Adaptive Signal Processing

EE22S044

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Sparse LMS System Identification

1 Introduction

They proposed a new approach to adaptive system identification when the system model is sparse.
Two new algorithms

1. zero-attracting LMS (ZA-LMS)
2. reweighted zero-attracting LMS (RZA-LMS)

The ZA-LMS is derived via combining a l_1 norm penalty on the coefficients into the quadratic LMS cost function, which generates a zero attractor in the LMS iteration. The zero attractor promotes sparsity in taps during the filtering process, and therefore accelerates convergence when identifying sparse systems. To further improve the filtering performance, the RZA-LMS is developed using a reweighted zero attractor

2 Review of Standard LMS

$$y(n) = W^T X(n) + v(n) \tag{1}$$

- y(n) be a sample of the observed output signal
- W = [W_0, W_1, ... , W_{N-1}]^T is the filter coefficient vector
- X(n)=$[x(n), x(n-1), ..., x(n-N+1)]^T$ denotes the vector of input signal X(n)
- v(n) is the observation noises assumed to be independent with X(n)

Cost function:

$$L(n) = \frac{1}{2}e(n)^2 \tag{2}$$

where

$$e(n) = y(n) - W^T X(n) \tag{3}$$

The filter coefficient vector is then updated by

$$W(n+1) = W(n) + \mu e(n) X(n) \tag{4}$$

The well known condition for convergence of LMS is

$$0 < \mu < \frac{1}{\lambda_{max}} \tag{5}$$

Steady State excess MSE is

$$P_{\text{ex}}(\infty) = \lim_{n\to\infty} E\left[\left((\mathbf{w}(n) - \mathbf{w})^T \mathbf{x}(n)\right)^2\right] = \frac{\eta}{2-\eta} P_0 \tag{6}$$

here P_0 is the power of observation noise

$$P_0 = E\left[v^2(n)\right], \tag{7}$$

$$\eta = tr\left(\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right) \tag{8}$$

3 ZA-LMS

$$L(n) = \frac{1}{2} e(n)^2 + \gamma ||W(n)||_1 \tag{9}$$

The filter coefficient vector is then updated by

$$W(n+1) = W(n) + \mu e(n) X(n) - \rho sgn W(n) \tag{10}$$

where sgn($\cdot$) is a component-wise sign function and $\rho = \mu\gamma$

- The ZA-LMS has an additional term sgn W(n), which always attracts the tap coefficients to zero.
- This the zero attractor, whose strength is controlled by ρ

If μ satisfies (5), the mean coefficient vector E[W(n)] converges to

$$E[W(\infty)] = W - \frac{\rho}{\mu} R^{-1} E[sgn W(\infty)] \tag{11}$$

- The convergence condition of the ZA-LMS and the standard LMS is the same, which is independent of ρ.
- Eq. (8) implies the ZA-LMS filter returns a biased estimate of the true coefficient vector.
- At the end we showed that with appropriate ρ, the ZALMS is able to yield lower MSE than the standard LMS for truly sparse systems

4 RZA-LMS

All the taps are forced to zero uniformly, its performance would deteriorate for less sparse systems.
The RZA-LMS derived using new cost function

$$L(n) = \frac{1}{2}e(n)^2 + \gamma^{'} \sum_{i=1}^{N} log(1 + |w_i|/\epsilon^{'}) \tag{12}$$

The filter coefficient vector is then updated by

$$W(n+1) = W(n) + \mu e(n)X(n) - \rho \frac{sgnW(n)}{1 + |W(n)|} \tag{13}$$

where

$$\rho = \mu\gamma^{'}/\epsilon^{'} \tag{14}$$

$$\epsilon = 1/\epsilon^{'} \tag{15}$$

5 Results

- Three experiments have been designed to demonstrate their tracking and steady-state performance.

5.1 Experiment 1

- In the first experiment, there are 16 coefficients in the time varying system. Initially, we set the 5th tap with value 1 and the others to zero, making the system have a sparsity of 1/16. After 500 iterations, all the odd taps are set to 1, while all the even taps remains to be zero, i.e., a sparsity of 8/16. After 1000 iterations all the even taps are set with value -1 while all the odd taps are maintained to be 1, leaving a completely non-sparse system.

- The input signal and the observed noise are white Gaussian random sequences with variance of 1 and 103, respectively. The three filters (LMS, ZA-LMS, and RZA-LMS) are run 200 times. The parameters are set as μ = 0.05, ρ = 5×104 and ϵ= 10. Note that we use the same and for the three filters.

- As we can see from the MSD results, when the system is very sparse (before the 500th iteration), both the ZA-LMS and the RZA-LMS yield faster convergence and better steady-state performance than the standard LMS.

- The RZA-LMS achieves lower MSD than the ZA-LMS.

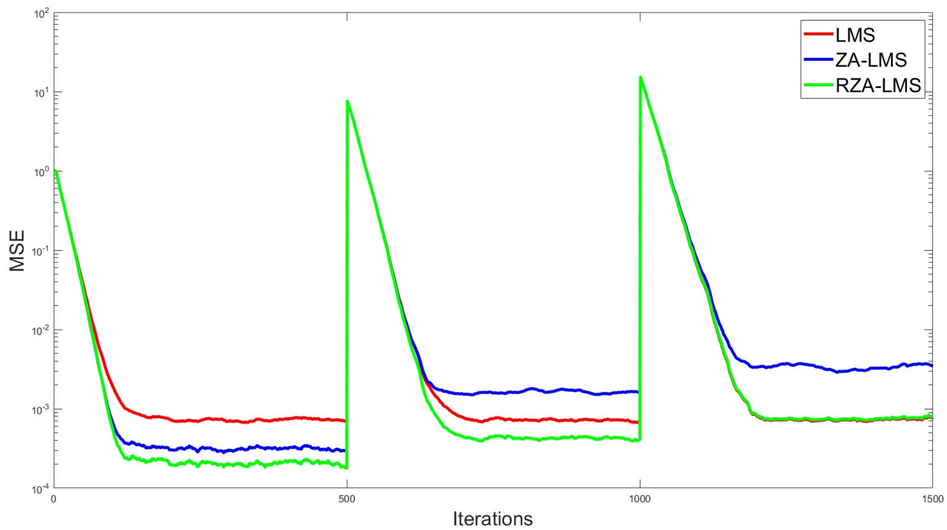

Figure 1: Tracking and steady-state behaviors of 16-order adaptive filters, driven by white input signal

- After the 500th iteration, as the number of non-zero taps increases to 8, the performance of the ZA-LMS deteriorates while the RZA-LMS maintains the best performance among the three filters.
- After 1000 iterations, the RZA-LMS still performs comparably to the standard LMS, even though the system is now completely non-sparse

5.2 Experiment 2

- The system in the second experiment is the same as the first one, except the coefficient switching times are set to the 7000th iteration and the 14000th iteration, respectively.
- The input signal x(n) is now a correlated signal generated by x(n)=0.8x(n 1) + u(n) and then normalized to variance 1, where u(n) is a white Gaussian noise. The variance of the observed noise is set to 103.
- The filter parameters are set as $\mu = 0.015$, $\rho = 3 \times 105$, and $\epsilon = 10$.
- Fig. 2 shows the MSD of the three filters, and similar performance trends are observed as in the first experiment.
- Observe that at the beginning of the iterations (e.g., from iteration 7000 to 8500), all the three filters converge at a nearly the same rate. After the

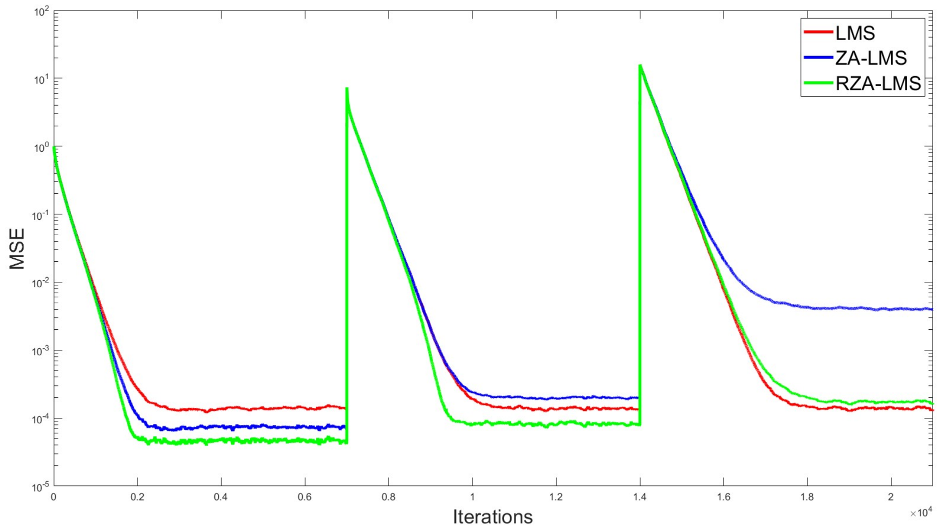

Figure 2: Tracking and steady-state behaviors of 16-order adaptive filters, driven by correlated input signal

8500th iteration, the convergence of the RZA-LMS accelerates, due to its selective shrink

5.3 Experiment 3

- The third experiment simulates a 256-tap system with 28 nonzero coefficients.
- The impulse response is shown in Fig. 3
- The driving signal and observed noise are the same as in the first experiment. μ is set to 5×103 in the three filters, and ϵ is set to 10.
- This time we select different values of ρ for the ZA-LMS and the RZA-LMS to yield the best MSE, where ρ is 2.5×106 for the ZALMS and 105 for the RZA-LMS. The simulations are performed 200 times and the average excess MSE is shown in Fig. 4. One sees that for this long sparse system, the zero-attracting algorithms significantly outperform the standard LMS, as measured by faster convergence rate and lower steady-state MSE.

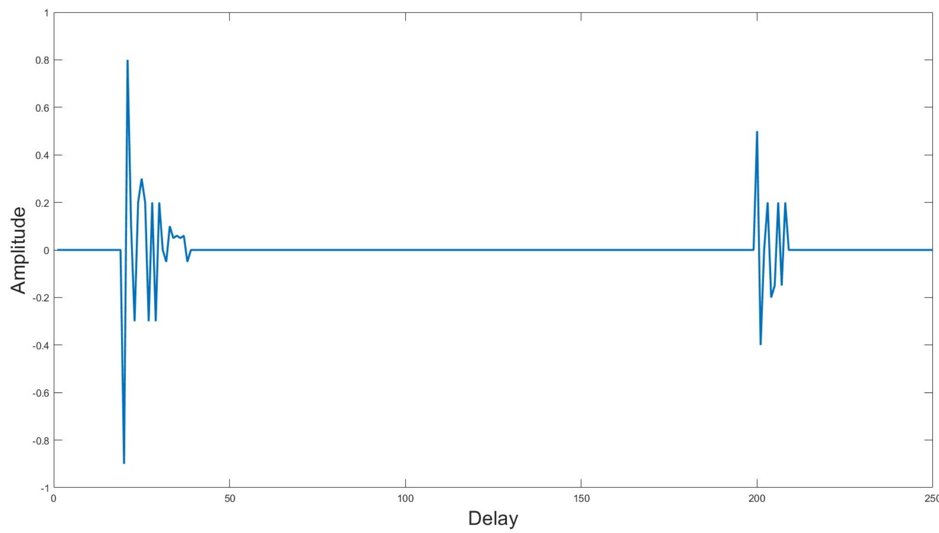

Figure 3: The impulse response of the system in the third experiment

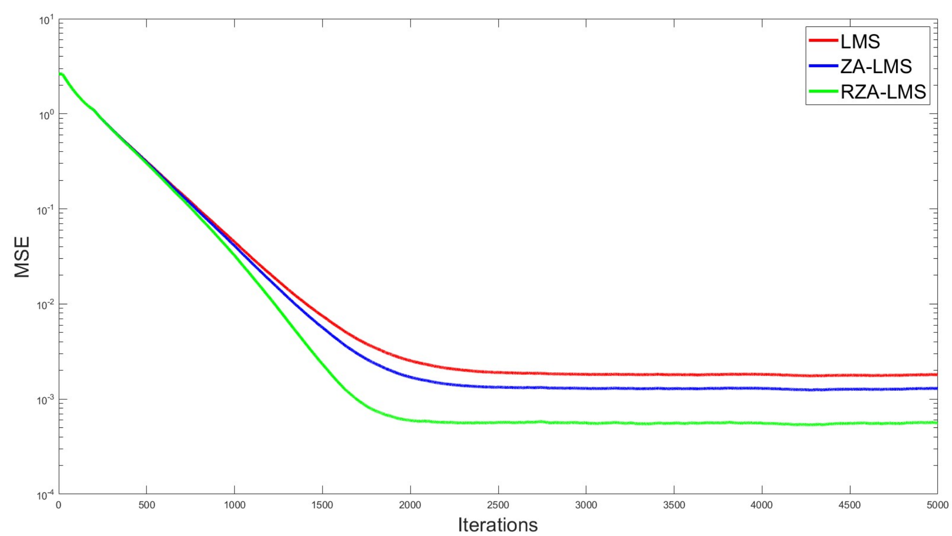

Figure 4: Tracking and steady-state behaviors of 256-order adaptive filters, driven by white input signal.