

Module D

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## Module D: Discontinuous functions in ODEs

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**How can we solve and apply ODEs involving functions that are not continuous?**

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At the end of this module, students will be able to...

**D1. Laplace Transform.** ...compute the Laplace transform of a function

**D2. Discontinuous ODEs.** ...solve initial value problems for ODEs with discontinuous coefficients

**D3. Modeling non-smooth motion.** ...model the motion of an object undergoing discontinuous acceleration

**D4. Modeling non-smooth oscillators.** ...model mechanical oscillators undergoing discontinuous acceleration

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## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Compute integrals by using integration by parts
- Evaluate improper integrals
- Use a partial fraction decomposition to rewrite a rational expression
- Model a mass-spring system (Standard C6)

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The following resources will help you prepare for this module.

- Compute integrals by using integration by parts  
<https://youtu.be/S1Bp9hZBqaQ>, <https://youtu.be/bZ8YAHDTFJ8>
- Evaluate improper integrals <https://youtu.be/qv7DM5Ph0vU>
- Use a partial fraction decomposition to rewrite a rational expression  
<https://youtu.be/HZTv4zCgEnA>, <https://youtu.be/ofQt2N0UpCg>
- Model a mass-spring system (Standard C6)

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# Module D Section 1

## Module D

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## Observation D.1.1

In this module, we want to learn how to model (and solve) situations with **discontinuous** force, such as

- Collisions
- Thrust that can be turned on and off instantly
- Applied voltages that can be turned on and off instantly

Today we will learn how to model these forces, and introduce a tool called them **Laplace Transform** that we will use to solve the resulting IVPs.

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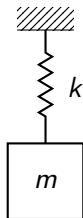
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**Activity D.1.2** ( $\sim 5$  min)

A 4 kg mass is hung from a spring with spring constant  $k = 16$  N/m. The mass is at rest, when it is hit with a hammer imparting 3Ns of upward impulse.



Write an initial value problem modelling this system.



### Definition D.1.3

The **Dirac delta distribution**  $\delta(t)$  models the application of instantaneous force. **It is not a function**, but makes sense in definite integrals:

If  $a, b$  is any open interval containing 0, then

$$\int_a^b f(t)\delta(t)dt = f(0)$$

for any function  $f(t)$  that is continuous around 0.

Thus, we can model the situation in the previous activity by

$$4y'' + 16y = 3\delta(t), \quad y(0) = 0, \quad y'(0) = 0$$

## Definition D.1.4

We can make sense of  $\delta$  in another way: as the “derivative” of a non-differentiable function.

The **unit impulse function**  $u(t)$  is given by

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0. \end{cases}$$

Note that  $u(s) = \int_{-\infty}^s \delta(t) dt$ ; in this fuzzy sense,  $\delta$  is the “derivative” of  $u(t)$  (which is not differentiable everywhere!)

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**Activity D.1.5** (*~10 min*)

Try to solve the IVP

$$y'' + y = \delta(t)$$

Where does our existing technique break down?

## Observation D.1.6

To get around this difficulty, we will apply an **integral transform** called the **Laplace Transform** to our ODE.

- We want to use a definite integral to handle things like  $\delta$ , which we can only understand via a definite integral.
- Since we are focused on IVPs, we can integrate starting at 0, but need to go to  $\infty$
- But now we need to worry about convergence—thus we will multiply by a suitable function that decays fast enough to make most functions converge.

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**Activity D.1.7** ( $\sim 5$  min)

Arrange the following functions in order of how fast they decay to zero in the limit at infinity:

- (A)  $x^{-n}$  for a positive integer  $n$
- (B)  $e^{-ax}$  for a positive integer  $a$
- (C)  $\frac{1}{\ln(ax)}$  for a positive integer  $a$
- (D)  $\frac{1}{\ln(x^n)}$  for a positive integer  $n$

## Definition D.1.8

The **Laplace Transform** of a function  $f(t)$  is the function

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

Note that the Laplace transform turns a function of  $t$  into a function of  $s$ .

Moreover,  $\mathcal{L}$  is linear:  $\mathcal{L}\{f + g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$ , and  $\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$  for constants  $c$ .

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**Activity D.1.9** (*~5 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) \, dt.$$

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**Activity D.1.9** ( $\sim 5$  min)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Compute  $\mathcal{L}\{\delta(t)\}$



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**Activity D.1.9** (*~5 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Compute  $\mathcal{L}\{\delta(t)\}$

*Part 2:* If  $a > 0$ , compute  $\mathcal{L}\{\delta(t - a)\}$

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**Activity D.1.10** (*~5 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) \, dt.$$

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**Activity D.1.10** (*~5 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Compute  $\mathcal{L}\{e^t\}$

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**Activity D.1.10** (*~5 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) \, dt.$$

*Part 1:* Compute  $\mathcal{L}\{e^t\}$

*Part 2:* If  $a > 0$ , compute  $\mathcal{L}\{e^{at}\}$

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## Module D Section 2

## Observation D.2.1

Last week, we encountered the **Laplace Transform** of a function  $f(t)$ :

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

Recall that the Laplace transform turns a function of  $t$  into a function of  $s$ .

Moreover,  $\mathcal{L}$  is linear:  $\mathcal{L}\{f + g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$ , and  $\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$  for constants  $c$ .

Our goal for today is to develop a few more properties of  $\mathcal{L}$ , and then see how to use it to solve IVPs (standards D1,D2).

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## Observation D.2.2

We computed a few Laplace Transforms:

- $\mathcal{L}\{\delta(t)\} = 1$
- $\mathcal{L}\{\delta(t - a)\} = e^{-as}$  for any  $a > 0$ .
- $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

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**Activity D.2.3** (*~15 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) \, dt.$$



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**Activity D.2.3** (*~15 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) \, dt.$$

*Part 1:* Compute  $\mathcal{L}\{1\}$

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**Activity D.2.3** (*~15 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) \, dt.$$

*Part 1:* Compute  $\mathcal{L}\{1\}$ *Part 2:* Compute  $\mathcal{L}\{t\}$

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**Activity D.2.3** (*~15 min*)

Recall that

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Compute  $\mathcal{L}\{1\}$ *Part 2:* Compute  $\mathcal{L}\{t\}$ *Part 3:* Compute  $\mathcal{L}\{t^2\}$

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**Activity D.2.4** (*~10 min*)

Recall

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

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**Activity D.2.4** (*~10 min*)

Recall

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Compute  $\mathcal{L}\{\sin(t)\}$  (**Hint:**

$$\int e^{ax} \sin(bx) \, dx = \frac{1}{a^2 + b^2} e^{ax} (a \sin(bx) - b \cos(bx)) + C.)$$

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**Activity D.2.4** (*~10 min*)

Recall

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Compute  $\mathcal{L}\{\sin(t)\}$  (**Hint:**

$$\int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} (a \sin(bx) - b \cos(bx)) + C.)$$

*Part 2:* Compute  $\mathcal{L}\{\cos(t)\}$ .

## Observation D.2.5

So now our list of Laplace transforms is:

- $\mathcal{L}\{\delta(t - a)\} = e^{-as}$  for any  $a > 0$ .
- $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$
- $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$  for any positive integer  $n$ .
- $\mathcal{L}\{1\} = \frac{1}{s}$
- $\mathcal{L}\{\sin(t)\} = \frac{1}{s^2+1}$
- $\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}$

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**Activity D.2.6** (*~10 min*)

Suppose we want to apply the Laplace transform to an IVP: we will need to know how  $\mathcal{L}\{y'\}$  is related to  $\mathcal{L}\{y\}$ . Recall

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$



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**Activity D.2.6** (*~10 min*)

Suppose we want to apply the Laplace transform to an IVP: we will need to know how  $\mathcal{L}\{y'\}$  is related to  $\mathcal{L}\{y\}$ . Recall

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Use integration by parts to relate  $\mathcal{L}\{y'\}$  to  $\mathcal{L}\{y\}$ .

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**Activity D.2.6** ( $\sim 10$  min)

Suppose we want to apply the Laplace transform to an IVP: we will need to know how  $\mathcal{L}\{y'\}$  is related to  $\mathcal{L}\{y\}$ . Recall

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

*Part 1:* Use integration by parts to relate  $\mathcal{L}\{y'\}$  to  $\mathcal{L}\{y\}$ .

*Part 2:* Use the fact that  $\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$  to relate  $\mathcal{L}\{y''\}$  to  $\mathcal{L}\{y\}$ .

## Observation D.2.7

We have

$$\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$$

$$\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0)$$

This allows us to easily rewrite expressions like  $ay'' + by' + cy$  in terms of  $\mathcal{L}\{y\}$ .

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**Activity D.2.8** (*~15 min*)

Consider the simple IVP

$$y'' + y = \delta(t), \quad y(0) = 0, y'(0) = 0.$$

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**Activity D.2.8** (*~15 min*)

Consider the simple IVP

$$y'' + y = \delta(t), \quad y(0) = 0, y'(0) = 0.$$

*Part 1:* Apply the Laplace transform to this IVP, and simplify. Solve for  $\mathcal{L}\{y\}$ .

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**Activity D.2.8** (*~15 min*)

Consider the simple IVP

$$y'' + y = \delta(t), \quad y(0) = 0, y'(0) = 0.$$

*Part 1:* Apply the Laplace transform to this IVP, and simplify. Solve for  $\mathcal{L}\{y\}$ .

*Part 2:* Find a function  $y$  satisfying  $\mathcal{L}\{y\} = \frac{1}{s^2+1}$ . We write  $y = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$ .

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**Observation D.3.1**

To solve a linear IVP using Laplace transforms:

- 1) Apply  $\mathcal{L}$  to the ODE. Use the initial condition(s) in computing  $\mathcal{L}\{y'\}$ ,  $\mathcal{L}\{y''\}$ , etc.
- 2) Solve for  $\mathcal{L}\{y\}$ .
- 3) Take the inverse transform (using a table) to find the solution  $y$ .

Today our goal is to practice the last step (taking the inverse transform), and then practice solving IVPs this way.



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**Activity D.3.2** ( $\sim 5$  min)Compute  $\mathcal{L}^{-1} \left\{ \frac{1}{s+5} \right\}$ .

(a)  $u(t-5)$

(b)  $\delta(t-5)$

(c)  $e^{5t}$

(d)  $e^{-5t}$

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**Activity D.3.3** ( $\sim 5$  min)Compute  $\mathcal{L}^{-1} \left\{ \frac{e^{-10s}}{s} \right\}$ .

(a)  $u(t - 10)$

(b)  $\delta(t - 10)$

(c)  $u(t - 10)e^{-t}$

(d)  $\delta(t - 10)e^{-t}$

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**Activity D.3.4** ( $\sim 5$  min)Compute  $\mathcal{L}^{-1} \left\{ \frac{2e^{-2s}}{s^2+4} \right\}$ .

(a)  $u(t) \sin(2t)$

(b)  $u(t-2) \sin(2t)$

(c)  $u(t-2) \sin(2t-2)$

(d)  $u(t-2) \sin(2t-4)$

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**Activity D.3.5** ( $\sim 5$  min)Compute  $\mathcal{L}^{-1} \left\{ \frac{e^{-100s}}{s^2} \right\}$ .

(a)  $u(t)t$

(b)  $u(t)(t - 100)$

(c)  $u(t - 100)t$

(d)  $u(t - 100)(t - 100)$

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**Activity D.3.6** (*~15 min*)

Solve the IVP

$$500v' = 20(u(t) - u(t - 100)), \quad v(0) = 50.$$

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**Activity D.3.7** (*~15 min*)

Solve the IVP

$$y'' + 4y = \delta(t - 2),$$

$$y(0) = 0, \quad y'(0) = 1$$

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## Observation D.4.1

Today we will practice modeling an object undergoing discontinuous acceleration (standard D3).



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**Activity D.4.2** ( $\sim 30$  min)

A spacecraft weighing 500 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 20 N of force) are turned on and burn for 100 s.

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**Activity D.4.2** ( $\sim 30$  min)

A spacecraft weighing 500 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 20 N of force) are turned on and burn for 100 s.

*Part 1:* Write down a function modelling the thrust force on the spacecraft.

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**Activity D.4.2** ( $\sim 30$  min)

A spacecraft weighing 500 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 20 N of force) are turned on and burn for 100 s.

*Part 1:* Write down a function modelling the thrust force on the spacecraft.

*Part 2:* Write down an IVP modelling the velocity of the spacecraft.

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**Activity D.4.2** ( $\sim 30$  min)

A spacecraft weighing 500 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 20 N of force) are turned on and burn for 100 s.

*Part 1:* Write down a function modelling the thrust force on the spacecraft.

*Part 2:* Write down an IVP modelling the velocity of the spacecraft.

*Part 3:* Solve the IVP:

$$500v' = 20(u(t) - u(t - 100)), \quad v(0) = 50.$$

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**Activity D.4.2** ( $\sim 30$  min)

A spacecraft weighing 500 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 20 N of force) are turned on and burn for 100 s.

*Part 1:* Write down a function modelling the thrust force on the spacecraft.

*Part 2:* Write down an IVP modelling the velocity of the spacecraft.

*Part 3:* Solve the IVP:

$$500v' = 20(u(t) - u(t - 100)), \quad v(0) = 50.$$

*Part 4:* What is its velocity after 200 s?

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## Observation D.5.1

Last week we saw how to use the Laplace transform to model a spacecraft undergoing discontinuous acceleration.

Today we will model springs undergoing discontinuous acceleration (standard D4).

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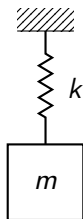
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**Activity D.5.2** ( $\sim 10$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 1$  N/m. The mass is at rest, when it is hit with a hammer imparting 3Ns of upward impulse.





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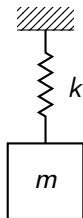
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**Activity D.5.2** ( $\sim 10$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 1$  N/m. The mass is at rest, when it is hit with a hammer imparting 3Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

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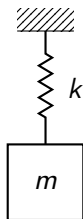
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**Activity D.5.2** ( $\sim 10$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 1$  N/m. The mass is at rest, when it is hit with a hammer imparting 3Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

*Part 2:* Use the Laplace transform to solve this IVP.

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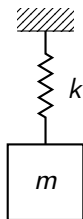
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**Activity D.5.2** ( $\sim 10$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 1$  N/m. The mass is at rest, when it is hit with a hammer imparting 3Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

*Part 2:* Use the Laplace transform to solve this IVP.

*Part 3:* When will the mass first return to equilibrium?

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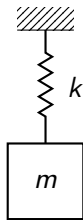
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**Activity D.5.3** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m. The mass is pulled down 1 m and released from rest. 10 seconds later, it is hit with a hammer imparting 5Ns of upward impulse.



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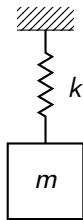
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**Activity D.5.3** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m. The mass is pulled down 1 m and released from rest. 10 seconds later, it is hit with a hammer imparting 5Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

## Module D

Section D.1

Section D.2

Section D.3

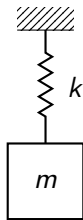
Section D.4

Section D.5

Section D.6

**Activity D.5.3** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m. The mass is pulled down 1 m and released from rest. 10 seconds later, it is hit with a hammer imparting 5Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

*Part 2:* Use the Laplace transform to solve this IVP.

## Module D

Section D.1

Section D.2

Section D.3

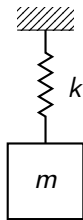
Section D.4

Section D.5

Section D.6

**Activity D.5.3** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m. The mass is pulled down 1 m and released from rest. 10 seconds later, it is hit with a hammer imparting 5Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

*Part 2:* Use the Laplace transform to solve this IVP.

*Part 3:* Where is the mass after 15 s?

## Module D

Section D.1

Section D.2

Section D.3

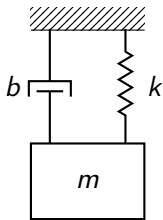
Section D.4

Section D.5

Section D.6

**Activity D.5.4** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m and a  $b = 4$  kg/s<sup>2</sup> linear damper. The mass is pulled down 1 m and released from rest. 5 seconds later, it is hit with a hammer imparting 10Ns of upward impulse.





## Module D

Section D.1

Section D.2

Section D.3

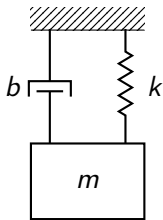
Section D.4

Section D.5

Section D.6

**Activity D.5.4** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m and a  $b = 4$  kg/s<sup>2</sup> linear damper. The mass is pulled down 1 m and released from rest. 5 seconds later, it is hit with a hammer imparting 10Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

## Module D

Section D.1

Section D.2

Section D.3

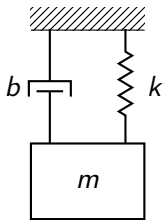
Section D.4

Section D.5

Section D.6

**Activity D.5.4** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m and a  $b = 4$  kg/s<sup>2</sup> linear damper. The mass is pulled down 1 m and released from rest. 5 seconds later, it is hit with a hammer imparting 10Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

*Part 2:* Use the Laplace transform to solve this IVP.

## Module D

Section D.1

Section D.2

Section D.3

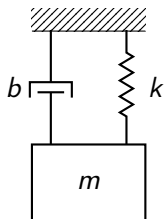
Section D.4

Section D.5

Section D.6

**Activity D.5.4** ( $\sim 15$  min)

A 1 kg mass is hung from a spring with spring constant  $k = 4$  N/m and a  $b = 4$  kg/s<sup>2</sup> linear damper. The mass is pulled down 1 m and released from rest. 5 seconds later, it is hit with a hammer imparting 10Ns of upward impulse.



*Part 1:* Write an initial value problem modelling this system.

*Part 2:* Use the Laplace transform to solve this IVP.

*Part 3:* Where is the mass after 15 s?

Module D

Section D.1

Section D.2

Section D.3

Section D.4

Section D.5

**Section D.6**

## Module D Section 6