Pershitz family resulted in unrealistically high overshoots. At the same time the original Pershitz + Pershitz combination only relatively slightly underpredicts the first overshoot and the second overshoot can be even treated as quite acceptable. However, inspection of the plotted heading angle histories in Figs. 8–11 shows also a somewhat delayed reaction of the Pershitz family models. Very peculiar is the behaviour of the Pershitz–Tumashik + Pershitz–Tumashik model: in the 10° – 10° zigzag a quite normal value of the first overshoot is combined with a pathologically large value of the second one. In the 20° – 20° zigzag, however, the same model, on the contrary, combines an anomalously large first overshoot with a more or less normal second one although all the following overshoots are too large. All empiric model combinations overpredict the zizag periods and in some cases—to a very large degree.

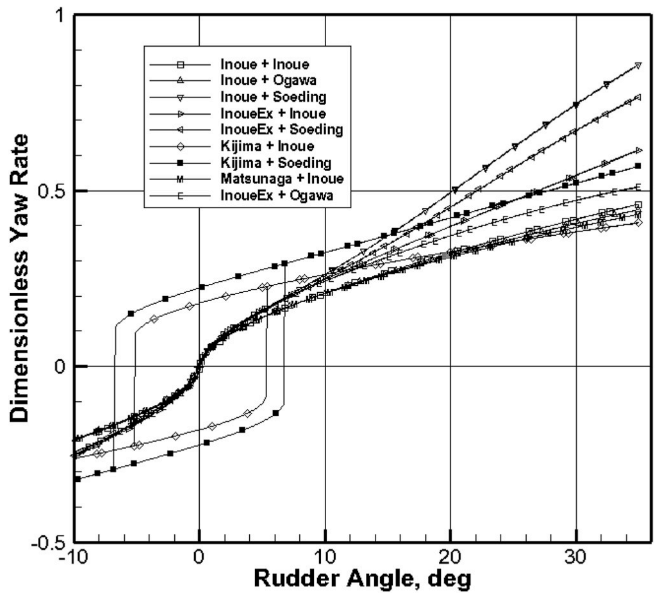

Fig. 6. Spiral curves: MMG hull models.

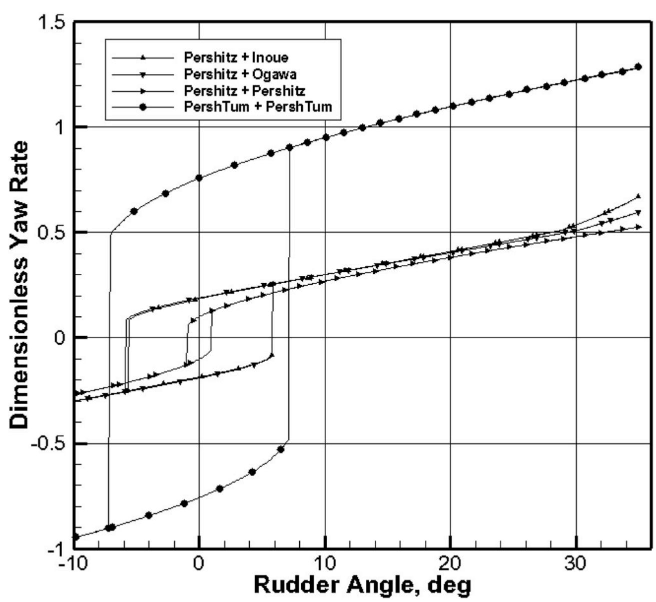

Fig. 7. Spiral curves: Pershitz hull models.

3.5. Hull force coefficients

Plots of the hull force coefficients computed according to various empiric models are presented in Figs. 12–14, where the graphs for the sway force and yaw moment coefficients are augmented with tank experimental data obtained in oblique towing and estimated from oscillatory PMM tests (Sung et al., 2018). These data correspond to the values of r' varying from -0.5 to $+0.5$ for $|\beta| \leq 3^{\circ}$ with the step 0.1. This range reduces to $[0,\ 0.5\mathrm{sign}\beta]$ for $|\beta| = 6^{\circ},\ 9^{\circ}$. Qualitatively, all responses for the surge force are very similar but, except for the pair Inoue—Matsunaga, are rather different quantitatively. In particular, they are much smaller by the absolute value for the Kijima model. In all models the linear dependence on the rate of yaw and drift angle is accounted for through the reduced sway-sway added mass and the reduction coefficient recommended by the Kijima model is much smaller than the average value 0.625 used here for the Inoue model.

The plots for the sway force coefficient show a somewhat greater variety in behaviour. The Matsunaga model is the only one where the sign of the influence of yaw does not change while for the Inoue and Kijima models it may change for the "unnatural" drift angles i.e. for those which do not correspond to the direction of turn. It can be suspected and is confirmed by the data from (Sung et al., 2018) that these combinations were given less attention in planning and execution of the underlying tests as long as such regimes, though not impossible, are much less likely in normal manoeuvring. The plots for both Persitz models are in compliance with their structure neglecting any dependence on the rate of yaw.

Considering the plots for the yaw moment coefficient it is easy to notice that extension of the Inoue model lead to amplification of the yaw damping role while similar effect for the sway force was opposite. It can be said that the extension procedure is closing the Inoue model to the Kijima one at least for the considered range of the kinematic parameters. The classic Pershitz model is characterized by full structural linearity while the responses of its extended version looks quite unusual which apparently confirms our suspicion that this model had been developed for simulating hard low-speed manoeuvres and its uniform validity is not guaranteed.

Finally, in Table 4 presented are values of the regression coefficients for the main models explored. The Inoue Extended and Pershitz–Tumashik have rather different trigonometric structures and it was not found reasonable to present all those specific coefficients. However, the values of the "linear derivatives" in these two methods are the same as in the Inoue and Tumashik methods respectively which is caused by asymptotic equivalence of each pair. Comparison of "nonlinear derivatives" makes in general little sense for non-orthogonal regressions of different structure but values of the "linear derivatives" may give some insight although they are also dependent on the remaining parts of the models. The differences for the MMG group of models are moderate except for $Y_r^{'}$ which is especially difficult to estimate in physical modelling. At the same time, the coefficient $N_v^{'}$ shows relatively small variations for all models.

3.6. Rudder forces

Responses of the rudder in all empiric models depend on a considerable number of parameters and it was not possible to perform their exhaustive investigation within the framework of the present contribution, so the rudder surge and sway forces were computed and plotted for two cases: (1) straight run at the approach speed and the corresponding propeller rotation frequency, and (2) motion with the drift angle 20°, $r^{'} = 0.5$ and at 70% of the approach speed. The plots for the surge force are presented in Fig. 15. The Inoue, Ogawa and Soeding models show zero surge force for the undeflected rudder as their responses are based entirely on the rudder normal force. This is not the case for the both Pershitz models accounting for the profile resistance of the rudder but, apparently, the Tumashik model underpredicts the absolute value of surge force. The Inoue and Ogawa models give very close results in straight run but diverge more in turn where they also show a positive surge force within some range of the deflection angle where the rudder attack angle becomes negative. This does not happen with other models presuming stronger straightening effects from the hull and slipstream.

Regarding the sway force all models excluding the Pershitz–Tumashik model show rather close responses in straight run but the differences increase significantly in a more general case of curvilinear motion.

3.7. Discussion of observed uncertainties

Inspection of the whole set of obtained numerical results presented in the previous subsections confirms the well-known fact that reliable prediction of responses and standard manoeuvring measures with existing un-tuned empirical methods may present a prohibitively

Table 3
Overshoots in zigzag manoeuvres.

Hull model	Rudder model	Zigzag $10^\circ - 10^\circ$, 1st overshoot, deg	Zigzag $10^\circ - 10^\circ$, 2nd overshoot, deg	Zigzag $20^\circ - 20^\circ$, 1st overshoot, deg	Zigzag $20^\circ - 20^\circ$, 2nd overshoot, deg
Inoue	Inoue	5.3	8.5	9.4	11.5
Inoue	Ogawa	5.2	8.5	9.4	11.5
Inoue	Soeding	5.7	10.1	10.7	14.3
Matsunaga	Inoue	5.3	8.7	9.2	11.2
Inoue Extended	Inoue	5.8	10.1	10.5	14.7
Inoue Extended	Soeding	5.85	10.0	10.2	13.7
Kijima	Inoue	20.4	31.7	16.3	15.7
Kijima	Soeding	37.8	43.6	21.0	18.2
Pershitz	Pershitz	5.1	11.7	10.7	16.8
Pershitz	Inoue	18.8	44.2	18.0	20.3
Pershitz	Ogawa	20.5	46.4	19.7	22.0
Pershitz–Tumashik	Pershitz–Tumashik	12.0	200.0	139.0	27.7
Yasukawa et al. (2015)		5.6–8.2	10.5–16.7	no data	no data
Stern & Agdrup (2009)		7.0–9.0	13.0–20.0	no data	no data
FRMT: SIMMAN (2018)		8.2	not reached	13.8	14.9

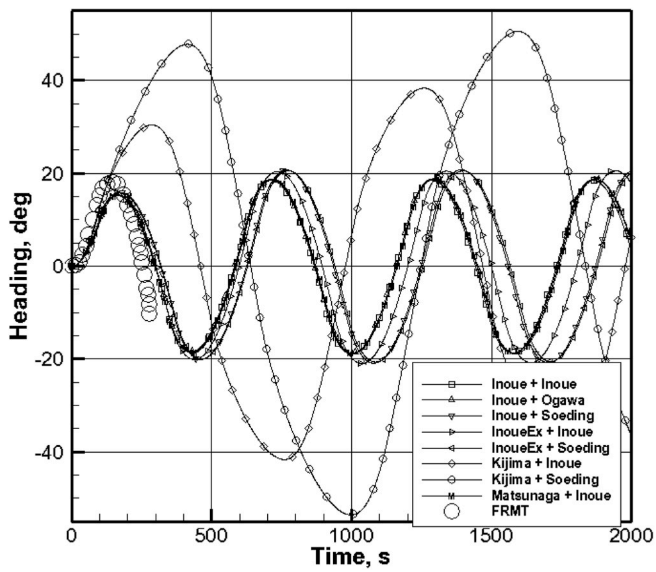

Fig. 8. Time history for $10^\circ - 10^\circ$zigzag manoeuvre: MMG hull models.

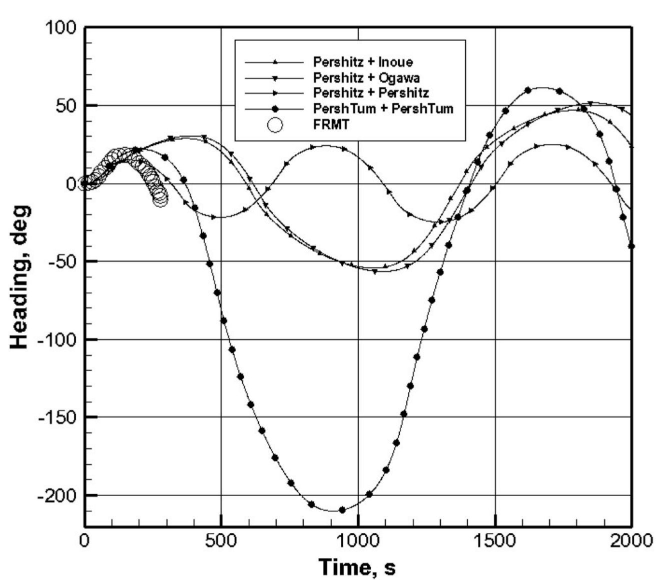

Fig. 9. Time history for $10^\circ - 10^\circ$zigzag manoeuvre: Pershitz hull models.

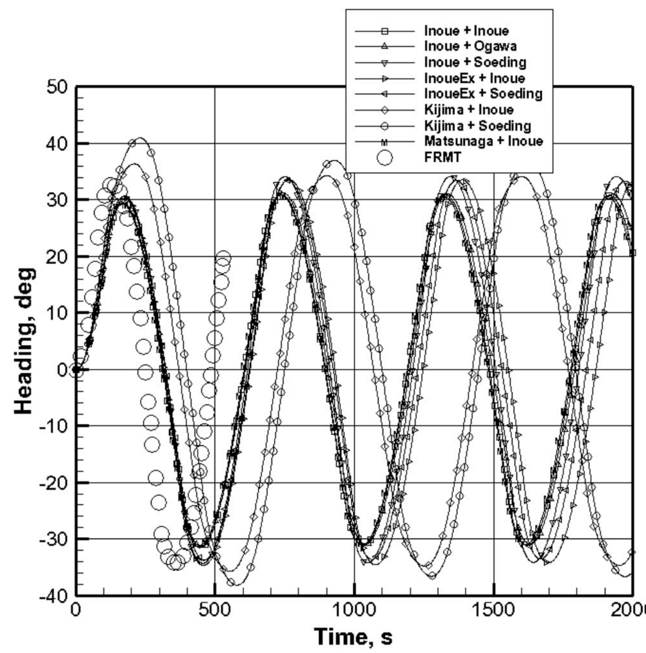

Fig. 10. Time history for $20^\circ - 20^\circ$zigzag manoeuvre: MMG hull models.

difficult task, at least for full-bodied vessels let alone using such methods for construction of fully adequate manoeuvring mathematical models. The latter remark is essential as even when some two different ship mathematical models produce, within reasonable tolerance, identical estimates of such manoeuvring measures as the tactical diameter and advance in a turn at 35° helm, and all zigzag overshoots, this still does not imply closeness of responses to arbitrary control programmes and under action of exogenous factors e.g. in wind. Although the mentioned set of numerical measures has a long history and is generally recognized as consistent for defining manoeuvring performance, there is no proof, either analytical or numerical that this or even some more extended set is sufficient for defining unambiguously and fully the mathematical model of some particular ship. Often, however, empiric methods do not pretend to form adequate mathematical models but are only aiming at prediction of the manoeuvring parameters required by IMO Standards ((IMO 2002a,b) although even this task may be problematic.

Discussing the observed uncertainties the following comments can be formulated:

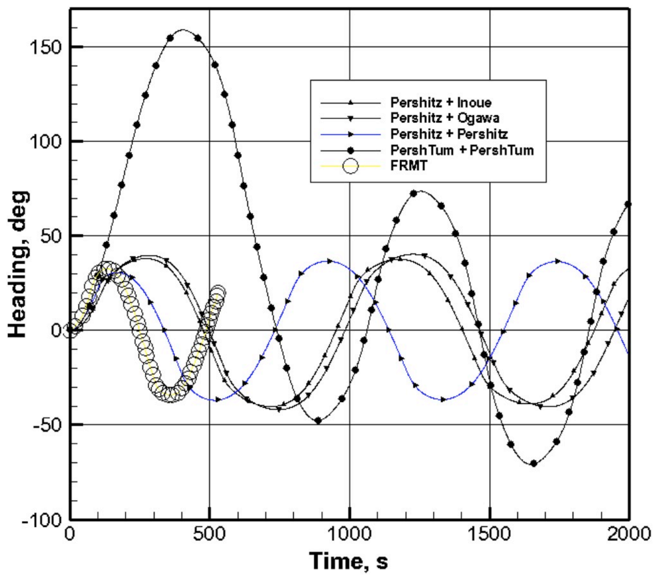

Fig. 11. Time history for 20° − 20°zigzag manoeuvre: Pershitz hull models.

1. In this particular study, the FRMT or CMT/PMM data can be treated as "true" ones although they are also prone to some uncertainties, primarily of random nature. These uncertainties, however, are reasonably small when the data are obtained on high-quality facilities in reputed hydrodynamic centres. All existing empiric methods are based on some previously conducted serial model tests and as long as the underlying experimental data are not free from random errors, the empiric methods could have been designed in a more flexible way embedding uncertainty estimates and intervals for the hull-regression coefficients, hull-rudder interaction coefficients among others. However, this either was never realized or is not documented in published descriptions of the methods. The only known exception is definition of the parameter C_m in (6) for which in (Inoue et al., 1981a) it was proposed to select any value between 0.5 and 0.75. However, comparative simulations performed by the authors had shown that the ship responses are weakly sensitive with respect to this coefficient and the mean value $C_m = 0.625$ was used ever since. However, variations of values of many other coefficients may influence the responses of the models much stronger. These variations can be approximately estimated using the plots given in (Inoue et al., 1981b). Bulian et al. (2006) used this graphical scatter to demonstrate by means of statistical modelling that the scatter of the turning and zigzag measures mentioned above can be quite considerable and even very large when each of the coefficient varies inside the interval $\pm 10\%$ of the base value. In particular, this means that through appropriate adjustment an empiric method can be tuned to produce a desired output. However, in reality all empiric methods were positioned as deterministic by their developers and their responses are supposed to be free of random errors for any defined hull shape and rudder arrangement. So, any disagreement with the "true" responses should be interpreted as a method bias (systematic error). This makes any statistical analysis of the responses very limited as, unlike, random errors, the biases are hardly Gauss distributed especially when the number of various methods is small. However, such statistics as the mean and median value, and the mean square error can be found anyway.
2. Analysis of data on the relative tactical diameter from Table 2 obtained with empiric methods, first, confirmed that distribution of residuals is close to uniform within the observed range from 1.764 to 5.11, the average value is 3.6 and the approximate median value is 3.75, both values being somewhat distant from the FRMT value 3.32. The mean square error (MSE) was obtained to be equal to 1.223 which is comparable to maximum deviations from the average value equal to 1.51 and −1.84. Of course, it is impossible to establish a confidence interval using the standard procedure based on the Student distribution which can only be applied if the true data distribution is Gaussian, at least approximately.
3. It seemed, however, reasonable to exclude from the analysis "artificial" model combinations, and extended versions not envisaged by the developers of the methods. This leaves with only 6 "legitimate" combinations: all MMG–MMG and Pershitz–Pershitz. In this case the average relative tactical diameter is 4.44 and the median is 4.95. The both numbers are even farther away from the "true" value although the estimated MSE error was reduced to 0.923.
4. Data for the relative advance and for the first overshoot showed similar properties. So it can be concluded that averaging over various methods may improve the required estimate only occasionally. While this conclusion is in no way unexpected, it also indicates that the "assimilation" approach suggested by Burnay and Ankudinov (2003) is in no way a panacea.
5. Although some methods can provide good estimates for some parameters, one is able to estimate well all of them. For instance, the tactical diameter is estimated with error 1.6% by the Kijima–Ogawa method, 2.7%—by the Pershitz–Pershitz method and 0.8%—by the Pershitz–Inoue combination. However, the errors in estimating the advance for the same methods are 13.2%, 17.6% and 40.7% respectively where the last error is absolutely unacceptable. However, the first combination, as already commented, produces an absolutely inacceptable spiral curve and is not able to simulate the 10–10 zigzag. Further, the Pershitz method underpredicts the first overshoot in 10-10 zigzag by 38% and the first overshoot in 20–20 zigzag—by 22%. This makes this method also unreliable in predicting standard manoeuvring measures, let alone doubtful shapes of produced trajectories.

It makes sense to discuss in some detail possible causes of the observed poor performance of the tested empiric methods and to outline ways to their improvement.

1. As long as the basic equations of motion (1)–(2) can be viewed as practically exact, at least for low-speed ships, and the values of the involved added masses affect the manoeuvring motion relatively weekly, all possible inaccuracies are related to the prediction of the quasi-steady hull and rudder forces and moments. Also, it goes mainly about the sway force and the yaw moment because the surge force mainly affects the speed drop. This also indicates at a relatively weak influence of the propeller model as in all tested methods the propeller sway force was neglected because previous estimates showed that this force may become significant and the propellers can act as additional stabilizers only on large diameter high-thrust twin-screw propellers typical for naval combatants.
2. Observations of the plots for the hull sway force (Fig. 13) show that in all main methods, except that of Pershitz, the sway force is somewhat overestimated in pure drift as compared to the tank data. Influence of the rate of yaw is captured more or less accurately by the Inoue method and slightly overestimated by the Kijima method. However, as was mentioned earlier, this dependence is of minor importance and even its complete neglection by the Pershitz method should not be fatal.
3. For the hull yaw moment (Fig. 14) the Inoue method shows the best agreement with the available experimental data although it remains satisfactory for the Matsunaga and Kijima methods which show slight overestimation. Overestimation for the Pershitz method is more significant at small drift angles due to full neglection of nonlinearity in that model.
4. Although, it was not possible to trace large divergences with the available tank data, the latter are of limited range and apparently much more substantial divergences could have been observed for

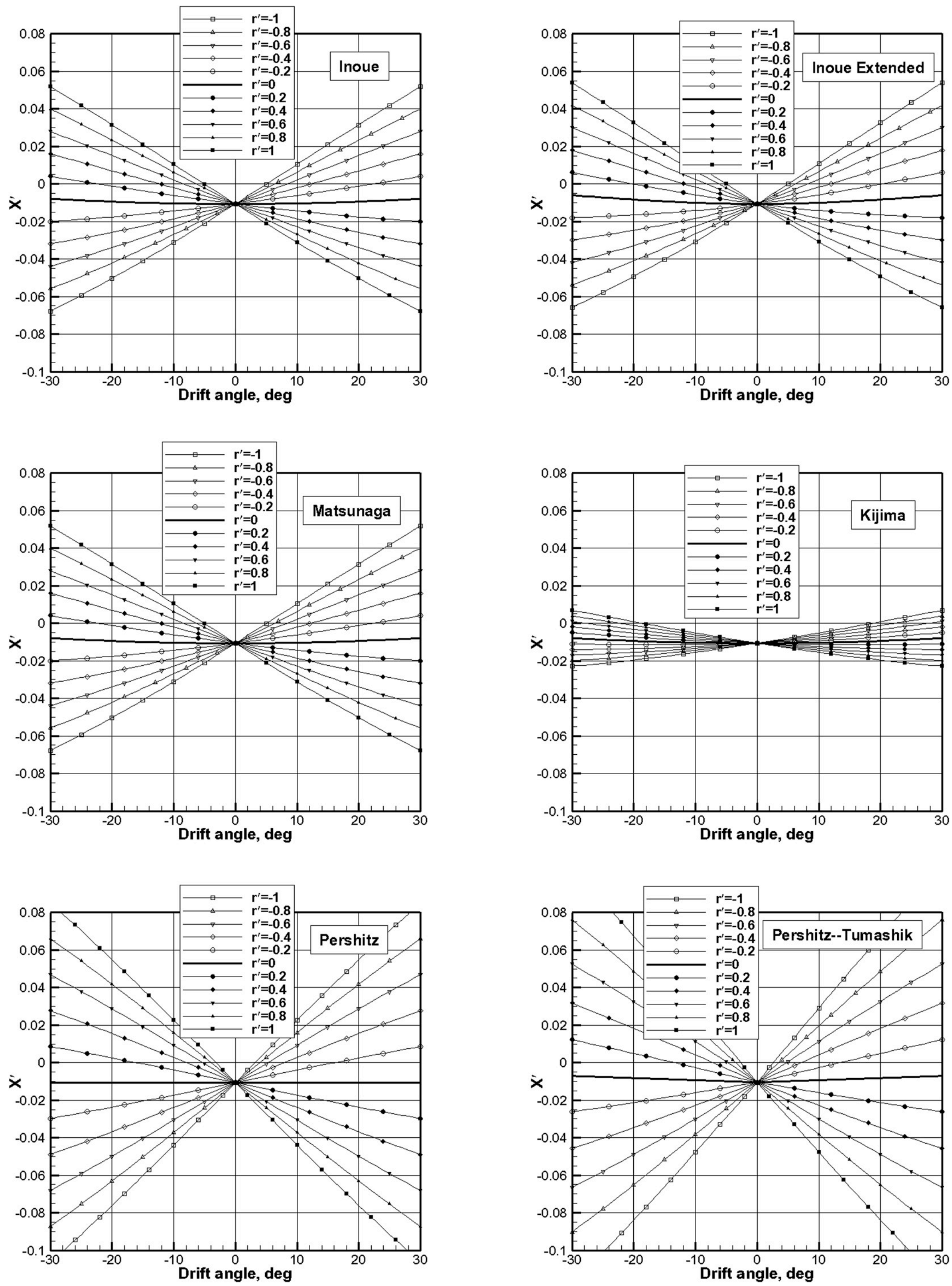

Fig. 12. Response curves of hull surge force coefficient.

combinations of large drift angles and large yaw rate which are rather common for full-bodied ships. It is remarkable that the disagreement between various empiric methods is larger in those domains. Regarding large differences observed for kinematic parameters in simulations performed with the same rudder model but different hull models it can be concluded that even moderate inaccuracies in prediction of the hull sway force and yaw moment can ruin final kinematical predictions at least for unstable or marginally stable ships for which the balance of various force components is rather vulnerable.

5. One of the main reasons for possible errors in prediction of the hull forces by empiric methods is, beyond any doubts, insufficient set of form parameters. In particular, when the Inoue method was published, many experts were surprised by absence of accounting for the specifics of the shape of the afterbody while it was already known that those specifics may affect seriously the hull forces and the earliest versions of the Pershitz method were using the "i"-parameter described in Appendix B. Finally, this deficiency was overcome in the Kijima method where two afterbody shape parameters were introduced (see C_{PA} and C_{WA} in Table 1). Unlike the "i"-parameter, these two as the whole Kijima method were aimed primarily at full-bodied

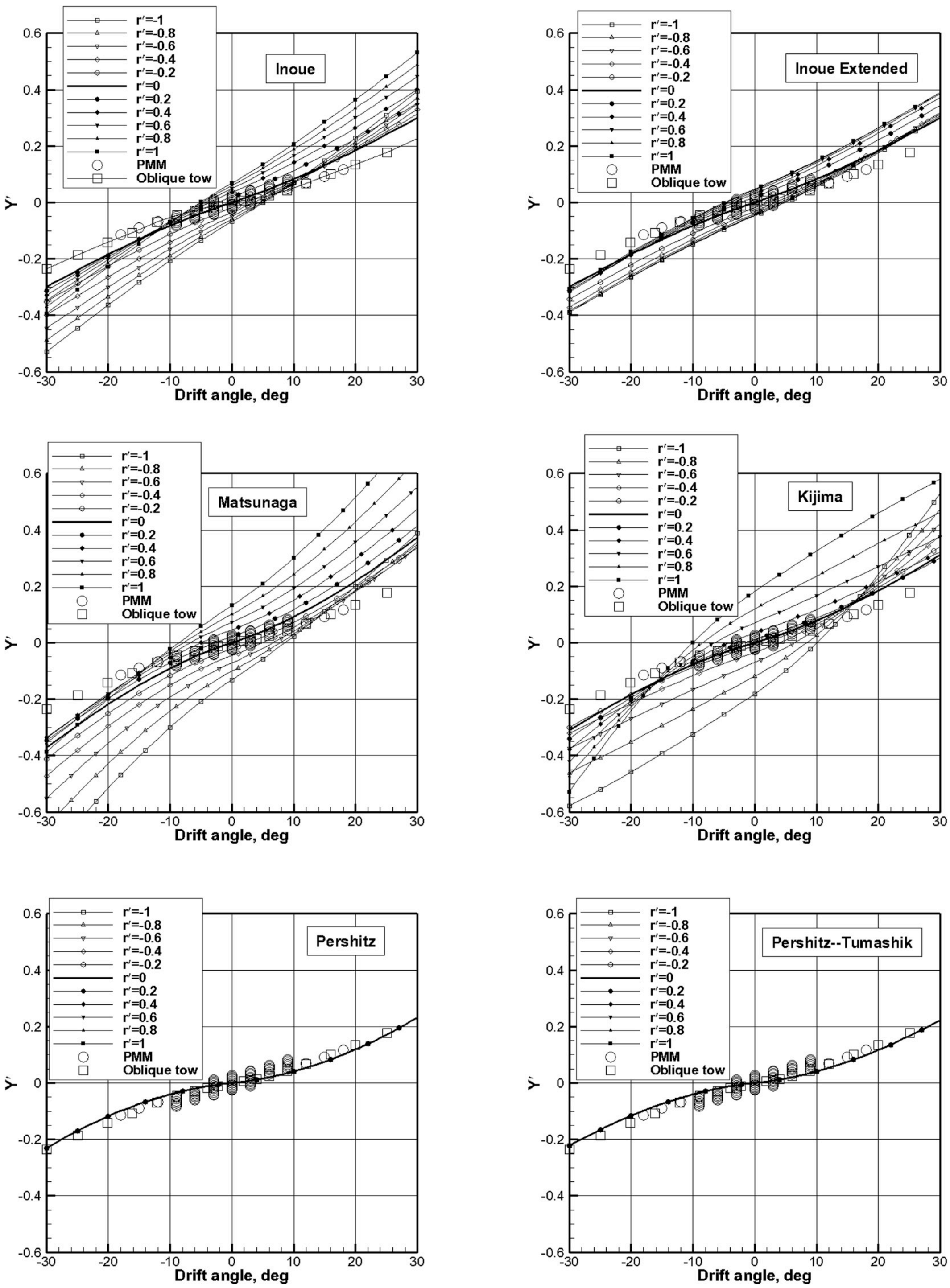

Fig. 13. Response curves of hull sway force coefficient.

vessels. Regarding this, a somewhat poorer performance of the Kijima method for the KVLCC2 ship form discussed above is surprising although it is maybe caused by a limited range of comparison. It is likely, however, that much more afterbody form parameters than 1 or 2 are required to satisfactorily predict sway forces and yaw moments as these depend strongly on many local effects which include flow separation and formation of free vortices in curvilinear motion. Number of physical scaled models and tests required for working out those parameters and describing their influence may become prohibitively large and it may happen that future empiric methods will be rather based on systematic CFD computations.

6. An additional source of uncertainties related to the prediction of hull forces with empiric methods, which is typically overlooked, is related to methods used for establishing regression models finally constituting part of this or that method. Details of these methods are typically not published but it can be understood e.g. from (Inoue et al., 1981b) that the regression analysis is performed in two stages. First, the "manoeuvring derivatives" are estimated for each of the tested hull forms, and, second, each of those "derivatives" is approximated with some regression over the selected form parameters. As the primary regression models are polynomial, non-orthogonal and not always minimized through analysis of

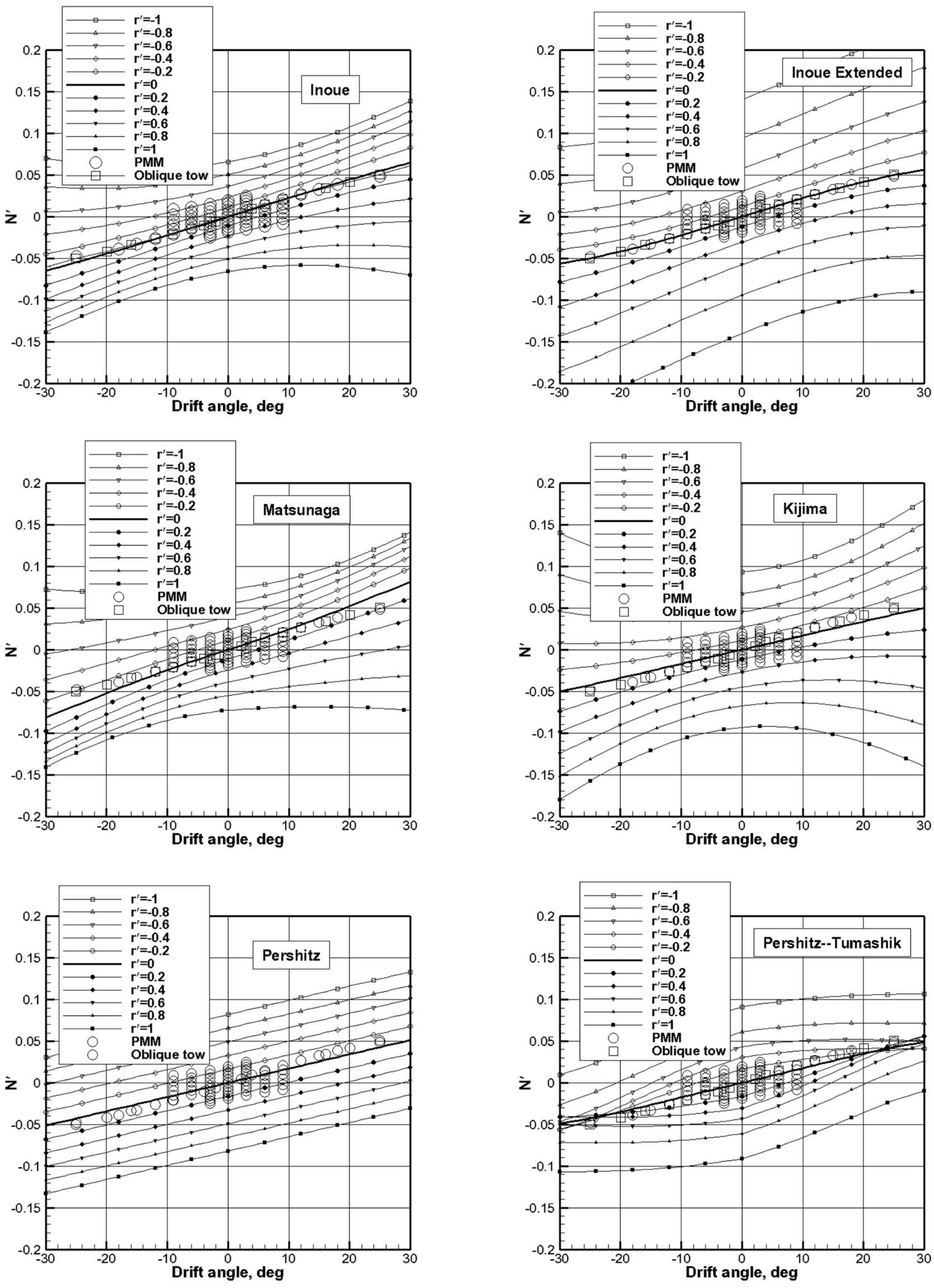

Fig. 14. Response curves of hull yaw moment coefficient.

variances and rejection of less significant regressors, they may be prone to multicollinearity and to excessive sensitivity to the values of higher-order coefficients. If the latter are estimated by means of secondary regressions, the final response with such interpolated "derivatives" may deviate far away from expected and will not provide fair prediction. An indication to a possibility of such unstable behaviour of a polynomial model can be found in (Woodward, 2014) where a paradoxically high uncertainty linked with the coefficient N'_{rrr} was discovered while the nonlinearity of the response surface for the yaw moment with respect to the rate of yaw was evidently very weak. Of course, extremely simple polynomial models like those defined by (9) are much less likely to suffer from those effects. It is clear that when the primary array of captive model tests results laid in foundation of this or that method is available it is always possible to reduce the regression-related uncertainty performing a regular one-stage analysis using compound linear regression models for both kinematical and form parameters. After this analysis is completed, it will be possible to represent approximations in the same form as before but with different values of numerical coefficients.

Table 4
Values of "hydrodynamic derivatives".

Coefficient	Inoue	Matsunaga	Kijima	Pershitz
X'_0	−0.01066	−0.01066	−0.01066	−0.01066
X'_{vr}	0.120	0.120	0.030	0.192
Y'_v	−0.410	−0.410	−0.353	−0.131
Y'_r	0.102	0.102	0.020	–
Y'_{vv}	−0.380	−0.671	−0.538	−0.606
Y'_{vr}	−0.325	–	–	–
Y'_{rr}	−0.0341	0.0296	0.161	–
Y'_{vvr}	–	−0.226	−0.625	–
Y'_{vrr}	–	−0.406	−0.486	–
N'_v	−0.13	−0.13	−0.096	−0.0994
N'_r	−0.053	−0.053	−0.0487	−0.083
N'_{vv}	–	−0.0669	−0.001	–
N'_{rr}	−0.0126	−0.0196	−0.0445	–
N'_{vvr}	−0.155	−0.136	−0.268	–
N'_{vrr}	0.0612	0.0952	0.0609	–

7. Reliable estimates of the rudder sway force and yaw moment are also very important for successful prediction of the manoeuvring performance of the ship. The plots in Fig. 16 are very alarming: while in straight run different rudder models agree fairly well one with another, this is much less so in more general situations. Regarding that most tests involving the rudder were performed mainly in straight run and often without the hull, it can be concluded that the existing empiric or semi-empiric rudder models involving a number or rather roughly defined coefficients accounting for the sidewash in curvilinear motion are another serious source of uncertainties in manoeuvring predictions. It is not clear whether a really dependable and simple rudder model is possible, but necessity of further extensive work in that direction is evident.

4. Conclusion

During the last 5–6 decades a number of universal empiric methods have been published in such form that they could be coded and used by any researcher although sometimes necessary information was spread over various publications. Most published applied studies based on these methods were, however, limited in the sense that only one somehow favourite, chosen or preferred method was used. Outputs of various empiric methods were assembled at SIMMAN workshops but there typically different methods were implemented within different computer codes by different institutions which could introduce additional uncertainties let alone possibilities for tuning the methods. An attempt to evaluate inter-laboratory uncertainties on the basis of SIMMAN 2008 data was undertaken by Woodward (2014) whose analysis included two empiric methods but also Free-Running Model Test, Computational Fluid Dynamics and captive-model test data and was based on an old methodology proposed by Youden (1959). One of the first attempts to compare various empiric methods implemented within the same code was undertaken in (Sutulo and Guedes Soares, 2018a,b) where,

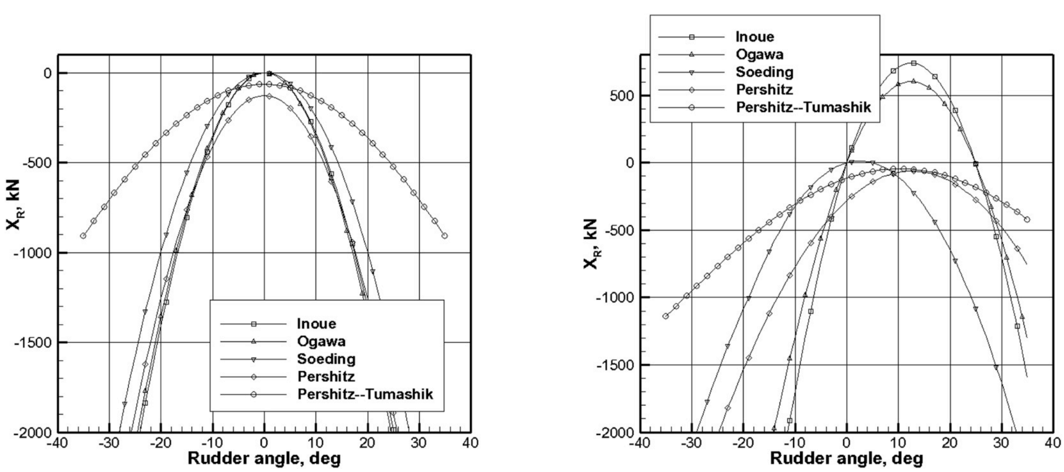

Fig. 15. Rudder surge force curves: left—straight run, right—curved motion with drift.

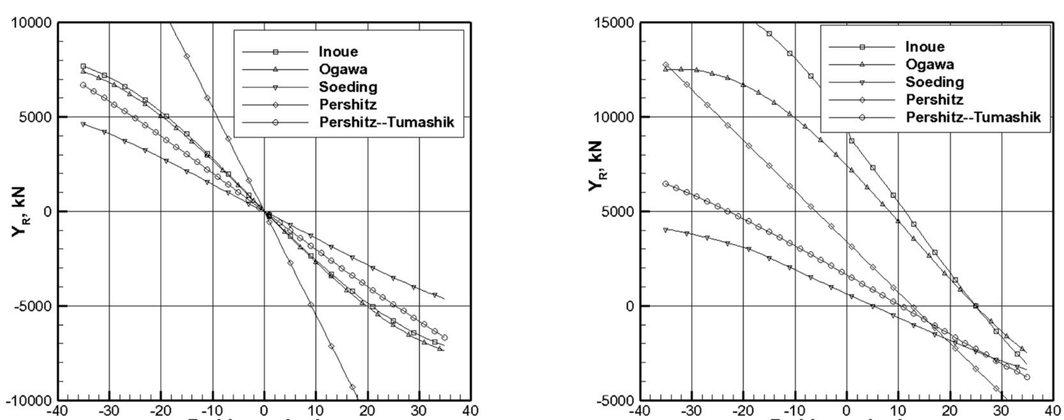

Fig. 16. Rudder sway force curves: left—straight run, right—curved motion with drif

however, the comparison was carried out in a very limited scale and some implementation inaccuracies were later discovered. For the first time, comparisons included the Pershitz method very popular during many decades in Russian ship hydrodynamics but so far inaccessible to most researchers in the remaining world. However, no full description of the method was given in the cited publication due to space limitations. Full description of this method in its original and extended forms is provided in the present article.

In the present study the multi-model object-oriented offline manoeuvring simulation code MANIST was revised and augmented with the implementation of the extended version of the Pershitz method associated also with Tumashik. As all universal empiric methods are modular, it was possible to test not only “pure” methods but also hybridized combinations when the hull model belongs to one method and the rudder model—to another. Altogether 16 combinations were tested through simulation of the 35° helm turning manoeuvre, the Dieudonné spiral manoeuvre and the standard zigzag manoeuvres. Inspection of the obtained results permits to draw the following conclusions:

1. Most combinations fail to predict more or less accurately the main measures of the turning manoeuvre i.e. the tactical diameter and the transfer. Of all 4 cases which were evaluated as successful in this respect, only one was “pure”: the Pershitz + Pershitz combination. Two combinations (Kijima + Ogawa and Pershitz + Söding) must be, however, rejected according to other criteria: first of them showed an absolutely unacceptable spiral curve corresponding to an uncontrollable ship while the second one gave unacceptable values of the dimensionless yaw rate and drift angle in steady turn. The second acceptable combination was a hybridized one: Inoue + Soeding. Additional consideration of the zigzag overshoots did not alter this conclusion.
2. All combinations based on the Inoue-type hull model correspond to directionally stable ship models, which is clearly seen from the spiral curves. On the contrary, all combinations based on the Pershitz hull model lead to unstable simulated vessels. However, the degree of instability is insignificant in the case of the original Pershitz + Pershitz combination.
3. Although extensions of the polynomial models to the case of arbitrary (low-speed) manoeuvres in theory must provide results similar to their base models, the deviations may become substantial. This is true for both the transition from the Pershitz to Pershitz–Tumashik and from the Inoue to Extended Inoue hull models. In the latter case, the causes of such effects are clear: the extension was performed by using trigonometric polynomials asymptotically equivalent to the original algebraic ones but the interval of asymptotic equivalence turned out to be smaller than is desirable for full equivalence in the standard turning manoeuvre.
4. Some very unexpected phenomena related to hybridization of various models were discovered. In particular, while two rather similar rudder models (Inoue and Ogawa) gave very close results when combined with the Inoue hull model, while the results turned very different even qualitatively when the same two ruder models were coupled with the Kijima hull model. Exact causes of such differences are still not quite clear but, apparently some shine will be dropped after the latest MMG variant of the rudder model (Yasukawa and Yoshimura, 2015) is implemented and tested. This is planned as future work but so far has not been realized because the structure of the new model is somewhat different from that of earlier versions and immediate embedding turned out impossible.
5. The plotted responses of the hull forces/moment coefficients provide some additional insight into the peculiarities of various models but still cannot give immediate explanation of their impact on ship dynamics. Likely, such explanation could be obtained through introducing some artificial partial variations inspired by the presented plots into each specific manoeuvring model.
6. The performed comparison of the rudder outputs according to various models showed that while most models provide relatively close estimates of the forces in straight run, this is much less so when the sway and yaw are present. This means that the often practised comparison and validation for the straight run only is in no way sufficient.
7. It can be dangerous to assess the validity of a certain method only on the basis of the turning manoeuvre and especially when only the standard measures i.e. the tactical diameter and the advance are considered. Probably the set of manoeuvres used in this study is the reasonable minimum for comparisons of this kind. In particular, the spiral manoeuvre, though the most time consuming, is very informative and likely should be made mandatory for all comparative studies like those performed in SIMMAN workshops.

In general, the present research has demonstrated that the existing empiric methods normally require some tuning using available results of free-running model tests and of full-scale trials to be useable as effective simulation tool for existing ships. This, however, makes somewhat doubtful their application in early design stages when no independent information about the ship manoeuvring qualities is available. It is even difficult to indicate some method that would be more dependable than the others, at least for full-bodied merchant ships although one could make some judgements analysing the results presented above and any attempts to “improve” this or that method by substitution of hull and rudder modules may become very risky. Apparently, any improvements should be undertaken by the original developers of the methods who are better familiar with implicit assumptions and limitations of initial databases. Special attention should be probably given to all aspects of hull–rudder and propeller–rudder interaction. At the same time, empirical methods of modular type are relatively physically consistent and can serve as a good platform for developing more accurate tuned models. When it goes about early design stages, it could be suggested to select any modular method and to calibrate it against some existing prototype ship which is close by its characteristics to the ship under design but for which reliable and rather detailed data on manoeuvring performance are available. After such calibration, the method can be applied to the newly designed configuration with much higher certainty as small changes with respect to the prototype can be modelled reliably enough. Regarding the choice of the method to be calibrated, apparently any one is suitable but the authors would avoid using the Perszitz method for full-bodied ships as there the used mathematical model is schematized more heavily limiting tuning options.

At the same time, it must not be forgotten that the present study was only performed for one ship characterized by the hull shape which is not beneficial for accurate manoeuvrability predictions. Carrying out similar studies for other ship forms is highly desirable. From this viewpoint, naval combatants may be of special interest as e.g. the Pershitz method had been primarily developed for this kind of ships and its better performance could be then expected.

Acknowledgements

This work was performed within the project SHOPERA—Energy Efficient Safe SHip OPERAtion, which was partially funded by the EU 7th Framework Programme under contract 605221.

Appendix A. Approximations of the "nonlinear derivatives" of Inoue method

The authors of the Inoue method have published data on the "nonlinear manoeuvring derivatives" in the form of plots where, besides experimental points, approximating curves were traced (Inoue et al., 1981b). The following formulae rather accurately approximate those traced curves, so they cannot be treated as regressions. At the same time, inspection of the mentioned plots demonstrates that some particular experimental values can deviate substantially from the approximated ones.

The approximations are:

$$
\begin{aligned}
&Y'_{vv} = -6.65C_{BT1} + 0.0735,\\
&Y'_{vr} = 1.73C_{BT1} - 0.443,\\
&Y'_{rr} = -\frac{1}{2}C_{BT1},\\
&N'_{rr} = \begin{cases} 0.675C_{BL} - 0.1015 & \text{at} \quad C_{BL} \le 0.113 \\ -6.9(C_{BL} - 0.156)^2 - 0.012 & \text{at} \quad C_{BL} > 0.113 \end{cases},\\
&N'_{vvr} = \begin{cases} 23.7C_{BL} - 2.23 & \text{at} \quad C_{BL} \in [0.071,\ 0.088] \\ (-91.5C_{BL} + 21.15)C_{BL} - 1.294 & \text{at} \quad C_{BL} \in (0.088,\ 0.143] \\ -2.88C_{BL} + 0.268 & \text{at} \quad C_{BL} > 0.143 \end{cases},\\
&N'_{vrr} = 0.43C_{BT2} - 0.0637,
\end{aligned}
\tag{A1}
$$

where $C_{BL} = C_B B/L$, $C_{BT1} = (1 - C_B)T/B$ and $C_{BT2} = C_B T/B$. Unless a tighter condition is imposed in (A1), these parameters must stay within the following intervals: $C_{BT1} \in [0.02,\ 0.15]$, $C_{BT2} \in [0.078,\ 0.4]$, $C_{BL} \in [0.0615,\ 0.2]$.

Appendix B. Definition of parameters of the Pershitz and Pershitz–Tumashik models

The approximations described below are rather bulky and look very different from those used in other empiric methods. This is explained by the fact that all those approximations constructed by Dr Melkozerova are secondary, i.e. approximating certain plots and nomograms. No details about development of the latter have ever been published and all comments available in the literature are of general nature. Apparently, it is now impossible to recover the tricks and assumptions lying behind those data. The authors have verified the approximations against the original nomograms which permitted to trace and correct some misprints present in (Voytkunski, 1985). All the constants in the present Appendix were extracted directly from the used computer code which minimized probability of new misprints.

Coefficients related to hull forces in the Pershitz model

The description of the forces contains many auxiliary coefficients. In order to avoid introduction of an excessively large number of different symbols, these must be treated as local i.e. one and the same symbol may have more than one different definitions.

The following common hull shape parameters are used: the hull prismatic coefficient $C_P = C_B/C_M$, where C_M is the midship section area coefficient; the hull aspect ratio $k_{H2} = T/L$, and the length-to-beam ratio L/B. All following approximations are valid for $C_P \in [0.55,\ 0.85]$, for $k_{H2} \in [0.02,\ 0.08]$ and for $L/B \in [4.0,\ 10.0]$. If the value of some parameter falls out of the indicated interval, it must be forced to the limit to avoid unpredictable behaviour of approximations.

Definition of the effective centerplane area coefficient

The general formula is:

$$C_L = 1.0 - \frac{3.0}{20.0 - i} \cdot \frac{A_{CO}}{LT} + C_{L0}(\theta), \tag{A2}$$

where i is the number (not necessarily integer) of the theoretical section at which transition from U- to V-shaped sections occurs (the sections are numbered from 0 at the fore perpendicular to 20 at the aft perpendicular), A_{CO} is the stern cut-off area of the centerplane (when it is defined, presence of the rudder, horn, rudder post, propeller struts are ignored but the bossings are reducing this area), $C_{L0} = 0.054T\theta/L$, where $\theta = \theta_1 + \theta_2$ is the total trim angle, θ_1 is the static trim angle, and θ_2 is the dynamic trim estimated for the straight run approach phase as function of the Froude number Fn and the dimensionless abscissa of the centre of mass $x'_G = x_G/L$, which must satisfy the condition $x'_G \in [-0.04,\ 0.02]$.

For ships with a bulbous stern:

$$C_L = 0.975 + C_{L0}, \tag{A3}$$

and for ships with a thick skeg:

$$C_L = 0.962 + C_{L0}. \tag{A4}$$

In any case, the used coefficient must satisfy the condition $C_L \in [0.93,\ 1.0]$.

The dynamic trim is estimated using the following formulae where it is assumed $f \equiv Fn$:

$$\theta_2 = \begin{cases} 0 & \text{at} \quad f < 0.34 \\ a_1^1 f^2 + b_1^1 f + c_1^1 & \text{at} \quad f \in [0.34, \ 0.42) \\ a_1^2 f^2 + b_1^2 & \text{at} \quad f \in [0.42, \ 0.46) \\ a_1^3 f^2 + b_1^3 & \text{at} \quad f \in [0.46, \ 0.54) \\ a_1^4 f^2 + b_1^4 f + c_1^4 & \text{at} \quad f > 0.54 \end{cases} \tag{A5}$$

where

$$\begin{array}{l}
\left.\begin{array}{l} a_1^1 = 525.0 x_G'^2 + 32.35 x_G' + 1.032; \\ b_1^1 = -345.0 x_G'^2 + 22.95 x_G' - 0.661; \\ c_1^1 = 50.7 x_G'^2 + 3.725 x_G' + 0.1017; \end{array}\right\} \text{at} \quad x_G' \le -0.015 \\
\left.\begin{array}{l} a_1^1 = 810.0 x_G'^2 + 24.60 x_G' + 0.853; \\ b_1^1 = -620.0 x_G'^2 + 19.90 x_G' - 0.565; \\ c_1^1 = 121.7 x_G'^2 + 3.84 x_G' + 0.0925; \end{array}\right\} \text{at} \quad x_G' > -0.015 \\
\left.\begin{array}{l} a_1^2 = -5.0 x_G'^2 - 0.550 x_G' + 0.151; \\ b_1^2 = 2.5 x_G'^2 - 0.055 x_G' - 0.058; \end{array}\right\} \text{at} \quad x_G' \le -0.02 \\
\left.\begin{array}{l} a_1^2 = -0.375 x_G' + 0.153; \\ b_1^2 = 2.0 x_G'^2 + 0.03 x_G' - 0.0583; \end{array}\right\} \text{at} \quad x_G' \in (-0.02, \ 0.0] \\
\left.\begin{array}{l} a_1^2 = -0.375 x_G' + 0.153; \\ b_1^2 = 1.5 x_G'^2 + 0.005 x_G' - 0.0583; \end{array}\right\} \text{at} \quad x_G' > 0.0 \\
\left.\begin{array}{l} a_1^3 = 0.75 x_G' + 0.12; \\ b_1^3 = -0.55 x_G' - 0.043; \end{array}\right\} \text{at} \quad x_G' \le -0.04 \\
\left.\begin{array}{l} a_1^3 = 0.5 x_G' + 0.112; \\ b_1^3 = -0.042 x_G' - 0.0391; \end{array}\right\} \text{at} \quad x_G' \in (-0.04, \ -0.01] \\
\left.\begin{array}{l} a_1^3 = 0.25 x_G' + 0.11; \\ b_1^3 = -0.2967 x_G' - 0.0379; \end{array}\right\} \text{at} \quad x_G' > -0.01 \\
a_1^4 = -1.875 x_G' - 0.1917; \\
b_1^4 = 2.4 x_G' + 0.277; \\
c_1^4 = -0.9083 x_G' - 0.073;
\end{array} \tag{A6}$$

It is clear that for most merchant ships the dynamic trim will be insignificant and will be neglected what agrees with the formulae (A3) and (A4).

Definition of Y_v' and c_1 :

$$\begin{array}{l}
Y_v' = -c_1, \quad c_1 = a_3 B + b_3, \ \text{where:} \\
B = a_2 U + b_2, \quad U = a_1 \dfrac{L}{B} + b_1, \\
a_1 = \left(a_1^2 C_P + a_1^1\right) C_P + a_1^0, \\
b_1 = \left(b_1^2 C_P + b_1^1\right) C_P + b_1^0, \\
a_2 = (16.67 k_{H2} - 11.92) k_{H2} + 0.06; \\
b_2 = (-261.1 k_{H2} + 213.6) k_{H2} - 2.468; \\
a_3 = (0.2392 C_P - 0.4009) C_P + 0.1815, \\
b_3 = (0.4033 C_P - 0.6965) C_P + 0.3263,
\end{array} \tag{A7}$$

where:

$$\begin{array}{lll}
a_1^2 = 375.0;\ \ a_1^1 = -725.8;\ \ a_1^0 = 349.5; & \text{at} & L/B \in [4,\ \ 6]\ \& \\
b_1^2 = 3000.0;\ \ b_1^1 = -5702.0;\ \ b_1^0 = 2723.0 & & C_L \in [0.93,\ \ 0.95] \\
a_1^2 = 150.0;\ \ a_1^1 = -293.5;\ \ a_1^0 = 141.97; & \text{at} & L/B \in [4,\ \ 6]\ \& \\
b_1^2 = 800.0;\ \ b_1^1 = -1560.0;\ b_1^0 = 773.5; & & C_L \in [0.95,\ \ 0.97] \\
a_1^2 = -1.137;\ \ a_1^1 = 0.24;\ \ a_1^0 = -0.753; & \text{at} & L/B \in [4,\ \ 6]\ \& \\
b_1^2 = 0.0;\ \ b_1^1 = -4.667;\ \ b_1^0 = 17.57; & & C_L > 0.97 \\
a_1^2 = 1000.0;\ \ a_1^1 = -1898.0;\ \ a_1^0 = 900.0; & \text{at} & L/B \in (6,\ \ 8]\ \& \\
b_1^2 = 1800.0;\ \ b_1^1 = -3494.0;\ \ b_1^0 = 1705.0; & & C_L \in [0.93,\ \ 0.95] \\
a_1^2 = 175.0;\ \ a_1^1 = -339.8;\ \ a_1^0 = 163.9; & \text{at} & L/B \in (6,\ \ 8]\ \& \\
b_1^2 = 0.0;\ \ b_1^1 = -30.0;\ \ b_1^0 = 38.42; & & C_L \in (0.95,\ \ 0.97] \\
a_1^2 = 0.0;\ \ a_1^1 = -0.5;\ \ a_1^0 = -0.485; & \text{at} & L/B \in (6,\ \ 8]\ \& \\
b_1^2 = 516.7;\ \ b_1^1 = -1032.0;\ \ b_1^0 = 523.7; & & C_L > 0.97 \\
a_1^2 = 350.0;\ \ a_1^1 = -664.5;\ \ a_1^0 = 314.96; & \text{at} & L/B \in (8,\ \ 10]\ \& \\
b_1^2 = 3600.0;\ \ b_1^1 = -6928.0;\ \ b_1^0 = 3339.0; & & C_L \in [0.93,\ \ 0.95] \\
a_1^2 = 0.0;\ \ a_1^1 = -1.5;\ \ a_1^0 = 0.985; & \text{at} & L/B \in (8,\ \ 10]\ \& \\
b_1^2 = 2000.0;\ \ b_1^1 = -3894.0;\ \ b_1^0 = 1901.0; & & C_L \in [0.95,\ \ 0.97] \\
a_1^2 = 0.0;\ \ a_1^1 = 1.5;\ \ a_1^0 = 0.985; & \text{at} & L/B \in (8,\ \ 10]\ \& \\
b_1^2 = 316.67;\ \ b_1^1 = -629.5;\ \ b_1^0 = 318.0; & & C_L > 0.97
\end{array} \tag{A8}$$

Definition of Y'_{vv} and c_2 :

$$\begin{array}{l}
Y'_{vv} = -c_2,\quad c_2 = a_3 B + b_3,\ \text{ where:} \\
B = a_2 U + b_2,\quad U = a_1 C_L + b_1, \\
a_1 = 54.46 C_P - 59.43, \\
b_1 = -31.44 C_P + 46.8, \\
a_2 = \left(a_2^2 U + a_2^1\right) U + a_2^0, \\
b_2 = \left(b_2^2 U + b_2^1\right) U + b_2^0, \\
a_3 = \left(a_3^2 k_{H2} + a_3^1\right) k_{H2} + a_2^0, \\
b_3 = \left(b_3^2 k_{H2} + b_3^1\right) k_{H2} + b_3^0,
\end{array} \tag{A9}$$

where:

$$\begin{array}{lll}
a_2^2 = -0.0105;\ \ a_2^1 = -0.0585;\ \ a_2^0 = 0.985; & \text{at} & U \le 2.0 \\
b_2^2 = 0.06;\ \ b_2^1 = -0.65;\ \ b_2^0 = 2.91; & & \\
a_2^2 = 0.001;\ \ a_2^1 = -0.079;\ \ a_2^0 = 0.98; & \text{at} & U \in (2.0,\ \ 5.0] \\
b_2^2 = -0.0267;\ \ b_2^1 = -0.41;\ \ b_2^0 = 2.78; & & \\
a_2^2 = -0.005;\ \ a_2^1 = -0.015;\ \ a_2^0 = 0.81; & \text{at} & U > 5.0 \\
b_2^2 = 0.03;\ \ b_2^1 = -0.89;\ \ b_2^0 = 3.76; & & \\
a_3^2 = 1.0;\ \ a_3^1 = 0.85;\ \ a_3^0 = 0.0311; & \text{at} & k_{H2} \le 0.04 \\
b_3^2 = -55.0;\ \ b_3^1 = 7.85;\ \ b_3^0 = 0.124; & & \\
a_3^2 = 1.0;\ \ a_3^1 = 0.615;\ \ a_3^0 = 0.0405; & \text{at} & k_{H2} \in (0.04, 0.06] \\
b_3^2 = 40.0;\ \ b_3^1 = -0.1;\ \ b_3^0 = 0.29; & & \\
a_3^2 = -5.0;\ \ a_3^1 = 1.05;\ \ a_3^0 = 0.036; & \text{at} & k_{H2} > 0.06 \\
b_3^2 = -10.0;\ \ b_3^1 = 2.5;\ \ b_3^0 = 0.314; & &
\end{array} \tag{A10}$$

Definition of c_3 :

$$\begin{array}{l}
c_3 = a_2 U + b_2,\ \text{ where:} \\
a_2 = (2.569 k_{H2} - 0.5805) k_{H2} + 0.00183; \\
b_2 = (-27.7 k_{H2} + 6.428) k_{H2} - 0.01749; \\
U = a_1 \dfrac{L}{B} + b_1,\ \text{ where:} \\
a_1 = \begin{cases} (24.65 C_P - 29.67) C_P + 7.547 & \text{at}\ \ C_P < 0.72 \\ 5.917 C_P - 5.3 & \text{at}\ \ C_P \ge 0.72 \end{cases}, \\
b_1 = \begin{cases} (-60.44 C_P + 74.61) C_P - 9.255 & \text{at}\ \ C_P < 0.68 \\ -10.08 C_P + 20.34 & \text{at}\ \ C_P \ge 0.68 \end{cases},
\end{array} \tag{A11}$$

Definition of $m_1,\ m_2,\ N'_v$:

$$
\begin{aligned}
&N'_v = -(2m_1 + m_2),\\
&m_2 = -\frac{\log(1.023C_L)}{11.6C_L - 9.29},\\
&m_1 = a_1 C + b_1, \quad \text{where}\\
&a_1 = (-0.1317k_{H2} + 0.05358)k_{H2} + 0.000181;\\
&b_1 = (-2.361k_{H2} + 0.8653)k_{H2} - 0.000161;\\
&C = C_U + C_V,\\
&\left.\begin{aligned} &C_U = -1.3U + 2.6;\\ &C_V = \left(0.01792\frac{L}{B} + 0.1275\right)\frac{L}{B} + 6.113; \end{aligned}\right\} \text{at} \quad U \le 4.0\\
&\left.\begin{aligned} &C_U = -1.3U + 7.8;\\ &C_V = \left(0.02333\frac{L}{B} - 0.045\right)\frac{L}{B} + 1.187; \end{aligned}\right\} \text{at} \quad U > 4.0\\
&U = U_0 + C_\Phi,\\
&\left.\begin{aligned} &U_0 = (-235.0C_L + 474.2)C_L - 235.8;\\ &C_\Phi = (-74.67C_p + 110.9)C_p - 39.64; \end{aligned}\right\} \text{at} \quad C_P \le 0.7\\
&\left.\begin{aligned} &U_0 = (-210.0C_L + 422.9)C_L - 207.2;\\ &C_\Phi = (12.0C_p - 8.8)C_p - 0.64; \end{aligned}\right\} \text{at} \quad C_P > 0.7
\end{aligned}
\tag{A12}
$$

Definition of m_3 :

$$
\begin{aligned}
&m_3 = a_2 U + b_2, \quad \text{where:}\\
&a_2 = 0.001\left(0.7728\cdot 10^{-3}\exp(8.20939C_P) - 1.873\right);\\
&b_2 = 0.01\left(-0.4404\cdot 10^{-2}\exp(7.47893C_P) + 5.709\right);\\
&U = \frac{a_1 C_L + b_1}{C_L - 1.029};\\
&a_1 = 31.26 - 9.0146\cdot\exp(0.066947L/B);\\
&b_1 = 8.6245\cdot\exp(0.071419L/B) - 31.26;
\end{aligned}
\tag{A13}
$$

Definition of m_4 :

$$
\begin{aligned}
&m_4 = C_4 + C_U \quad \text{where:}\\
&C_4 = \begin{cases} (-71.88k_{H2} + 4.238)k_{H2} - 0.066 & \text{at} \quad k_{H2} \le 0.028,\\ (-9.375k_{H2} + 0.8875)k_{H2} - 0.0212 & \text{at} \quad k_{H2} \in (0.028, 0.04],\\ (-3.833k_{H2} + 0.415)k_{H2} - 0.01117 & \text{at} \quad k_{H2} > 0.04; \end{cases}\\
&C_U = 0.00827U - 0.017,\\
&U = U_0 + C_s,\\
&U_0 = \begin{cases} (-140.62C_P + 180.62)C_P - 53.35 & \text{at} \quad C_P \le 0.64,\\ (-56.67C_P + 75.1)C_P - 20.2 & \text{at} \quad C_P \in (0.64, \ 0.74],\\ (-216.7C_P + 312.8)C_P - 108.51 & \text{at} \quad C_P > 0.74; \end{cases}\\
&C_s = \begin{cases} (1900.57C_L - 3696.0)C_L + 1796.0 & \text{at} \quad C_L \le 0.96,\\ (391.70C_L - 810.4)C_L + 416.4 & \text{at} \quad C_L > 0.96. \end{cases}
\end{aligned}
\tag{A14}
$$

Definition of A_1 and A_2 :

$$
\begin{aligned}
&A_1 = 0.09 - 0.0033(L/B - 7.0) - 20.0(k_{H2} - 0.05)^2\\
&\quad + 0.4(C_L - 0.9) + 0.05(C_M - 0.9);\\
&A_2 = 0.008\frac{L}{B} + 0.9(k_{H2} - 0.05) + 0.45(C_L - 0.955);
\end{aligned}
\tag{A15}
$$

Estimation of the zero surge force drift angle β_x:

$$
\beta_x = \begin{cases} \dfrac{\pi}{180.0}[(-67684.35 + 35415.0C_L)C_L + 32440.595] & \text{at} \quad C_L < 0.95,\\ \dfrac{\pi}{180.0}[(-7518.96 + 3732.8C_L)C_L + 3876.66] & \text{at} \quad C_L \ge 0.95. \end{cases}
\tag{A16}
$$

References

Barr, R.A., 1993. A review and comparison of ship maneuvering simulation methods. Trans. SNAME 101, 609–635.

Barras, C.B., 2004. Ship Design and Performance for Masters and Mates. Elesevier Butterworth–Heinemann, Oxford, UK.

Betram, V., Schneekluth, H., 1998. Ship Design for Efficiency and Economy. Elesevier Butterworth–Heinemann, Oxford, UK.

Brix, J. (Ed.), 1993. Manoeuvring Technical Manual. Seehafen Verlag, Hamburg.

Bulian, G., Nicolosi, R.G., Francescutto, A., 2006. On the effect of uncertainty modelling in the hydrodynamic derivatives of a ship manoeuvring mathematical model. In: 7th International Conference on Hydrodynamics: Theory and Applications. ISCIA, Italy, p. 10p, 4–6 October 2006.

Burnay, S., Ankudinov, V., 2003. The prediction of hydrodynamic forces acting on the hull of a maneuvring ship based upon a database of prediction methods. In: International Conference on Marine Simulation and Ship Maneuverability MARSIM'03, 25–28 August 2003, Kanazawa, Japan, pp. RC-2-1–RC-2-12.

Crane, C.L., Eda, H., Landsburg, A., 1989. Controllability. In: Lewis, E.V. (Ed.), Principles of Naval Architecture, vol. 3. SNAME, Jersey City, NJ, pp. 191–365.

Fukui, Y., Yoshimura, Y., Yano, H., Kondo, M., 2015. Investigation of effect of roll motion on manoeuvrability using 4-DOF mathematical model, MARSIM 2015. In: International Conference on Ship Manoeuvrability and Maritime Simulation. Newcastle University, United Kingdom, 8–11 September 2015, Paper 3-1-5, 12p.

Hirano, M., 1980. On calculation method of ship manoeuvring motion (in Japanese). J. Soc. Nav. Archit. Jpn. 147, 144–153, 6.

IMO, 1993. Interim Standards for Ship Manoeuvrability. IMO. Resolution A.751(18) adopted on 4 November 1993.

IMO, 2002. Standards for Ship Manoeuvrability. International Maritime Organization (IMO). Resolution MSC.137(76)4.

IMO, 2002. Explanatory notes to the standards for ship manoeuvrability. MSC/Circ. 1053, 16 December 2002, 41p.

Inoue, S., Hirano, M., Kijima, K., Takashina, J., 1981. A practical calculation method of ship maneuvering motion. Int. Shipbuild. Prog. 28, 207–222, 325.

Inoue, S., Hirano, M., Kijima, K., 1981. Hydrodynamic derivatives on ship maneuvering. Int. Shipbuild. Prog. 28, 112–125, 321.

Ishiguro, T., Tanaka, S., Yoshimura, Y., 1996. A study on the accuracy of the recent prediction technique of ship's manoeuvrability at early design stage. In: Chislett, M. S. (Ed.), Marine Simulation and Ship Manoeuvrability: Proceedings MARSIM'96, Copenhagen, September 9–13, 1996, pp. 547–561. Rotterdam, Balkema.

Kijima, K., 2003. Some studies on the prediction for ship manoeuvrability. In: International Conference on Marine Simulation and Ship Maneuverability MARSIM'03, 25–28 August 2003, Kanazawa, Japan. KN-3-1–KN-3-10.

Kijima, K., Nakiri, Y., 2003. On the practical prediction method for ship manoeuvring characteristics. In: International Conference on Marine Simulation and Ship Maneuverability MARSIM'03, 25–28 August 2003, Kanazawa, Japan. RC-6-1–RC-2-10.

Matsunaga, M., 1993. Method of Predicting Ship Manoeuvrability in Deep and Shallow Waters as a Function of Loading Condition. NK Technical Bulletin, pp. 51–59.

Ogawa, A., Kasai, H., 1978. On the mathematical model of manoeuvring motion of ships. Int. Shipbuild. Prog. 25, 306–319, 292.

Ogawa, A., Hasegawa, K., Yoshimura, Y., 1980. Mathematical modeling of the ship's maneuvering. Nihon Zosen Gakkai-shi (Techno Marine) 616, 565–576 (in Japanese).

Oosterveld, M.W.C., van Oossanen, P., 1975. Further computer-analysed data of the Wageningen B-screw series. Int. Shipbuild. Prog. 22, 251–262.

Papanikolaou, A., Zaraphonitis, G., Bitner-Gregersen, E., Shigunov, V., El Moctar, O., Guedes Soares, C., Reddy, D.N., Sprenger, F., 2016. Energy efficient safe ship operation (SHOPERA). Transp. Res. Procedia 14, 820–829.

Pershitz, R.Y., 1983. Manoeuvrability and Ship Steering. Sudostroyeniye, Leningrad (in Russian).

Quadvlieg, F., Simonsen, C., Otzen, J., Stern, F., 2015. Review of the SIMMAN2014 workshop on the state of the art of prediction techniques for ship manoeuvrability, MARSIM 2015. In: International Conference on Ship Manoeuvrability and Maritime Simulation. Newcastle University, United Kingdom, 8–11 September 2015, Paper 4-2-3, 16p.

SIMMAN, 2018. In: Quadvlieg, F., Stern, F., Simonsen, C.D., Otzen, J.F. (Eds.), Workshop on Verification and Validation of Ship Manoeuvring Simulation Methods, Copenhagen, December 8–10, 2014, Workshop Proceedings, pp. 1–112, 2018, Part A: Summary,.

Söding, H., 1982. Prediction of ship steering capabilities. Schiffstechnik 29, 3–29.

Stern, F., Agdrup, K. (Eds.), 2009. SIMMAN2008: Workshop on Verification and Valdation of Ship Manoeuvring Simulation Methods, Copenhagen, 14-16 April 2008. Force Technology Publ.

Sung, Y.J., Park, S.-H., Ahn, K.-S., Chung, S.-H., Shin, S.S., Jun, J.-H., 2018. In: Quadvlieg, F., Stern, F., Simonsen, C.D., Otzen, J.F. (Eds.), Evaluation of Deep-Water Manoeuvring Performances of KVLCC2 Based on PMM Test and RANS Simulation, SIMMAN 2014: Workshop on Verification and Validation of Ship Manoeuvring Simulation Methods, Copenhagen, December 8–10, 2014, Workshop Proceedings, pp. C105–C110, 2018.

Sutulo, S.V., 1994. Extension of polynomial ship mathematical models to arbitrary manoeuvres. In: Proceedings of the International Shipbuilding Conference in Commemoration of the Centenary of the Krylov Ship Research Institute (ISC), St. Petersburg, 8-12 October 1994, Section B: Ship Hydrodynamics. St. Petersburg, 1994, pp. 291–298.

Sutulo, S., Guedes Soares, C., 2005. An object-oriented manoeuvring simulation code for surface displacement ships. In: Guedes Soares, C., Garbatove, Y., Fonseca, N. (Eds.), Maritime Transportation and Exploitation of Ocean and Coastal Resources. Taylor & Francis, Leiden, The Netherlands, pp. 287–294 (3/1).

Sutulo, S., Guedes Soares, C., 2011. Mathematical models for simulation of manoeuvring performance of ships. In: Guedes Soares, C., et al. (Eds.), Marine Technology and Engineering. Taylor & Francis Group, London, pp. 661–698.

Sutulo, S., Guedes Soares, C., 2015. Development of a core mathematical model for arbitrary manoeuvres of a shuttle tanker. Appl. Ocean Res. 51, 293–308.

Sutulo, S., Guedes Soares, C., 2015. Preliminary analysis of ship manoeuvrability criteria in wind. In: Guedes Soares, C., Santos, T.A. (Eds.), Maritime Technology and Engineering,: 933–946. Taylor & Francis Group, London, UK, pp. 933–946.

Sutulo, S., Guedes Soares, C., 2016. In: Guedes Soares, C., Santos, T.A. (Eds.), "Analysis of Manoeuvrability Criteria and Standards in View of Environmental Factors and EEDI Impact", Maritime Technology and Engineering 3. Taylor & Francis Group, London, UK, pp. 371–383.

Sutulo, S., Guedes Soares, C., 2018. In: Quadvlieg, F., Stern, F., Simonsen, C.D., Otzen, J. F. (Eds.), Prediction of Manoeuvring Characteristics of the KVLCC2 Ship with a Hybrid System-Based Code, SIMMAN 2014: Workshop on Verification and Validation of Ship Manoeuvring Simulation Methods, Copenhagen, December 8–10, 2014, Workshop Proceedings, pp. C111–C116, 2018.

Sutulo, S., Guedes Soares, C., 2018. In: Comparative Simulation of Definitive Manoeuvres of the KVLCC2 Benchmark Ship Using Different Empiric Mathematical Models", Maritime Transportation and Harvesting of Sea Resources, Guedes Soares C. And Teixeira A.P, vol. 1. CRC Press, Taylor & Francis Group, London, UK, pp. 571–579.

Vassilyev, A.V., 1989. Manoevrability of Ships. Sudostroyeniye, Leningrad (in Russian).

Voytkunsky, Y.I. (Ed.), 1985. Handbook on Ship Hydrodynamics (In Russian), vol. 3. Sudostroyeniye, Leningrad.

Woodward, M.D., 2014. Evaluation of inter-facility uncertainty of ship manoeuvring performance prediction. Ocean Eng. 88, 598–606.

Yasukawa, H., Yoshimura, Y., 2015. Introduction of MMG standard method for ship maneuvering predictions. J. Mar. Sci. Technol. 20, 37–52.

Yasukawa, H., Hirata, N., Yonemasu, I., Terada, D., Matsuda, A., 2015. Maneuvering Simulation of a KVLCC2 Tanker in Irregular Seas, MARSIM 2015: International Conference on Ship Manoeuvrability and Maritime Simulation. Newcastle University, United Kingdom, 8–11 September 2015, Paper 2-2-3, 16p.

Youden, W.J., 1959. Graphical diagnosis of interlaboratory test results. In: Ku, H.H. (Ed.), Precision Measurement and Calibration: Selected NBS Papers on Statistical Concepts and Procedures, vol. 1. NBS Special Publication 300, Washington, D.C., pp. 133–137, 1969.