

Exercise 1. Let $f : G \rightarrow H$ be a group homomorphism and suppose $\ker f < N < G$ for some subgroup N . Show that if H is abelian, then N must be normal.

Exercise 2. Consider the set $N \leq S_4$ given by

$$N = \{(1), (12)(34), (13)(24), (14)(23)\}.$$

- (a) Verify that N is a subgroup.
- (b) Show that $N \triangleleft S_4$.
- (c) Conclude that A_4 is not simple.

Exercise 3. Show that A_4 has no subgroup of order 6.

Exercise 4. Show that A_n is the only subgroup of S_n that has index 2 (*Hint: show that an index 2 subgroup must contain a 3-cycle*).

Exercise 5. The dihedral group D_6 and the alternating group A_4 both have 12 elements. Determine if they are isomorphic or not.