GREEN'S THEOREM PROJECT REPORT

Subject: **Calculus**

Project Title:

GREEN'S THEOREM — EXPLANATION AND VERIFICATION

Group Members:

1. Yash – 25BCE10747

2. Sivi – 25MIP10153

3. Sanskar – 25MIP10125

Slot :

C11-C12-C13

Submitted to :

Dr. Juhi Kesarwani

ABSTRACT

This project explains Green's Theorem in simple terms and verifies it using a worked example and a Python program. Green's Theorem relates a line integral around a closed curve to a double integral over the region inside the curve. We use a specific vector field and the unit square region to demonstrate the theorem. Manual calculations and numerical verification confirm that the theorem holds for the chosen example.

INTRODUCTION

Green's Theorem is a key result in vector calculus. It connects the circulation (line integral) around a closed boundary to the behavior (curl) inside the region. This theorem helps convert complicated line integrals into often simpler double integrals, and is useful in physics and engineering for analyzing fluid flow and fields. The purpose of this project is to explain the theorem clearly, show step-by-step calculations for a chosen example, and verify the result numerically using Python.

STATEMENT OF GREEN'S THEOREM

If a simple closed curve C encloses a region R, and P(x,y) and Q(x,y) have continuous partial derivatives on an open region containing R, then:

Line integral of (P dx + Q dy) around the boundary C

is equal to the double integral over R of (dQ/dx − dP/dy).

CONDITIONS FOR APPLICATION

1. Curve C must be closed (forms a full loop).

2. Orientation must be positive (anticlockwise).

3. Region R must be simple and connected (no holes).

4. Functions P and Q must have continuous partial derivatives inside and on the boundary of R.

APPLICATIONS OF GREEN'S THEOREM

- Area calculation: Green's Theorem can be used to compute the area of a plane region using line integrals.

- Fluid circulation: Relates circulation around a closed path to rotation within the area.

- Electromagnetism: Useful in converting boundary integrals to area integrals for field computations.

- Foundation for higher-dimensional theorems: Green's Theorem is a special case of the more general Stokes' Theorem.

EXAMPLE USED IN THIS PROJECT

Vector field chosen:

$P(x, y) = -y$

$Q(x, y) = x$

Region chosen:

Unit square with corners at:

(0, 0), (1, 0), (1, 1), (0, 1)

That is: $0 \le x \le 1$ and $0 \le y \le 1$

We will compute:

1. The double integral of (dQ/dx − dP/dy) over the square.

2. The line integral of (P dx + Q dy) around the square in anticlockwise direction.

DOUBLE INTEGRAL CALCULATION

1. Compute partial derivatives:

- dQ/dx = derivative of Q(x,y)=x with respect to x = 1

- dP/dy = derivative of P(x,y)=-y with respect to y = -1

2. Compute integrand:

dQ/dx − dP/dy = 1 − (−1) = 2

3. Area of the region:

Area of unit square = 1 × 1 = 1

4. Double integral value:

Double integral = (integrand) × (area) = 2 × 1 = 2

LINE INTEGRAL CALCULATION (P dx + Q dy) AROUND THE SQUARE

We compute the line integral by summing contributions from four sides in anticlockwise order.

SIDE 1: Bottom edge (0,0) → (1,0)

- ***$y = 0$, $dy = 0$***
- ***$P = -y = 0$, $Q = x$***
- ***Contribution = integral of ($0\,dx + x\,dy$) = 0***

SIDE 2: Right edge (1,0) → (1,1)

- ***$x = 1$, $dx = 0$, y goes 0 → 1***
- ***$P = -y$, $Q = 1$***
- ***Contribution = integral of ($P\,dx + Q\,dy$) = integral of ($1\,dy$) from 0 to 1 = 1***

SIDE 3: Top edge (1,1) → (0,1)

- ***$y = 1$, $dy = 0$, x goes 1 → 0***
- ***$P = -1$, $Q = x$***
- ***Contribution = integral of ($P\,dx + Q\,dy$) = integral of ($-1\,dx$) from 1 to 0***

  $= -(0 - 1) = 1$

SIDE 4: Left edge (0,1) → (0,0)

- ***$x = 0$, $dx = 0$, y goes 1 → 0***
- ***$P = -y$, $Q = 0$***
- ***Contribution = integral of ($0\,dx + 0\,dy$) = 0***

Total line integral = 0 + 1 + 1 + 0 = 2

NUMERICAL VERIFICATION (PYTHON)

We also verify the result using a Python program.

Program description:

- The program computes the line integral numerically around the four edges of the square.
- It computes the double integral directly (since the integrand is constant).
- The program prints both values and their difference.

Expected output (approx):

Line Integral Result: 2.0 (or close to 2.0)

Double Integral Result: 2

Difference: 0.0 (or very small)

This numerical check confirms the manual calculation and verifies Green's Theorem for the chosen example.

CONCLUSION

Both the line integral and the double integral give the value 2. This confirms that Green’s Theorem is true for the chosen vector field and region. The project successfully shows the connection between boundary integrals and area integrals.