

Module S: Systems of ODEs

How can we solve and apply systems of linear ODEs?

At the end of this module, students will be able to...

- S1. Solving systems.** ...solve systems of constant coefficient ODEs
- S2. Modeling interacting populations.** ...model the populations of two interacting populations with a system of ODEs
- S3. Modeling coupled oscillators.** ...model systems of coupled mechanical oscillators using a system of ODEs

Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Add Euclidean vectors and multiply Euclidean vectors by scalars.
- Perform basic manipulations of augmented matrices and linear systems **E1,E2,E3**.
- Apply linear combinations and spanning sets **V3,V4**.

The following resources will help you prepare for this module.

- Adding and subtracting Euclidean vectors (Khan Academy):
<http://bit.ly/2y8A0wa>
- Linear combinations of Euclidean vectors (Khan Academy):
<http://bit.ly/2nK3wne>
- Adding and subtracting complex numbers (Khan Academy):
<http://bit.ly/1PE3ZMQ>
- Adding and subtracting polynomials (Khan Academy):
<http://bit.ly/2d5SLGZ>

Module S Section 1

Activity S.1.1 (*~10 min*)

Consider the countries of Transia and Wakanda: each year, 8% of people living in Transia move to Wakanda, and 3% of Wakandans move to Transia.

Let T be the population of Transia, and W the population of Wakanda (both are functions of time, t).

Write down two differential equations modelling the population changes $\frac{dT}{dt}$ and $\frac{dW}{dt}$.

Activity S.1.2 (~ 5 min)

This problem resulted in a **system of linear differential equations**, namely

$$T' = 0.03W - 0.08T$$

$$W' = 0.08T - 0.03W$$

Rewrite this system using differential operators.

Activity S.1.3 (*~15 min*)

Solve the system

$$(D + 0.08)T - (0.03)W = 0$$

$$-0.08T + (D + 0.03)W = 0$$

Observation S.1.4

Because D is linear, $a(D + b) = (D + b)a$ for constants a, b . This is not true in general!

Thus, for any **constant coefficient linear systems of differential equations**, we can use our typical elimination technique.

Activity S.1.5 (*~15 min*)

Solve the system

$$x' = 5x - 2y$$

$$y' = 6y - 3x$$

with initial conditions $x(0) = 2$, $y(0) = -1$.

Activity S.1.6 (*~10 min*)

Solve the system

$$x' = x + 4y - t$$

$$y' = 2y - 3x + t$$