VS1: Vector spaces

Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = \left(x_1 + x_2 + 5,\ \sqrt{y_1^2 + y_2^2}\right)$$

$$c \odot (x, y) = (cx,\ cy)\,.$$

(a) Show that vector addition is associative, that is:

$$((x_1, y_1) \oplus (x_2, y_2)) \oplus (x_3, y_3) = (x_1, y_1) \oplus ((x_2, y_2) \oplus (x_3, y_3))\,.$$

(b) Explain why V nonetheless is not a vector space.

VS2: Linear combinations

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- $\begin{bmatrix} -6 \\ 1 \\ -4 \\ -4 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -2 \\ 0 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 1 \\ -2 \\ -2 \end{bmatrix}$.
- $\begin{bmatrix} -7 \\ 0 \\ -5 \\ -3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -2 \\ 0 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 1 \\ -2 \\ -2 \end{bmatrix}$.

(b) Use these statements to determine if each vector is or is not a linear combination. If it is, give an example of such a linear combination.