Exercise 1. If G is a group, the *center* of G is the group

$$Z(G) = \{ a \in G \mid ax = xa \text{ for all } x \in G \}.$$

Show that the center is a normal subgroup of G.

Exercise 2. Consider the subgroup $N = \langle (123) \rangle$ of S_3 .

- (a) Show that N is a normal subgroup.
- (b) Describe S_3/N .

Exercise 3. Let $f: G \to H$ be a group homomorphism, and set $N = \ker f$. Let K < G be any subgroup.

- (a) Show that $f^{-1}(f(K)) = KN$.
- (b) Show that $f^{-1}(f(K)) = K$ if and only if N < K.

Exercise 4. Consider the subgroups $H = \langle 7 \rangle$ and $K = \langle 42 \rangle$ of \mathbb{Z} . Note that $K \triangleleft H$; describe H/K.

Exercise 5. Let G be a group. Show that if G/Z(G) is cyclic, then G is abelian.