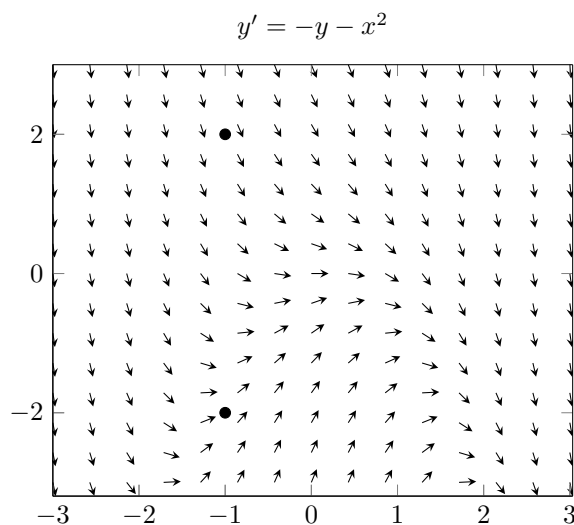


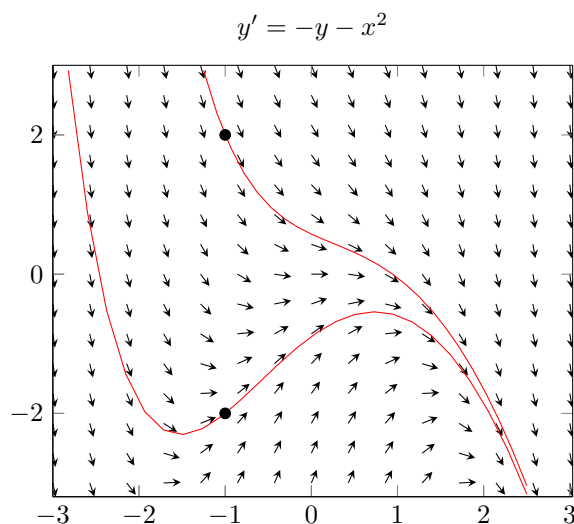
## Sample Assessment Exercises

This document contains one exercise and solution for each standard. The goal is to give you an idea of what the exercises might look like, and what the expectations for a complete solution are.

**C1.** Sketch a solution curve through each point marked in the slope field.



**Solution:**



□

**C2.** Find the general solution to

$$y' + y = -t^2.$$

**Solution:** First, we find a general solution to the homogeneous equation

$$y' + y = 0.$$

This has auxiliary equation  $r + 1 = 0$ , which has a single root at  $r = -1$ , so  $ce^{-t}$  is a solution. We can find a particular solution  $y_p$  to the given equation by using undetermined coefficients; since  $-t^2$  is a polynomial,

we let  $y_p = At^2 + Bt + D$  and determine the coefficients  $A$ ,  $B$ , and  $D$ .

$$\begin{aligned} y_p' + y_p &= (2At + B) + (At^2 + Bt + D) \\ &= At^2 + (2A + B)t + (B + D) \end{aligned}$$

So if  $y_p$  is a solution, we must have  $y_p' + y_p = -t^2$ , giving us the system of equations

$$\begin{aligned} A &= -1 \\ 2A + B &= 0 \\ B + D &= 0 \end{aligned}$$

Thus we easily deduce that  $A = -1$ ,  $B = 2$ , and  $D = -2$ , giving  $y_p = -t^2 + 2t - 2$ . Thus, the general solution is

$$y = -t^2 + 2t - 2 + ce^{-t}.$$

□

**C3.** Find the general solution to

$$y'' + 6y' + 13y = 0.$$

**Solution:** We begin by writing the auxilliary equation  $r^2 + 6r + 13 = 0$  and finding the roots. There are many ways to do this; here, we complete the square:

$$0 = r^2 + 6r + 13 = r^2 + 6r + 9 + 4 = (r + 3)^2 + 4.$$

Thus, we can easily solve to obtain  $r = -3 \pm 2i$ . Thus the general solution is

$$y = c_1 e^{-3t} \cos(2t) + c_2 e^{-3t} \sin(2t).$$

□

**C4.** Find the general solution to

$$y'' + 6y' + 13y = 13t^2 - t - 4.$$

**Solution:** First, we find a general solution to the homogenous equation  $y'' + 6y' + 13y = 0$ . We begin by writing the auxilliary equation  $r^2 + 6r + 13 = 0$  and finding the roots. There are many ways to do this; here, we complete the square:

$$0 = r^2 + 6r + 13 = r^2 + 6r + 9 + 4 = (r + 3)^2 + 4.$$

Thus, we can easily solve to obtain  $r = -3 \pm 2i$ . Thus the general solution to the homogeneous equation is

$$y = c_1 e^{-3t} \cos(2t) + c_2 e^{-3t} \sin(2t).$$

To find a particular solution to  $y'' + 6y' + 13y = 13t^2 - t - 4$ , we set  $y_p = At^2 + Bt + D$  and determine the coefficients  $A$ ,  $B$ ,  $D$ .

$$\begin{aligned} y_p'' + 6y_p' + 13y_p &= (2A) + 6(2At + B) + 13(At^2 + Bt + D) \\ &= (13A)t^2 + (12A + 13B)t + (2A + 6B + 13D) \end{aligned}$$

This gives us the system of equations

$$\begin{aligned} 13A &= 13 \\ 12A + 13B &= -1 \\ 2A + 6B + 13D &= -4 \end{aligned}$$

Then we easily deduce  $A = 1$ ,  $B = -1$ , and  $D = 0$ , so that  $y_p = t^2 - t$ . Thus, the general solution to the nonhomogeneous equation is

$$y = c_1 e^{-3t} \cos(2t) + c_2 e^{-3t} \sin(2t) + t^2 - t.$$

□

**C5.** Find the solution to

$$y'' + 10y' + 24y = 0$$

when  $y(0) = -3$  and  $y'(0) = 2$ .

**Solution:** The auxilliary equation is  $r^2 + 10r + 24 = 0$ , which has roots  $r = -6$  and  $r = -4$ . Thus, the general solution is of the form  $y = c_1e^{-4t} + c_2e^{-6t}$ .

$$\begin{aligned} -3 &= y(0) & &= c_1 + c_2 \\ 2 &= y'(0) & &= -4c_1 + 6c_2 \end{aligned}$$

Solving this system yields  $c_1 = -2$  and  $c_2 = -1$ , so the solution to the IVP is

$$y = -2e^{-4t} - e^{-6t}$$

□