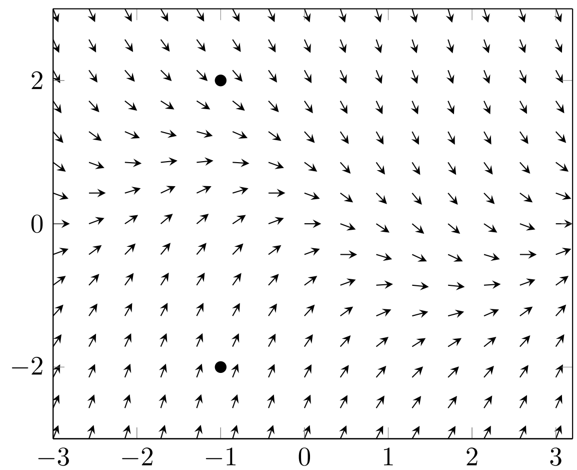
Module C

Standard C1

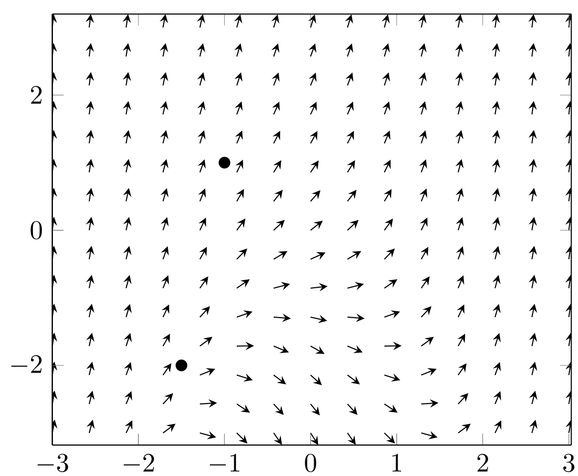
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = -y - \sin(x)$$



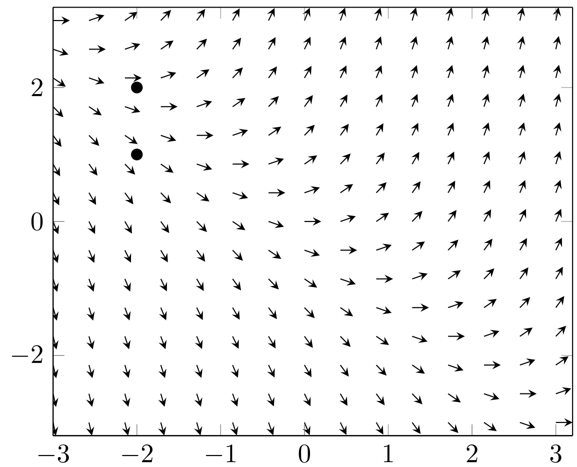
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 + y + 1$$



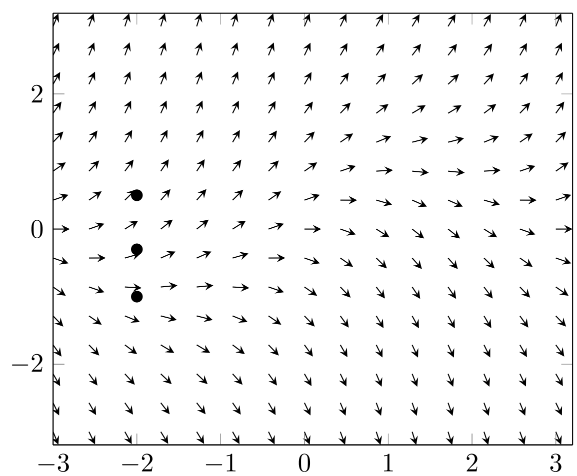
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = x + y$$



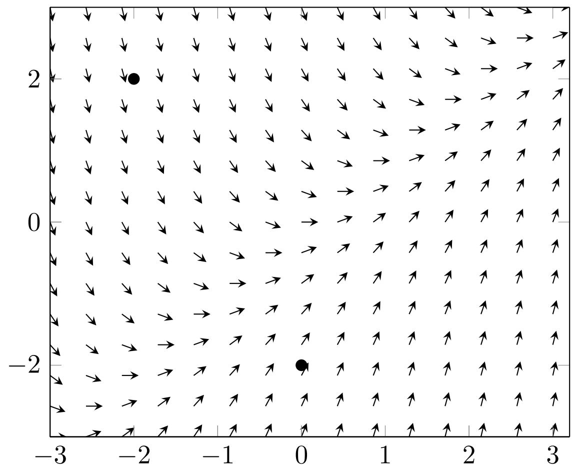
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = y - \sin(x)$$



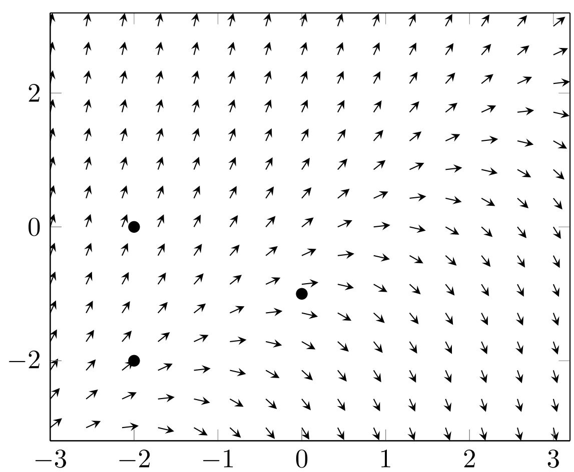
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = x - y$$



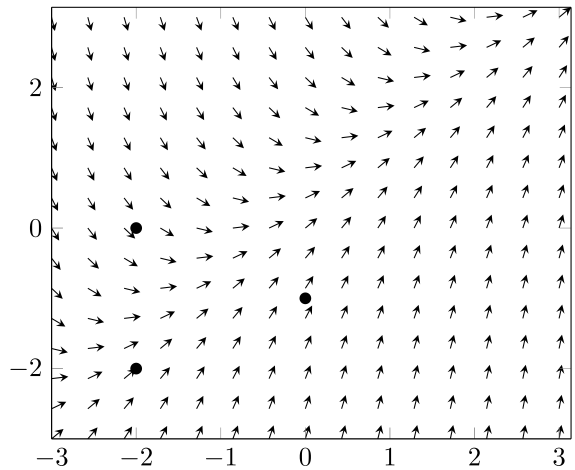
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = y - x + 1$$



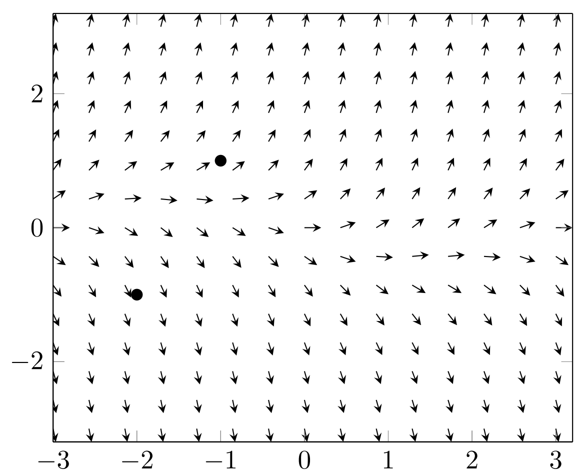
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = x - y + 1$$

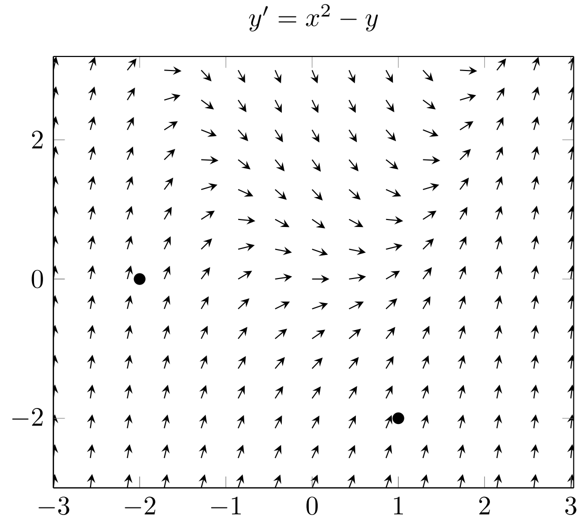


C1. Sketch a solution curve through each point marked in the slope field.

$$y' = \sin(x) + 2y$$



C1. Sketch a solution curve through each point marked in the slope field.


$y' = x^2 - y$
2
0
−2
−3
−2
−1
0
1
2
3

C2. Find the general solution to
$$y' + 3y = 6t + 5.$$

C2. Find the general solution to
$$y' + 4y = 4.$$

C2. Find the general solution to
$$y' + 2y = 6t - 1.$$

C2. Find the general solution to
$$y' - y = e^t.$$

C2. Find the general solution to
$$y' + y = e^t.$$

C2. Find the general solution to
$$y' - y = e^{-t}.$$

C2. Find the general solution to
$$y' + y = e^{-t}.$$

C2. Find the general solution to
$$y' + 3y = 10\sin(t).$$

C2. Find the general solution to
$$y' + 2y = 10\sin(t).$$

C2. Find the general solution to
$$y' + 2y = 5\sin(t).$$

C2. Find the general solution to
$$y' + 3y = 10\cos(t).$$

C2. Find the general solution to
$$y' + 2y = 10\cos(t).$$

C2. Find the general solution to

$$y' + 2y = 5\cos(t).$$

C3. Find the general solution to
$$y'' + 2y' + y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 8y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 3y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' - 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' - 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' + 5y' + 6y = 0.$$

C3. Find the general solution to
$$y'' - 2y' + 2y = 0.$$

C3. Find the general solution to
$$y'' + 2y' + 2y = 0.$$

C3. Find the general solution to
$$y'' - 6y' + 10y = 0.$$

C3. Find the general solution to
$$y'' + 6y' + 10y = 0.$$

C3. Find the general solution to

$$y'' - 2y' + 5y = 0.$$

C3. Find the general solution to

$$y'' + 2y' + 5y = 0.$$

C3. Find the general solution to

$$y'' - 4y' + 5y = 0.$$

C3. Find the general solution to

$$y'' + 4y' + 5y = 0.$$

C4. Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

C4. Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2\sin(3x)$$

C4. Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^{x}$$

C4. Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

C4. Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

C4. Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

C4. Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

C4. Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

C4. Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

C4. Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$

C4. Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

C5. Find the solution to
$$y'' + 2y' + y = 0$$
when $y(0) = 0$ and $y'(0) = 2$.

C5. Find the solution to
$$y'' + 2y' + y = 0$$
when $y(0) = 2$ and $y'(0) = 0$.

C5. Find the solution to
$$y'' + 2y' - 8y = 0$$
when $y(0) = 3$ and $y'(0) = -6$.

C5. Find the solution to
$$y'' + 4y' + 3y = 0$$
when $y(0) = 1$ and $y'(0) = 5$.

C5. Find the solution to
$$y'' + 2y' - 3y = 0$$
when $y(0) = 5$ and $y'(0) = 1$.

C5. Find the solution to
$$y'' + 2y' - 3y = 0$$
when $y(0) = 2$ and $y'(0) = 2$.

C5. Find the solution to
$$y'' - 2y' - 3y = 0$$
when $y(0) = 2$ and $y'(0) = 2$.

C5. Find the solution to
$$y'' + 4y' + 4y = 0$$
when $y(0) = 1$ and $y'(0) = 3$.

C5. Find the solution to
$$y'' - 4y' + 4y = 0$$
when $y(0) = 1$ and $y'(0) = 3$.

C5. Find the solution to
$$y'' + 4y' + 4y = 0$$
when $y(0) = 3$ and $y'(0) = 1$.

C5. Find the solution to
$$y'' - 4y' + 4y = 0$$
when $y(0) = 3$ and $y'(0) = 1$.

C5. Find the solution to
$$y'' + 5y' + 6y = 0$$

when $y(0) = 3$ and $y'(0) = 1$.

C5. Find the solution to

$$y'' + 5y' + 6y = 0$$

when $y(0) = 1$ and $y'(0) = 2$.

Module S

Standard S1

S1. Find the general solution of the system

$$x' = x + y,$$
$$y' = 4x + y.$$

S1. Find the general solution of the system

$$x' = x + 2y,$$
$$y' = 3x + 2y.$$

S1. Find the general solution of the system

$$x' = 2x + y,$$
$$y' = x + 2y.$$

S1. Find the general solution of the system

$$x' = 2x + y,$$
$$y' = 2x + 3y.$$

S1. Find the general solution of the system

$$x' = 3x + y,$$
$$y' = x + 3y.$$

S1. Find the general solution of the system

$$x' = 3x + y,$$
$$y' = 2x + 2y.$$

S1. Find the general solution of the system

$$\begin{aligned} x' &= 4x + y, \\ y' &= 2x + 3y. \end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned} x' &= 4x + 3y, \\ y' &= x + 2y. \end{aligned}$$

Module F

Standard F1

F1. Find the general solution to $\frac{dy}{dx} + 3xy = 0$.

F1. Find the general solution to $y' - y\sin(x) = 0$.

F1. Find the general solution to $y' = \frac{x+2}{y}$.

F1. Find the general solution to $\frac{dy}{dx} = \frac{1+x}{1+y}$.

F1. Find the general solution to $xy' = y$.

F1. Find the general solution to $y\frac{dy}{dx} = y^2\cos(x)$.

F1. Find the general solution to $xy\frac{dy}{dx} = 1$.

F1. Find the general solution to $x\cos(y)y' = 1$.

F2. Consider the autonomous equation
$$\frac{dx}{dt} = x - 3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F2. Consider the autonomous equation
$$\frac{dx}{dt} = 1 - x.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F2. Consider the autonomous equation
$$\frac{dx}{dt} = (x - 3)^2.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(1) = 2$.

F2. Consider the autonomous equation
$$\frac{dx}{dt} = (x + 4)^2.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(4) = 0$.

F2. Consider the autonomous equation
$$\frac{dx}{dt} = (4 - x)^3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(3) = 2$.

F2. Consider the autonomous equation
$$\frac{dx}{dt} = (5 - x)^3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F2. Consider the autonomous equation
$$\frac{dx}{dt} = x^2 - 7x + 10.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 3$.

F2. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(3) = 0$.

F2. Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(5) = 1$.

F2. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F2. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F2. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F3. Find the general solution to $xy' + 4y = 2x$.

F3. Find the general solution to $xy' + 4y = \sqrt{x}$ (for $x > 0$).

F3. Find the general solution to $xy' + 2y = x^2$.

F3. Find the general solution to $y' = 2 + x + 2y + xy$.

F3. Find the general solution to $y' = 1 + 2x + y + 2xy$.

F4. One of the two ODEs below is exact. Identify which one, and solve it.

$$(x + 2y)y' + y = 2x$$
$$(x + 2y)y' - y = -2x$$

F4. One of the two ODEs below is exact. Identify which one, and solve it.

$$(3x + 2y)y' + 3y = 2x$$
$$(3x + 2y)y' - 3y = -2x$$

F4. One of the two ODEs below is exact. Identify which one, and solve it.

$$(x^2 + 3y^2)y' - 2xy = -3x^2$$
$$(x^2 + 3y^2)y' + 2xy = 3x^2$$

F4. One of the two ODEs below is exact. Identify which one, and solve it.

$$(2xy + 3y^2)y' + y^2 = 3x^2$$
$$(2xy + 3y^2)y' - y^2 = -3x^2$$

F4. One of the two ODEs below is exact. Identify which one, and solve it.

$$\cos(x)\cos(y)y' = \sin(x)\sin(y)$$
$$\cos(x)\cos(y)y' = \sin(x) + \sin(y)$$

F4. One of the two ODEs below is exact. Identify which one, and solve it.

$$\sin(x)\sin(y)y' = \cos(x) + \cos(y)$$
$$\sin(x)\sin(y)y' = \cos(x)\cos(y)$$

F4. One of the two ODEs below is exact. Identify which one, and solve it.

$$(y^3e^x + xe^x)y' + 3e^xy^2 = 3x^2$$
$$(2ye^x + e^y)y' + e^xy^2 = 3x^2$$

Module N

Standard N1

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x^2y + xy^2; \qquad y(1) = 3$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = 2x^2 + xy + 3y^2; \qquad y(1) = -1$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x + \ln(y); \qquad y(1) = 2$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt{x+y}; \qquad y(1) = 1$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt[3]{x-y}; \qquad y(2) = 2$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \frac{y}{x}; \qquad y(2) = 1$$

N2. Consider the differential equation
$$xy'' + y' = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$xy'' - y' = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$x^2y'' - 4xy' + 4y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$x^2y'' - xy' - 3y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$x^2y'' + xy' + 4y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$y'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$e^x y'' - 2y' + 4e^{4x}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$y'' + y' - e^{-2x}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{3}{t}x + 2y,$$
$$y' = 2\ln(t)x + y + 1$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{2}{t}x + y,$$
$$y' = x + \ln(t)y + 2$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -x + \sqrt{t},$$
$$y' = 2x + ty + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + 2y + \sqrt{t},$$
$$y' = x + y + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + y + \sqrt[3]{t},$$
$$y' = x + 2y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = tx + 2y + \sqrt[3]{t},$$
$$y' = -y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + \ln(t)y + 2,$$
$$y' = -\frac{1}{t}y + 2t$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = 2\ln(t)x + y + 1,$$
$$y' = -\frac{2}{t}x + y$$

Module D

Standard D1

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t-5)\}(s) = \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{3t}\}(s) = \frac{1}{s-3}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4)+e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t)+u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{1+e^t\}(s) = \frac{1}{s} + \frac{1}{s-1}.$$