

Exercise 1. Let G be a group with two normal subgroups, N and H . Suppose that $G = N \rtimes H$.

- (a) Show that $G = H \rtimes N$.
- (b) Show that $G = N \times H$.
- (c) Show that G is abelian if and only if N and H are abelian.

Exercise 2. Let G, H be finite cyclic groups. Show that $G \times H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.

Exercise 3. Show that S_3 is **not** a direct product of any of its proper subgroups.

Exercise 4. Show that S_n is a semidirect product.

Exercise 5. Show that a free abelian group is a free group if and only if it is cyclic.

Exercise 6. Show that the direct sum of free abelian groups is a free abelian group (Note that this is not true for direct products).