Particle Swarm Optimization

Swarm Intelligence: Any attempt to design algorithms or distributed Problem solving devices inspired by the collective behaviour of Social insect colonies and other animal societies.

Steps:

* Initial position and velocity of particles are generated randomly within the search space.

* $$\boxed{V_i = \underbrace{wV_i}_{①} + \underbrace{C_1 r_1 (P_{best,i} - X_i)}_{②} + \underbrace{C_2 r_2 (g_{best} - X_i)}_{③}}$$

* Position of Particle is modified as $X_i = X_i + V_i$

* Evalute the objective function f_i and update the population, irrespective of the fitness.

$V_i \rightarrow$ velocity of i^{th} particle $w \rightarrow$ inertia of the particle

$C_1, C_2 \rightarrow$ Acceleration co-efficients $X_i \rightarrow$ Position of i^{th} particle

$r_1, r_2 \rightarrow$ Random Numbers $\in [0,1]$ of size $(1 \times D)$

$P_{best,i} \rightarrow$ Personal best of i^{th} Particle

$g_{best} \rightarrow$ Global best of ~~i^{th}~~ all particle

* Update P_{best} and g_{best} if

$$\boxed{\left.\begin{aligned} P_{best,i} &= X_i \\ f_{best,i} &= f_i \end{aligned}\right\} \text{ if } f_i < f_{P_{best,i}}}$$

$$\boxed{\left.\begin{aligned} g_{best} &= P_{best,i} \\ f_{g_{best}} &= f_{P_{best,i}} \end{aligned}\right\} \text{ if } f_{P_{best,i}} < f_{g_{best}}}$$

① $\longrightarrow$ Momentum Part

② $\longrightarrow$ Cognitive Part

③ $\longrightarrow$ Social Part

Possible Cases:

Cases	Better than it's Personal best	Better than Global best	Remarks
case 1	✗	✗	Do not update P_{best} & g_{best}
Case 2	✓	✗	update P_{best} Do not update g_{best}
Case 3	✓	✓	Update P_{best} and g_{best}
Case 4	✗	✓	Don't, Not Possible case

$$\therefore \min f(x) = \sum_{i=1}^{2} x_i^2 \; ; \; 0 \leq x_i \leq 10, \; [i=1,2].$$

Case 1:

Let $X = [5,6]$, $f = 61$

$P_{best} = [4,5]$, $f_{Pbest} = 41$

$g_{best} = [2,3]$, $f_{g_{best}} = 13$

$f > f_{Pbest}$

$f > f_{gbest}$

Case 2:

Let $x = [4, 3]$, $f = 25$

$P_{best} = [4 \; 5]$, $f_{Pbest} = 41$

$g_{best} = [2 \; 3]$, $f_{gbest} = 13$

$f < f_{Pbest}$

$f_{Pbest} = 25$ and $P_{best} = [4 \; 3]$

$f > f_{gbest}$

Case 3:

Let $x = [1 \; 3]$, $f = 10$

$P_{best} = [4 \; 5]$, $f_{Pbest} = 41$

$g_{best} = [2,3]$, $f_{gbest} = 13$

$f < f_{Pbest} \longrightarrow f_{Pbest} = 10$ and $P_{best} = [1 \; 3]$

$f < f_{gbest} \longrightarrow f_{gbest} = 10$ and $g_{best} = [1 \; 3]$

nce Again. Governing Equation

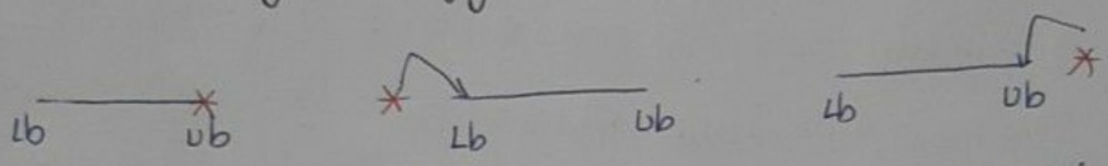
$$\boxed{\overrightarrow{X_i^{t+1}} = \overrightarrow{X_i^t} + \overrightarrow{V_i^{t+1}}}$$

$$\boxed{\overrightarrow{V_i^{t+1}} = w\overrightarrow{V_i^t} + c_1 r_1\left[\overrightarrow{P_i^t} - \overrightarrow{X_i^t}\right] + c_2 r_2\left[\overrightarrow{G^t} - \overrightarrow{X_i^t}\right]}$$

in time and iteration's Perspective

* "Meta-Heuristic" technique used in many fields. "TLBO" ← Mind this

Corner Bounding Strategy:



within bound — * to Shift to Lb — * Shift to Ub.

* No greedy selection in P_{best} - local best position.

greedy Selection in g_{best} - global best position

Parameter's character:

i. $C_1 = C_2 = 0$ ⟶ move in Same direction until hits SPB

ii. $C_1 > 0$ & $C_2 = 0$ ⟶ local Search, no global best

iii. $C_1 = 0$ & $C_2 > 0$ ⟶ global Search faster, but no optimal

iv. $C_1 = C_2$ ⟶ attracted towards $P_{best,i}$ & g_{best}

v. $C_1 >> C_2$ ⟶ Particle attracted towards its $P_{best,i}$ which result in excessive wandering

vi. $C_1 << C_2$ ⟶ towards g_{best} but Premature Convergence towards optima

vii. Low values of C_1 & C_2 ⟶ Smooth Particle trajectories

viii. high values of C_1 & C_2 ⟶ Abrupt Movements

Significance of Parameter:

Inertia weight: → Control the impact of Previous velocity in new dir.
→ Balancing exploration and exploitation
→ Large inertia weight result in exploration & Small inertia causes exploitation
(diverges) (decelerate)

Types:

cases
* it can be constant [C]
* multiplied with damping ratio in every iteration [$w = \alpha w$] $\alpha \rightarrow$ damping ratio
* Linearly decreased b/w W_{max} and W_{min}
→ w is linearly decreasing from 0.9 to 0.4
$w_{max,min}$ - user defined parameter

$$\boxed{w = W_{max} - \frac{[W_{max} - W_{min}]t}{T}}$$

* "Constriction co-efficients" for 'w' term (important method)

* Implemented to prevent explosion and also aid particles to converge to optima

$$\boxed{\chi = \frac{2K}{|2 - \phi - \sqrt{\phi^2 - 4\phi}|}}$$

$$\boxed{\begin{array}{c} 0 \leq K \leq 1 \\ \phi = \phi_1 + \phi_2 > 4 \end{array}}$$

Commonly used values
$K = 1,\ \phi_1 = 2.05,\ \phi_2 = 2.05$

constriction co-efficient rule
$w = \chi,\ C_1 = \chi\phi_1,\ C_2 = \chi\phi_2$