Module N: Numerical

How can we use numerical approximation methods to apply and solve unsolvable ODEs?

At the end of this module, students will be able to...

N1. **First Order Existence and Uniqueness.** ...determine when a unique solution exists for a first order ODE

N2. **Second Order Linear Existence and Uniqueness.** ...determine when a unique solution exists for a second order linear ODE

N3. **Systems Existence and Uniqueness.** ...determine when a unique solution exists for a system of first order ODEs

N4. **Euler's method for first order ODES.** ...use Euler's method to find approximate solution to first order ODEs

N5. **Euler's method for systems.** ...use Euler's method to find approximate solutions to systems of first order ODEs

Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- State the definition of a spanning set, and determine if a set of Euclidean vectors spans $\mathbb{R}^n$ **V4**.
- State the definition of linear independence, and determine if a set of Euclidean vectors is linearly dependent or independent **S1**.
- State the definition of a basis, and determine if a set of Euclidean vectors is a basis **S2,S3**.
- Find a basis of the solution space to a homogeneous system of linear equations **S6**.

Module N Section 1

Definition N.1.1

A **linear transformation** (also known as a **linear map**) is a map between vector spaces that preserves the vector space operations. More precisely, if V and W are vector spaces, a map $T : V \to W$ is called a linear transformation if

1. $T(\mathbf{v} + \mathbf{w}) = T(\mathbf{v}) + T(\mathbf{w})$ for any $\mathbf{v}, \mathbf{w} \in V$.
2. $T(c\mathbf{v}) = cT(\mathbf{v})$ for any $c \in \mathbb{R}, \mathbf{v} \in V$.

In other words, a map is linear when vector space operations can be applied before or after the transformation without affecting the result.