

## Module C

### Standard C1

**C1.** Set up and solve a differential equation to answer the following question:

A water droplet with a radius of  $100\ \mu\text{m}$  has a mass of about  $4 \times 10^{-12}\text{kg}$  and a terminal velocity of  $27\ \frac{\text{cm}}{\text{s}}$ . Such a droplet is dropped from rest. What is its velocity after  $0.01\ \text{s}$ ?

**C1.** Set up and solve a differential equation to answer the following question:

A water droplet with a radius of  $100\ \mu\text{m}$  has a mass of about  $4 \times 10^{-12}\text{kg}$ . Such a droplet is dropped from rest. After  $0.02\ \text{s}$  it has reached half of its terminal velocity. What is its terminal velocity?

**C1.** Set up and solve a differential equation to answer the following question:

A water droplet with a radius of  $10\ \mu\text{m}$  has a mass of about  $4 \times 10^{-15}\text{kg}$ . Such a droplet is dropped from rest. After  $0.002\ \text{s}$  it has reached half of its terminal velocity. What is its terminal velocity?

**C1.** Set up and solve a differential equation to answer the following question:

A single grain of corn pollen has a radius of  $50\ \mu\text{m}$  and a terminal velocity of  $27\ \frac{\text{cm}}{\text{s}}$ . Such a droplet is dropped from rest. What is its velocity after  $0.01\ \text{s}$ ?

**C2.** Find the general solution to

$$y' + 3y = 6t + 5.$$

**C2.** Find the general solution to

$$y' + 4y = 4.$$

**C2.** Find the general solution to

$$y' + 2y = 6t - 1.$$

**C2.** Find the general solution to

$$y' - y = e^t.$$

**C2.** Find the general solution to

$$y' + y = e^t.$$

**C2.** Find the general solution to

$$y' - y = e^{-t}.$$

**C2.** Find the general solution to

$$y' + y = e^{-t}.$$

**C2.** Find the general solution to

$$y' + 3y = 10 \sin(t).$$

**C2.** Find the general solution to

$$y' + 2y = 10 \sin(t).$$

**C2.** Find the general solution to

$$y' + 2y = 5 \sin(t).$$

**C2.** Find the general solution to

$$y' + 3y = 10 \cos(t).$$

**C2.** Find the general solution to

$$y' + 2y = 10 \cos(t).$$

**C2.** Find the general solution to

$$y' + 2y = 5 \cos(t).$$

C4. Find the general solution to

$$y'' + 2y' + y = 0.$$

C4. Find the general solution to

$$y'' + 2y' - 8y = 0.$$

C4. Find the general solution to

$$y'' + 4y' + 3y = 0.$$

C4. Find the general solution to

$$y'' + 2y' - 3y = 0.$$

C4. Find the general solution to

$$y'' - 2y' - 3y = 0.$$

C4. Find the general solution to

$$y'' + 4y' + 4y = 0.$$

C4. Find the general solution to

$$y'' - 4y' + 4y = 0.$$

C4. Find the general solution to

$$y'' + 5y' + 6y = 0.$$

C4. Find the general solution to

$$y'' - 2y' + 2y = 0.$$

C4. Find the general solution to

$$y'' + 2y' + 2y = 0.$$

C4. Find the general solution to

$$y'' - 6y' + 10y = 0.$$

C4. Find the general solution to

$$y'' + 6y' + 10y = 0.$$

**C4.** Find the general solution to

$$y'' - 2y' + 5y = 0.$$

**C4.** Find the general solution to

$$y'' + 2y' + 5y = 0.$$

**C4.** Find the general solution to

$$y'' - 4y' + 5y = 0.$$

**C4.** Find the general solution to

$$y'' + 4y' + 5y = 0.$$

**C5.** Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

**C5.** Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2 \sin(3x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^x$$

**C5.** Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

**C5.** Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

**C5.** Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

**C5.** Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

**C6.** Find the solution to

$$y'' + 2y' + y = 0$$

when  $y(0) = 0$  and  $y'(0) = 2$ .

**C6.** Find the solution to

$$y'' + 2y' + y = 0$$

when  $y(0) = 2$  and  $y'(0) = 0$ .

**C6.** Find the solution to

$$y'' + 2y' - 8y = 0$$

when  $y(0) = 3$  and  $y'(0) = -6$ .

**C6.** Find the solution to

$$y'' + 4y' + 3y = 0$$

when  $y(0) = 1$  and  $y'(0) = 5$ .

**C6.** Find the solution to

$$y'' + 2y' - 3y = 0$$

when  $y(0) = 5$  and  $y'(0) = 1$ .

**C6.** Find the solution to

$$y'' + 2y' - 3y = 0$$

when  $y(0) = 2$  and  $y'(0) = 2$ .

**C6.** Find the solution to

$$y'' - 2y' - 3y = 0$$

when  $y(0) = 2$  and  $y'(0) = 2$ .

**C6.** Find the solution to

$$y'' + 4y' + 4y = 0$$

when  $y(0) = 1$  and  $y'(0) = 3$ .

**C6.** Find the solution to

$$y'' - 4y' + 4y = 0$$

when  $y(0) = 1$  and  $y'(0) = 3$ .

**C6.** Find the solution to

$$y'' + 4y' + 4y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C6.** Find the solution to

$$y'' - 4y' + 4y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C6.** Find the solution to

$$y'' + 5y' + 6y = 0$$



when  $y(0) = 3$  and  $y'(0) = 1$ .

**C6.** Find the solution to

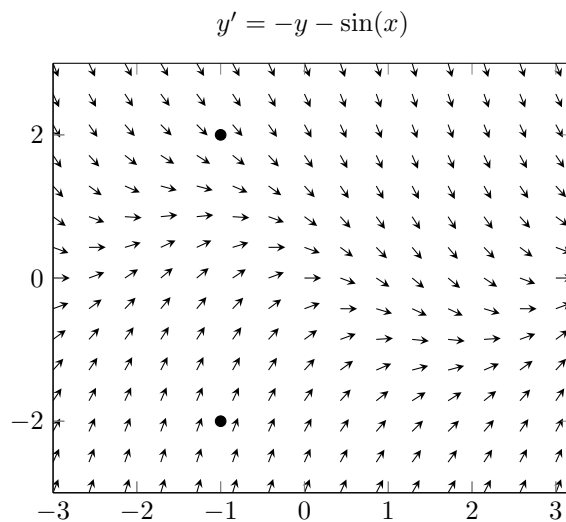
$$y'' + 5y' + 6y = 0$$

when  $y(0) = 1$  and  $y'(0) = 2$ .

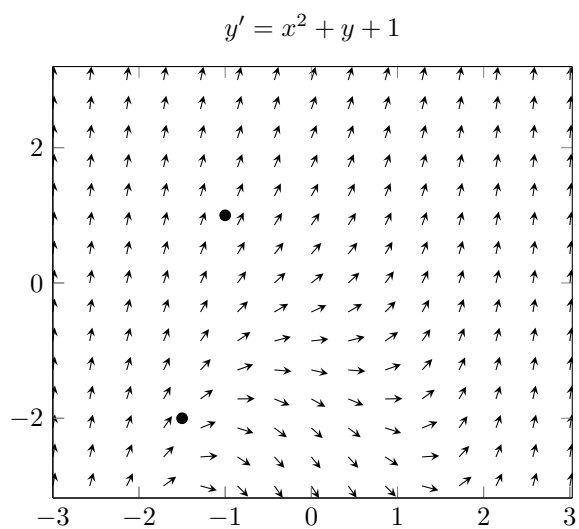
## Module F

### Standard F1

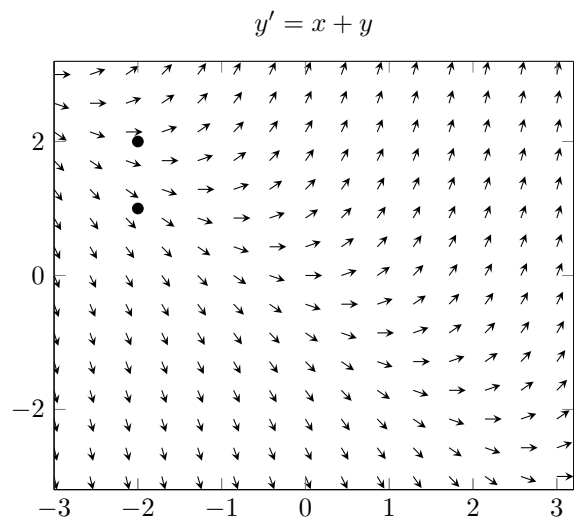
**F1.** Sketch a solution curve through each point marked in the slope field.



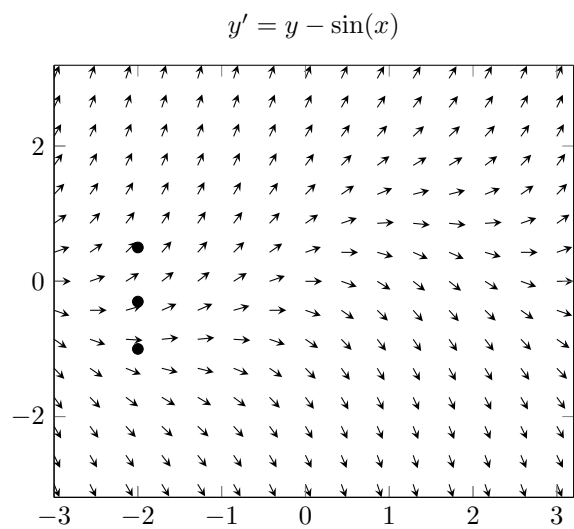
**F1.** Sketch a solution curve through each point marked in the slope field.



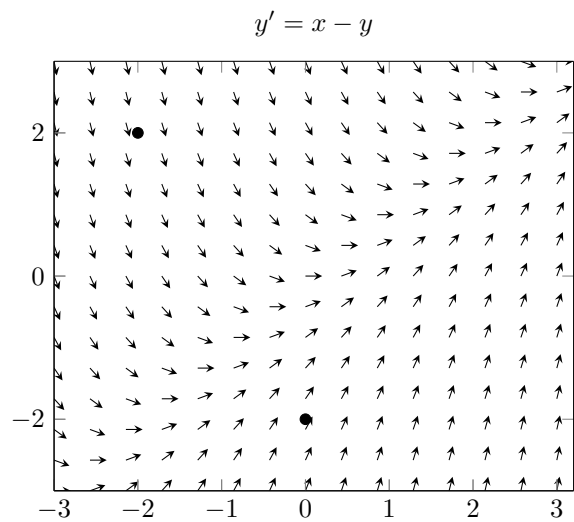
**F1.** Sketch a solution curve through each point marked in the slope field.



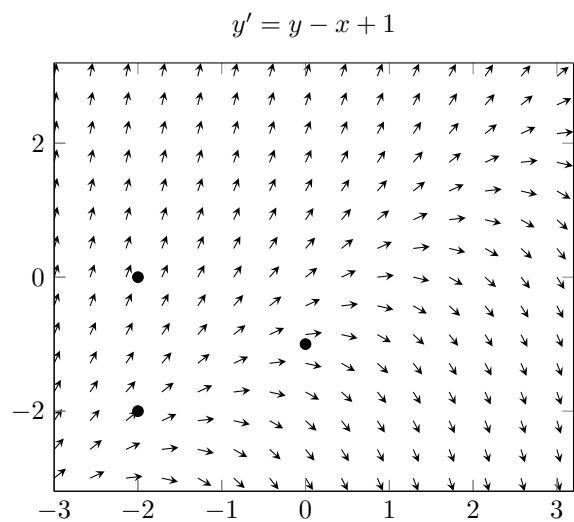
**F1.** Sketch a solution curve through each point marked in the slope field.



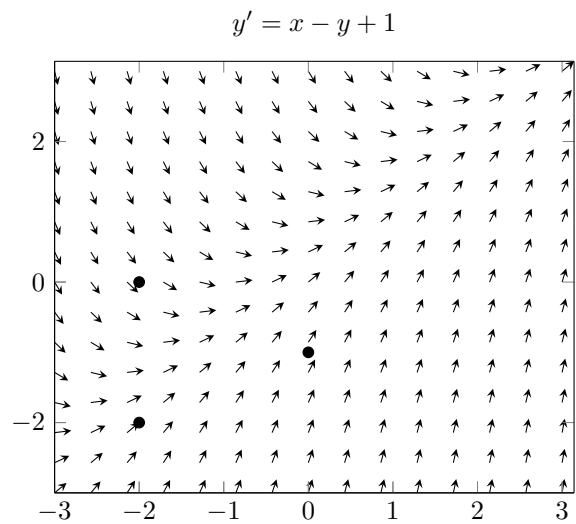
**F1.** Sketch a solution curve through each point marked in the slope field.



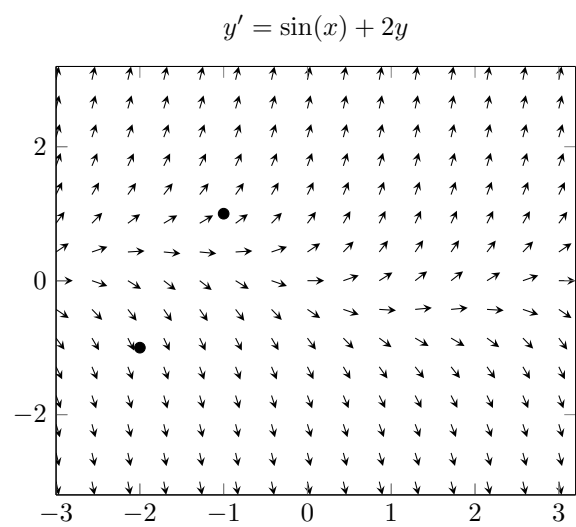
**F1.** Sketch a solution curve through each point marked in the slope field.



**F1.** Sketch a solution curve through each point marked in the slope field.

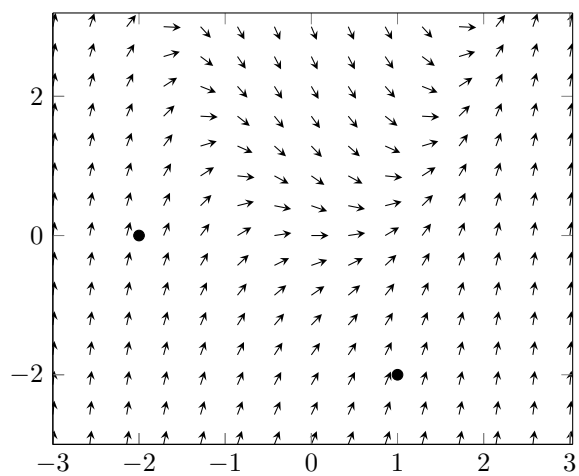


**F1.** Sketch a solution curve through each point marked in the slope field.



**F1.** Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 - y$$



**F2.** Find the general solution to  $\frac{dy}{dx} + 3xy = 0$ .

**F2.** Find the general solution to  $y' - y \sin(x) = 0$ .

**F2.** Find the general solution to  $y' = \frac{x+2}{y}$ .

**F2.** Find the general solution to  $\frac{dy}{dx} = \frac{1+x}{1+y}$ .

**F2.** Find the general solution to  $xy' = y$ .

**F2.** Find the general solution to  $y \frac{dy}{dx} = y^2 \cos(x)$ .

**F2.** Find the general solution to  $xy \frac{dy}{dx} = 1$ .

**F2.** Find the general solution to  $x \cos(y)y' = 1$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = x - 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 4$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = 1 - x.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = (x - 3)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(1) = 2$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = (x + 4)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(4) = 0$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = (4 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(3) = 2$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = (5 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 4$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 7x + 10.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 3$ .



**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(3) = 0$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(5) = 1$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F3.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F4.** Find the general solution to  $xy' + 4y = 2x$ .

**F4.** Find the general solution to  $xy' + 4y = \sqrt{x}$  (for  $x > 0$ ).

**F4.** Find the general solution to  $xy' + 2y = x^2$ .

**F4.** Find the general solution to  $y' = 1 + 2x + y + 2xy$ .

## Module S

### Standard S1

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= x + y, \\y' &= 4x + y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= x + 2y, \\y' &= 3x + 2y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= x + 2y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= 2x + 3y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 3x + y, \\y' &= x + 3y.\end{aligned}$$

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**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 4x + y, \\y' &= 2x + 3y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 4x + 3y, \\y' &= x + 2y.\end{aligned}$$

## Module N

### Standard N1

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x^2y + xy^2; \quad y(1) = 3$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = 2x^2 + xy + 3y^2; \quad y(1) = -1$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x + \ln(y); \quad y(1) = 2$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt{x+y}; \quad y(1) = 1$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt[3]{x-y}; \quad y(2) = 2$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \frac{y}{x}; \quad y(2) = 1$$

**N2.** Consider the differential equation

$$xy'' + y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$xy'' - y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' - 4xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' - xy' - 3y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' + xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$e^xy'' - 2y' + 4e^{4x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' + y' - e^{-2x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{3}{t}x + 2y, \\y' &= 2\ln(t)x + y + 1\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{2}{t}x + y, \\y' &= x + \ln(t)y + 2\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -x + \sqrt{t}, \\y' &= 2x + ty + \sqrt[3]{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + 2y + \sqrt{t}, \\y' &= x + y + \sqrt[3]{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + y + \sqrt[3]{t}, \\y' &= x + 2y + \sqrt{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= tx + 2y + \sqrt[3]{t}, \\y' &= -y + \sqrt{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + \ln(t)y + 2, \\y' &= -\frac{1}{t}y + 2t\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = 2 \ln(t)x + y + 1,$$

$$y' = -\frac{2}{t}x + y$$



## Module D

### Standard D1

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{u(t-5)\}(s) = \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{e^{3t}\}(s) = \frac{1}{s-3}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4) + e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t) + u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{1 + e^t\}(s) = \frac{1}{s} + \frac{1}{s-1}.$$