Exercise 1. If G is a group, the *center* of G is the group

$$Z(G) = \{a \in G \mid ax = xa \text{ for all } x \in G\}.$$

Show that the center is a normal subgroup of G.

Exercise 2. Consider the subgroup $N = \langle (123) \rangle$ of S_3.

(a) Show that N is a normal subgroup.

(b) Describe S_3/N.

Exercise 3. Let $f : G \to H$ be a group homomorphism, and set $N = \ker f$. Let $K < G$ be any subgroup.

(a) Show that $f^{-1}(f(K)) = KN$.

(b) Show that $f^{-1}(f(K)) = K$ if and only if $N < K$.

Exercise 4. Consider the subgroups $H = \langle 7 \rangle$ and $K = \langle 42 \rangle$ of $\mathbb{Z}$. Note that $K \triangleleft H$; describe H/K.

Exercise 5. Let G be a group. Show that if $G/Z(G)$ is cyclic, then G is abelian.