### Module C

### Standard C1

C1. Find the general solution to

$$y' + 3y = 6t + 5.$$

C1. Find the general solution to

$$y' + 4y = 4.$$

C1. Find the general solution to

$$y' + 2y = 6t - 1.$$

C1. Find the general solution to

$$y' - y = e^t.$$

C1. Find the general solution to

$$y' + y = e^t.$$

C1. Find the general solution to

$$y' - y = e^{-t}.$$

C1. Find the general solution to

$$y' + y = e^{-t}.$$

C1. Find the general solution to

$$y' + 3y = 10\sin(t).$$

C1. Find the general solution to

$$y' + 2y = 10\sin(t).$$

C1. Find the general solution to

$$y' + 2y = 5\sin(t).$$

C1. Find the general solution to

$$y' + 3y = 10\cos(t).$$

 ${f C1.}$  Find the general solution to

$$y' + 2y = 10\cos(t).$$

C1. Find the general solution to

$$y' + 2y = 5\cos(t).$$

- C2. A water droplet with a radius of 100  $\mu$ m has a mass of about  $4 \times 10^{-12}$ kg and a terminal velocity of 27  $\frac{\text{cm}}{\text{s}}$ . Such a droplet is dropped from rest.
- (a) Write down an IVP modelling the velocity.
- (b) What is its velocity after 0.01 s?
- C2. A water droplet with a radius of 100  $\mu$ m has a mass of about  $4 \times 10^{-12}$ kg. Such a droplet is dropped from rest.
- (a) Write down an IVP modelling the velocity.
- (b) After 0.02 s it has reached half of its terminal velocity. What is its terminal velocity?
- C2. A water droplet with a radius of 10  $\mu$ m has a mass of about  $4 \times 10^{-15}$ kg and a terminal velocity of 270  $\frac{\mu m}{s}$ . Such a droplet is dropped from rest.
- (a) Write down an IVP modelling the velocity.
- (b) What is its velocity after 0.001 s?
- C2. A water droplet with a radius of 10  $\mu$ m has a mass of about  $4 \times 10^{-15}$ kg. Such a droplet is dropped from rest.
- (a) Write down an IVP modelling the velocity.
- (b) After 0.002 s it has reached half of its terminal velocity. What is its terminal velocity?
- C2. A single grain of corn pollen with a radius of 50  $\mu$ m and a mass of about  $5 \times 10^{-13}$ kg has a terminal velocity of 27  $\frac{\text{cm}}{\text{s}}$ . Such a pollen grain is dropped from rest.
- (a) Write down an IVP modelling the velocity.
- (b) What is its velocity after 0.01 s?
- C2. A single grain of corn pollen with a radius of 50  $\mu$ m and a mass of about  $5 \times 10^{-13}$ kg. Such a pollen grain is dropped from rest.
- (a) Write down an IVP modelling the velocity.
- (b) After 0.01 s it has reached half of its terminal velocity. What is its terminal velocity?

$$y'' + 2y' + y = 0.$$

$$y'' + 2y' - 8y = 0.$$

$$y'' + 4y' + 3y = 0.$$

$$y'' + 2y' - 3y = 0.$$

$$y'' - 2y' - 3y = 0.$$

$$y'' + 4y' + 4y = 0.$$

$$y'' - 4y' + 4y = 0.$$

$$y'' + 5y' + 6y = 0.$$

$$y'' - 2y' + 2y = 0.$$

$$y'' + 2y' + 2y = 0.$$

$$y'' - 6y' + 10y = 0.$$

$$y'' + 6y' + 10y = 0.$$

$$y'' - 2y' + 5y = 0.$$

$$y'' + 2y' + 5y = 0.$$

$${f C3.}$$
 Find the general solution to

$$y'' - 4y' + 5y = 0.$$

$$y'' + 4y' + 5y = 0.$$

$$y'' + 2y' + y = 0$$

when 
$$y(0) = 0$$
 and  $y'(0) = 2$ .

$$y'' + 2y' + y = 0$$

when 
$$y(0) = 2$$
 and  $y'(0) = 0$ .

$$y'' + 2y' - 8y = 0$$

when 
$$y(0) = 3$$
 and  $y'(0) = -6$ .

$$y'' + 4y' + 3y = 0$$

when 
$$y(0) = 1$$
 and  $y'(0) = 5$ .

$$y'' + 2y' - 3y = 0$$

when 
$$y(0) = 5$$
 and  $y'(0) = 1$ .

$$y'' + 2y' - 3y = 0$$

when 
$$y(0) = 2$$
 and  $y'(0) = 2$ .

$$y'' - 2y' - 3y = 0$$

when 
$$y(0) = 2$$
 and  $y'(0) = 2$ .

$$y'' + 4y' + 4y = 0$$

when 
$$y(0) = 1$$
 and  $y'(0) = 3$ .

$$y'' - 4y' + 4y = 0$$

when 
$$y(0) = 1$$
 and  $y'(0) = 3$ .

$$C4.$$
 Find the solution to

$$y'' + 4y' + 4y = 0$$

when 
$$y(0) = 3$$
 and  $y'(0) = 1$ .

$$y'' - 4y' + 4y = 0$$

when 
$$y(0) = 3$$
 and  $y'(0) = 1$ .

$$y'' + 5y' + 6y = 0$$

when y(0) = 3 and y'(0) = 1.

C4. Find the solution to

$$y'' + 5y' + 6y = 0$$

when y(0) = 1 and y'(0) = 2.

C5. Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2\sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^x$$

C5. Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

- **C6.** Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant 4kg/s<sup>2</sup>). The mass is pulled down 1m from its equilibrium position and released from rest.
- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?
- **C6.** Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant  $4kg/s^2$ ). The mass is pushed up 0.5m from its equilibrium position and released from rest.
- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?
- **C6.** Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant 1kg/s<sup>2</sup>). The mass is pulled down 1m from its equilibrium position and released from rest.
- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 3s?
- **C6.** Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant  $1 \text{kg/s}^2$ ). The mass is pushed up 0.5m from its equillibrium position and released from rest.
- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?
- **C6.** Consider the following scenario: A 2 kg mass is suspended by a spring (with spring constant  $4 \text{kg/s}^2$ ). A linear damper is attached to the system (with constant 6 kg/s. The mass is pulled down 1 m from its equillibrium position and released from rest.
- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?
- **C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s<sup>2</sup>). A linear damper is attached to the system (with constant 1kg/s. The mass is pulled down 1m from its equillibrium position and released from rest.
- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?
- **C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4kg/s^2$ ). A linear damper is attached to the system (with constant 6kg/s. An external force is applied, modelled by the function  $F(t) = \sin(t)$ . The mass is pulled down 1m from its equilibrium position and released from rest.
- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?
- **C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4kg/s^2$ ). A linear damper is attached to the system (with constant 6kg/s). An external force is applied, modelled by the function  $F(t) = \cos(t)$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

# Module F

## Standard F1

F1. Sketch a solution curve through each point marked in the slope field.

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F1. Sketch a solution curve through each point marked in the slope field.



- **F2.** Find the general solution to  $\frac{dy}{dx} + 3xy = 0$ .
- **F2.** Find the general solution to  $y' y\sin(x) = 0$ .
- **F2.** Find the general solution to  $y' = \frac{x+2}{y}$ .
- **F2.** Find the general solution to  $\frac{dy}{dx} = \frac{1+x}{1+y}$ .
- **F2.** Find the general solution to xy' = y.
- **F2.** Find the general solution to  $y\frac{dy}{dx} = y^2 \cos(x)$ .
- **F2.** Find the general solution to  $xy\frac{dy}{dx} = 1$ .
- **F2.** Find the general solution to  $x\cos(y)y'=1$ .

- **F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take the ball to travel across a tennis court, which is 24m long?
- **F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 0.5s?
- **F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take the ball to travel across a tennis court, which is 24m long?
- **F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 0.5s?
- **F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take for the ball to reach a defender standing 30m away?
- **F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 1s?
- **F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 45m/s.
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

- (b) How long does it take for the ball to reach a defender standing 30m away?
- **F3.** A baseball has a mass of  $0.145 \,\mathrm{kg}$  and a drag coefficient of  $0.0009 \,\mathrm{kg/m}$ . A batter hits a line drive; the ball leaves the bat travelling  $45 \,\mathrm{m/s}$ .
- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 1s?

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x - 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(0) = 4.
- **F4.** Consider the autonomous equation

$$\frac{dx}{dt} = 1 - x.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(2) = 2.
- **F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (x-3)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(1) = 2.
- F4. Consider the autonomous equation

$$\frac{dx}{dt} = (x+4)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(4) = 0.
- **F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (4 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(3) = 2.
- F4. Consider the autonomous equation

$$\frac{dx}{dt} = (5 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(0) = 4.
- **F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 7x + 10.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(0) = 3.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(3) = 0.
- F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(5) = 1.
- **F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(2) = 2.
- **F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(2) = 2.
- **F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point x(2) = 2.

- **F5.** Find the general solution to xy' + 4y = 2x.
- **F5.** Find the general solution to  $xy' + 2y = x^2$ .
- **F5.** Find the general solution to  $xy' + 2y = 4x^2 3x$ .
- **F5.** Find the general solution to  $xy' + 2y = x^2 3x$ .
- **F5.** Find the general solution to  $\cos(x)y' + \sin(x)y = x + \sin(x)\cos(x)$ .
- **F5.** Find the general solution to  $cos(x)y' + sin(x)y = x cos^2(x)$ .
- **F5.** Find the general solution to  $(x^2 + 1)y' 2xy = 1$ .
- **F5.** Find the general solution to  $(x^2 + 1)y' 2xy = x + 1$ .

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$(x+2y)y' + y = 2x$$
$$(x+2y)y' - y = -2x$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$(3x + 2y)y' + 3y = 2x$$
$$(3x + 2y)y' - 3y = -2x$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$(x^{2} + 3y^{2})y' - 2xy = -3x^{2}$$
$$(x^{2} + 3y^{2})y' + 2xy = 3x^{2}$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(2xy + 3y^2)y' + y^2 = 3x^2$$
$$(2xy + 3y^2)y' - y^2 = -3x^2$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\cos(x)\cos(y)y' = \sin(x)\sin(y)$$
$$\cos(x)\cos(y)y' = \sin(x) + \sin(y)$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\sin(x)\sin(y)y' = \cos(x) + \cos(y)$$
$$\sin(x)\sin(y)y' = \cos(x)\cos(y)$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$(y^3e^x + xe^x)y' + 3e^xy^2 = 3x^2$$
$$(2ye^x + e^y)y' + e^xy^2 = 3x^2$$

# Module S

#### Standard S1

**S1.** Find the general solution of the system

$$x' = x + y,$$
  
$$y' = 4x + y.$$

S1. Find the general solution of the system

$$x' = x + 2y,$$
  
$$y' = 3x + 2y.$$

**S1.** Find the general solution of the system

$$x' = 2x + y,$$
$$y' = x + 2y.$$

**S1.** Find the general solution of the system

$$x' = 2x + y,$$
  
$$y' = 2x + 3y.$$

**S1.** Find the general solution of the system

$$x' = 3x + y,$$
  
$$y' = x + 3y.$$

 ${f S1.}$  Find the general solution of the system

$$x' = 3x + y,$$
  
$$y' = 2x + 2y.$$

 ${f S1.}$  Find the general solution of the system

$$x' = 4x + y,$$
  
$$y' = 2x + 3y.$$

 ${f S1.}$  Find the general solution of the system

$$x' = 4x + 3y,$$
  
$$y' = x + 2y.$$

### Module N

#### Standard N1

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x^2y + xy^2;$$
  $y(1) = 3$ 

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = 2x^2 + xy + 3y^2;$$
  $y(1) = -1$ 

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x + \ln(y);$$
  $y(1) = 2$ 

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt{x+y}; \qquad y(1) = 1$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt[3]{x - y}; \quad y(2) = 2$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \frac{y}{x}; \qquad y(2) = 1$$

N2. Consider the differential equation

$$xy'' + y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$xy'' - y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' - 4xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$x^2y'' - xy' - 3y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' + xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$y'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$e^x y'' - 2y' + 4e^{4x} y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' + y' - e^{-2x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{3}{t}x + 2y,$$
  
$$y' = 2\ln(t)x + y + 1$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{2}{t}x + y,$$
  
$$y' = x + \ln(t)y + 2$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -x + \sqrt{t},$$
  
$$y' = 2x + ty + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + 2y + \sqrt{t},$$
  
$$y' = x + y + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + y + \sqrt[3]{t},$$
  
$$y' = x + 2y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = tx + 2y + \sqrt[3]{t},$$
  
$$y' = -y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + \ln(t)y + 2,$$
  
$$y' = -\frac{1}{t}y + 2t$$

 ${f N3.}$  Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = 2\ln(t)x + y + 1,$$

$$y' = -\frac{2}{t}x + y$$

### Module D

#### Standard D1

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\lbrace u(t-5)\rbrace(s) = \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\lbrace e^{3t}\rbrace(s) = \frac{1}{s-3}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4) + e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t) + u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{1 + e^t\}(s) = \frac{1}{s} + \frac{1}{s - 1}.$$