

## Module C

### Standard C1

**C1.** Find the general solution to

$$y' + 3y = 6t + 5.$$

**C1.** Find the general solution to

$$y' + 4y = 4.$$

**C1.** Find the general solution to

$$y' + 2y = 6t - 1.$$

**C1.** Find the general solution to

$$y' - y = e^t.$$

**C1.** Find the general solution to

$$y' + y = e^t.$$

**C1.** Find the general solution to

$$y' - y = e^{-t}.$$

**C1.** Find the general solution to

$$y' + y = e^{-t}.$$

**C1.** Find the general solution to

$$y' + 3y = 10e^{-3t} \sin(t).$$

**C1.** Find the general solution to

$$y' + 2y = 10e^{-2t} \sin(t).$$

**C1.** Find the general solution to

$$y' + 2y = 5e^{-2t} \sin(t).$$

**C1.** Find the general solution to

$$y' + 3y = 10e^{-3t} \cos(t).$$

**C1.** Find the general solution to

$$y' + 2y = 10e^{-2t} \cos(t).$$

**C1.** Find the general solution to

$$y' + 2y = 5e^{-2t} \cos(t).$$

**C2.** A water droplet with a radius of  $100\ \mu\text{m}$  has a mass of about  $4 \times 10^{-12}\text{kg}$  and a terminal velocity of  $27\ \frac{\text{cm}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C2.** A water droplet with a radius of  $50\ \mu\text{m}$  has a mass of about  $5 \times 10^{-13}\text{kg}$  and a terminal velocity of  $3.5\ \frac{\text{cm}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C2.** A water droplet with a radius of  $10\ \mu\text{m}$  has a mass of about  $4 \times 10^{-15}\text{kg}$  and a terminal velocity of  $270\ \frac{\mu\text{m}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.001\ \text{s}$ ?

**C2.** A water droplet with a radius of  $5\ \mu\text{m}$  has a mass of about  $5 \times 10^{-16}\text{kg}$  and a terminal velocity of  $35\ \frac{\mu\text{m}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.001\ \text{s}$ ?

**C2.** A single grain of corn pollen with a radius of  $50\ \mu\text{m}$  and a mass of about  $5 \times 10^{-13}\text{kg}$  has a terminal velocity of  $27\ \frac{\text{cm}}{\text{s}}$ . Such a pollen grain is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C2.** A single grain of spruce pollen with a radius of  $25\ \mu\text{m}$  and a mass of about  $6 \times 10^{-14}\text{kg}$  has a terminal velocity of  $3\ \frac{\text{cm}}{\text{s}}$ . Such a pollen grain is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C3.** Find the general solution to

$$y'' + 2y' + y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' - 8y = 0.$$

**C3.** Find the general solution to

$$y'' + 4y' + 3y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' - 3y = 0.$$

**C3.** Find the general solution to

$$y'' - 2y' - 3y = 0.$$

**C3.** Find the general solution to

$$y'' + 4y' + 4y = 0.$$

**C3.** Find the general solution to

$$y'' - 4y' + 4y = 0.$$

**C3.** Find the general solution to

$$y'' + 5y' + 6y = 0.$$

**C3.** Find the general solution to

$$y'' - 2y' + 2y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' + 2y = 0.$$

**C3.** Find the general solution to

$$y'' - 6y' + 10y = 0.$$

**C3.** Find the general solution to

$$y'' + 6y' + 10y = 0.$$

**C3.** Find the general solution to

$$y'' - 2y' + 5y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' + 5y = 0.$$

**C3.** Find the general solution to

$$y'' - 4y' + 5y = 0.$$

**C3.** Find the general solution to

$$y'' + 4y' + 5y = 0.$$

**C4.** Find the solution to

$$y'' + 2y' + y = 0$$

when  $y(0) = 0$  and  $y'(0) = 2$ .

**C4.** Find the solution to

$$y'' + 2y' + y = 0$$

when  $y(0) = 2$  and  $y'(0) = 0$ .

**C4.** Find the solution to

$$y'' + 2y' - 8y = 0$$

when  $y(0) = 3$  and  $y'(0) = -6$ .

**C4.** Find the solution to

$$y'' + 4y' + 3y = 0$$

when  $y(0) = 1$  and  $y'(0) = 5$ .

**C4.** Find the solution to

$$y'' + 2y' - 3y = 0$$

when  $y(0) = 5$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' + 2y' - 3y = 0$$

when  $y(0) = 2$  and  $y'(0) = 2$ .

**C4.** Find the solution to

$$y'' - 2y' - 3y = 0$$

when  $y(0) = 2$  and  $y'(0) = 2$ .

**C4.** Find the solution to

$$y'' + 4y' + 4y = 0$$

when  $y(0) = 1$  and  $y'(0) = 3$ .

**C4.** Find the solution to

$$y'' - 4y' + 4y = 0$$

when  $y(0) = 1$  and  $y'(0) = 3$ .

**C4.** Find the solution to

$$y'' + 4y' + 4y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' - 4y' + 4y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' + 5y' + 6y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' + 5y' + 6y = 0$$

when  $y(0) = 1$  and  $y'(0) = 2$ .

**C5.** Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

**C5.** Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2 \sin(3x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^x$$

**C5.** Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

**C5.** Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

**C5.** Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

**C5.** Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

**C5.** Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$



**C5.** Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

**C6.** Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?

**C6.** Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). The mass is pushed up 0.5m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?

**C6.** Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant  $1\text{kg/s}^2$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 3s?

**C6.** Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant  $1\text{kg/s}^2$ ). The mass is pushed up 0.5m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $6\text{kg/s}$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $1\text{kg/s}$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $6\text{kg/s}$ ). An external force is applied, modelled by the function  $F(t) = \sin(t)$ . The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $6\text{kg/s}$ ). An external force is applied, modelled by the function  $F(t) = \cos(t)$ . The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

## Module F

### Standard F1

**F1.** Sketch a solution curve through each point marked in the slope field.



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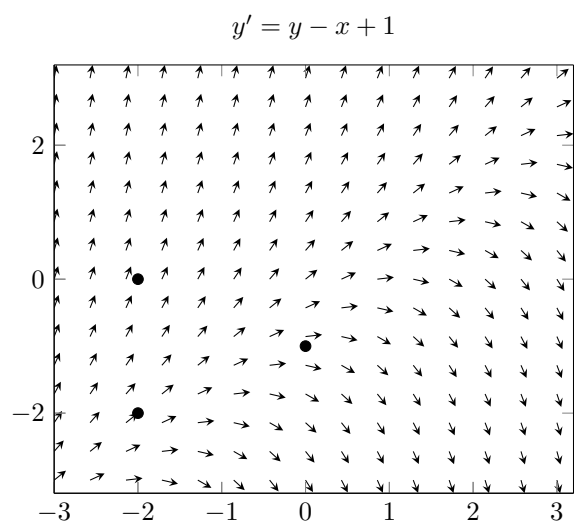
**F1.** Sketch a solution curve through each point marked in the slope field.



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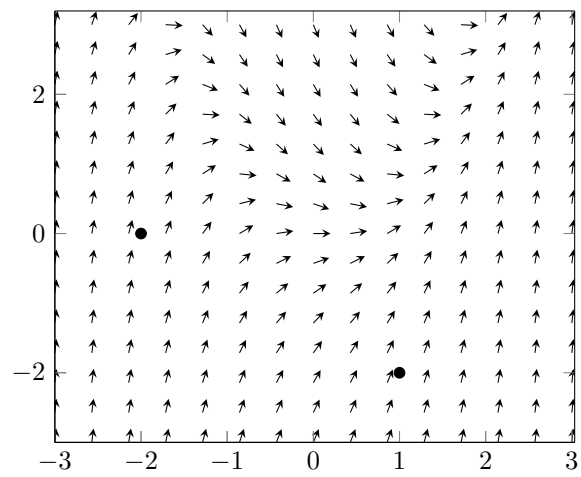


**F1.** Sketch a solution curve through each point marked in the slope field.



**F1.** Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 - y$$





**F2.** Find the general solution to  $\frac{dy}{dx} + 3xy = 0$ .

**F2.** Find the general solution to  $y' - y \sin(x) = 0$ .

**F2.** Find the general solution to  $y' - y^2 e^x = 0$ .

**F2.** Find the general solution to  $y' = \frac{x+2}{y^2}$ .

**F2.** Find the general solution to  $xy' = y$ .

**F2.** Find the general solution to  $y \frac{dy}{dx} = y^2 \cos(x)$ .

**F2.** Find the general solution to  $xy^2 \frac{dy}{dx} = 1$ .

**F2.** Find the general solution to  $x \cos(y) y' = 1$ .

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take the ball to travel across a tennis court, which is 24m long?

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 0.5s?

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take the ball to travel across a tennis court, which is 24m long?

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 0.5s?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take for the ball to reach a defender standing 30m away?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 1s?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 45m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How long does it take for the ball to reach a defender standing 30m away?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 45m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How far has the ball gone after 1s?

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x - 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 4$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = 1 - x.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (x - 3)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(1) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (x + 4)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(4) = 0$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (4 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(3) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (5 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 4$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 7x + 10.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 3$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(3) = 0$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(5) = 1$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F5.** Find the general solution to  $xy' + 4y = 2x$ .

**F5.** Find the general solution to  $xy' + 2y = x^2$ .

**F5.** Find the general solution to  $xy' + 2y = 4x^2 - 3x$ .

**F5.** Find the general solution to  $xy' + 2y = x^2 - 3x$ .

**F5.** Find the general solution to  $\cos(x)y' + \sin(x)y = \sin(x)\cos(x)$ .

**F5.** Find the general solution to  $\cos(x)y' + \sin(x)y = x\cos^2(x)$ .

**F5.** Find the general solution to  $(x^2 + 1)y' - 2xy = x^2 + 1$ .

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(x + 2y)y' + y &= 2x \\ (x + 2y)y' - y &= -2x\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(3x + 2y)y' + 3y &= 2x \\ (3x + 2y)y' - 3y &= -2x\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(x^2 + 3y^2)y' - 2xy &= -3x^2 \\ (x^2 + 3y^2)y' + 2xy &= 3x^2\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(2xy + 3y^2)y' + y^2 &= 3x^2 \\ (2xy + 3y^2)y' - y^2 &= -3x^2\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}\cos(x) \cos(y)y' &= \sin(x) \sin(y) \\ \cos(x) \cos(y)y' &= \sin(x) + \sin(y)\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}\sin(x) \sin(y)y' &= \cos(x) + \cos(y) \\ \sin(x) \sin(y)y' &= \cos(x) \cos(y)\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(y^3 e^x + x e^x)y' + 3e^x y^2 &= 3x^2 \\ (2y e^x + e^y)y' + e^x y^2 &= 3x^2\end{aligned}$$

## Module S

### Standard S1

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= x + y, \\y' &= 4x + y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= x + 2y, \\y' &= 3x + 2y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= x + 2y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= 2x + 3y.\end{aligned}$$

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**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 4x + 3y, \\y' &= x + 2y.\end{aligned}$$

**S2.** Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\begin{aligned}\frac{dG}{dt} &= 0.1G - 0.002G^2 - 0.005BG \\ \frac{dB}{dt} &= 0.2B - 0.003B^2 - 0.005BG.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a lake is stocked with 25 of each fish, what will happen to the two populations in the long term?

**S2.** Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\begin{aligned}\frac{dG}{dt} &= 0.1G - 0.002G^2 - 0.005BG \\ \frac{dB}{dt} &= 0.2B - 0.004B^2 - 0.005BG.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a lake is stocked with 25 bluegills and 30 greenfish, what will happen to the two populations in the long term?

**S2.** Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\begin{aligned}\frac{dG}{dt} &= 0.1G - 0.005G^2 - 0.004BG \\ \frac{dB}{dt} &= 0.2B - 0.002B^2 - 0.004BG.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a lake is stocked with 25 of each fish, what will happen to the two populations in the long term?

**S2.** Two populations of competing species of birds, bluebirds and redbirds, are modelled by the system

$$\begin{aligned}\frac{dB}{dt} &= 0.1B - 0.005B^2 - 0.004BR \\ \frac{dR}{dt} &= 0.2R - 0.02R^2 - 0.004BR.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a forest contains 50 bluebirds and 20 redbirds, what will happen to the two populations in the long term?

**S2.** Two populations of competing species of birds, bluebirds and redbirds, are modelled by the system

$$\begin{aligned}\frac{dB}{dt} &= 0.1B - 0.005B^2 - 0.004BR \\ \frac{dR}{dt} &= 0.2R - 0.02R^2 - 0.004BR.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a forest contains 100 bluebirds and 30 redbirds, what will happen to the two populations in the long term?

**S2.** Two populations of competing species of birds, bluebirds and redbirds, are modelled by the system

$$\begin{aligned}\frac{dB}{dt} &= 0.2B - 0.005B^2 - 0.004BR \\ \frac{dR}{dt} &= 0.2R - 0.002R^2 - 0.01BR.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a forest contains 50 bluebirds and 20 redbirds, what will happen to the two populations in the long term?

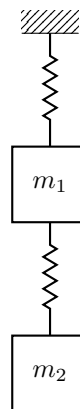
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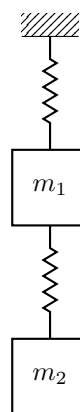
**S3.** Consider the dual mass-spring system at the right. Both masses are 1kg. The upper spring has a spring constant of 1N/m while the lower spring constant is 6N/m. The lower mass is pulled down 1m without disturbing the upper mass, and released from rest.

- Write down an IVP modelling the motion of the two masses.
- Where will the two masses be after 2s?



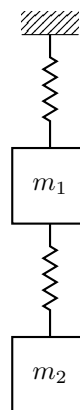
**S3.** Consider the dual mass-spring system at the right. Both masses are 1kg. The upper spring has a spring constant of 1N/m while the lower spring constant is 6N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.

- Write down an IVP modelling the motion of the two masses.
- Where will the two masses be after 3s?



**S3.** Consider the dual mass-spring system at the right. Both masses are 1kg. The upper spring has a spring constant of 1N/m while the lower spring constant is 6N/m. The upper mass is pushed up 0.5m without disturbing the lower mass, and released from rest.

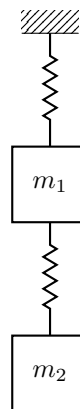
- Write down an IVP modelling the motion of the two masses.
- Where will the two masses be after 4s?



**S3.** Consider the dual mass-spring system at the right. Both masses are 3kg. The upper spring has a spring constant of 8N/m while the lower spring constant is 6N/m. The upper mass is pushed up 0.5m without disturbing the lower mass, and released from rest.

(a) Write down an IVP modelling the motion of the two masses.

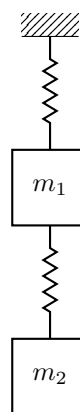
(b) Where will the two masses be after 2s?



**S3.** Consider the dual mass-spring system at the right. Both masses are 3kg. The upper spring has a spring constant of 8N/m while the lower spring constant is 6N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.

(a) Write down an IVP modelling the motion of the two masses.

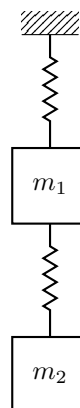
(b) Where will the two masses be after 2s?



**S3.** Consider the dual mass-spring system at the right. The upper mass is 2kg while the lower mass is 1kg. The upper spring has a spring constant of 6N/m while the lower spring constant is 4N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.

(a) Write down an IVP modelling the motion of the two masses.

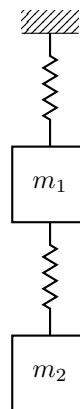
(b) Where will the two masses be after 3s?



**S3.** Consider the dual mass-spring system at the right. The upper mass is 2kg while the lower mass is 1kg. The upper spring has a spring constant of 6N/m while the lower spring constant is 4N/m. The lower mass is pulled down 1m without disturbing the upper mass, and released from rest.

(a) Write down an IVP modelling the motion of the two masses.

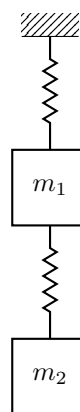
(b) Where will the two masses be after 2s?



**S3.** Consider the dual mass-spring system at the right. The upper mass is 1kg while the lower mass is 2kg. The upper spring has a spring constant of 3N/m while the lower spring constant is 4N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.

(a) Write down an IVP modelling the motion of the two masses.

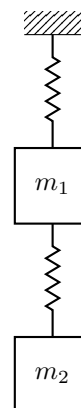
(b) Where will the two masses be after 3s?



**S3.** Consider the dual mass-spring system at the right. The upper mass is 1kg while the lower mass is 2kg. The upper spring has a spring constant of 3N/m while the lower spring constant is 4N/m. The lower mass is pulled down 1m without disturbing the upper mass, and released from rest.

(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 2s?



## Module N

### Standard N1

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = x^2y + xy^2; \quad y(1) = 3$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = 2x^2 + xy + 3y^2; \quad y(1) = -1$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = x + \ln(y); \quad y(1) = 2$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = x + \ln(y); \quad y(2) = 1$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt{x+y}; \quad y(1) = 1$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt{x+y}; \quad y(1) = -1$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt[3]{x-y}; \quad y(2) = 2$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt[3]{x-y}; \quad y(2) = 3$$

**N1.** Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \frac{y}{x}; \quad y(2) = 1$$

**N2.** Consider the differential equation

$$xy'' + y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$xy'' - y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' - 4xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' - xy' - 3y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' + xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$xy'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$e^xy'' - 2xy' + 4e^{4x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$xy'' + y' - e^{-2x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.



**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{3}{t}x + 2y, \\y' &= 2\ln(t)x + y + 1\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{2}{t}x + y, \\y' &= x + \ln(t)y + 2\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -x + \sqrt{t}, \\y' &= 2x + ty + \sqrt[3]{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + 2y + \sqrt{t}, \\y' &= x + y + \sqrt[3]{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + y + \sqrt[3]{t}, \\y' &= x + 2y + \sqrt{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= tx + 2y + \sqrt[3]{t}, \\y' &= -y + \sqrt{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + \ln(t)y + 2, \\y' &= -\frac{1}{t}y + 2t\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = 2 \ln(t)x + y + 1,$$

$$y' = -\frac{2}{t}x + y$$

**N4.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $y(1.8)$ , where  $y$  is a solution to the IVP

$$y' + y^2 = x, \quad y(1) = 1.$$

**N4.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $y(1.8)$ , where  $y$  is a solution to the IVP

$$y' + y^2 = x, \quad y(1) = 2.$$

**N4.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $y(2)$ , where  $y$  is a solution to the IVP

$$y' + y^2 = x, \quad y(1.5) = 1.$$

**N4.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $y(1.8)$ , where  $y$  is a solution to the IVP

$$y' + 2y^2 = x, \quad y(1) = 1.$$

**N4.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $y(2)$ , where  $y$  is a solution to the IVP

$$y' + 2y^2 = x, \quad y(1.5) = 1.$$

**N4.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $y(2)$ , where  $y$  is a solution to the IVP

$$y' + 2y^2 = x, \quad y(1.5) = 2.$$

**N4.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $y(1.8)$ , where  $y$  is a solution to the IVP

$$y' + y^2 = x - 1, \quad y(1) = 1.$$

**N4.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $y(2)$ , where  $y$  is a solution to the IVP

$$y' + y^2 = x - 1, \quad y(1.5) = 1.$$

**N4.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $y(1.8)$ , where  $y$  is a solution to the IVP

$$y' + xy^2 = x - 1, \quad y(1) = 1.$$

**N4.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $y(1.8)$ , where  $y$  is a solution to the IVP

$$y' + xy^2 = x - 1, \quad y(1) = 2.$$

**N4.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $y(2)$ , where  $y$  is a solution to the IVP

$$y' + xy^2 = x - 1, \quad y(1.5) = 1.$$

**N5.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $x(2.3)$  and  $y(2.3)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 3x - ty & x(2) &= 1 \\y' &= 2x - y^2 & y(2) &= 0\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $x(2.3)$  and  $y(2.3)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 3x - ty & x(2) &= 1 \\y' &= 2x - y^2 & y(2) &= 1\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.1$  to estimate  $x(2.3)$  and  $y(2.3)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 2x - ty & x(2) &= 1 \\y' &= tx - y^2 & y(2) &= 1\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $x(2.6)$  and  $y(2.6)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 3x - ty & x(2) &= 1 \\y' &= 2x - y^2 & y(2) &= 0\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $x(2.6)$  and  $y(2.6)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 3x - ty & x(2) &= 1 \\y' &= 2x - y^2 & y(2) &= 1\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $x(2.6)$  and  $y(2.6)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 2x - ty & x(2) &= 1 \\y' &= tx - y^2 & y(2) &= 1\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $x(2.6)$  and  $y(2.6)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 2x - ty & x(2) &= 1 \\y' &= 2x - y^2 & y(2) &= 0\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $x(2.6)$  and  $y(2.6)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 2x - 2y & x(2) &= 1 \\y' &= 2x - y^2 & y(2) &= 1\end{aligned}$$

**N5.** Use Euler's method with stepsize  $h = 0.2$  to estimate  $x(2.6)$  and  $y(2.6)$ , where  $x$  and  $y$  are solutions to the IVP

$$\begin{aligned}x' &= 2x - ty & x(2) &= 1 \\y' &= 2tx - y^2 & y(2) &= 1\end{aligned}$$

## Module D

### Standard D1

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{u(t-5)\}(s) = \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{e^{3t}\}(s) = \frac{1}{s-3}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4) + e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t) + u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{1 + e^t\}(s) = \frac{1}{s} + \frac{1}{s-1}.$$

**D2.** Solve the IVP

$$y'' + 9y = 4\delta(t - 2), \quad y(0) = 1, \quad y'(0) = 1$$

**D2.** Solve the IVP

$$y'' + 9y = 2\delta(t - 3), \quad y(0) = 1, \quad y'(0) = 0$$

**D2.** Solve the IVP

$$y'' + 4y = 2\delta(t - 1), \quad y(0) = 1, \quad y'(0) = 2$$

**D2.** Solve the IVP

$$y'' + 4y = 2\delta(t - 7), \quad y(0) = 1, \quad y'(0) = 2$$

**D2.** Solve the IVP

$$y'' + 9y = 2\delta(t - 7), \quad y(0) = 1, \quad y'(0) = 2$$

**D2.** Solve the IVP

$$y'' + 4y = 4u(t - 10), \quad y(0) = 3, \quad y'(0) = 0$$

**D2.** Solve the IVP

$$y'' + 4y = 12u(t - 7), \quad y(0) = 0, \quad y'(0) = 5$$

**D2.** Solve the IVP

$$y'' + 4y = 8u(t - 3), \quad y(0) = 0, \quad y'(0) = 5$$

**D2.** Solve the IVP

$$y'' + 4y = 8u(t - 3), \quad y(0) = -2, \quad y'(0) = -2$$

**D2.** Solve the IVP

$$y'' + 9y = 36u(t - 2), \quad y(0) = 1, \quad y'(0) = -2$$



**D3.** A spacecraft weighing 100 kg is travelling 30 m/s. At time  $t = 0$ , its thrusters (which provide 30 N of force) are turned on and burn for 100 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 150 s?

**D3.** A spacecraft weighing 1000 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 30 N of force) are turned on and burn for 100 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 150 s?

**D3.** A spacecraft weighing 1000 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 50 N of force) are turned on and burn for 60 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 150 s?

**D3.** A spacecraft weighing 150 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 30 N of force) are turned on and burn for 100 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 150 s?

**D3.** A spacecraft weighing 150 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 30 N of force) are turned on and burn for 150 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 200 s?

**D3.** A spacecraft weighing 1500 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 50 N of force) are turned on and burn for 150 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 200 s?

**D3.** A spacecraft weighing 1500 kg is travelling 75 m/s. At time  $t = 0$ , its thrusters (which provide 50 N of force) are turned on and burn for 100 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 200 s?

**D3.** A spacecraft weighing 2000 kg is travelling 75 m/s. At time  $t = 0$ , its thrusters (which provide 100 N of force) are turned on and burn for 100 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 200 s?

**D3.** A spacecraft weighing 2000 kg is travelling 50 m/s. At time  $t = 0$ , its thrusters (which provide 100 N of force) are turned on and burn for 75 s.

(a) Write down an IVP modelling the velocity of the spacecraft.

(b) What is its velocity after 100 s?

**D4.** A 4 kg mass is suspended on a spring with spring constant 16 N/m. It is pulled down 1m and released from rest. Then, after 5 s, it is struck by a hammer imparting 6Ns of upward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 8 s? Is this above or below the equilibrium position?

**D4.** A 4 kg mass is suspended on a spring with spring constant 36 N/m. It is pulled down 1m and released from rest. Then, after 5 s, it is struck by a hammer imparting 6Ns of upward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 10 s? Is this above or below the equilibrium position?

**D4.** A 1 kg mass is suspended on a spring with spring constant 36 N/m. It is pushed up 1m and released from rest. Then, after 5 s, it is struck by a hammer imparting 3Ns of downward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 10 s? Is this above or below the equilibrium position?

**D4.** A 1 kg mass is suspended on a spring with spring constant 16 N/m. It is pushed up 1m and released from rest. Then, after 5 s, it is struck by a hammer imparting 4Ns of downward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 7 s? Is this above or below the equilibrium position?

**D4.** A 1 kg mass is suspended on a spring with spring constant 16 N/m. It is pushed up 2m and released from rest. Then, after 3 s, it is struck by a hammer imparting 5Ns of downward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 5 s? Is this above or below the equilibrium position?

**D4.** A 1 kg mass is suspended on a spring with spring constant 9 N/m. It is pushed up 2m and released from rest. Then, after 5 s, it is struck by a hammer imparting 6Ns of upward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 8 s? Is this above or below the equilibrium position?

**D4.** A 3 kg mass is suspended on a spring with spring constant 3 N/m. It is pushed up 2m and released from rest. Then, after 5 s, it is struck by a hammer imparting 6Ns of upward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 10 s? Is this above or below the equilibrium position?

**D4.** A 3 kg mass is suspended on a spring with spring constant 3 N/m. It is pushed up 2m and released from rest. Then, after 5 s, it is struck by a hammer imparting 6Ns of downward impulse.

- (a) Write down an IVP modelling the position of the mass.
- (b) What is the position of the mass after 7 s? Is this above or below the equilibrium position?