Actor-Critic Methods

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*Slides are adopted from Deep Reinforcement Learning by Sergey Levine & Policy Gradients by David Silver

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Types of RL algorithms

$$\theta^* = \arg\max_\theta R_{\tau \sim p_\theta(\tau)} \left[\sum_t r(s_t, a_t) \right]$$

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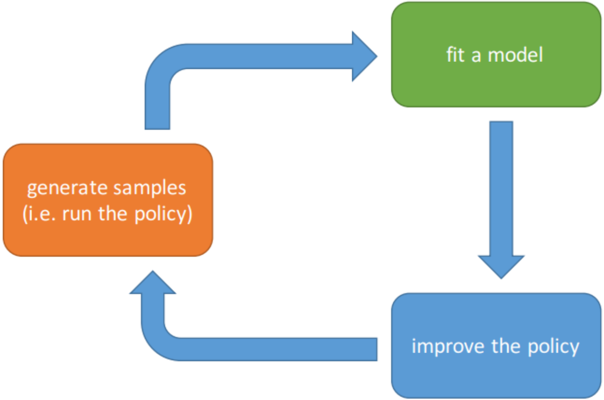
- Actor-critic: estimate value function or Q-function of the current policy, use it to improve the policy

Model-based Algorithms



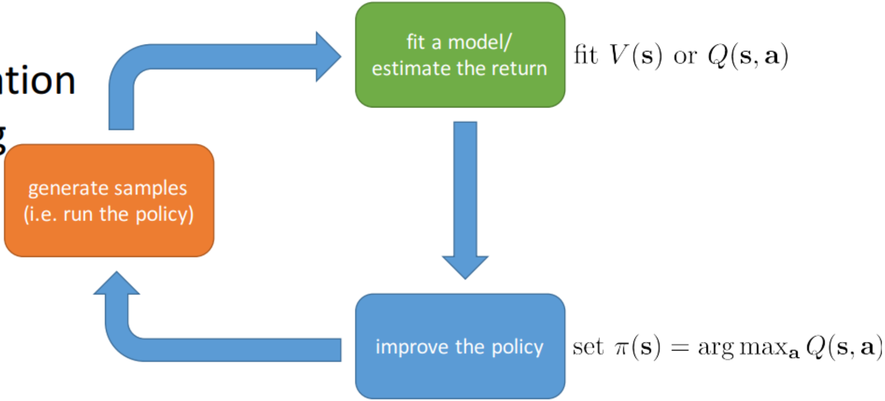
learn $p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

a few options

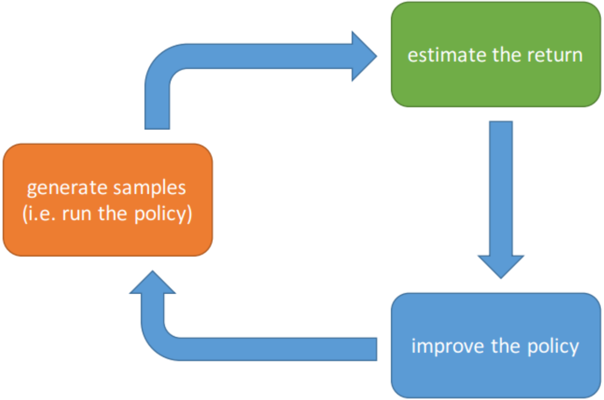
Value Based Algorithms

Examples:

- Value-Iteration
- Q-Learning
- DQN



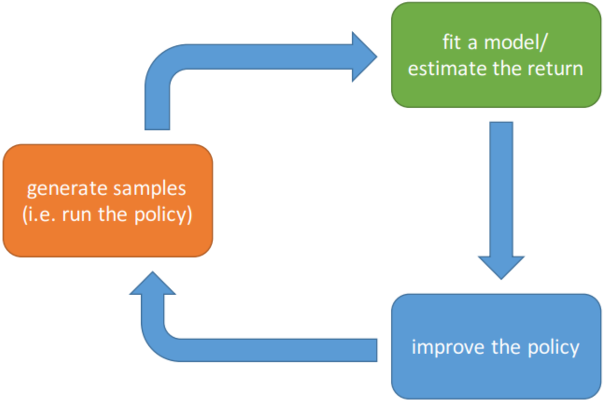
Direct Policy Gradient



evaluate returns
$R_\tau = \sum_t r(\mathbf{s}_t, \mathbf{a}_t)$

$\theta \leftarrow \theta + \alpha \nabla_\theta E[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)$

Actor-critic: Value Function + Policy Gradients



fit $V(\mathbf{s})$ or $Q(\mathbf{s}, \mathbf{a})$

evaluate returns using V or Q!

$\theta \leftarrow \theta + \alpha \nabla_\theta E[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)$

Comparison: Sample Efficiency

- **Sample efficiency:** How many samples do we need to get a good policy?

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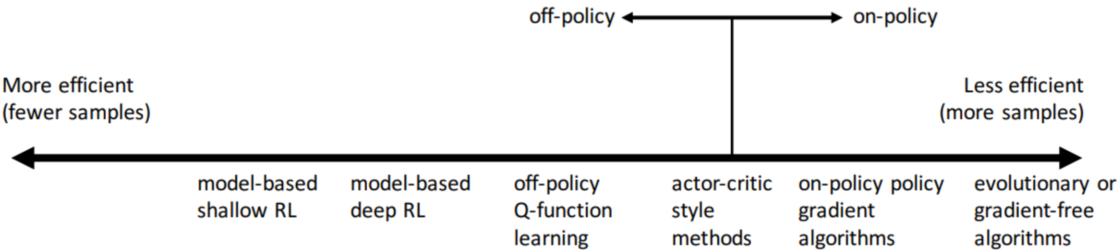
  - On policy: each time the policy is changed, even a little bit, we need to generate new samples

Comparison: Sample Efficiency



REINFORCE (Monte-Carlo Policy Gradient)

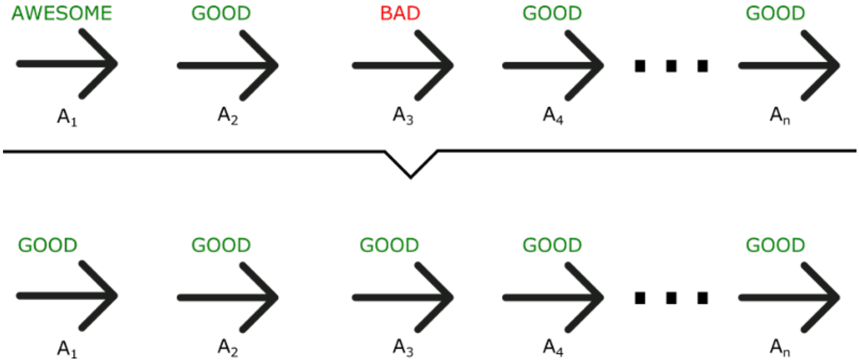
- Update parameters by stochastic gradient ascent
- Using policy gradient theorem
- Using return G_t as an unbiased sample of $Q^{\pi_\theta}(s_t, a_t)$

$$\Delta\theta_t = \alpha G_t \nabla_\theta \log \pi_\theta(s_t, a_t)$$

REINFORCE, A Monte-Carlo Policy-Gradient Method (episodic)

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta}), \forall a \in \mathcal{A}, s \in \mathcal{S}, \boldsymbol{\theta} \in \mathbb{R}^n$
Initialize policy weights $\boldsymbol{\theta}$
Repeat forever:
    Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot,\boldsymbol{\theta})$
    For each step of the episode $t = 0, \ldots, T-1$:
        $G_t \leftarrow$ return from step t
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha\gamma^t G_t \nabla_{\boldsymbol{\theta}} \log \pi(A_t|S_t,\boldsymbol{\theta})$

REINFORCE: Problem



Solution

Policy Update: $\Delta\theta = \alpha * \nabla_\theta * (log\, \pi(S_t, A_t, \theta)) * \cancel{R(t)}$

New update: $\Delta\theta = \alpha * \nabla_\theta * (log\, \pi(S_t, A_t, \theta)) * \boxed{Q(S_t, A_t)}$

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- We can use a critic to estimate the action-value function:

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$$\nabla_\theta J(\theta) \approx E_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s, a) Q_w(s, a)]$$

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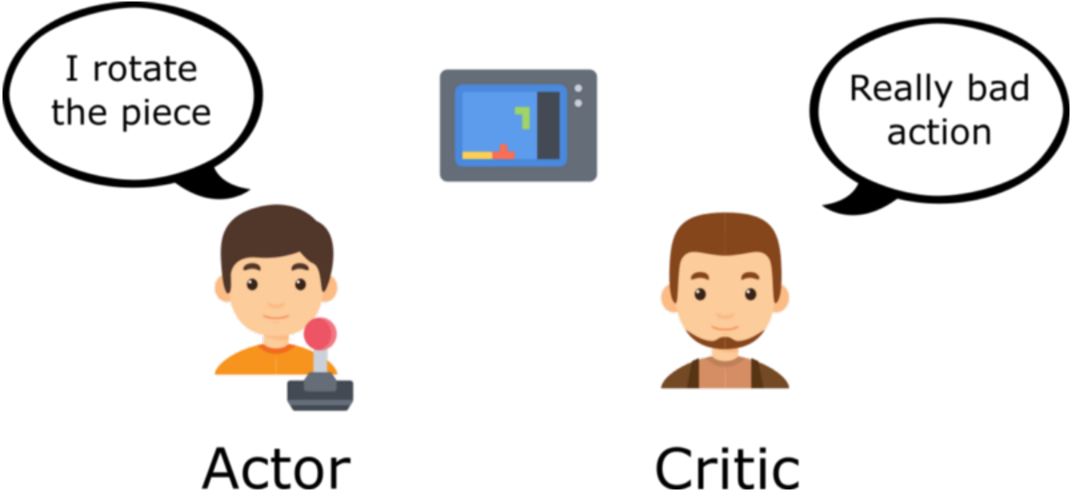

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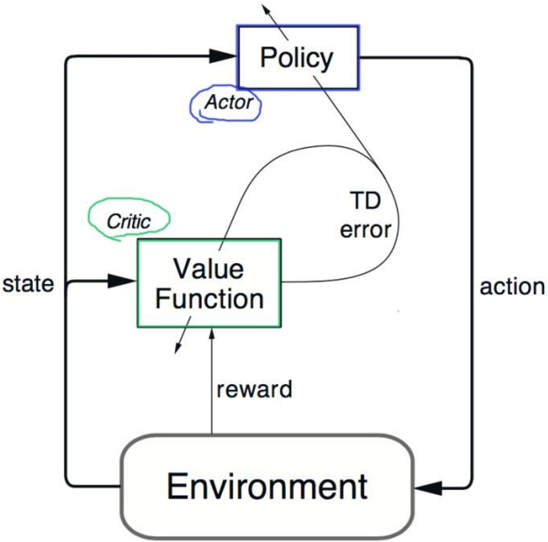
- The critic computes value functions to help assist the actor in learning. These are usually the state value, state-action value, or advantage value, denoted as $V(s)$, $Q(s, a)$, and $A(s, a)$, respectively.


Policy
Actor
TD
error
Critic
Value
Function
state
action
reward
Environment

Actor-Critic

- The critic is solving a familiar problem: policy evaluation
- How good is policy π_θ for current parameters θ?

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- The critic is solving a familiar problem: policy evaluation
- How good is policy π_θ for current parameters θ?
- To estimate, use any policy evaluation method:
  - Monte-Carlo policy evaluation
  - Temporal-Difference learning
  - Least-squares policy evaluation

Estimating the TD Error

- For the true value function $V_{\pi_\theta}(s)$, the TD error δ_{π_θ}

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- In practice we can use an approximate TD error, that requires one set of parameters w

$$\delta_w = r + \gamma V_w(s') - V_w(s)$$

Actor-Critic: Critic (Linear TD(0)) + Actor (policy gradient)

One-step Actor–Critic (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)
Loop forever (for each episode):
    Initialize S (first state of episode)
    $I \leftarrow 1$
    Loop while S is not terminal (for each time step):
        $A \sim \pi(\cdot|S,\boldsymbol{\theta})$
        Take action A, observe S', R
        $\delta \leftarrow R + \gamma\hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})$ (if S' is terminal, then $\hat{v}(S',\mathbf{w}) \doteq 0$)
        $\boxed{\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}}\delta\nabla\hat{v}(S,\mathbf{w})}$
        $\boxed{\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} I\delta\nabla\ln\pi(A|S,\boldsymbol{\theta})}$
        $I \leftarrow \gamma I$
        $S \leftarrow S'$

Recap: REINFORCE with Baseline

REINFORCE with Baseline (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
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Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ $\quad (G_t)$
    $\delta \leftarrow G - \hat{v}(S_t,\mathbf{w})$
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S_t,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

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A(s, a) &= Q_w(s, a) - V_v(s)
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- And updating *both* value functions by e.g. TD learning

Summary of Policy Gradient Algorithms

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&= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) Q_w(s,a)] && \text{Q Actor-Critic} \\
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$= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) A_w(s,a)]$	Advantage Actor-Critic (A2C)
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&= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) A_w(s,a)] && \text{Advantage Actor-Critic (A2C)} \\
&= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) \delta] && \text{TD Actor-Critic}
\end{aligned}
$$

- Each leads a stochastic gradient ascent algorithm
- Critic uses policy evaluation (e.g. MC or TD learning) to estimate $Q_\pi(s,a)$, $A_\pi(s,a)$ or $V_\pi(s)$.