



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2 - 5)$$

$$c \odot (x, y) = (cx, cy).$$

(a) Show that vector addition is associative, that is:

$$((x_1, y_1) \oplus (x_2, y_2)) \oplus (x_3, y_3) = (x_1, y_1) \oplus ((x_2, y_2) \oplus (x_3, y_3)).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (4x_1x_2, y_1 + 2y_2)$$

$$c \odot (x, y) = (cx, 0).$$

(a) Show that there exists an additive identity element, that is:

$$\text{There exists } (w, z) \in V \text{ such that } (x, y) \oplus (w, z) = (x, y).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 + 3, y_1 + y_2)$$

$$c \odot (x, y) = (cx, y^c).$$

(a) Show that vector addition is associative, that is:

$$((x_1, y_1) \oplus (x_2, y_2)) \oplus (x_3, y_3) = (x_1, y_1) \oplus ((x_2, y_2) \oplus (x_3, y_3)).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (3x_1 + x_2, y_1 + y_2)$$

$$c \odot (x, y) = (cx, cy).$$

(a) Show that scalar multiplication distributes over vector addition, that is:

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x, y) = (cx, cy - 7c + 7).$$

(a) Show that scalar multiplication is associative, that is:

$$a \odot (b \odot (x, y)) = (ab) \odot (x, y).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (2x_1 + 2x_2, 5y_1 + 5y_2)$$

$$c \odot (x, y) = (cx, cy).$$

(a) Show that scalar multiplication distributes over vector addition, that is:

$$c \odot ((x_1, y_1) \oplus (x_2, y_2)) = c \odot (x_1, y_1) \oplus c \odot (x_2, y_2).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2 - 6)$$

$$c \odot (x, y) = (cx, cy).$$

(a) Show that vector addition is associative, that is:

$$((x_1, y_1) \oplus (x_2, y_2)) \oplus (x_3, y_3) = (x_1, y_1) \oplus ((x_2, y_2) \oplus (x_3, y_3)).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x, y) = (4cx, 3cy).$$

(a) Show that scalar multiplication distributes over scalar addition, that is:

$$(c + d) \odot (x, y) = c \odot (x, y) \oplus d \odot (x, y).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x, y) = (cx, cy - 6c + 6).$$

(a) Show that scalar multiplication is associative, that is:

$$a \odot (b \odot (x, y)) = (ab) \odot (x, y).$$

(b) Explain why V nonetheless is not a vector space.



VS1

Vector spaces



Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$c \odot (x, y) = (3cx, 4cy).$$

(a) Show that scalar multiplication distributes over scalar addition, that is:

$$(c + d) \odot (x, y) = c \odot (x, y) \oplus d \odot (x, y).$$

(b) Explain why V nonetheless is not a vector space.



VS2



Linear combinations

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- $\begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ -4 \\ 6 \end{bmatrix}$.
- $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -3 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ -4 \\ 6 \end{bmatrix}$.

(b) Use these statements to determine if each vector is or is not a linear combination. If it is, give an example of such a linear combination.



VS2



Linear combinations

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- $\begin{bmatrix} 13 \\ -4 \\ 13 \\ -19 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -4 \\ 1 \\ -4 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 2 \\ 0 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 2 \\ 8 \\ -4 \end{bmatrix}$.
- $\begin{bmatrix} 14 \\ -5 \\ 12 \\ -18 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -4 \\ 1 \\ -4 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 2 \\ 0 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 2 \\ 8 \\ -4 \end{bmatrix}$.

(b) Use these statements to determine if each vector is or is not a linear combination. If it is, give an example of such a linear combination.



VS2



Linear combinations

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- $\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -6 \\ 12 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 5 \\ -8 \end{bmatrix}$.
- $\begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -6 \\ 12 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 5 \\ -8 \end{bmatrix}$.

(b) Use these statements to determine if each vector is or is not a linear combination. If it is, give an example of such a linear combination.



VS2



Linear combinations

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- $\begin{bmatrix} -1 \\ 2 \\ -5 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 10 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ -9 \\ 15 \end{bmatrix}$.
- $\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 10 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ -9 \\ 15 \end{bmatrix}$.

(b) Use these statements to determine if each vector is or is not a linear combination. If it is, give an example of such a linear combination.



VS3

Spanning sets



(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- The set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 3 \\ -5 \end{bmatrix} \right\}$ **spans** \mathbb{R}^4 .
- The set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 3 \\ -5 \end{bmatrix} \right\}$ does **not** span \mathbb{R}^4 .

(b) Explain how to determine which of these statements is true.



VS3



Spanning sets

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- The set of vectors $\left\{ \begin{bmatrix} -3 \\ -5 \\ 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 15 \\ 18 \\ -18 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ 2 \\ -14 \end{bmatrix}, \begin{bmatrix} -8 \\ -11 \\ 10 \\ -6 \end{bmatrix} \right\}$ **spans** \mathbb{R}^4 .
- The set of vectors $\left\{ \begin{bmatrix} -3 \\ -5 \\ 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 15 \\ 18 \\ -18 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ 2 \\ -14 \end{bmatrix}, \begin{bmatrix} -8 \\ -11 \\ 10 \\ -6 \end{bmatrix} \right\}$ does **not** span \mathbb{R}^4 .

(b) Explain how to determine which of these statements is true.



VS3

Spanning sets



(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- The set of vectors $\left\{ \begin{bmatrix} 5 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ -1 \\ 4 \end{bmatrix} \right\}$ **spans** \mathbb{R}^4 .
- The set of vectors $\left\{ \begin{bmatrix} 5 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ -1 \\ 4 \end{bmatrix} \right\}$ does **not** span \mathbb{R}^4 .

(b) Explain how to determine which of these statements is true.



VS3

Spanning sets



(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- The set of vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ -11 \\ -11 \\ -13 \end{bmatrix} \right\}$ **spans** \mathbb{R}^4 .
- The set of vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ -4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ -11 \\ -11 \\ -13 \end{bmatrix} \right\}$ does **not** span \mathbb{R}^4 .

(b) Explain how to determine which of these statements is true.



VS3

Spanning sets



(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

- The set of vectors $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -3 \\ 1 \end{bmatrix} \right\}$ **spans** \mathbb{R}^4 .
- The set of vectors $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -3 \\ 1 \end{bmatrix} \right\}$ does **not** span \mathbb{R}^4 .

(b) Explain how to determine which of these statements is true.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 7x + 6y = 0 \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 7xy^2 = 0 \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^2 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \middle| 2w + 3x = 5z \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \middle| 5x = z^2 - 2w + 2y \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^4 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 5x = 3y \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| x^2 + 5y = 0 \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^2 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| x^3 + 3y + 5z = 0 \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| 4x + 5y = 5z \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^3 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 3xy^2 = 0 \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 5x + 4y = 0 \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^2 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 3x = 3y \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 6xy^2 = 0 \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^2 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \left[\begin{array}{c} x \\ y \\ z \\ w \end{array} \right] \middle| -6w + 6y = 4x - 4z \right\} \quad W = \left\{ \left[\begin{array}{c} x \\ y \\ z \\ w \end{array} \right] \middle| x^3 + 4y + 4z = 2w \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^4 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \mid x + 4y + 2z = 5w \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \mid 6x = z^2 - 5w + 5y \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^4 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 5xy^2 = 0 \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 7x + 6y = 0 \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^2 and one is not.



VS4

Subspaces



Consider the following two sets of Euclidean vectors:

$$U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| 4x^3y + 5z = 0 \right\} \quad W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| 2x + y = 4z \right\}$$

Explain why one of these sets is a subspace of \mathbb{R}^3 and one is not.



VS5



Linear independence

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

• The set of vectors $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -11 \\ 2 \\ -1 \end{bmatrix} \right\}$ is linearly **independent**.

• The set of vectors $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -11 \\ 2 \\ -1 \end{bmatrix} \right\}$ is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS5



Linear independence

(a) Write a statement involving the solutions of a vector equation that's equivalent to each claim below.

• The set of vectors $\left\{ \begin{bmatrix} -3 \\ 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 5 \\ -4 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 8 \\ -6 \end{bmatrix} \right\}$ is linearly **independent**.

• The set of vectors $\left\{ \begin{bmatrix} -3 \\ 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 5 \\ -4 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 8 \\ -6 \end{bmatrix} \right\}$ is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS6



Basis identification

(a) Write a statement involving spanning and independence properties that's equivalent to each claim below.

- The set of vectors $\left\{ \begin{bmatrix} 0 \\ -3 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -9 \\ -3 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 6 \\ -2 \end{bmatrix} \right\}$ is a **basis** of \mathbb{R}^4 .
- The set of vectors $\left\{ \begin{bmatrix} 0 \\ -3 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -9 \\ -3 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 6 \\ -2 \end{bmatrix} \right\}$ is **not** a basis of \mathbb{R}^4 .

(b) Explain how to determine which of these statements is true.



VS6



Basis identification

(a) Write a statement involving spanning and independence properties that's equivalent to each claim below.

- The set of vectors $\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -6 \\ 6 \\ -7 \end{bmatrix} \right\}$ is a **basis** of \mathbb{R}^4 .
- The set of vectors $\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -6 \\ 6 \\ -7 \end{bmatrix} \right\}$ is **not** a basis of \mathbb{R}^4 .

(b) Explain how to determine which of these statements is true.



VS7



Basis of a subspace

Consider the following subspace W of \mathbb{R}^4 :

$$W = \text{span} \left\{ \begin{bmatrix} -2 \\ -1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -6 \\ -2 \\ 4 \\ -2 \end{bmatrix} \right\}.$$

- (a) Explain how to find a basis of W .
- (b) Explain how to find the dimension of W .



VS7



Basis of a subspace

Consider the following subspace W of \mathbb{R}^4 :

$$W = \text{span} \left\{ \begin{bmatrix} 0 \\ 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -6 \\ -6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -14 \\ -7 \\ 14 \end{bmatrix} \right\}.$$

- (a) Explain how to find a basis of W .
- (b) Explain how to find the dimension of W .



VS8

Polynomial and matrix spaces



(a) Given the set

$$\{x^3 - x^2 - x - 1, -x^3 - 2x^2 - 3x - 2, 4x^3 + 5x^2 + 8x + 5\}$$

write a statement involving the solutions to a polynomial equation that's equivalent to each claim below.

- The set of polynomials is linearly **independent**.
- The set of polynomials is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\{-2x^2 + 1, 6x^2 - 3, 2x^2 - 1, -4x^2 + 2, -2x^3 + 3x^2 + x - 5\}$$

write a statement involving the solutions to a polynomial equation that's equivalent to each claim below.

- The set of polynomials **spans** \mathcal{P}_3
- The set of polynomials does **not** span \mathcal{P}_3

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\{-2x^3 - 3x^2 - 4x + 4, 2x^3 + 3x^2 + x - 3, 12x^3 + 18x^2 + 15x - 21\}$$

write a statement involving the solutions to a polynomial equation that's equivalent to each claim below.

- The set of polynomials is linearly **independent**.
- The set of polynomials is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\{x^3 - 2x^2 + x + 2, 2x^2 - 1, -x^3 + 3x^2 + 3x - 2, x^3 - 6x^2 + 9x + 5\}$$

write a statement involving the solutions to a polynomial equation that's equivalent to each claim below.

- The set of polynomials is linearly **independent**.
- The set of polynomials is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\{-x + 1, -x + 1, 5x^3 + 4x^2 - 4x + 5, 15x^3 + 12x^2 - 13x + 16\}$$

write a statement involving the solutions to a polynomial equation that's equivalent to each claim below.

- The set of polynomials **spans** \mathcal{P}_3
- The set of polynomials does **not** span \mathcal{P}_3

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\{x^3 - 2x^2 + x + 2, 2x^2 - 1, -x^3 + 3x^2 + 3x - 2, x^3 - 6x^2 + 9x + 5\}$$

write a statement involving the solutions to a polynomial equation that's equivalent to each claim below.

- The set of polynomials is linearly **independent**.
- The set of polynomials is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\{-3x^3 + 2x^2 + 2x + 3, -x^3 + x^2 + x + 1, -2x^3 + 2x^2 + x, 4x^3 - x^2 - 2x - 6\}$$

write a statement involving the solutions to a polynomial equation that's equivalent to each claim below.

- The set of polynomials is linearly **independent**.
- The set of polynomials is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\left\{ \begin{bmatrix} -3 & -2 \\ -2 & -4 \end{bmatrix}, \begin{bmatrix} -3 & -2 \\ -2 & -4 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ -1 & -3 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 11 & 12 \\ 8 & 18 \end{bmatrix} \right\}$$

write a statement involving the solutions to a matrix equation that's equivalent to each claim below.

- The set of matrices **spans** $M_{2,2}$
- The set of matrices does **not** span $M_{2,2}$

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\left\{ \begin{bmatrix} -2 & -4 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ -5 & 2 \end{bmatrix}, \begin{bmatrix} 6 & 13 \\ -9 & -1 \end{bmatrix} \right\}$$

write a statement involving the solutions to a matrix equation that's equivalent to each claim below.

- The set of matrices is linearly **independent**.
- The set of matrices is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS8

Polynomial and matrix spaces



(a) Given the set

$$\left\{ \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}, \begin{bmatrix} 4 & -3 \\ 1 & -3 \end{bmatrix}, \begin{bmatrix} -2 & 3 \\ -4 & -2 \end{bmatrix} \right\}$$

write a statement involving the solutions to a matrix equation that's equivalent to each claim below.

- The set of matrices is linearly **independent**.
- The set of matrices is linearly **dependent**.

(b) Explain how to determine which of these statements is true.



VS9



Homogeneous systems

Consider the following homogeneous system of equations.

$$\begin{array}{rcccccl} x_1 & + & 4x_2 & - & 2x_3 & = & 0 \\ & & x_2 & & & = & 0 \\ & & 2x_2 & & & = & 0 \\ x_1 & + & 5x_2 & - & 2x_3 & = & 0 \\ & - & 3x_2 & & & = & 0 \end{array}$$

- (a) Find the solution space of this system.
- (b) Find a basis of the solution space.



VS9



Homogeneous systems

Consider the following homogeneous system of equations.

$$\begin{array}{rcccccc} & - & x_2 & - & x_3 & = & 0 \\ x_1 & + & 5x_2 & + & 7x_3 & = & 0 \\ -x_1 & - & 4x_2 & - & 6x_3 & = & 0 \\ -2x_1 & - & 4x_2 & - & 8x_3 & = & 0 \\ x_1 & + & 2x_2 & + & 4x_3 & = & 0 \end{array}$$

- (a) Find the solution space of this system.
- (b) Find a basis of the solution space.



VS9



Homogeneous systems

Consider the following homogeneous system of equations.

$$\begin{array}{rcccccccl} -x_1 & - & x_2 & + & 3x_3 & = & 0 \\ & & & & x_3 & = & 0 \\ -x_1 & - & x_2 & - & 3x_3 & = & 0 \\ -x_1 & - & x_2 & - & 5x_3 & = & 0 \\ x_1 & + & x_2 & + & x_3 & = & 0 \end{array}$$

- (a) Find the solution space of this system.
- (b) Find a basis of the solution space.



VS9



Homogeneous systems

Consider the following homogeneous system of equations.

$$\begin{array}{rcccccl} -3x_1 & - & 4x_2 & - & 2x_3 & = & 0 \\ -x_1 & - & 3x_2 & - & 4x_3 & = & 0 \\ -x_1 & & & + & 2x_3 & = & 0 \\ 2x_1 & + & 4x_2 & + & 4x_3 & = & 0 \\ x_1 & & & - & 2x_3 & = & 0 \end{array}$$

- (a) Find the solution space of this system.
- (b) Find a basis of the solution space.



VS9



Homogeneous systems

Consider the following homogeneous system of equations.

$$\begin{array}{rcccccccl} x_1 & - & 2x_2 & - & x_3 & & & = & 0 \\ -2x_1 & + & 4x_2 & + & 3x_3 & + & 4x_4 & = & 0 \\ -3x_1 & + & 6x_2 & + & 2x_3 & - & 3x_4 & = & 0 \\ -3x_1 & + & 6x_2 & + & 2x_3 & - & 2x_4 & = & 0 \end{array}$$

- (a) Find the solution space of this system.
- (b) Find a basis of the solution space.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(h(x)) = 2h(x) - 2h'(4) \quad \text{and} \quad T(h(x)) = 2h(x^3) - 4$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(g(x)) = 3g(x)^2 - 2g'(-4) \quad \text{and} \quad T(g(x)) = g(-4) + 4g(x^3)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(h(x)) = -4h(-1) - h(x^3) \quad \text{and} \quad T(h(x)) = -3h(x)^2 - 4h(x)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(f(x)) = -2f(-4) + 5f'(x) \quad \text{and} \quad T(f(x)) = -f(x)^2 - 4f(x)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(f(x)) = -f(x)^2 + 2f(x^3) \quad \text{and} \quad T(f(x)) = 5x^3f(x) - 4f'(-4)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(h(x)) = -3h(x^3) - 5h'(x) \quad \text{and} \quad T(h(x)) = -2h(x)^2 - h'(3)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(g(x)) = g'(x) + 1 \quad \text{and} \quad T(g(x)) = -5g(4) + 3g'(-1)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(h(x)) = -5h(-4) + 5h'(x) \quad \text{and} \quad T(h(x)) = 4x^2 - xh(x)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(h(x)) = -3h(x)^2 + h(5) \quad \text{and} \quad T(h(x)) = 2h(x^2) - h'(-3)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT1

Linear maps



Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(h(x)) = -2x - 2h(-5) \quad \text{and} \quad T(h(x)) = x^3 h(x) + 5h(x^2)$$

Explain why one these maps is a linear transformation and why the other map is not.



AT2



Standard matrices

- (a) Find the standard matrix for the linear transformation $S : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ given by

$$S \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} -2x - 3y - 5z - 6w \\ y + 4z - w \\ x + y + 4w \end{bmatrix}.$$

- (b) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} -2 & -1 & 4 \\ 3 & 1 & -7 \\ -5 & -4 & 8 \end{bmatrix}.$$

Compute $T \left(\begin{bmatrix} -4 \\ -3 \\ 0 \end{bmatrix} \right)$.



AT2

Standard matrices

- (a) Find the standard matrix for the linear transformation $S : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ given by

$$S \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} -2x + 2y + 6z \\ x + 6y + 8z \\ x + 2y + z \\ x + y \end{bmatrix}.$$

- (b) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation given by the standard matrix

$$\begin{bmatrix} -4 & 1 & 7 \\ -5 & 1 & 8 \end{bmatrix}.$$

Compute $T \left(\begin{bmatrix} -8 \\ -1 \\ -6 \end{bmatrix} \right)$.



AT3



Image and kernel

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} -x + y - 2z + 2w \\ x - y + z - 2w \\ -3x + 3y + 2z + 6w \end{bmatrix}.$$

- (a) Explain how to find the image of T and the kernel of T .
- (b) Explain how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain how to find the rank and nullity of T , and why the rank-nullity theorem holds for T .



AT3



Image and kernel

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} -y - 4z - 7w \\ x - y - 2w \\ -y - 3z - 6w \end{bmatrix}.$$

- (a) Explain how to find the image of T and the kernel of T .
- (b) Explain how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain how to find the rank and nullity of T , and why the rank-nullity theorem holds for T .



AT3



Image and kernel

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} x_1 - x_2 + x_3 - 5x_4 \\ x_2 + x_3 + 2x_4 \\ x_1 - x_2 + 2x_3 - 6x_4 \end{bmatrix}.$$

- (a) Explain how to find the image of T and the kernel of T .
- (b) Explain how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain how to find the rank and nullity of T , and why the rank-nullity theorem holds for T .



AT3



Image and kernel

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} x + 2y - 7w \\ -2x - 3y + 2z + 7w \\ x - y - 5z + 12w \end{bmatrix}.$$

- (a) Explain how to find the image of T and the kernel of T .
- (b) Explain how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain how to find the rank and nullity of T , and why the rank-nullity theorem holds for T .



AT3



Image and kernel

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} -x_1 + 2x_2 + 2x_4 \\ x_1 - 2x_2 - 4x_3 + 6x_4 \\ -2x_1 + 4x_2 + 5x_3 - 6x_4 \end{bmatrix}.$$

- (a) Explain how to find the image of T and the kernel of T .
- (b) Explain how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain how to find the rank and nullity of T , and why the rank-nullity theorem holds for T .



AT3



Image and kernel

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} x + y + z + 3w \\ z + w \\ -x - y - 5z - 7w \end{bmatrix}.$$

- (a) Explain how to find the image of T and the kernel of T .
- (b) Explain how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain how to find the rank and nullity of T , and why the rank-nullity theorem holds for T .



AT3



Image and kernel

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} x - 2y - 3z + 4w \\ 2x - 4y - 5z + 6w \\ x - 2y + 2z - 6w \end{bmatrix}.$$

- (a) Explain how to find the image of T and the kernel of T .
- (b) Explain how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain how to find the rank and nullity of T , and why the rank-nullity theorem holds for T .



AT4



Injectivity and surjectivity

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^5$ be the linear transformation given by the standard matrix
$$\begin{bmatrix} 0 & 0 & 3 \\ 0 & 1 & 5 \\ 3 & -3 & -5 \\ -1 & 1 & 5 \\ 1 & -1 & 2 \end{bmatrix}.$$

- (a) Explain why T is or is not injective.
- (b) Explain why T is or is not surjective.



AT4



Injectivity and surjectivity

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^5$ be the linear transformation given by the standard matrix
$$\begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 0 \\ -1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \end{bmatrix}.$$

- (a) Explain why T is or is not injective.
- (b) Explain why T is or is not surjective.



AT4



Injectivity and surjectivity

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation given by the standard matrix $\begin{bmatrix} -1 & 1 & 1 \\ -1 & 0 & -2 \\ 2 & 0 & 5 \end{bmatrix}$.

- (a) Explain why T is or is not injective.
- (b) Explain why T is or is not surjective.



AT4



Injectivity and surjectivity

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by the standard matrix $\begin{bmatrix} 1 & 1 & 1 & -2 \\ 4 & 5 & 1 & -6 \\ -3 & -4 & 1 & 3 \end{bmatrix}$.

- (a) Explain why T is or is not injective.
- (b) Explain why T is or is not surjective.



AT4



Injectivity and surjectivity

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be the linear transformation given by the standard matrix $\begin{bmatrix} -3 & 2 & 1 & 4 \\ 4 & -3 & 0 & -6 \\ 4 & -4 & 5 & -9 \\ 0 & 2 & -4 & 0 \end{bmatrix}$.

- (a) Explain why T is or is not injective.
- (b) Explain why T is or is not surjective.



MX1



Multiplying matrices

Of the following three matrices, only two may be multiplied.

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 4 \\ -1 & -4 \\ 0 & -5 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 2 & -4 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \\ 1 & -1 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -4 \\ 3 & -5 \\ 0 & 1 \end{bmatrix}$$

Explain which two can be multiplied and why. Then show how to find their product.



MX1



Multiplying matrices

Of the following three matrices, only two may be multiplied.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 1 & -2 \\ -1 & -2 \\ -1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 2 & -5 \\ 1 & -1 & 3 & -2 \end{bmatrix}$$

Explain which two can be multiplied and why. Then show how to find their product.



MX1

Multiplying matrices

Of the following three matrices, only two may be multiplied.

$$A = \begin{bmatrix} 1 & 0 & 1 & -3 \\ -1 & 1 & -1 & 6 \\ -2 & 1 & -1 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \\ -3 & -5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & -3 & -5 \\ -1 & 0 & 1 & 2 \end{bmatrix}$$

Explain which two can be multiplied and why. Then show how to find their product.



MX1

Multiplying matrices

Of the following three matrices, only two may be multiplied.

$$A = \begin{bmatrix} -3 & 1 & -2 \\ -4 & 1 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & -3 \\ 5 & 1 & -2 \\ 5 & -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 6 & -2 \\ 0 & 1 & 4 & -1 \end{bmatrix}$$

Explain which two can be multiplied and why. Then show how to find their product.



MX1



Multiplying matrices

Of the following three matrices, only two may be multiplied.

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 4 & -1 & 6 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 2 & -5 & -1 \\ -1 & 1 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -6 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Explain which two can be multiplied and why. Then show how to find their product.



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix Q that may be used to perform the row operation $R_2 - 5R_4 \rightarrow R_2$.
- (b) Give a 4×4 matrix N that may be used to perform the row operation $5R_2 \rightarrow R_2$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $5R_2 \rightarrow R_2$ and then $R_2 - 5R_4 \rightarrow R_2$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix B that may be used to perform the row operation $R_4 \leftrightarrow R_2$.
- (b) Give a 4×4 matrix N that may be used to perform the row operation $5R_4 \rightarrow R_4$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $5R_4 \rightarrow R_4$ and then $R_4 \leftrightarrow R_2$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix P that may be used to perform the row operation $R_3 + 4R_2 \rightarrow R_3$.
- (b) Give a 4×4 matrix N that may be used to perform the row operation $2R_3 \rightarrow R_3$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $2R_3 \rightarrow R_3$ and then $R_3 + 4R_2 \rightarrow R_3$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix Q that may be used to perform the row operation $-5R_1 \rightarrow R_1$.
- (b) Give a 4×4 matrix B that may be used to perform the row operation $R_1 - 4R_3 \rightarrow R_1$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $-5R_1 \rightarrow R_1$ and then $R_1 - 4R_3 \rightarrow R_1$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix M that may be used to perform the row operation $R_3 \leftrightarrow R_1$.
- (b) Give a 4×4 matrix Q that may be used to perform the row operation $3R_3 \rightarrow R_3$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $R_3 \leftrightarrow R_1$ and then $3R_3 \rightarrow R_3$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix N that may be used to perform the row operation $R_4 - 2R_1 \rightarrow R_4$.
- (b) Give a 4×4 matrix C that may be used to perform the row operation $R_2 \leftrightarrow R_4$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $R_2 \leftrightarrow R_4$ and then $R_4 - 2R_1 \rightarrow R_4$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix B that may be used to perform the row operation $4R_4 \rightarrow R_4$.
- (b) Give a 4×4 matrix M that may be used to perform the row operation $R_4 + 2R_1 \rightarrow R_4$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $4R_4 \rightarrow R_4$ and then $R_4 + 2R_1 \rightarrow R_4$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix M that may be used to perform the row operation $R_1 \leftrightarrow R_3$.
- (b) Give a 4×4 matrix C that may be used to perform the row operation $-4R_1 \rightarrow R_1$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $-4R_1 \rightarrow R_1$ and then $R_1 \leftrightarrow R_3$ to A (note the order).



MX2



Row operations as matrix multiplication

Let A be a 4×4 matrix.

- (a) Give a 4×4 matrix B that may be used to perform the row operation $R_4 \leftrightarrow R_2$.
- (b) Give a 4×4 matrix N that may be used to perform the row operation $R_2 - 2R_1 \rightarrow R_2$.
- (c) Use matrix multiplication to describe the matrix obtained by applying $R_4 \leftrightarrow R_2$ and then $R_2 - 2R_1 \rightarrow R_2$ to A (note the order).



MX3



The inverse of a matrix

Explain why each of the following matrices is or is not invertible by discussing its corresponding linear transformation. If the matrix is invertible, explain how to find its inverse.

$$B = \begin{bmatrix} 1 & -2 & -3 & 1 \\ 3 & -5 & -6 & 4 \\ 0 & -2 & -6 & -2 \\ 0 & 3 & 9 & 3 \end{bmatrix} \quad C = \begin{bmatrix} -5 & 3 & 5 & -1 \\ -2 & 1 & 2 & -1 \\ -2 & -1 & 3 & -4 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$



MX3



The inverse of a matrix

Explain why each of the following matrices is or is not invertible by discussing its corresponding linear transformation. If the matrix is invertible, explain how to find its inverse.

$$Q = \begin{bmatrix} 3 & -4 & 2 & -12 \\ 1 & -1 & 0 & -3 \\ -3 & 4 & -2 & 12 \\ -2 & 4 & -4 & 12 \end{bmatrix} \quad L = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & -3 & 0 & -3 \\ 0 & 4 & -3 & 0 \\ 0 & -1 & -4 & 1 \end{bmatrix}$$



MX3



The inverse of a matrix

Explain why each of the following matrices is or is not invertible by discussing its corresponding linear transformation. If the matrix is invertible, explain how to find its inverse.

$$A = \begin{bmatrix} 1 & 1 & -2 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 4 \\ 0 & 5 & -1 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 1 & 2 & -1 & 6 \\ 2 & -3 & 5 & -9 \\ -4 & 0 & -4 & 0 \end{bmatrix}$$



MX3



The inverse of a matrix

Explain why each of the following matrices is or is not invertible by discussing its corresponding linear transformation. If the matrix is invertible, explain how to find its inverse.

$$D = \begin{bmatrix} -1 & 1 & 1 & -1 \\ -2 & 2 & 1 & -4 \\ 2 & -2 & -3 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} \quad A = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 4 & -3 & 0 & -3 \\ -4 & 2 & 1 & 3 \\ -4 & 3 & 0 & 4 \end{bmatrix}$$



MX3



The inverse of a matrix

Explain why each of the following matrices is or is not invertible by discussing its corresponding linear transformation. If the matrix is invertible, explain how to find its inverse.

$$M = \begin{bmatrix} 0 & -1 & -2 & 2 \\ 0 & 1 & 1 & -4 \\ 1 & 0 & -3 & 1 \\ 1 & 0 & -3 & 2 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & -4 & 1 & 8 \\ 1 & 5 & -2 & -10 \\ -1 & -3 & 0 & 6 \end{bmatrix}$$



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant -4 .

- (a) Let C be the matrix obtained from A by applying the row operation $R_2 \leftrightarrow R_3$. What is $\det C$?
- (b) Let B be the matrix obtained from A by applying the row operation $-4R_1 \rightarrow R_1$. What is $\det B$?
- (c) Let M be the matrix obtained from A by applying the row operation $R_4 + 4R_3 \rightarrow R_4$. What is $\det M$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant 2.

- (a) Let P be the matrix obtained from A by applying the row operation $-3R_1 \rightarrow R_1$. What is $\det P$?
- (b) Let M be the matrix obtained from A by applying the row operation $R_4 + 2R_1 \rightarrow R_4$. What is $\det M$?
- (c) Let N be the matrix obtained from A by applying the row operation $R_3 \leftrightarrow R_1$. What is $\det N$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant -7 .

- (a) Let B be the matrix obtained from A by applying the row operation $R_3 \leftrightarrow R_4$. What is $\det B$?
- (b) Let M be the matrix obtained from A by applying the row operation $R_1 + 4R_4 \rightarrow R_1$. What is $\det M$?
- (c) Let C be the matrix obtained from A by applying the row operation $-4R_3 \rightarrow R_3$. What is $\det C$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant -7 .

- (a) Let C be the matrix obtained from A by applying the row operation $2R_3 \rightarrow R_3$. What is $\det C$?
- (b) Let P be the matrix obtained from A by applying the row operation $R_1 \leftrightarrow R_3$. What is $\det P$?
- (c) Let B be the matrix obtained from A by applying the row operation $R_3 + 2R_2 \rightarrow R_3$. What is $\det B$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant -7 .

- (a) Let M be the matrix obtained from A by applying the row operation $2R_3 \rightarrow R_3$. What is $\det M$?
- (b) Let Q be the matrix obtained from A by applying the row operation $R_1 \leftrightarrow R_2$. What is $\det Q$?
- (c) Let B be the matrix obtained from A by applying the row operation $R_4 + 5R_1 \rightarrow R_4$. What is $\det B$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant -5 .

- (a) Let N be the matrix obtained from A by applying the row operation $-5R_1 \rightarrow R_1$. What is $\det N$?
- (b) Let P be the matrix obtained from A by applying the row operation $R_4 + 2R_1 \rightarrow R_4$. What is $\det P$?
- (c) Let C be the matrix obtained from A by applying the row operation $R_3 \leftrightarrow R_4$. What is $\det C$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant -4 .

- (a) Let B be the matrix obtained from A by applying the row operation $-5R_2 \rightarrow R_2$. What is $\det B$?
- (b) Let C be the matrix obtained from A by applying the row operation $R_2 \leftrightarrow R_3$. What is $\det C$?
- (c) Let P be the matrix obtained from A by applying the row operation $R_4 + 2R_1 \rightarrow R_4$. What is $\det P$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant 5.

- (a) Let P be the matrix obtained from A by applying the row operation $R_4 \leftrightarrow R_1$. What is $\det P$?
- (b) Let B be the matrix obtained from A by applying the row operation $R_4 - 5R_1 \rightarrow R_4$. What is $\det B$?
- (c) Let M be the matrix obtained from A by applying the row operation $4R_3 \rightarrow R_3$. What is $\det M$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant -2 .

- (a) Let C be the matrix obtained from A by applying the row operation $R_2 + 2R_3 \rightarrow R_2$. What is $\det C$?
- (b) Let P be the matrix obtained from A by applying the row operation $-5R_3 \rightarrow R_3$. What is $\det P$?
- (c) Let N be the matrix obtained from A by applying the row operation $R_4 \leftrightarrow R_3$. What is $\det N$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant 4.

- (a) Let Q be the matrix obtained from A by applying the row operation $R_1 \leftrightarrow R_3$. What is $\det Q$?
- (b) Let M be the matrix obtained from A by applying the row operation $-4R_1 \rightarrow R_1$. What is $\det M$?
- (c) Let B be the matrix obtained from A by applying the row operation $R_2 - 2R_3 \rightarrow R_2$. What is $\det B$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant 2.

- (a) Let Q be the matrix obtained from A by applying the row operation $R_1 \leftrightarrow R_2$. What is $\det Q$?
- (b) Let M be the matrix obtained from A by applying the row operation $-5R_4 \rightarrow R_4$. What is $\det M$?
- (c) Let C be the matrix obtained from A by applying the row operation $R_3 - 3R_4 \rightarrow R_3$. What is $\det C$?



GT1

Row operations and determinants



Let A be a 4×4 matrix with determinant 5.

- (a) Let C be the matrix obtained from A by applying the row operation $R_2 \leftrightarrow R_1$. What is $\det C$?
- (b) Let N be the matrix obtained from A by applying the row operation $R_1 + 2R_4 \rightarrow R_1$. What is $\det N$?
- (c) Let Q be the matrix obtained from A by applying the row operation $-3R_2 \rightarrow R_2$. What is $\det Q$?



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} -3 & 1 & 5 & 4 \\ -2 & 5 & 5 & -1 \\ 5 & 0 & 2 & -3 \\ 2 & 1 & 0 & 0 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 3 & 0 & -1 & 2 \\ 5 & 0 & 0 & 2 \\ 3 & 3 & -1 & 5 \\ 3 & 1 & 5 & 4 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} -5 & -4 & -3 & -1 \\ 1 & 0 & 1 & 4 \\ 3 & 0 & -1 & 0 \\ -5 & 3 & -1 & 1 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & -2 & -3 & 4 \\ -3 & 5 & 4 & -2 \\ 2 & 1 & 0 & 0 \\ 1 & 5 & 2 & -2 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 5 & 2 & 3 & 0 \\ -3 & -4 & 1 & -1 \\ 0 & -1 & 0 & -3 \\ 3 & -4 & -1 & -4 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} -5 & -4 & 4 & 4 \\ 3 & 0 & -1 & 0 \\ -2 & 2 & -4 & 4 \\ -2 & -2 & 0 & 1 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 2 & 5 & 0 & 0 \\ 4 & 4 & 4 & -2 \\ 3 & 3 & -4 & 0 \\ -3 & -4 & -4 & -1 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 3 & 4 & 0 & -5 \\ -5 & 2 & -2 & -5 \\ -4 & 1 & -1 & 4 \\ -5 & 3 & 0 & 4 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 2 & 3 & 5 & -4 \\ -2 & 1 & 3 & -3 \\ -5 & 2 & -1 & -2 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} -5 & 0 & -5 & -4 \\ 5 & -2 & -1 & -5 \\ 0 & 2 & 0 & -1 \\ -5 & 2 & -3 & 5 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 3 & 0 & 1 & -2 \\ 2 & 4 & -5 & -1 \\ -4 & 4 & -5 & 0 \\ -3 & -3 & 3 & 0 \end{bmatrix}.$$



GT2

Determinants



Show how to compute the determinant of the matrix

$$A = \begin{bmatrix} 3 & 0 & -4 & 0 \\ 3 & -1 & -1 & -1 \\ 5 & 0 & 3 & -2 \\ 3 & -2 & 2 & 4 \end{bmatrix}.$$



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 8 & 2 \\ -14 & -8 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -1 & 1 \\ -2 & -4 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 5 & 2 \\ -9 & -6 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -1 & 1 \\ -2 & -4 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -3 & 1 \\ 6 & 2 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -1 & 1 \\ -8 & -7 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 10 & 2 \\ -21 & -3 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 9 & 1 \\ -15 & 1 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 5 & 1 \\ -2 & 2 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -1 & 2 \\ 6 & 3 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -3 & 1 \\ 6 & 2 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -2 & 2 \\ -4 & -8 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 5 & 2 \\ -9 & -6 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 12 & 2 \\ -51 & -11 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} -1 & 2 \\ 6 & -2 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 3 & 2 \\ -15 & -8 \end{bmatrix}$.



GT3

Eigenvalues



Explain how to find the eigenvalues of the matrix $\begin{bmatrix} 5 & 1 \\ -11 & -7 \end{bmatrix}$.



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -1 in the matrix

$$\begin{bmatrix} 2 & 4 & -1 & -5 \\ 0 & 0 & -3 & -9 \\ 1 & 1 & 0 & 2 \\ -2 & -2 & 3 & 5 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 2 in the matrix

$$\begin{bmatrix} 0 & -4 & 4 & 2 \\ -2 & -2 & 4 & 2 \\ 0 & 0 & 2 & 0 \\ -3 & -6 & 6 & 5 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -4 in the matrix

$$\begin{bmatrix} -3 & -1 & -3 & 3 \\ 2 & -5 & -2 & -2 \\ 2 & -2 & -9 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -2 in the matrix

$$\begin{bmatrix} -1 & 4 & 10 & 1 \\ 3 & -7 & -21 & 3 \\ 0 & -3 & -11 & 0 \\ 0 & 2 & 6 & -2 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -2 in the matrix

$$\begin{bmatrix} 2 & -4 & 4 & 12 \\ -1 & -1 & -1 & -3 \\ -4 & 4 & -6 & -12 \\ -4 & 4 & -4 & -14 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 2 in the matrix

$$\begin{bmatrix} 2 & 0 & -4 & -12 \\ 1 & 1 & -4 & -13 \\ 0 & 0 & 3 & 3 \\ 1 & -1 & -1 & -2 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 2 in the matrix

$$\begin{bmatrix} 3 & 1 & 0 & 1 \\ -1 & 2 & 2 & -2 \\ 3 & 0 & -3 & 5 \\ -5 & -2 & 2 & -2 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -2 in the matrix

$$\begin{bmatrix} -1 & -4 & -11 & 1 \\ -1 & -1 & 5 & -5 \\ -1 & -4 & -7 & -4 \\ 0 & 4 & 8 & 2 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -3 in the matrix

$$\begin{bmatrix} -3 & 0 & -1 & -2 \\ 1 & -6 & -3 & -18 \\ 1 & -4 & -7 & -23 \\ 0 & -1 & -1 & -8 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -2 in the matrix

$$\begin{bmatrix} -2 & -1 & 2 & 1 \\ 1 & -3 & 4 & -1 \\ 3 & -2 & 8 & -4 \\ -2 & 5 & -14 & -3 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 3 in the matrix

$$\begin{bmatrix} -1 & 8 & -12 & -4 \\ 2 & -1 & 6 & 2 \\ -5 & 10 & -12 & -5 \\ -3 & 6 & -9 & 0 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -3 in the matrix

$$\begin{bmatrix} -4 & -2 & 2 & -3 \\ -4 & -11 & 8 & -12 \\ -1 & -2 & -1 & -3 \\ -3 & -6 & 6 & -12 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -2 in the matrix

$$\begin{bmatrix} -1 & 1 & -2 & -1 \\ 5 & 3 & -10 & -5 \\ -4 & -4 & 6 & 4 \\ 5 & 5 & -10 & -7 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 2 in the matrix

$$\begin{bmatrix} 3 & -2 & 2 & -1 \\ 5 & -8 & 10 & -5 \\ 4 & -8 & 10 & -4 \\ -2 & 4 & -4 & 4 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 2 in the matrix

$$\begin{bmatrix} 3 & -1 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 4 & -4 & 10 & 4 \\ 2 & -2 & 4 & 4 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue -4 in the matrix

$$\begin{bmatrix} -1 & 2 & -5 & 3 \\ 1 & -3 & -4 & 2 \\ 1 & 1 & -7 & 2 \\ 2 & 0 & -3 & -6 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 3 in the matrix

$$\begin{bmatrix} 4 & 0 & 3 & -2 \\ 1 & 3 & 3 & -3 \\ 0 & 1 & 5 & -4 \\ 5 & -3 & 9 & 8 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 3 in the matrix

$$\begin{bmatrix} 6 & -9 & 9 & -3 \\ -4 & 15 & -12 & 4 \\ 2 & -6 & 9 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 1 in the matrix

$$\begin{bmatrix} 2 & 3 & 1 & 2 \\ -2 & -5 & -2 & -4 \\ 3 & 9 & 4 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$



GT4

Eigenvectors



Explain how to find a basis for the eigenspace associated to the eigenvalue 3 in the matrix

$$\begin{bmatrix} 5 & -2 & -5 & -4 \\ -1 & 4 & 5 & 7 \\ -2 & 2 & 7 & 2 \\ -1 & 1 & 1 & 2 \end{bmatrix}.$$