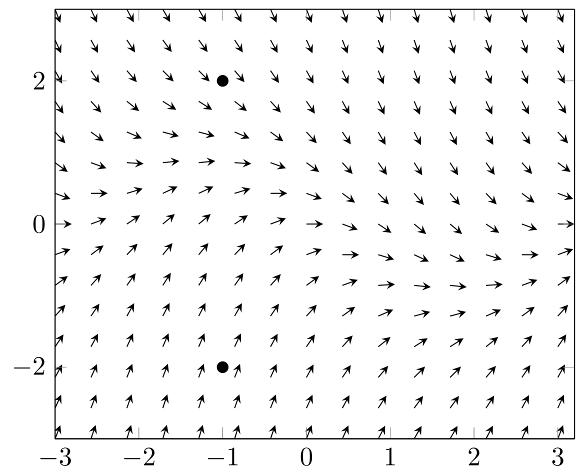
Module C

Standard C1

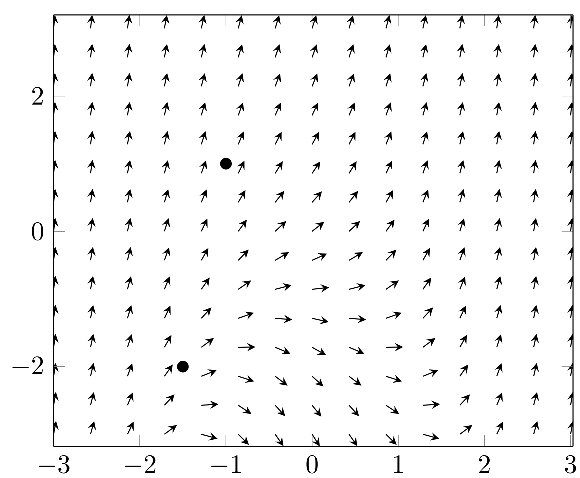
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = -y - \sin(x)$$



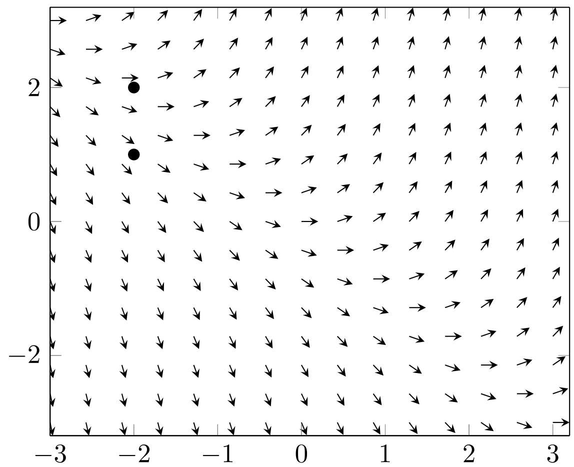
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 + y + 1$$



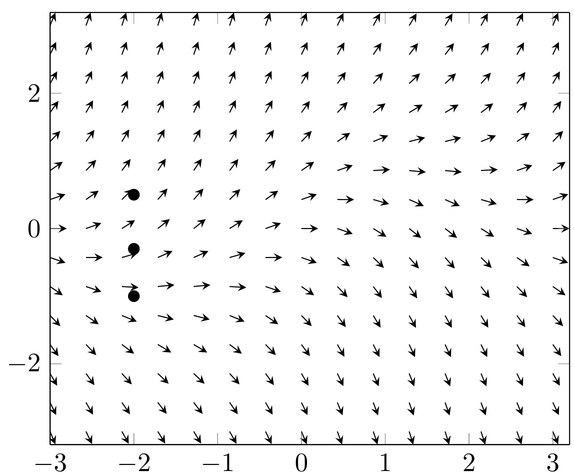
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = x + y$$



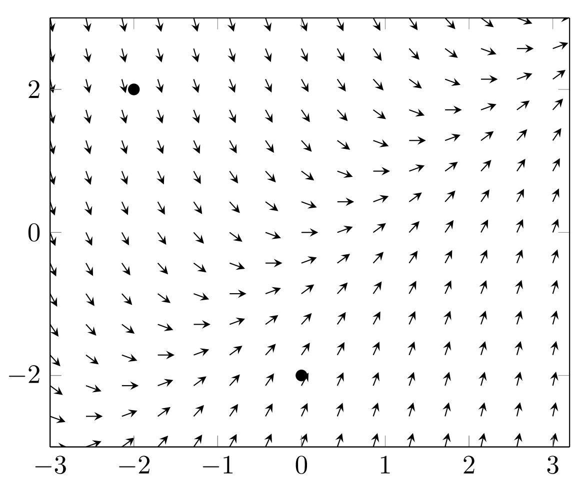
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = y - \sin(x)$$

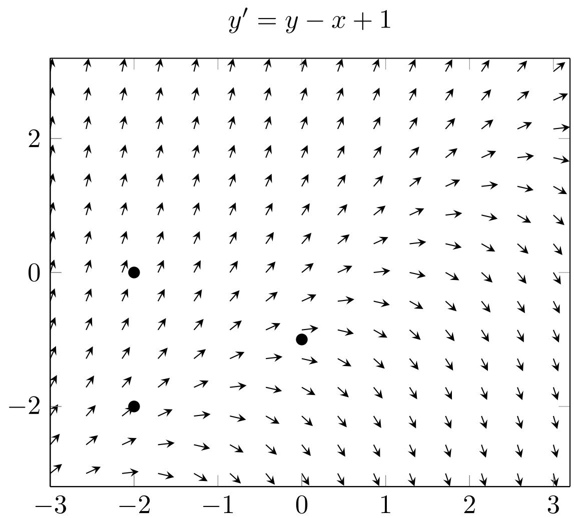


C1. Sketch a solution curve through each point marked in the slope field.

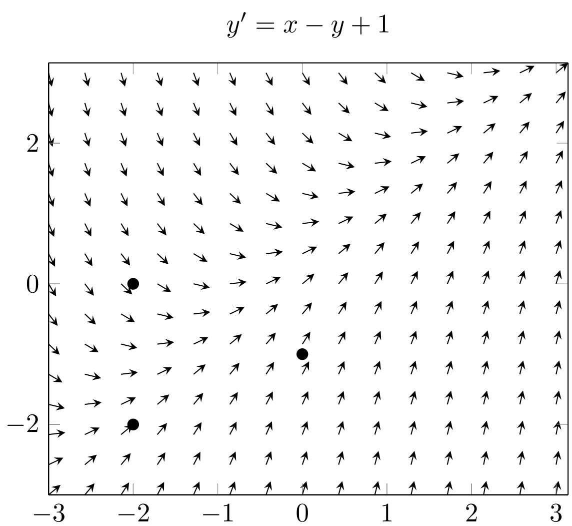
$$y' = x - y$$



C1. Sketch a solution curve through each point marked in the slope field.

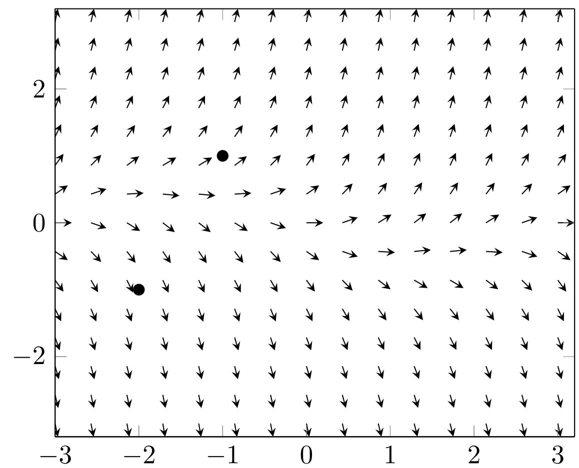


C1. Sketch a solution curve through each point marked in the slope field.



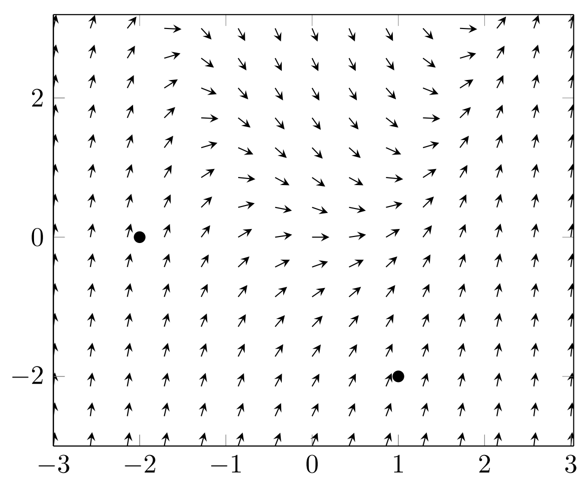
C1. Sketch a solution curve through each point marked in the slope field.

$$y' = \sin(x) + 2y$$



C1. Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 - y$$



C3. Find the general solution to
$$y'' + 2y' + y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 8y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 3y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' - 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' - 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' + 5y' + 6y = 0.$$

C3. Find the general solution to
$$y'' - 2y' + 2y = 0.$$

C3. Find the general solution to
$$y'' + 2y' + 2y = 0.$$

Module S

Standard S1

S1. Determine if the vectors in the set $\left\{\begin{bmatrix}1\\1\\-1\end{bmatrix}, \begin{bmatrix}3\\-1\\1\end{bmatrix}, \begin{bmatrix}2\\0\\-2\end{bmatrix}\right\}$ are linearly dependent or linearly independent.
Solution.

$$\text{RREF}\left(\begin{bmatrix}1 & 3 & 2\\1 & -1 & 0\\-1 & 1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since each column is a pivot column, the vectors are linearly independent. □

S1. Determine if the vectors in the set $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\2\\-1\end{bmatrix}, \begin{bmatrix}1\\3\\-2\end{bmatrix}\right\}$ are linearly dependent or linearly independent
Solution.

$$\text{RREF}\left(\begin{bmatrix}1 & 1 & 1\\0 & 2 & 3\\1 & -1 & -2\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & -\frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$$

Since there is a nonpivot column, the set is linearly dependent. □

S1. Determine if the vectors in the set $\left\{\begin{bmatrix}-3\\8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}$ are linearly dependent or linearly independent
Solution.

$$\text{RREF}\left(\begin{bmatrix}-3 & 1 & 0\\8 & 2 & -1\\0 & 2 & 3\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Every column is a pivot column, therefore the set is linearly independent. □

S1. Determine if the vectors in the set $\left\{\begin{bmatrix}-3\\-8\\0\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}0\\-1\\3\end{bmatrix}\right\}$ are linearly dependent or linearly independent
Solution.

$$\text{RREF}\left(\begin{bmatrix}-3 & 1 & 0\\-8 & 2 & -1\\0 & 2 & 3\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & \frac{1}{2}\\0 & 1 & \frac{3}{2}\\0 & 0 & 0\end{bmatrix}$$

This has a non pivot column, therefore the set is linearly dependent. □

S1. Determine if the vectors in the set $\left\{\begin{bmatrix}3\\-1\\0\\4\end{bmatrix}, \begin{bmatrix}1\\2\\-2\\1\end{bmatrix}, \begin{bmatrix}3\\-8\\6\\5\end{bmatrix}\right\}$ are linearly dependent or linearly independent.
Solution.

$$\text{RREF}\left(\begin{bmatrix}3 & 1 & 3\\-1 & 2 & -8\\0 & -2 & 6\\4 & 1 & 5\end{bmatrix}\right) = \begin{bmatrix}1 & 0 & 2\\0 & 1 & -3\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Since the reduced row echelon form has a nonpivot column, the vectors are linearly dependent. □

S1. Determine if the vectors in the set $\left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 6 \\ 5 \end{bmatrix} \right\}$ are linearly dependent or linearly independent.

Solution.

$$\mathrm{RREF}\left(\begin{bmatrix} 3 & 1 & 3 \\ -1 & 2 & -8 \\ 1 & -2 & 6 \\ 4 & 1 & 5 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the reduced row echelon form has only pivot columns, the vectors are linearly independent. $\square$

S1. Determine if the vectors in the set $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix} \right\}$ are linearly dependent or linearly independent.

Solution.

$$\mathrm{RREF} \begin{bmatrix} 2 & -1 & 2 \\ 1 & -3 & -4 \\ -1 & 1 & 0 \\ 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the third column is not a pivot column, the set is linearly dependent. $\square$

S1. Determine if the vectors in the set $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} \right\}$ are linearly dependent or linearly independent.

Solution.

$$\mathrm{RREF} \begin{bmatrix} 2 & -1 & 2 & 1 \\ 1 & -3 & -4 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the third column is not a pivot column, the set is linearly dependent. $\square$

Module F

Standard F1

V1. Let V be the set of all real numbers together with the operations $\oplus$ and $\odot$ defined by, for any $x, y \in V$ and $c \in \mathbb{R}$,

$$\begin{aligned} x \oplus y &= x + y \\ c \odot x &= cx - 3(c-1) \end{aligned}$$

(a) Show that **scalar multiplication** is **associative**: $a \odot (b \odot x) = (ab) \odot x$ for all scalars $a, b \in \mathbb{R}$ and $x \in V$.

(b) Show that scalar multiplication does not distribute over vector addition, i.e. for some scalar $a \in \mathbb{R}$ and $x, y \in V$, $a \odot (x \oplus y) \neq a \odot x \oplus a \odot y$.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2 + 2x_1y_1) \\ c \odot (x_1, x_2) &= (cx_1, cx_2) \end{aligned}$$

(a) Show that the vector **addition** $\oplus$ is **associative**:
$(x_1, x_2) \oplus ((y_1, y_2) \oplus (z_1, z_2)) = ((x_1, x_2) \oplus (y_1, y_2)) \oplus (z_1, z_2)$ for all $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in V$.

(b) Show that scalar multiplication does not distribute over scalar addition, i.e. for some scalars $a, b \in \mathbb{R}$ and $(x_1, x_2) \in V$, $(a + b) \odot (x_1, x_2) \neq a \odot (x_1, x_2) \oplus b \odot (x_1, x_2)$.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1 - 1, x_2 + y_2 - 1) \\ c \odot (x_1, x_2) &= (cx_1, cx_2) \end{aligned}$$

(a) Show that this vector space has an **additive identity** element, i.e. an element $\mathbf{z} \in V$ satisfying $(x, y) \oplus \mathbf{z} = (x, y)$ for every $(x, y) \in V$.

(b) Show that scalar multiplication does not distribute over vector addition, i.e. for some $a \in \mathbb{R}$ and $(x_1, x_2), (y_1, y_2) \in V$, $a \odot ((x_1, x_2) \oplus (y_1, y_2)) \neq a \odot (x_1, x_2) \oplus a \odot (y_1, y_2)$.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2) \\ c \odot (x_1, x_2) &= (0, cx_2) \end{aligned}$$

(a) Show that **scalar multiplication distributes over scalar addition**, i.e. that $(c + d) \odot (x_1, x_2) = c \odot (x_1, x_2) \oplus d \odot (x_1, x_2)$ for every $c, d \in \mathbb{R}$ and $(x_1, x_2) \in V$.

(b) Show that 1 is not a scalar multiplicative identity.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$\begin{aligned} (x_1, x_2) \oplus (y_1, y_2) &= (x_1 + y_1, x_2 + y_2) \\ c \odot (x_1, x_2) &= (c^2x_1, c^3x_2) \end{aligned}$$

(a) Show that **scalar multiplication distributes over vector addition**, i.e. that $c \odot ((x_1, x_2) \oplus (y_1, y_2)) = c \odot (x_1, x_2) \oplus c \odot (y_1, y_2)$ for all $c \in \mathbb{R}$ and $(x_1, x_2), (y_1, y_2) \in V$.

(b) Show that scalar multiplication does not distribute over scalar addition, i.e. for some $c, d \in \mathbb{R}$ and $(x_1, x_2) \in V$ $(c + d) \odot (x_1, x_2) \neq c \odot (x_1, x_2) \oplus d \odot (x_1, x_2)$.

V1. Let V be the set of all real numbers with the operations, for any $x, y \in V$, $c \in \mathbb{R}$,

$$x \oplus y = \sqrt{x^2 + y^2}$$
$$c \odot x = cx$$

(a) Show that the **vector addition** $\oplus$ **is associative**, i.e. that $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ for all $x, y, z \in V$.

(b) Show that there is no additive identity element, i.e. there is no element $\mathbf{z} \in V$ such that $x \oplus \mathbf{z} = x$ for all $x \in V$.

V1. Let V be the set of all pairs of real numbers with the operations, for any $(x_1, x_2), (y_1, y_2) \in V$, $c \in \mathbb{R}$,

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, x_2 y_2)$$
$$c \odot (x_1, x_2) = (cx_1, cx_2)$$

(a) Show that there is an **additive identity element**, i.e. an element $\mathbf{z} \in V$ such that $(x_1, x_2) \oplus \vec{z} = (x_1, x_2)$ for any $(x_1, x_2) \in V$.

(b) Show that scalar multiplication does not distribute over vector addition, i.e. for some $a \in \mathbb{R}$ and $(x_1, x_2), (y_1, y_2) \in V$, that $a \odot ((x_1, x_2) \oplus (y_1, y_2)) \neq a \odot (x_1, x_2) \oplus a \odot (y_1, y_2)$.

Module N

Standard N1

A1. Consider the following maps of polynomials $S : \mathcal{P}^6 \to \mathcal{P}^6$ and $T : \mathcal{P}^6 \to \mathcal{P}^6$ defined by

$$S(f(x)) = f(x) + 3 \text{ and } T(f(x)) = f(x) + f(3).$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. T is linear, S is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^4 \to \mathcal{P}^5$ and $T : \mathcal{P}^4 \to \mathcal{P}^5$ defined by

$$S(f(x)) = xf(x) - f(1) \text{ and } T(f(x)) = xf(x) - x.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = f'(x) - f''(x) \text{ and } T(f(x)) = f(x) - (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^4$ and $T : \mathcal{P}^2 \to \mathcal{P}^4$ defined by

$$S(f(x)) = x^2 f(x) \text{ and } T(f(x)) = (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = (f(x))^2 + 1 \text{ and } T(f(x)) = (x^2 + 1)f(x).$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. T is linear, S is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^2$ and $T : \mathcal{P}^2 \to \mathcal{P}^2$ defined by

$$S(ax^2 + bx + c) = cx^2 + bx + a \text{ and } T(ax^2 + bx + c) = a^2x^2 + b^2x + c^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^1$ and $T : \mathcal{P}^2 \to \mathcal{P}^1$ defined by

$$S(ax^2 + bx + c) = 2ax + b \text{ and } T(ax^2 + bx + c) = a^2x + b.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^3$ and $T : \mathcal{P}^2 \to \mathcal{P}^3$ defined by

$$S(ax^2 + bx + c) = ax^3 + bx^2 + cx \text{ and } T(ax^2 + bx + c) = abcx^3.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

Module D

Standard D1

A1. Consider the following maps of polynomials $S : \mathcal{P}^6 \to \mathcal{P}^6$ and $T : \mathcal{P}^6 \to \mathcal{P}^6$ defined by

$$S(f(x)) = f(x) + 3 \text{ and } T(f(x)) = f(x) + f(3).$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. T is linear, S is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^4 \to \mathcal{P}^5$ and $T : \mathcal{P}^4 \to \mathcal{P}^5$ defined by

$$S(f(x)) = xf(x) - f(1) \text{ and } T(f(x)) = xf(x) - x.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = f'(x) - f''(x) \text{ and } T(f(x)) = f(x) - (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^4$ and $T : \mathcal{P}^2 \to \mathcal{P}^4$ defined by

$$S(f(x)) = x^2 f(x) \text{ and } T(f(x)) = (f(x))^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P} \to \mathcal{P}$ and $T : \mathcal{P} \to \mathcal{P}$ defined by

$$S(f(x)) = (f(x))^2 + 1 \text{ and } T(f(x)) = (x^2 + 1)f(x).$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. T is linear, S is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^2$ and $T : \mathcal{P}^2 \to \mathcal{P}^2$ defined by

$$S(ax^2 + bx + c) = cx^2 + bx + a \text{ and } T(ax^2 + bx + c) = a^2x^2 + b^2x + c^2.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^1$ and $T : \mathcal{P}^2 \to \mathcal{P}^1$ defined by

$$S(ax^2 + bx + c) = 2ax + b \text{ and } T(ax^2 + bx + c) = a^2x + b.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □

A1. Consider the following maps of polynomials $S : \mathcal{P}^2 \to \mathcal{P}^3$ and $T : \mathcal{P}^2 \to \mathcal{P}^3$ defined by

$$S(ax^2 + bx + c) = ax^3 + bx^2 + cx \text{ and } T(ax^2 + bx + c) = abcx^3.$$

Show that one of these maps is a linear transformation, and that the other map is not.
Solution. S is linear, T is not. □