Exercise 1. Let G be a group. Show that the following are equivalent:

(i) $|G|$ is prime

(ii) G has exactly two subgroups (G and the trivial subgroup).

(iii) $G \cong \mathbb{Z}_p$ for some prime p.

Exercise 2. Let G be a group, and let H and K subgroups of finite index. Show that if $[G : H]$ and $[G : K]$ are coprime, then $G = HK$.

Exercise 3. Let H, K, and N be subgroups of a group G. Show that if $H < N$, then $HK \cap N = H(K \cap N)$.

Exercise 4. Show that every subgroup of index 2 is normal.

Exercise 5. Let $N = \{\sigma \in S_4 \mid \sigma(4) = 4\}$. Determine if N is a normal subgroup of S_4 or not.

Exercise 6. Let $Q = \langle i, j \mid i^4 = e, i^2 = j^2, iji = j\rangle$ (this is called the *quaternion group*). Show that every subgroup of Q is normal.