Review

Draw a flowchart connecting related standards in the class.

GT3

Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 7 & 1 \\ -18 & -4 \end{bmatrix}$.

GT4

Find a basis for the eigenspace associated to the eigenvalue 3 in the matrix

$$\begin{bmatrix} -7 & -8 & 2 \\ 8 & 9 & -1 \\ \frac{13}{2} & 5 & 2 \end{bmatrix}.$$

VS4

Consider the following two sets of Euclidean vectors.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, x + y = z \right\} \qquad U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \, x + y = z^2 \right\}$$

Show that one of these sets is a subspace of $\mathbb{R}^3$, and that one of the sets is not.

AT1

Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = f(x) - 3x \text{ and } T(f(x)) = f(x) - 3f'(x).$$

Show that one of these maps is a linear transformation, and that the other map is not.

VS1

Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 x_2,\ y_1 y_2)$$
$$c \odot (x, y) = (x^c,\ y^c)\,.$$

a. Show that there exists an additive identity element, that is:

$$\text{There exists } (w, z) \in V \text{ such that } (x, y) \oplus (w, z) = (x, y).$$

holds.

b. Show why V is not a vector space.

GT2

Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 1 & 1 & 2 & 4 \\ 1 & 1 & 1 & 3 \\ -3 & 1 & 2 & -5 \end{bmatrix}$$

MX2

Let A be a 4×4 matrix.

a. Give a 4×4 matrix C that may be used to perform the row operation $R_4 \to R_4 + 5R_2$.
b. Give a 4×4 matrix P that may be used to perform the row operation $R_4 \to 3R_4$.
c. Use matrix multiplication to describe the matrix obtained by applying $R_4 \to 3R_4$ and then $R_4 \to R_4 + 5R_2$ to A (note the order).

GT1

Let A be a 4×4 matrix with determinant 4.

a. Let M be the matrix obtained from A by applying the row operation $R_3 \leftrightarrow R_2$. What is $\det M$?

b. Let P be the matrix obtained from A by applying the row operation $R_1 \to 5R_1$. What is $\det P$?

c. Let C be the matrix obtained from A by applying the row operation $R_1 \to R_1 + 5R_4$. What is $\det C$?

VS8

(a) Given the set

$$\{x^3 + 3\,x^2 + 1, -x^3 + x^2 + x, -3\,x^3 - 2\,x^2 + 3\,x - 2, -6\,x^3 - 9\,x^2 + 6\,x - 6\}$$

write a statement involving a polynomial equation that's equivalent to each claim below.

- The set of polynomials **spans** $\mathcal{P}_3$
- The set of polynomials does **not** span $\mathcal{P}_3$

(b) Explain how to determine which of these statements is true.

AT3

Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3y + 2z - 3w \\ 2x + 4y + 6z - 10w \\ x + 6y - z + 3w \end{bmatrix}$$

Compute a basis for the kernel and a basis for the image of T.