Standard V1

V1.1 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

$\square$

V1.2 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not associative

$\square$

V1.3 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

$\square$

V1.4 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because Scalar multiplication does not distribute over scalar addition

□

V1.5 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 1y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.6 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.7 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not associative

□

V1.8 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because an additive identity element does not exist

□

V1.9 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because an additive identity element does not exist

□

V1.10 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 1y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because an additive identity element does not exist

□

V1.11 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not associative

□

V1.12 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.13 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not associative

□

V1.14 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.15 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + x_2, y_1 + 1y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because Scalar multiplication does not distribute over scalar addition

□

V1.16 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.17 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.18 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not associative

□

V1.19 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because an additive identity element does not exist

□

V1.20 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because Scalar multiplication does not distribute over scalar addition

□

V1.21 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because an additive identity element does not exist

□

V1.22 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + x_2, y_1 + 1y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.23 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + x_2, y_1 + 2y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.24 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + x_2, y_1 + 1y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not commutative

□

V1.25 Let V be the set of all pairs of numbers $(x_1, y_1) \in \mathbb{R}^2$ together with the operations

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + 3y_2)$$
$$c \odot (x_1, y_1) = (cx_1, cy_1)$$

Show that scalar multiplication distributes over vector addition, i.e.

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

. Show that V nonetheless is not a vector space.

Solution. V is not a vector space because vector addition is not associative

$\square$

Standard V2

V2.1 Explain why the vector $\begin{bmatrix} 4 \\ -5 \\ 3 \\ -5 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 0 \\ -3 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ -5 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 2 \\ 2 \\ -3 \end{bmatrix}$.

Solution. $\begin{bmatrix} 4 \\ -5 \\ 3 \\ -5 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 0 \\ -3 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ -5 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 2 \\ 2 \\ -3 \end{bmatrix}$. □

V2.2 Explain why the vector $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 3 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 0 \\ -1 \\ -5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 11 \\ 20 \\ 13 \end{bmatrix}$, anc

Solution. $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 3 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} 0 \\ -1 \\ -5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 11 \\ 20 \\ 13 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 4 \\ 19 \\ -4 \end{bmatrix}$. □

V2.3 Explain why the vector $\begin{bmatrix} -7 \\ 9 \\ 4 \\ 1 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -1 \\ -3 \\ -2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -2 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -13 \\ -17 \\ 8 \end{bmatrix}$, a

Solution. $\begin{bmatrix} -7 \\ 9 \\ 4 \\ 1 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} -1 \\ -3 \\ -2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -2 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -13 \\ -17 \\ 8 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 10 \\ 10 \\ 0 \end{bmatrix}$. □

V2.4 Explain why the vector $\begin{bmatrix} 5 \\ -1 \\ 2 \\ -4 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -2 \\ 2 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Solution. $\begin{bmatrix} 5 \\ -1 \\ 2 \\ -4 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -2 \\ 2 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. □

V2.5 Explain why the vector $\begin{bmatrix} -2 \\ -6 \\ 7 \\ 5 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 9 \\ 6 \\ 3 \\ -2 \end{bmatrix}$

Solution. $\begin{bmatrix} -2 \\ -6 \\ 7 \\ 5 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 9 \\ 6 \\ 3 \\ -2 \end{bmatrix}$. □

V2.6 Explain why the vector $\begin{bmatrix} -6 \\ 6 \\ 5 \\ -5 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 0 \\ -2 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -2 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ 2 \\ 0 \\ -4 \end{bmatrix}$.

Solution. $\begin{bmatrix} -6 \\ 6 \\ 5 \\ -5 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} 0 \\ -2 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -2 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ 2 \\ 0 \\ -4 \end{bmatrix}$. □

V2.7 Explain why the vector $\begin{bmatrix} 2 \\ -8 \\ -12 \\ 8 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -3 \\ -3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix}$, anc

Solution. $\begin{bmatrix} 2 \\ -8 \\ -12 \\ 8 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ -1 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -3 \\ -3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 1 \\ -5 \\ 0 \end{bmatrix}$. □

V2.8 Explain why the vector $\begin{bmatrix} -9 \\ -5 \\ -15 \\ -7 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -2 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$, and

Solution. $\begin{bmatrix} -9 \\ -5 \\ -15 \\ -7 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -2 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -3 \\ 3 \\ 2 \end{bmatrix}$. □

V2.9 Explain why the vector $\begin{bmatrix} -24 \\ -2 \\ 13 \\ 11 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 4 \\ 0 \\ -4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 1 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -2 \\ 2 \\ -3 \end{bmatrix}$, a

Solution. $\begin{bmatrix} -24 \\ -2 \\ 13 \\ 11 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 4 \\ 0 \\ -4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 1 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -2 \\ 2 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ -4 \\ 1 \end{bmatrix}$. □

V2.10 Explain why the vector $\begin{bmatrix} -9 \\ 0 \\ -4 \\ -4 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 0 \\ 3 \\ 0 \end{bmatrix}$

Solution. $\begin{bmatrix} -9 \\ 0 \\ -4 \\ -4 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 0 \\ 3 \\ 0 \end{bmatrix}$. □

V2.11 Explain why the vector $\begin{bmatrix} 2 \\ 10 \\ 2 \\ -2 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -1 \\ -1 \\ -5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 2 \\ 2 \end{bmatrix}$, and

Solution. $\begin{bmatrix} 2 \\ 10 \\ 2 \\ -2 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -1 \\ -1 \\ -5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ 4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 2 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}$. □

V2.12 Explain why the vector $\begin{bmatrix} 8 \\ -1 \\ 17 \\ 6 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -1 \\ -3 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -5 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ -2 \\ 0 \end{bmatrix}$

Solution. $\begin{bmatrix} 8 \\ -1 \\ 17 \\ 6 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -1 \\ -3 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -5 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ -2 \\ 0 \end{bmatrix}$. □

V2.13 Explain why the vector $\begin{bmatrix} -9 \\ 2 \\ -6 \\ 0 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -2 \\ 3 \\ 1 \end{bmatrix}$

Solution. $\begin{bmatrix} -9 \\ 2 \\ -6 \\ 0 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -2 \\ 3 \\ 1 \end{bmatrix}$. □

V2.14 Explain why the vector $\begin{bmatrix} 9 \\ 15 \\ 2 \\ 3 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 3 \\ -3 \\ 2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ -1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ -3 \\ 1 \\ -2 \end{bmatrix}$.

Solution. $\begin{bmatrix} 9 \\ 15 \\ 2 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ -3 \\ 2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ -1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ -3 \\ 1 \\ -2 \end{bmatrix}$. □

V2.15 Explain why the vector $\begin{bmatrix} -7 \\ 7 \\ 5 \\ 9 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -4 \\ 2 \\ -3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -8 \\ 2 \\ -4 \\ -1 \end{bmatrix}$.

Solution. $\begin{bmatrix} -7 \\ 7 \\ 5 \\ 9 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} -4 \\ 2 \\ -3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -8 \\ 2 \\ -4 \\ -1 \end{bmatrix}$. □

V2.16 Explain why the vector $\begin{bmatrix} 4 \\ -5 \\ 4 \\ 8 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ -3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ 2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ 4 \\ -4 \end{bmatrix}$, a

Solution. $\begin{bmatrix} 4 \\ -5 \\ 4 \\ 8 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ -3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ 2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ 4 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 6 \\ -1 \\ 2 \end{bmatrix}$. □

V2.17 Explain why the vector $\begin{bmatrix} 9 \\ 6 \\ -14 \\ 3 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 0 \\ -3 \\ 2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -3 \\ -4 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -3 \\ 4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ - \\ 1 \\ - \end{bmatrix}$

Solution. $\begin{bmatrix} 9 \\ 6 \\ -14 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 0 \\ -3 \\ 2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -3 \\ -4 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -3 \\ 4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -2 \\ 1 \\ -1 \end{bmatrix}$. □

V2.18 Explain why the vector $\begin{bmatrix} 8 \\ -7 \\ -9 \\ 2 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -2 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 4 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -1 \\ 2 \\ -3 \end{bmatrix}$

Solution. $\begin{bmatrix} 8 \\ -7 \\ -9 \\ 2 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -2 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 4 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -1 \\ 2 \\ -3 \end{bmatrix}$. □

V2.19 Explain why the vector $\begin{bmatrix} 8 \\ 4 \\ -7 \\ -2 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 7 \\ -3 \\ 18 \\ 5 \end{bmatrix}$, a

Solution. $\begin{bmatrix} 8 \\ 4 \\ -7 \\ -2 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 7 \\ -3 \\ 18 \\ 5 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ 12 \\ 9 \end{bmatrix}$. $\square$

V2.20 Explain why the vector $\begin{bmatrix} -3 \\ -12 \\ -5 \\ 5 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 4 \\ 2 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} - \\ 1 \\ 2 \\ 1 \end{bmatrix}$

Solution. $\begin{bmatrix} -3 \\ -12 \\ -5 \\ 5 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 4 \\ 2 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 1 \\ 2 \\ 1 \end{bmatrix}$. $\square$

V2.21 Explain why the vector $\begin{bmatrix} -8 \\ 6 \\ -9 \\ -4 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 0 \\ -9 \\ -2 \end{bmatrix}$.

Solution. $\begin{bmatrix} -8 \\ 6 \\ -9 \\ -4 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 0 \\ -9 \\ -2 \end{bmatrix}$. $\square$

V2.22 Explain why the vector $\begin{bmatrix} 14 \\ 5 \\ -7 \\ -2 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -5 \\ -2 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 0 \\ -1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ 3 \\ 2 \end{bmatrix}$.

Solution. $\begin{bmatrix} 14 \\ 5 \\ -7 \\ -2 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -5 \\ -2 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 0 \\ -1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ 3 \\ 2 \end{bmatrix}$. $\square$

V2.23 Explain why the vector $\begin{bmatrix} 7 \\ 3 \\ -3 \\ 4 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -1 \\ -3 \\ -5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -2 \\ 1 \end{bmatrix}$.

Solution. $\begin{bmatrix} 7 \\ 3 \\ -3 \\ 4 \end{bmatrix}$ is not a linear combination of the vectors $\begin{bmatrix} -1 \\ -3 \\ -5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -2 \\ 1 \end{bmatrix}$. □

V2.24 Explain why the vector $\begin{bmatrix} 4 \\ -3 \\ 8 \\ 2 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} -4 \\ -3 \\ -2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 0 \\ -5 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -3 \\ -1 \end{bmatrix}$.

Solution. $\begin{bmatrix} 4 \\ -3 \\ 8 \\ 2 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -4 \\ -3 \\ -2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 0 \\ -5 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -3 \\ -1 \end{bmatrix}$. □

V2.25 Explain why the vector $\begin{bmatrix} -2 \\ 2 \\ 3 \\ 7 \end{bmatrix}$ is or is not a linear combination of the vectors $\begin{bmatrix} 3 \\ -2 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -2 \\ -1 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 1 \\ -5 \\ -2 \end{bmatrix}$

Solution. $\begin{bmatrix} -2 \\ 2 \\ 3 \\ 7 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 3 \\ -2 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -2 \\ -1 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 1 \\ -5 \\ -2 \end{bmatrix}$. □

Standard V3

V3.1 Explain why the vectors $\begin{bmatrix} 0 \\ -5 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2 \\ -5 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 2 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ -6 \\ 5 \\ -11 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 0 \\ -5 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2 \\ -5 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 2 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ -6 \\ 5 \\ -11 \end{bmatrix}$ do not span $\mathbb{R}^4$.

□

V3.2 Explain why the vectors $\begin{bmatrix} 0 \\ -1 \\ -5 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -4 \\ -4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 4 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -3 \\ -5 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 0 \\ -1 \\ -5 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -4 \\ -4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 4 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -3 \\ -5 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.3 Explain why the vectors $\begin{bmatrix} -1 \\ -4 \\ -2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 4 \\ 3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ -4 \\ 3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ -3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -4 \\ 4 \\ 2 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -1 \\ -4 \\ -2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 4 \\ 3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ -4 \\ 3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ -3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -4 \\ 4 \\ 2 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.4 Explain why the vectors $\begin{bmatrix} 3 \\ 3 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -6 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -5 \\ -2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 2 \\ -1 \\ 1 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 3 \\ 3 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -6 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -5 \\ -2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 2 \\ -1 \\ 1 \end{bmatrix}$ do not span $\mathbb{R}^4$.

□

V3.5 Explain why the vectors $\begin{bmatrix} -2 \\ -4 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -3 \\ -4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ -3 \\ 3 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -2 \\ -4 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -3 \\ -4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ -3 \\ 3 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.6 Explain why the vectors $\begin{bmatrix} 0 \\ -3 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -9 \\ 3 \\ -2 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 6 \\ 4 \\ 0 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 0 \\ -3 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -9 \\ 3 \\ -2 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 6 \\ 4 \\ 0 \end{bmatrix}$ do not span $\mathbb{R}^4$.

□

V3.7 Explain why the vectors $\begin{bmatrix} 3 \\ -2 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -5 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 4 \\ 1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ -5 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -2 \\ -4 \\ 3 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 3 \\ -2 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -5 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 4 \\ 1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ -5 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -2 \\ -4 \\ 3 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.8 Explain why the vectors $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -6 \\ 0 \\ -12 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 12 \\ 0 \\ 24 \\ 12 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -5 \\ -4 \\ 2 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -6 \\ 0 \\ -12 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 12 \\ 0 \\ 24 \\ 12 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -5 \\ -4 \\ 2 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.9 Explain why the vectors $\begin{bmatrix} 4 \\ 0 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 1 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ -4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -2 \\ 10 \\ -7 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -4 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ -4 \\ -2 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 4 \\ 0 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 1 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ -4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -2 \\ 10 \\ -7 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -4 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ -4 \\ -2 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.10 Explain why the vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 4 \\ 1 \\ -5 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 4 \\ 1 \\ -5 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.11 Explain why the vectors $\begin{bmatrix} -1 \\ -1 \\ -5 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -4 \\ 2 \\ -5 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -1 \\ -1 \\ -5 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -4 \\ 2 \\ -5 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.12 Explain why the vectors $\begin{bmatrix} -1 \\ -4 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 7 \\ 0 \\ -2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ 4 \\ -5 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -1 \\ -4 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 7 \\ 0 \\ -2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ 4 \\ -5 \end{bmatrix}$ do not span $\mathbb{R}^4$.

□

V3.13 Explain why the vectors $\begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -9 \\ 9 \\ -18 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -6 \\ 3 \\ -3 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 3 \\ -3 \\ -1 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -9 \\ 9 \\ -18 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -6 \\ 3 \\ -3 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 3 \\ -3 \\ -1 \end{bmatrix}$ do not span $\mathbb{R}^4$.

□

V3.14 Explain why the vectors $\begin{bmatrix} 3 \\ -4 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -5 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -4 \\ 1 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 3 \\ -4 \\ -4 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 3 \\ -4 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -5 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -4 \\ 1 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 3 \\ -4 \\ -4 \end{bmatrix}$ do span $\mathbb{R}^4$. $\square$

V3.15 Explain why the vectors $\begin{bmatrix} -4 \\ 3 \\ -3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 0 \\ 3 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -4 \\ 3 \\ 2 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -4 \\ 3 \\ -3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 0 \\ 3 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -4 \\ 3 \\ 2 \end{bmatrix}$ do span $\mathbb{R}^4$. $\square$

V3.16 Explain why the vectors $\begin{bmatrix} 1 \\ -4 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -5 \\ 2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 2 \\ -3 \\ 2 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 1 \\ -4 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -5 \\ 2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 2 \\ -3 \\ 2 \end{bmatrix}$ do span $\mathbb{R}^4$. $\square$

V3.17 Explain why the vectors $\begin{bmatrix} 0 \\ -5 \\ 2 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ -4 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 5 \\ -8 \\ 10 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -2 \\ 2 \\ -3 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 0 \\ -5 \\ 2 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ -4 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 5 \\ -8 \\ 10 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -2 \\ 2 \\ -3 \end{bmatrix}$ do not span $\mathbb{R}^4$. $\square$

V3.18 Explain why the vectors $\begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 12 \\ -10 \\ -8 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 2 \\ 1 \\ -5 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 12 \\ -10 \\ -8 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 2 \\ 1 \\ -5 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.19 Explain why the vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ -4 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 19 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ -3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -5 \\ 4 \\ 2 \\ -4 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ -4 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 19 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ -3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -5 \\ 4 \\ 2 \\ -4 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.20 Explain why the vectors $\begin{bmatrix} 4 \\ 4 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -4 \\ -3 \\ -5 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 4 \\ 4 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -4 \\ -3 \\ -5 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.21 Explain why the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -2 \\ 1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -4 \\ 3 \\ -5 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -2 \\ 1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -4 \\ 3 \\ -5 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.22 Explain why the vectors $\begin{bmatrix} -5 \\ -3 \\ 1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ -1 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -4 \\ -4 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -5 \\ -3 \\ 1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ -1 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -4 \\ -4 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.23 Explain why the vectors $\begin{bmatrix} -1 \\ -4 \\ -5 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -3 \\ -2 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 3 \\ 1 \\ -2 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -1 \\ -4 \\ -5 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -3 \\ -2 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 3 \\ 1 \\ -2 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

V3.24 Explain why the vectors $\begin{bmatrix} -4 \\ -4 \\ -2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 0 \\ -5 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 8 \\ -11 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} 20 \\ -4 \\ 28 \\ -3 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} -4 \\ -4 \\ -2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 0 \\ -5 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 8 \\ -11 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} 20 \\ -4 \\ 28 \\ -3 \end{bmatrix}$ do not span $\mathbb{R}^4$.

□

V3.25 Explain why the vectors $\begin{bmatrix} 3 \\ -2 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ -1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ -5 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 0 \\ -4 \\ -2 \end{bmatrix}$ span or don't span $\mathbb{R}^4$.

Solution. The vectors $\begin{bmatrix} 3 \\ -2 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ -1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ -5 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 0 \\ -4 \\ -2 \end{bmatrix}$ do span $\mathbb{R}^4$.

□

Standard V5

V5.1 Explain why the vectors $\begin{bmatrix} 0 \\ -6 \\ -3 \\ 0 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -6 \\ 4 \\ 0 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ 4 \\ 4 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -1 \\ -2 \\ 4 \\ -2 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 0 \\ -6 \\ -3 \\ 0 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -6 \\ 4 \\ 0 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ 4 \\ 4 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -1 \\ -2 \\ 4 \\ -2 \end{bmatrix}$ are linearly independent . $\square$

V5.2 Explain why the vectors $\begin{bmatrix} 1 \\ -1 \\ -6 \\ 5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ -5 \\ -5 \\ -4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ -2 \\ 5 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 12 \\ 8 \\ -2 \\ 18 \\ 6 \end{bmatrix}$, and $\begin{bmatrix} 19 \\ 22 \\ 34 \\ -22 \\ -4 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 1 \\ -1 \\ -6 \\ 5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ -5 \\ -5 \\ -4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ -2 \\ 5 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 12 \\ 8 \\ -2 \\ 18 \\ 6 \end{bmatrix}$, and $\begin{bmatrix} 19 \\ 22 \\ 34 \\ -22 \\ -4 \end{bmatrix}$ are linearly dependent . $\square$

V5.3 Explain why the vectors $\begin{bmatrix} -1 \\ -5 \\ -2 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 4 \\ -3 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ -6 \\ -5 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 1 \\ 4 \\ -3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -4 \\ 5 \\ 2 \\ 0 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -1 \\ -5 \\ -2 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 4 \\ -3 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ -6 \\ -5 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 1 \\ 4 \\ -3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -4 \\ 5 \\ 2 \\ 0 \end{bmatrix}$ are linearly independent . $\square$

V5.4 Explain why the vectors $\begin{bmatrix} 4 \\ 3 \\ -1 \\ 3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -4 \\ 5 \\ 5 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -10 \\ 7 \\ -1 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -10 \\ 0 \\ -4 \\ -14 \\ 0 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 4 \\ 3 \\ -1 \\ 3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -4 \\ 5 \\ 5 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -10 \\ 7 \\ -1 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -10 \\ 0 \\ -4 \\ -14 \\ 0 \end{bmatrix}$ are linearly dependent . $\square$

V5.5 Explain why the vectors $\begin{bmatrix} -3 \\ -5 \\ 0 \\ 4 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -4 \\ -4 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 1 \\ 2 \\ -4 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -1 \\ -1 \\ -4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ -3 \\ -1 \\ 1 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -3 \\ -5 \\ 0 \\ 4 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -4 \\ -4 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 1 \\ 2 \\ -4 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -1 \\ -1 \\ -4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ -3 \\ -1 \\ 1 \end{bmatrix}$ are linearly independent . $\square$

V5.6 Explain why the vectors $\begin{bmatrix} 0 \\ -4 \\ -2 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ -3 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -4 \\ -1 \\ -4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ 3 \\ -4 \\ 4 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 0 \\ -4 \\ -2 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -3 \\ -3 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -4 \\ -1 \\ -4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ 3 \\ -4 \\ 4 \end{bmatrix}$ are linearly independent . $\square$

V5.7 Explain why the vectors $\begin{bmatrix} 4 \\ -2 \\ -3 \\ 2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 4 \\ -6 \\ 1 \\ -6 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 5 \\ 3 \\ -6 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -5 \\ 4 \\ 5 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 4 \\ -2 \\ -3 \\ 2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 4 \\ -6 \\ 1 \\ -6 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 5 \\ 3 \\ -6 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -2 \\ -5 \\ 4 \\ 5 \end{bmatrix}$ are linearly independent . $\square$

V5.8 Explain why the vectors $\begin{bmatrix} 3 \\ 0 \\ 5 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 0 \\ 4 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 4 \\ -1 \\ 5 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -6 \\ 4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -6 \\ -5 \\ 3 \\ -6 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 3 \\ 0 \\ 5 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 0 \\ 4 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 4 \\ -1 \\ 5 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -6 \\ 4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -6 \\ -5 \\ 3 \\ -6 \end{bmatrix}$ are linearly independent . ☐

V5.9 Explain why the vectors $\begin{bmatrix} 4 \\ 1 \\ -5 \\ 5 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ -5 \\ 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ -6 \\ -3 \\ -3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -2 \\ 10 \\ -1 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -4 \\ -5 \\ -3 \\ -4 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 4 \\ 1 \\ -5 \\ 5 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ -5 \\ 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ -6 \\ -3 \\ -3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -2 \\ 10 \\ -1 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -4 \\ -5 \\ -3 \\ -4 \end{bmatrix}$ are linearly dependent . ☐

V5.10 Explain why the vectors $\begin{bmatrix} -3 \\ 0 \\ -3 \\ -4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 5 \\ 3 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 4 \\ 5 \\ 1 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 5 \\ -1 \\ 1 \\ 8 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -3 \\ 0 \\ -3 \\ -4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 5 \\ 3 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 4 \\ 5 \\ 1 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 5 \\ -1 \\ 1 \\ 8 \end{bmatrix}$ are linearly dependent . ☐

V5.11 Explain why the vectors $\begin{bmatrix} -2 \\ -1 \\ -6 \\ 4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -1 \\ 2 \\ -1 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \\ 4 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 3 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -5 \\ 3 \\ -6 \\ -4 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -2 \\ -1 \\ -6 \\ 4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -1 \\ 2 \\ -1 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \\ 4 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 3 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -5 \\ 3 \\ -6 \\ -4 \end{bmatrix}$ are linearly independent . □

V5.12 Explain why the vectors $\begin{bmatrix} -1 \\ -5 \\ -5 \\ -3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -2 \\ -2 \\ -6 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -4 \\ -1 \\ -2 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -7 \\ -9 \\ -3 \\ 5 \\ -11 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -1 \\ -5 \\ -5 \\ -3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -2 \\ -2 \\ -6 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -4 \\ -1 \\ -2 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -7 \\ -9 \\ -3 \\ 5 \\ -11 \end{bmatrix}$ are linearly dependent . □

V5.13 Explain why the vectors $\begin{bmatrix} 1 \\ -1 \\ 3 \\ -2 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 7 \\ -9 \\ -1 \\ -10 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 12 \\ -21 \\ -11 \\ -14 \\ -12 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 1 \\ -1 \\ 3 \\ -2 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 7 \\ -9 \\ -1 \\ -10 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 12 \\ -21 \\ -11 \\ -14 \\ -12 \end{bmatrix}$ are linearly dependent . □

V5.14 Explain why the vectors $\begin{bmatrix} 4 \\ -5 \\ 2 \\ -5 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -1 \\ 1 \\ 0 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -4 \\ 5 \\ 3 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 6 \\ 18 \\ -9 \\ 15 \\ 21 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 4 \\ -5 \\ 2 \\ -5 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -1 \\ 1 \\ 0 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -4 \\ 5 \\ 3 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 6 \\ 18 \\ -9 \\ 15 \\ 21 \end{bmatrix}$ are linearly dependent . □

V5.15 Explain why the vectors $\begin{bmatrix} -5 \\ 4 \\ -4 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ 2 \\ -4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ -3 \\ -4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 16 \\ -13 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -5 \\ 4 \\ -4 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ 2 \\ -4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ -3 \\ -4 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 16 \\ -13 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ are linearly dependent . $\square$

V5.16 Explain why the vectors $\begin{bmatrix} 2 \\ -4 \\ 2 \\ 1 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -4 \\ 1 \\ -3 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ -2 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 2 \\ 4 \\ -5 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 2 \\ -3 \\ 2 \\ 4 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 2 \\ -4 \\ 2 \\ 1 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -4 \\ 1 \\ -3 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ -2 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 2 \\ 4 \\ -5 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 2 \\ -3 \\ 2 \\ 4 \end{bmatrix}$ are linearly dependent . $\square$

V5.17 Explain why the vectors $\begin{bmatrix} 0 \\ -6 \\ 3 \\ -6 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -5 \\ 0 \\ -3 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 0 \\ 3 \\ -4 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ 7 \\ -6 \\ 9 \\ -14 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 0 \\ -6 \\ 3 \\ -6 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -6 \\ -5 \\ 0 \\ -3 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 0 \\ 3 \\ -4 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ 7 \\ -6 \\ 9 \\ -14 \end{bmatrix}$ are linearly dependent . $\square$

V5.18 Explain why the vectors $\begin{bmatrix} -1 \\ -1 \\ 4 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 4 \\ -6 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -2 \\ -4 \\ -2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -19 \\ 11 \\ 0 \\ -5 \\ 4 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -1 \\ -1 \\ 4 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 4 \\ -6 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -2 \\ -4 \\ -2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -19 \\ 11 \\ 0 \\ -5 \\ 4 \end{bmatrix}$ are linearly dependent . □

V5.19 Explain why the vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 5 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -5 \\ -5 \\ 4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \\ -3 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 0 \\ 9 \\ -15 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ 5 \\ 2 \\ -4 \\ -1 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 5 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -5 \\ -5 \\ 4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \\ -3 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 0 \\ 9 \\ -15 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ 5 \\ 2 \\ -4 \\ -1 \end{bmatrix}$ are linearly dependent . □

V5.20 Explain why the vectors $\begin{bmatrix} 5 \\ 5 \\ 0 \\ -3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -2 \\ -1 \\ -1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 2 \\ -5 \\ 2 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 4 \\ -5 \\ -3 \\ -6 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 5 \\ 5 \\ 0 \\ -3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -2 \\ -1 \\ -1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 2 \\ -5 \\ 2 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 4 \\ -5 \\ -3 \\ -6 \end{bmatrix}$ are linearly independent . □

V5.21 Explain why the vectors $\begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 4 \\ 1 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 2 \\ -5 \\ -3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -5 \\ -4 \\ -7 \\ 3 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 4 \\ 1 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 2 \\ -5 \\ -3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -5 \\ -4 \\ -7 \\ 3 \end{bmatrix}$ are linearly dependent . □

V5.22 Explain why the vectors $\begin{bmatrix} -6 \\ -4 \\ 1 \\ 5 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ -4 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 5 \\ 3 \\ -2 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 15 \\ 6 \\ -10 \\ -14 \\ 13 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -6 \\ -4 \\ 1 \\ 5 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ -4 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 5 \\ 3 \\ -2 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 15 \\ 6 \\ -10 \\ -14 \\ 13 \end{bmatrix}$ are linearly dependent . □

V5.23 Explain why the vectors $\begin{bmatrix} -1 \\ -5 \\ -5 \\ 5 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ -4 \\ 4 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 2 \\ -3 \\ -3 \\ -3 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -1 \\ -5 \\ -5 \\ 5 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ -4 \\ 4 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 2 \\ -3 \\ -3 \\ -3 \end{bmatrix}$ are linearly independent . □

V5.24 Explain why the vectors $\begin{bmatrix} -5 \\ -5 \\ -2 \\ 4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -6 \\ 1 \\ 4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -12 \\ 2 \\ 8 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} 17 \\ 23 \\ -15 \\ -18 \\ 23 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} -5 \\ -5 \\ -2 \\ 4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -6 \\ 1 \\ 4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -12 \\ 2 \\ 8 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} 17 \\ 23 \\ -15 \\ -18 \\ 23 \end{bmatrix}$ are linearly dependent . □

V5.25 Explain why the vectors $\begin{bmatrix} 4 \\ -3 \\ 0 \\ 4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -6 \\ 4 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 3 \\ 3 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 4 \\ 0 \\ -5 \\ -3 \end{bmatrix}$ are linearly dependent or linearly independent.

Solution. The vectors $\begin{bmatrix} 4 \\ -3 \\ 0 \\ 4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ -6 \\ 4 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 3 \\ 3 \\ -6 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 4 \\ 0 \\ -5 \\ -3 \end{bmatrix}$ are linearly independent .

$\square$

Standard V6

V6.1 Explain why the vectors $\begin{bmatrix} 0 \\ -5 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -4 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^3$

Solution. The vectors $\begin{bmatrix} 0 \\ -5 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -4 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$ are a basis of $\mathbb{R}^3$.

□

V6.2 Explain why the vectors $\begin{bmatrix} 0 \\ -1 \\ -5 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -4 \\ -5 \\ -3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 8 \\ 11 \\ 25 \\ -6 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -3 \\ -5 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -9 \\ 1 \\ 0 \\ 2 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} 0 \\ -1 \\ -5 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -4 \\ -5 \\ -3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 8 \\ 11 \\ 25 \\ -6 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -3 \\ -5 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -9 \\ 1 \\ 0 \\ 2 \end{bmatrix}$ are not a basis of $\mathbb{R}^5$.

□

V6.3 Explain why the vectors $\begin{bmatrix} -1 \\ -4 \\ -2 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ -2 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ -5 \\ -4 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ 3 \\ -3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -4 \\ 4 \\ 2 \\ 0 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} -1 \\ -4 \\ -2 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ -2 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ -5 \\ -4 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ 3 \\ -3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -4 \\ 4 \\ 2 \\ 0 \end{bmatrix}$ are a basis of $\mathbb{R}^5$.

□

V6.4 Explain why the vectors $\begin{bmatrix} 3 \\ 3 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -3 \\ 1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 3 \\ -3 \\ -4 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} 3 \\ 3 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -3 \\ 1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 3 \\ -3 \\ -4 \end{bmatrix}$ are not a basis of $\mathbb{R}^4$.

□

V6.5 Explain why the vectors $\begin{bmatrix} -2 \\ -4 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -3 \\ -4 \\ -2 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} -2 \\ -4 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 0 \\ -3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -3 \\ -4 \\ -2 \end{bmatrix}$ are a basis of $\mathbb{R}^4$. $\square$

V6.6 Explain why the vectors $\begin{bmatrix} 0 \\ -3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -3 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^3$

Solution. The vectors $\begin{bmatrix} 0 \\ -3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -3 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix}$ are a basis of $\mathbb{R}^3$. $\square$

V6.7 Explain why the vectors $\begin{bmatrix} 3 \\ -2 \\ -3 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \\ -5 \\ 1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 3 \\ -5 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -2 \\ -4 \\ 3 \\ 4 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} 3 \\ -2 \\ -3 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \\ -5 \\ 1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 3 \\ -5 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -2 \\ -4 \\ 3 \\ 4 \end{bmatrix}$ are a basis of $\mathbb{R}^5$. $\square$

V6.8 Explain why the vectors $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 3 \\ -3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ -1 \\ 4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -5 \\ -4 \\ 2 \\ -5 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 3 \\ -3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ -1 \\ 4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -5 \\ -4 \\ 2 \\ -5 \end{bmatrix}$ are a basis of $\mathbb{R}^5$. $\square$

V6.9 Explain why the vectors $\begin{bmatrix} 4 \\ 0 \\ -4 \\ 4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ -4 \\ 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -5 \\ -3 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -8 \\ 5 \\ 15 \\ -9 \\ -14 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ -4 \\ -2 \\ -3 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} 4 \\ 0 \\ -4 \\ 4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ -4 \\ 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -5 \\ -3 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -8 \\ 5 \\ 15 \\ -9 \\ -14 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -3 \\ -4 \\ -2 \\ -3 \end{bmatrix}$ are not a basis of $\mathbb{R}^5$.

□

V6.10 Explain why the vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \end{bmatrix}$ are not a basis of $\mathbb{R}^4$.

□

V6.11 Explain why the vectors $\begin{bmatrix} -1 \\ -1 \\ -5 \\ 4 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ 2 \\ -1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 3 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -4 \\ 2 \\ -5 \\ -3 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} -1 \\ -1 \\ -5 \\ 4 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -1 \\ 2 \\ -1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 3 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -4 \\ 2 \\ -5 \\ -3 \end{bmatrix}$ are a basis of $\mathbb{R}^5$.

□

V6.12 Explain why the vectors $\begin{bmatrix} -1 \\ -4 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 1 \\ -5 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -2 \\ -2 \\ 1 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} -1 \\ -4 \\ -4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 1 \\ -5 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -2 \\ -2 \\ 1 \end{bmatrix}$ are a basis of $\mathbb{R}^4$.

□

V6.13 Explain why the vectors $\begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 3 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -3 \\ 3 \\ 1 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 3 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -3 \\ 3 \\ 1 \end{bmatrix}$ are a basis of $\mathbb{R}^4$.

□

V6.14 Explain why the vectors $\begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} -14 \\ 4 \\ -14 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^3$

Solution. The vectors $\begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -2 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} -14 \\ 4 \\ -14 \end{bmatrix}$ are not a basis of $\mathbb{R}^3$.

□

V6.15 Explain why the vectors $\begin{bmatrix} -4 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^3$

Solution. The vectors $\begin{bmatrix} -4 \\ 3 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$ are a basis of $\mathbb{R}^3$.

□

V6.16 Explain why the vectors $\begin{bmatrix} 1 \\ -4 \\ 1 \\ 1 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ 1 \\ -2 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ -2 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 12 \\ -6 \\ -4 \\ 15 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} 1 \\ -4 \\ 1 \\ 1 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ 1 \\ -2 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ -2 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 12 \\ -6 \\ -4 \\ 15 \end{bmatrix}$ are not a basis of $\mathbb{R}^5$.

□

V6.17 Explain why the vectors $\begin{bmatrix} 0 \\ -5 \\ 2 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ -4 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -5 \\ 4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -4 \\ 1 \\ -1 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} 0 \\ -5 \\ 2 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ -4 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -5 \\ 4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -4 \\ 1 \\ -1 \end{bmatrix}$ are a basis of $\mathbb{R}^4$.

$\square$

V6.18 Explain why the vectors $\begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -2 \\ 2 \\ -5 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 3 \\ 4 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -2 \\ 2 \\ -5 \end{bmatrix}$ are a basis of $\mathbb{R}^4$.

$\square$

V6.19 Explain why the vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -4 \\ -4 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \\ -3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 2 \\ -3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 12 \\ 12 \\ -9 \\ 3 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^5$

Solution. The vectors $\begin{bmatrix} -2 \\ 0 \\ -3 \\ 4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -4 \\ -4 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \\ -3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 2 \\ -3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 12 \\ 12 \\ -9 \\ 3 \end{bmatrix}$ are not a basis of $\mathbb{R}^5$.

$\square$

V6.20 Explain why the vectors $\begin{bmatrix} 4 \\ 4 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 2 \\ 2 \\ 1 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} 4 \\ 4 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ -2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 2 \\ 2 \\ 1 \end{bmatrix}$ are a basis of $\mathbb{R}^4$.

$\square$

V6.21 Explain why the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 1 \\ -12 \\ -11 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ 1 \\ -12 \\ -11 \end{bmatrix}$ are not a basis of $\mathbb{R}^4$.

$\square$

V6.22 Explain why the vectors $\begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ -8 \\ 3 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^3$

Solution. The vectors $\begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -6 \\ -8 \\ 3 \end{bmatrix}$ are not a basis of $\mathbb{R}^3$. $\square$

V6.23 Explain why the vectors $\begin{bmatrix} -1 \\ -4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^3$

Solution. The vectors $\begin{bmatrix} -1 \\ -4 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ are a basis of $\mathbb{R}^3$. $\square$

V6.24 Explain why the vectors $\begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^3$

Solution. The vectors $\begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix}$ are a basis of $\mathbb{R}^3$. $\square$

V6.25 Explain why the vectors $\begin{bmatrix} 3 \\ -2 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ -1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 2 \\ -5 \\ -2 \end{bmatrix}$ are or are not a basis of $\mathbb{R}^4$

Solution. The vectors $\begin{bmatrix} 3 \\ -2 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -3 \\ -1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 2 \\ -5 \\ -2 \end{bmatrix}$ are a basis of $\mathbb{R}^4$. $\square$

Standard V7

V7.1 Find a basis for the subspace

$$W = \operatorname{span}\left\{ \begin{bmatrix} 0 \\ -4 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -1 \\ 2 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 0 \\ -4 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -1 \\ 2 \end{bmatrix} \right\}$.

□

V7.2 Find a basis for the subspace

$$W = \operatorname{span}\left\{ \begin{bmatrix} 0 \\ -1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ -3 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 17 \\ -19 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -4 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 0 \\ -1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ -3 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 0 \\ -3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -4 \\ -3 \end{bmatrix} \right\}$.

□

V7.3 Find a basis for the subspace

$$W = \operatorname{span}\left\{ \begin{bmatrix} -1 \\ -4 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 5 \\ 18 \\ 7 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 3 \\ 1 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} -1 \\ -4 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ 2 \\ -4 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 3 \\ 1 \end{bmatrix} \right\}$.

□

V7.4 Find a basis for the subspace

$$W = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -10 \\ -8 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 7 \\ -10 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -2 \end{bmatrix} \right\}$. ☐

V7.5 Find a basis for the subspace

$$W = \text{span} \left\{ \begin{bmatrix} -2 \\ -3 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 16 \\ 6 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} -14 \\ -18 \\ -12 \\ 4 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} -2 \\ -3 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -3 \\ -1 \end{bmatrix} \right\}$. ☐

V7.6 Find a basis for the subspace

$$W = \text{span} \left\{ \begin{bmatrix} 0 \\ -3 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -4 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \\ 0 \\ -4 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 0 \\ -3 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -4 \\ -3 \end{bmatrix} \right\}$. ☐

V7.7 Find a basis for the subspace

$$W = \text{span} \left\{ \begin{bmatrix} 3 \\ -2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ -3 \\ 9 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 7 \\ -5 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 3 \\ -2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \\ 3 \end{bmatrix} \right\}$. ☐

V7.8 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}2\\0\\3\\2\end{bmatrix}, \begin{bmatrix}0\\2\\0\\3\end{bmatrix}, \begin{bmatrix}-3\\-2\\-2\\2\end{bmatrix}, \begin{bmatrix}0\\4\\-5\\-10\end{bmatrix}, \begin{bmatrix}0\\-4\\2\\3\end{bmatrix}, \begin{bmatrix}1\\-4\\-3\\2\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix}2\\0\\3\\2\end{bmatrix}, \begin{bmatrix}0\\2\\0\\3\end{bmatrix}, \begin{bmatrix}-3\\-2\\-2\\2\end{bmatrix}, \begin{bmatrix}0\\-4\\2\\3\end{bmatrix}\right\}$. □

V7.9 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}3\\0\\-3\\3\end{bmatrix}, \begin{bmatrix}3\\0\\1\\-4\end{bmatrix}, \begin{bmatrix}1\\1\\-3\\-4\end{bmatrix}, \begin{bmatrix}-2\\-2\\2\\-4\end{bmatrix}, \begin{bmatrix}0\\0\\-3\\1\end{bmatrix}, \begin{bmatrix}-6\\3\\8\\3\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix}3\\0\\-3\\3\end{bmatrix}, \begin{bmatrix}3\\0\\1\\-4\end{bmatrix}, \begin{bmatrix}1\\1\\-3\\-4\end{bmatrix}, \begin{bmatrix}-2\\-2\\2\\-4\end{bmatrix}\right\}$. □

V7.10 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}-2\\0\\-2\\-3\end{bmatrix}, \begin{bmatrix}-1\\0\\3\\2\end{bmatrix}, \begin{bmatrix}4\\0\\-12\\-8\end{bmatrix}, \begin{bmatrix}-2\\0\\3\\1\end{bmatrix}, \begin{bmatrix}1\\0\\10\\9\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix}-2\\0\\-2\\-3\end{bmatrix}, \begin{bmatrix}-1\\0\\3\\2\end{bmatrix}, \begin{bmatrix}-2\\0\\3\\1\end{bmatrix}\right\}$. □

V7.11 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}-1\\-1\\-4\\3\end{bmatrix}, \begin{bmatrix}-3\\-3\\-1\\1\end{bmatrix}, \begin{bmatrix}-1\\-4\\3\\-1\end{bmatrix}, \begin{bmatrix}6\\15\\-8\\2\end{bmatrix}, \begin{bmatrix}6\\6\\2\\-2\end{bmatrix}, \begin{bmatrix}-20\\-41\\13\\0\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} -1 \\ -1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 3 \\ -1 \end{bmatrix} \right\}$.

□

V7.12 Find a basis for the subspace

$$W = \text{span} \left\{ \begin{bmatrix} -1 \\ -3 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -4 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -9 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \\ -4 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} -1 \\ -3 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -4 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \\ -4 \end{bmatrix} \right\}$.

□

V7.13 Find a basis for the subspace

$$W = \text{span} \left\{ \begin{bmatrix} 0 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 11 \\ -11 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ -7 \\ 12 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 0 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -4 \end{bmatrix} \right\}$.

□

V7.14 Find a basis for the subspace

$$W = \text{span} \left\{ \begin{bmatrix} 3 \\ -4 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -17 \\ 16 \\ -6 \\ 16 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -3 \\ -4 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 3 \\ -4 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -3 \\ -4 \end{bmatrix} \right\}$.

□

V7.15 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix} -3 \\ 3 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -12 \\ 6 \\ -6 \\ -1 \end{bmatrix}, \begin{bmatrix} 30 \\ -16 \\ 16 \\ 3 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} -3 \\ 3 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}\right\}$.

□

V7.16 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 11 \\ -1 \\ 11 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -14 \\ 4 \\ -12 \\ 2 \end{bmatrix}, \begin{bmatrix} -18 \\ -1 \\ -17 \\ 8 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -1 \\ 0 \end{bmatrix}\right\}$.

□

V7.17 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix} 0 \\ -4 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} -10 \\ -8 \\ 19 \\ -4 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} 0 \\ -4 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ 3 \\ 0 \end{bmatrix}\right\}$.

□

V7.18 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix} -1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ -6 \\ -2 \\ 2 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 2 \\ -4 \end{bmatrix} \right\}$.

$\square$

V7.19 Find a basis for the subspace

$$W = \operatorname{span} \left\{ \begin{bmatrix} -2 \\ 0 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 1 \\ -3 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} -2 \\ 0 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 3 \\ -2 \end{bmatrix} \right\}$.

$\square$

V7.20 Find a basis for the subspace

$$W = \operatorname{span} \left\{ \begin{bmatrix} 3 \\ 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 10 \\ -1 \\ -7 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} -6 \\ -6 \\ 0 \\ 4 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 3 \\ 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ -1 \end{bmatrix} \right\}$.

$\square$

V7.21 Find a basis for the subspace

$$W = \operatorname{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 11 \\ 8 \\ -4 \\ 0 \end{bmatrix} \right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ -3 \end{bmatrix} \right\}$.

$\square$

V7.22 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}-4\\-3\\0\\3\end{bmatrix}, \begin{bmatrix}-2\\1\\-1\\-3\end{bmatrix}, \begin{bmatrix}24\\13\\2\\-9\end{bmatrix}, \begin{bmatrix}2\\-2\\-3\\-3\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix}-4\\-3\\0\\3\end{bmatrix}, \begin{bmatrix}-2\\1\\-1\\-3\end{bmatrix}, \begin{bmatrix}2\\-2\\-3\\-3\end{bmatrix}\right\}$.

□

V7.23 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}-1\\-3\\-4\\3\end{bmatrix}, \begin{bmatrix}3\\0\\1\\0\end{bmatrix}, \begin{bmatrix}-6\\9\\9\\-9\end{bmatrix}, \begin{bmatrix}-3\\3\\1\\-2\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix}-1\\-3\\-4\\3\end{bmatrix}, \begin{bmatrix}3\\0\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\3\\1\\-2\end{bmatrix}\right\}$.

□

V7.24 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}-3\\-3\\-2\\3\end{bmatrix}, \begin{bmatrix}-3\\0\\-4\\0\end{bmatrix}, \begin{bmatrix}3\\-2\\-3\\-1\end{bmatrix}, \begin{bmatrix}-3\\8\\-1\\-5\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix}-3\\-3\\-2\\3\end{bmatrix}, \begin{bmatrix}-3\\0\\-4\\0\end{bmatrix}, \begin{bmatrix}3\\-2\\-3\\-1\end{bmatrix}\right\}$.

□

V7.25 Find a basis for the subspace

$$W = \operatorname{span}\left\{\begin{bmatrix}3\\-2\\0\\2\end{bmatrix}, \begin{bmatrix}-2\\-2\\-1\\-4\end{bmatrix}, \begin{bmatrix}9\\4\\3\\14\end{bmatrix}, \begin{bmatrix}2\\2\\-4\\-2\end{bmatrix}, \begin{bmatrix}2\\12\\14\\24\end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{ \begin{bmatrix} 3 \\ -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -1 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -4 \\ -2 \end{bmatrix} \right\}$.

$\square$

Standard V8

V8.1 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} 0 \\ -4 \\ -2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -8 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -8 \\ 6 \\ 0 \\ 6 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 2. □

V8.2 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} 0 \\ -1 \\ -4 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ -4 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} -12 \\ -8 \\ -8 \\ -12 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -2 \\ -4 \\ -3 \end{bmatrix}, \begin{bmatrix} -16 \\ -12 \\ -16 \\ -12 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -4 \\ -3 \\ -4 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 4. □

V8.3 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} -1 \\ -4 \\ -1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 10 \\ 0 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 2 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -9 \\ -20 \\ 4 \\ -5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 17 \\ 6 \\ 6 \\ -2 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 3. □

V8.4 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} 2 \\ 2 \\ -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 3 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 8 \\ -10 \\ -16 \\ -12 \end{bmatrix}, \begin{bmatrix} -11 \\ 17 \\ -19 \\ -25 \\ -18 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -3 \\ 0 \\ -2 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 3. □

V8.5 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}-2\\-3\\0\\3\\-4\end{bmatrix}, \begin{bmatrix}0\\-3\\-3\\-1\\1\end{bmatrix}, \begin{bmatrix}2\\6\\3\\-2\\3\end{bmatrix}, \begin{bmatrix}0\\-3\\-3\\-1\\1\end{bmatrix}, \begin{bmatrix}2\\-2\\-2\\-1\\1\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3.

□

V8.6 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}0\\-3\\-2\\0\\-2\end{bmatrix}, \begin{bmatrix}-1\\-2\\-2\\2\\1\end{bmatrix}, \begin{bmatrix}-3\\6\\2\\6\\11\end{bmatrix}, \begin{bmatrix}2\\-2\\2\\-3\\3\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3.

□

V8.7 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}3\\-2\\-2\\1\\-1\end{bmatrix}, \begin{bmatrix}1\\3\\-4\\1\\-4\end{bmatrix}, \begin{bmatrix}-14\\13\\6\\-4\\1\end{bmatrix}, \begin{bmatrix}-16\\18\\4\\-4\\-2\end{bmatrix}, \begin{bmatrix}-26\\10\\24\\-10\\16\end{bmatrix}, \begin{bmatrix}1\\1\\-3\\3\\0\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3.

□

V8.8 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}2\\0\\3\\2\\0\end{bmatrix}, \begin{bmatrix}2\\0\\3\\-3\\-2\end{bmatrix}, \begin{bmatrix}-2\\2\\-1\\3\\-1\end{bmatrix}, \begin{bmatrix}6\\-6\\3\\-19\\-1\end{bmatrix}, \begin{bmatrix}-6\\2\\-7\\-6\\-3\end{bmatrix}, \begin{bmatrix}-20\\16\\-14\\50\\4\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3.

□

V8.9 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}3\\0\\-3\\3\\3\end{bmatrix}, \begin{bmatrix}0\\1\\-4\\1\\1\end{bmatrix}, \begin{bmatrix}-3\\-4\\-2\\-2\\2\end{bmatrix}, \begin{bmatrix}-4\\0\\0\\-3\\1\end{bmatrix}, \begin{bmatrix}2\\-3\\-3\\-2\\-3\end{bmatrix}, \begin{bmatrix}-1\\9\\3\\-1\\-5\end{bmatrix}\right\}.$$

Solution. The dimension of W is 5.

$\square$

V8.10 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}-2\\0\\-2\\-3\\-1\end{bmatrix}, \begin{bmatrix}0\\3\\2\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\-2\\0\\3\end{bmatrix}, \begin{bmatrix}0\\3\\-2\\1\\6\end{bmatrix}, \begin{bmatrix}1\\0\\-3\\-4\\1\end{bmatrix}\right\}.$$

Solution. The dimension of W is 4.

$\square$

V8.11 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}-1\\-1\\-4\\3\\-3\end{bmatrix}, \begin{bmatrix}-3\\-1\\1\\-1\\-4\end{bmatrix}, \begin{bmatrix}17\\9\\16\\-11\\31\end{bmatrix}, \begin{bmatrix}2\\2\\2\\1\\2\end{bmatrix}, \begin{bmatrix}6\\2\\10\\-12\\16\end{bmatrix}, \begin{bmatrix}25\\13\\16\\-8\\41\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3.

$\square$

V8.12 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}-1\\-3\\-3\\-2\\-3\end{bmatrix}, \begin{bmatrix}-2\\-1\\-2\\-4\\1\end{bmatrix}, \begin{bmatrix}-4\\-3\\-1\\-1\\-2\end{bmatrix}, \begin{bmatrix}1\\2\\-1\\3\\-4\end{bmatrix}, \begin{bmatrix}-4\\2\\1\\-2\\2\end{bmatrix}\right\}.$$

Solution. The dimension of W is 5.

$\square$

V8.13 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}0\\-1\\2\\-1\\3\end{bmatrix}, \begin{bmatrix}-2\\3\\1\\3\\1\end{bmatrix}, \begin{bmatrix}-4\\10\\-6\\10\\-10\end{bmatrix}, \begin{bmatrix}1\\2\\2\\-3\\-1\end{bmatrix}, \begin{bmatrix}7\\-27\\1\\-12\\16\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3.

$\square$

V8.14 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} 3 \\ -4 \\ 1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 3 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -15 \\ 4 \\ 5 \\ 13 \\ -8 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 3. □

V8.15 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} -3 \\ 3 \\ -3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 3 \\ 3 \\ 1 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 4. □

V8.16 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 13 \\ -7 \\ 0 \\ 16 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 4. □

V8.17 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} 0 \\ -4 \\ 2 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ -4 \\ 0 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 8 \\ -4 \\ 6 \\ -8 \\ 14 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -2 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ -3 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 3. □

V8.18 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} -1 \\ -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ -4 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -3 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 4 \\ 8 \\ -8 \end{bmatrix}, \begin{bmatrix} -4 \\ -4 \\ 2 \\ 2 \\ -2 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 4. $\square$

V8.19 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}-2\\0\\-2\\3\\-1\end{bmatrix},\begin{bmatrix}1\\-3\\-4\\2\\-1\end{bmatrix},\begin{bmatrix}0\\-1\\0\\-2\\3\end{bmatrix},\begin{bmatrix}3\\11\\18\\-11\\0\end{bmatrix},\begin{bmatrix}-4\\3\\1\\-3\\-1\end{bmatrix},\begin{bmatrix}-4\\-3\\-4\\0\\0\end{bmatrix}\right\}.$$

Solution. The dimension of W is 5. $\square$

V8.20 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}3\\3\\0\\-2\\-2\end{bmatrix},\begin{bmatrix}1\\-1\\-1\\-1\\3\end{bmatrix},\begin{bmatrix}3\\3\\0\\-2\\-2\end{bmatrix},\begin{bmatrix}-2\\2\\2\\2\\-6\end{bmatrix},\begin{bmatrix}3\\2\\1\\0\\2\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3. $\square$

V8.21 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}1\\0\\2\\2\\-4\end{bmatrix},\begin{bmatrix}-3\\3\\0\\2\\1\end{bmatrix},\begin{bmatrix}4\\-3\\2\\0\\-5\end{bmatrix},\begin{bmatrix}-3\\-3\\-3\\2\\-4\end{bmatrix},\begin{bmatrix}-19\\9\\-11\\2\\16\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3. $\square$

V8.22 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix}-4\\-3\\0\\3\\-2\end{bmatrix},\begin{bmatrix}1\\-1\\-3\\-1\\-1\end{bmatrix},\begin{bmatrix}2\\3\\2\\-2\\-3\end{bmatrix},\begin{bmatrix}11\\5\\-5\\-8\\8\end{bmatrix}\right\}.$$

Solution. The dimension of W is 3. $\square$

V8.23 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} -1 \\ -3 \\ -4 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ -5 \\ -4 \\ -1 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \\ -3 \\ -3 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 2. $\square$

V8.24 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} -3 \\ -3 \\ -2 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ 0 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 3 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \\ 3 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 4. $\square$

V8.25 Explain how to find the dimension of

$$W = \operatorname{span}\left\{\begin{bmatrix} 3 \\ -2 \\ 0 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -4 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 0 \\ -4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ -4 \\ 4 \\ 18 \end{bmatrix}\right\}.$$

Solution. The dimension of W is 4. $\square$

Standard V9

V9.1 Find a basis for the subspace of $\mathcal{P}^3$

$$W = \text{span}\ \left\{-3\,x^3 - 6\,x^2 - 5, -6\,x^3 + 2\,x^2 - 4\,x, 4\,x^3 - 2\,x^2 - x + 3, 0, 3\,x^3 + 4\,x^2 + 4\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{-3\,x^3 - 6\,x^2 - 5, -6\,x^3 + 2\,x^2 - 4\,x, 4\,x^3 - 2\,x^2 - x + 3, 3\,x^3 + 4\,x^2 + 4\right\}$. $\square$

V9.2 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\ \left\{\left[\begin{array}{cc} 5 & 2 \\ -5 & -5 \end{array}\right], \left[\begin{array}{cc} -3 & -2 \\ 5 & 0 \end{array}\right], \left[\begin{array}{cc} -8 & -5 \\ 0 & 2 \end{array}\right], \left[\begin{array}{cc} -5 & -4 \\ -1 & -4 \end{array}\right], \left[\begin{array}{cc} 3 & 1 \\ -1 & -6 \end{array}\right]\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\left[\begin{array}{cc} 5 & 2 \\ -5 & -5 \end{array}\right], \left[\begin{array}{cc} -3 & -2 \\ 5 & 0 \end{array}\right], \left[\begin{array}{cc} -8 & -5 \\ 0 & 2 \end{array}\right], \left[\begin{array}{cc} -5 & -4 \\ -1 & -4 \end{array}\right]\right\}$. $\square$

V9.3 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\ \left\{-6\,x^3 - 5\,x^2 - 3\,x + 1, -2\,x^3 - 5\,x^2 - x + 5, -2\,x^3 - 5\,x^2 - x + 5, 4\,x^3 + 5\,x^2 + 2\,x - 3, 24\,x^3 + 25\,x^2 + 12\,x - 11\right.$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{-6\,x^3 - 5\,x^2 - 3\,x + 1, -2\,x^3 - 5\,x^2 - x + 5\right\}$. $\square$

V9.4 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\ \left\{-7\,x^3 + x^2 + 2\,x + 7, 9\,x^3 - 7\,x^2 - 10\,x - 5, -4\,x^3 + 2\,x^2 + 3\,x + 3, -3\,x^3 + 4\,x^2 + 3\,x - 3, -x^3 + 3\,x^2 + 4\,x - 1\right\}$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{-7\,x^3 + x^2 + 2\,x + 7, 9\,x^3 - 7\,x^2 - 10\,x - 5, -3\,x^3 + 4\,x^2 + 3\,x - 3\right\}$. $\square$

V9.5 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\ \left\{8\,x^3 - 5\,x^2 + 9\,x - 7, -5\,x^2 - 3\,x + 1, -4\,x^3 - 6\,x + 4, -4\,x^3 - x^2 - x - 2, -4\,x^3 - 4\,x^2 + 2\,x + 1\right\}.$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{8\,x^3 - 5\,x^2 + 9\,x - 7, -5\,x^2 - 3\,x + 1, -4\,x^3 - x^2 - x - 2 - 4\,x^3 - 4\,x^2 + 2\,x + 1\right\}$. $\square$

V9.6 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\ \left\{\left[\begin{array}{cc} 0 & 9 \\ 3 & 9 \end{array}\right], \left[\begin{array}{cc} 6 & -18 \\ -2 & -16 \end{array}\right], \left[\begin{array}{cc} -6 & 0 \\ -4 & -2 \end{array}\right], \left[\begin{array}{cc} 0 & -3 \\ -1 & -3 \end{array}\right], \left[\begin{array}{cc} 0 & 3 \\ 1 & 3 \end{array}\right]\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} 0 & 9 \\ 3 & 9 \end{bmatrix}, \begin{bmatrix} 6 & -18 \\ -2 & -16 \end{bmatrix}\right\}$.

□

V9.7 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\left\{\begin{bmatrix} 10 & 6 \\ -2 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 6 \\ -4 & -7 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ -2 & -3 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} 10 & 6 \\ -2 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 6 \\ -4 & -7 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}\right\}$.

□

V9.8 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\left\{-2\,x^3 + 5\,x^2 - x + 4, -30\,x^3 + 30\,x^2 - 18\,x + 12, 5\,x^3 + 3\,x + 3, 3\,x^2 + 3, 5\,x^3 - 9\,x^2 + 3\,x - 6\right\}.$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{-2\,x^3 + 5\,x^2 - x + 4, -30\,x^3 + 30\,x^2 - 18\,x + 12, 5\,x^3 + 3\,x + 3\right\}$.

□

V9.9 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\left\{\begin{bmatrix} -6 & -6 \\ -1 & -7 \end{bmatrix}, \begin{bmatrix} -6 & -3 \\ -3 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 1 & -5 \end{bmatrix}, \begin{bmatrix} -5 & 1 \\ 0 & -4 \end{bmatrix}, \begin{bmatrix} 5 & 5 \\ 1 & 1 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} -6 & -6 \\ -1 & -7 \end{bmatrix}, \begin{bmatrix} -6 & -3 \\ -3 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 1 & -5 \end{bmatrix}, \begin{bmatrix} -5 & 1 \\ 0 & -4 \end{bmatrix}\right\}$.

□

V9.10 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\left\{\begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -4 & -1 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 0 & -3 \end{bmatrix}, \begin{bmatrix} -6 & 5 \\ 0 & 11 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & -15 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -4 & -1 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 0 & -3 \end{bmatrix}\right\}$.

□

V9.11 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\left\{-x^3 - 4\,x^2 - 5\,x + 4, 9\,x^3 + 13\,x^2 + 17\,x - 17, 17\,x^3 + 22\,x^2 + 29\,x - 30, -6\,x^3 - x^2 - 2\,x + 5, -3\,x^3 + 11\,x^2 + 13\right.$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{-x^3 - 4\,x^2 - 5\,x + 4, 9\,x^3 + 13\,x^2 + 17\,x - 17\right\}$.

$\square$

V9.12 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\,\left\{-2\,x^3 - 2\,x^2 - x - 4, -6\,x^3 + x^2 - 6\,x - 2, -13\,x^3 + 3\,x^2 - 3\,x + 1, -5\,x^3 - 5\,x^2 - x - 1, -2\,x^3 - 3\,x^2 - 5\,x - 3\right\}$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{-2\,x^3 - 2\,x^2 - x - 4, -6\,x^3 + x^2 - 6\,x - 2, -13\,x^3 + 3\,x^2 - 3\,x + 1, -5\,x^3 - 5\,x^2 - x - 1\right\}$.

$\square$

V9.13 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\,\left\{4\,x^3 - 3\,x^2 + 4\,x - 2, -15\,x^3 + 16\,x^2 - 7\,x + 13, 3\,x^3 + 2\,x^2 + 4\,x + 2, 3\,x^3 - x^2 + x - 1, 3\,x^3 - 4\,x^2 + 2\,x - 6\right\}$.

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{4\,x^3 - 3\,x^2 + 4\,x - 2, -15\,x^3 + 16\,x^2 - 7\,x + 13, 3\,x^3 + 2\,x^2 + 4\,x + 2, 3\,x^3 - 4\,x^2 + 2\,x - 6\right\}$.

$\square$

V9.14 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\,\left\{\begin{bmatrix} -5 & -2 \\ -6 & -1 \end{bmatrix}, \begin{bmatrix} -4 & 5 \\ 3 & -5 \end{bmatrix}, \begin{bmatrix} -4 & 4 \\ -5 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}, \begin{bmatrix} 4 & -16 \\ -7 & -6 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} -5 & -2 \\ -6 & -1 \end{bmatrix}, \begin{bmatrix} -4 & 5 \\ 3 & -5 \end{bmatrix}, \begin{bmatrix} -4 & 4 \\ -5 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}\right\}$.

$\square$

V9.15 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\,\left\{4\,x^3 - 4\,x^2 - 3\,x - 2, -4\,x^3 + 4\,x^2 - 5\,x - 6, 5\,x^3 - 5\,x^2 + 9\,x + 13, 3\,x^3 - 3\,x^2 + x - 1, 4\,x^3 + 5\,x^2 + 5\,x + 3\right\}$.

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{4\,x^3 - 4\,x^2 - 3\,x - 2, -4\,x^3 + 4\,x^2 - 5\,x - 6, 5\,x^3 - 5\,x^2 + 9\,x + 13, 4\,x^3 + 5\,x^2 + 5\,x + 3\right\}$.

$\square$

V9.16 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\,\left\{\begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -14 \\ -2 & -4 \end{bmatrix}, \begin{bmatrix} 1 & -4 \\ 2 & -4 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -4 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ -6 & 2 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -14 \\ -2 & -4 \end{bmatrix}, \begin{bmatrix} 1 & -4 \\ 2 & -4 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -4 & 2 \end{bmatrix}\right\}$.

□

V9.17 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\left\{\begin{bmatrix} 0 & -3 \\ -6 & 5 \end{bmatrix}, \begin{bmatrix} -6 & 4 \\ -6 & -5 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ -6 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 6 \\ 12 & -10 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ -4 & 1 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} 0 & -3 \\ -6 & 5 \end{bmatrix}, \begin{bmatrix} -6 & 4 \\ -6 & -5 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ -6 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ -4 & 1 \end{bmatrix}\right\}$.

□

V9.18 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\left\{4x^3 - 4x^2 + 2x - 1, 5x^3 + 4x^2 - 4x - 6, 4x^3 - x^2 - x - 1, x^3 + 2x^2 + 4x - 6, 3x^3 - 2x^2 - 4x - 2\right\}.$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{4x^3 - 4x^2 + 2x - 1, 5x^3 + 4x^2 - 4x - 6, 4x^3 - x^2 - x - 1, x^3 + 2x^2 + 4x - 6\right\}$.

□

V9.19 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\left\{14x^3 - 14x^2 - 7x, -4x^3 + 3x^2 + 2x - 3, -5x^3 + 2x^2 - x + 5, -x^3 + 4x^2 + 4x - 5, -3x^3 - 2x + 5\right\}.$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{14x^3 - 14x^2 - 7x, -4x^3 + 3x^2 + 2x - 3, -5x^3 + 2x^2 - x + 5\right\}$.

□

V9.20 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\left\{\begin{bmatrix} -1 & -1 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} -3 & -3 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} -4 & 10 \\ 10 & 0 \end{bmatrix}, \begin{bmatrix} -2 & 5 \\ 5 & 0 \end{bmatrix}, \begin{bmatrix} -4 & 10 \\ 10 & 0 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} -1 & -1 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} -3 & -3 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} -4 & 10 \\ 10 & 0 \end{bmatrix}\right\}$.

□

V9.21 Find a basis for the subspace of $\mathcal{P}^3$

$W = \text{span}\left\{4x^3 + 2x^2 + 4x + 1, 4x^3 - 5x^2 - 4x - 4, 3x^3 + x^2 + x - 2, 4x^3 - 4x^2 - 5x + 3, -3x^2 - 5x + 2\right\}.$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{4x^3 + 2x^2 + 4x + 1, 4x^3 - 5x^2 - 4x - 4, 3x^3 + x^2 + x - 2, 4x^3 - 4x^2 - 5x + 3\right\}$.

□

V9.22 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\left\{\begin{bmatrix} -4 & -6 \\ -4 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ -2 & -5 \end{bmatrix}, \begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} 21 & 16 \\ 14 & -8 \end{bmatrix}, \begin{bmatrix} 5 & -3 \\ 1 & -1 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} -4 & -6 \\ -4 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ -2 & -5 \end{bmatrix}, \begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} 21 & 16 \\ 14 & -8 \end{bmatrix}\right\}$. □

V9.23 Find a basis for the subspace of $M_{2,2}$

$$W = \text{span}\left\{\begin{bmatrix} 5 & 5 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} -3 & -1 \\ -5 & -5 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ -3 & -3 \end{bmatrix}, \begin{bmatrix} -11 & -5 \\ -11 & -15 \end{bmatrix}, \begin{bmatrix} -2 & 2 \\ -3 & -3 \end{bmatrix}\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{\begin{bmatrix} 5 & 5 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} -3 & -1 \\ -5 & -5 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ -3 & -3 \end{bmatrix}, \begin{bmatrix} -2 & 2 \\ -3 & -3 \end{bmatrix}\right\}$. □

V9.24 Find a basis for the subspace of $\mathcal{P}^3$

$$W = \text{span}\left\{20\,x^3 + 5\,x^2 + 20\,x - 7, -6\,x^3 - 5\,x + 4, -3\,x^3 + 4\,x^2 - x + 1, -4\,x^3 + 2\,x^2 + 5\,x - 2, -2\,x^3 - 5\,x^2 - 5\,x - 5\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{20\,x^3 + 5\,x^2 + 20\,x - 7, -6\,x^3 - 5\,x + 4, -3\,x^3 + 4\,x^2 - x + 1, -4\,x^3 + 2\,x^2 + 5\,x - 2\right\}$. □

V9.25 Find a basis for the subspace of $\mathcal{P}^3$

$$W = \text{span}\left\{3\,x^3 + 23\,x^2 - 2\,x - 12, 2\,x^3 - 4\,x^2 - 4\,x + 4, -3\,x^2 + 4\,x - 2, 5\,x^2 + 4\,x - 6, -x^3 - 3\,x^2 - 2\,x + 4\right\}.$$

Be sure to explain why your result is a basis.

Solution. A basis of W is $\left\{3\,x^3 + 23\,x^2 - 2\,x - 12, 2\,x^3 - 4\,x^2 - 4\,x + 4, -3\,x^2 + 4\,x - 2\right\}$. □

Standard V10

V10.1 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -4x_1 - 4x_2 &= 0 \\ -10x_1 + 2x_2 - 6x_3 &= 0 \\ -12x_1 - 6x_2 - 3x_3 &= 0 \\ 4x_1 + 4x_2 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \end{bmatrix} \right\}.$$

□

V10.2 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -1x_1 + 1x_2 - 4x_3 - 8x_4 + 2x_5 &= 0 \\ -4x_1 - 1x_2 + 10x_3 + 17x_4 - 5x_5 &= 0 \\ -3x_1 - 6x_2 + 10x_3 + 27x_4 - 5x_5 &= 0 \\ -2x_1 + 5x_2 + 10x_3 + 5x_4 - 5x_5 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 0 \\ 2 \\ -\frac{3}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□

V10.3 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -3x_1 - 1x_2 + 1x_3 + 2x_4 + 3x_5 &= 0 \\ -5x_2 + 5x_3 + 5x_4 + 15x_5 &= 0 \\ -6x_1 - 2x_2 + 2x_3 + 4x_4 + 6x_5 &= 0 \\ 18x_1 - 3x_2 + 3x_3 - 3x_4 + 9x_5 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□

V10.4 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 3x_1 + 4x_2 + 4x_3 + 3x_4 &= 0 \\ 4x_1 + 3x_2 + 3x_3 + 2x_4 &= 0 \\ -3x_1 - 1x_2 - 1x_3 - 4x_4 &= 0 \\ -5x_1 + 3x_2 + 3x_3 + 5x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

□

V10.5 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -4x_1 - 6x_2 + 1x_3 - 3x_4 &= 0 \\ -2x_1 + 4x_3 - 5x_4 &= 0 \\ -1x_1 - 4x_2 + 1x_3 &= 0 \\ -1x_1 - 4x_2 + 2x_3 + 4x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\{\}.$$

□

V10.6 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -3x_1 + 9x_2 &= 0 \\ -1x_1 + 3x_2 - 4x_3 &= 0 \\ -3x_1 + 9x_2 - 2x_3 &= 0 \\ -3x_1 + 9x_2 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

□

V10.7 Find a basis for the solution space of the homogeneous system

$$\begin{aligned}-4x_1 - 8x_2 - 2x_3 + 4x_4 + 4x_5 &= 0\\ 4x_1 + 2x_2 + 2x_3 - 2x_4 &= 0\\ 8x_1 + 1x_2 + 4x_3 - 3x_4 + 2x_5 &= 0\\ -12x_1 + 6x_2 - 6x_3 + 2x_4 - 8x_5 &= 0\end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□

V10.8 Find a basis for the solution space of the homogeneous system

$$\begin{aligned}-3x_2 + 3x_3 - 3x_4 - 9x_5 &= 0\\ 3x_1 - 6x_2 - 3x_4 + 6x_5 &= 0\\ -5x_2 + 5x_3 + 4x_4 - 15x_5 &= 0\\ 4x_1 - 11x_2 + 3x_3 - 1x_4 - 1x_5 &= 0\end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ -3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□

V10.9 Find a basis for the solution space of the homogeneous system

$$\begin{aligned}5x_1 - 11x_2 + 4x_3 - 10x_4 - 3x_5 &= 0\\ 1x_1 - 2x_2 + 1x_3 - 2x_4 - 3x_5 &= 0\\ 1x_1 - 8x_2 - 5x_3 - 2x_4 + 3x_5 &= 0\\ -5x_1 + 20x_2 + 5x_3 + 10x_4 - 5x_5 &= 0\end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

□

V10.10 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -3x_1 - 1x_3 + 3x_4 &= 0 \\ &= 0 \\ -3x_1 + 5x_3 + 3x_4 &= 0 \\ -4x_1 + 3x_3 + 4x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□

V10.11 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -12x_1 - 5x_2 - 2x_3 - 6x_4 - 7x_5 &= 0 \\ -9x_1 - 4x_2 - 1x_3 + 5x_4 - 5x_5 &= 0 \\ -8x_1 - 1x_2 - 6x_3 - 1x_4 - 7x_5 &= 0 \\ 8x_1 + 2x_2 + 4x_3 + 4x_4 + 6x_5 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□

V10.12 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -5x_1 + 1x_2 - 1x_3 - 2x_4 &= 0 \\ -3x_1 - 6x_2 - 5x_3 + 1x_4 &= 0 \\ -2x_1 - 4x_2 - 5x_3 + 4x_4 &= 0 \\ -2x_1 - 1x_2 - 3x_3 - 2x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\{\}.$$

□

V10.13 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 4x_1 + 1x_2 + 5x_3 + 2x_4 &= 0 \\ -3x_1 - 1x_2 - 4x_3 - 4x_4 &= 0 \\ 4x_1 + 3x_2 + 7x_3 + 3x_4 &= 0 \\ 2x_1 - 2x_2 + 2x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

□

V10.14 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -12x_1 - 2x_2 + 4x_3 &= 0 \\ 15x_1 - 6x_2 - 5x_3 &= 0 \\ -6x_1 - 1x_2 + 2x_3 &= 0 \\ 15x_1 + 1x_2 - 5x_3 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} \frac{1}{3} \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□

V10.15 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 1x_1 + 1x_2 - 5x_3 &= 0 \\ 3x_1 - 3x_2 + 4x_3 &= 0 \\ -2x_1 + 3x_2 - 4x_3 &= 0 \\ -3x_1 + 2x_2 - 1x_3 &= 0 \end{aligned}$$

Solution. A basis is

$$\{\}.$$

□

V10.16 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 2x_1 + 6x_2 - 6x_3 - 4x_4 - 1x_5 &= 0 \\ -4x_1 + 2x_3 + 2x_4 &= 0 \\ 2x_1 + 6x_2 - 3x_3 - 4x_4 - 1x_5 &= 0 \\ 1x_1 - 3x_2 - 2x_3 + 1x_4 + 3x_5 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

□

V10.17 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 4x_1 - 6x_2 + 1x_4 &= 0 \\ -6x_1 + 5x_2 - 6x_3 - 1x_4 &= 0 \\ -5x_1 + 3x_3 + 2x_4 &= 0 \\ +3x_2 - 6x_3 - 2x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\{\}.$$

□

V10.18 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 2x_1 + 3x_2 - 1x_3 + 4x_4 &= 0 \\ -4x_1 - 6x_2 - 1x_3 + 5x_4 &= 0 \\ 4x_1 + 4x_2 + 4x_3 - 2x_4 &= 0 \\ -6x_1 + 2x_2 - 1x_3 - 4x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\{\}.$$

□

V10.19 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -1x_2 - 2x_4 + 2x_5 &= 0 \\ -3x_1 + 2x_2 - 4x_3 &= 0 \\ 5x_1 - 5x_2 + 7x_3 - 3x_4 + 3x_5 &= 0 \\ -3x_1 - 5x_2 + 15x_3 + 5x_4 - 5x_5 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 0 \\ -2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

$\square$

V10.20 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -3x_1 + 4x_2 + 5x_3 + 2x_4 &= 0 \\ 1x_1 + 4x_2 + 5x_3 + 6x_4 &= 0 \\ -2x_1 + 2x_2 - 2x_4 &= 0 \\ -1x_1 - 5x_2 - 3x_3 - 4x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}.$$

$\square$

V10.21 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -5x_1 + 7x_2 + 1x_3 - 9x_4 &= 0 \\ -4x_1 + 5x_2 + 1x_3 - 7x_4 &= 0 \\ 4x_1 - 17x_2 + 3x_3 + 11x_4 &= 0 \\ 1x_1 - 11x_2 + 3x_3 + 5x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{4}{3} \\ \frac{1}{3} \\ 0 \\ 1 \end{bmatrix} \right\}.$$

$\square$

V10.22 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -1x_1 - 6x_2 - 3x_3 &= 0 \\ 4x_1 - 4x_2 + 1x_3 &= 0 \\ 5x_1 + 1x_2 - 1x_3 &= 0 \\ 3x_1 + 5x_2 - 4x_3 &= 0 \end{aligned}$$

Solution. A basis is

$$\{\}.$$

□

V10.23 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 5x_1 - 1x_2 - 8x_3 &= 0 \\ -5x_2 + 10x_3 &= 0 \\ 2x_1 - 5x_2 + 6x_3 &= 0 \\ 1x_1 + 5x_2 - 12x_3 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}.$$

□

V10.24 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} -5x_1 + 20x_2 - 5x_3 &= 0 \\ -5x_1 + 10x_2 &= 0 \\ -2x_1 + 16x_2 - 6x_3 &= 0 \\ 4x_1 - 10x_2 + 1x_3 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} 1 \\ \frac{1}{2} \\ 1 \end{bmatrix} \right\}.$$

□

V10.25 Find a basis for the solution space of the homogeneous system

$$\begin{aligned} 8x_1 - 2x_2 + 2x_3 + 4x_4 &= 0 \\ -6x_1 - 3x_2 - 6x_3 - 3x_4 &= 0 \\ -1x_2 - 1x_3 &= 0 \\ 8x_1 - 6x_2 - 2x_3 + 4x_4 &= 0 \end{aligned}$$

Solution. A basis is

$$\left\{ \begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

□