

Module C

Standard C1

C1. Find the general solution to

$$y' + 3y = 6t + 5.$$

C1. Find the general solution to

$$y' + 4y = 4.$$

C1. Find the general solution to

$$y' + 2y = 6t - 1.$$

C1. Find the general solution to

$$y' - y = e^t.$$

C1. Find the general solution to

$$y' + y = e^t.$$

C1. Find the general solution to

$$y' - y = e^{-t}.$$

C1. Find the general solution to

$$y' + y = e^{-t}.$$

C1. Find the general solution to

$$y' + 3y = 10 \sin(t).$$

C1. Find the general solution to

$$y' + 2y = 10 \sin(t).$$

C1. Find the general solution to

$$y' + 2y = 5 \sin(t).$$

C1. Find the general solution to

$$y' + 3y = 10 \cos(t).$$

C1. Find the general solution to

$$y' + 2y = 10 \cos(t).$$

C1. Find the general solution to

$$y' + 2y = 5 \cos(t).$$

C2. A water droplet with a radius of $100\ \mu\text{m}$ has a mass of about $4 \times 10^{-12}\text{kg}$ and a terminal velocity of $27\ \frac{\text{cm}}{\text{s}}$. Such a droplet is dropped from rest.

- (a) Write down an IVP modelling the velocity.
- (b) What is its velocity after $0.01\ \text{s}$?

C2. A water droplet with a radius of $100\ \mu\text{m}$ has a mass of about $4 \times 10^{-12}\text{kg}$. Such a droplet is dropped from rest.

- (a) Write down an IVP modelling the velocity.
- (b) After $0.02\ \text{s}$ it has reached half of its terminal velocity. What is its terminal velocity?

C2. A water droplet with a radius of $10\ \mu\text{m}$ has a mass of about $4 \times 10^{-15}\text{kg}$ and a terminal velocity of $270\ \frac{\mu\text{m}}{\text{s}}$. Such a droplet is dropped from rest.

- (a) Write down an IVP modelling the velocity.
- (b) What is its velocity after $0.001\ \text{s}$?

C2. A water droplet with a radius of $10\ \mu\text{m}$ has a mass of about $4 \times 10^{-15}\text{kg}$. Such a droplet is dropped from rest.

- (a) Write down an IVP modelling the velocity.
- (b) After $0.002\ \text{s}$ it has reached half of its terminal velocity. What is its terminal velocity?

C2. A single grain of corn pollen with a radius of $50\ \mu\text{m}$ and a mass of about $5 \times 10^{-13}\text{kg}$ has a terminal velocity of $27\ \frac{\text{cm}}{\text{s}}$. Such a pollen grain is dropped from rest.

- (a) Write down an IVP modelling the velocity.
- (b) What is its velocity after $0.01\ \text{s}$?

C2. A single grain of corn pollen with a radius of $50\ \mu\text{m}$ and a mass of about $5 \times 10^{-13}\text{kg}$. Such a pollen grain is dropped from rest.

- (a) Write down an IVP modelling the velocity.
- (b) After $0.01\ \text{s}$ it has reached half of its terminal velocity. What is its terminal velocity?

C3. Find the general solution to

$$y'' + 2y' + y = 0.$$

C3. Find the general solution to

$$y'' + 2y' - 8y = 0.$$

C3. Find the general solution to

$$y'' + 4y' + 3y = 0.$$

C3. Find the general solution to

$$y'' + 2y' - 3y = 0.$$

C3. Find the general solution to

$$y'' - 2y' - 3y = 0.$$

C3. Find the general solution to

$$y'' + 4y' + 4y = 0.$$

C3. Find the general solution to

$$y'' - 4y' + 4y = 0.$$

C3. Find the general solution to

$$y'' + 5y' + 6y = 0.$$

C3. Find the general solution to

$$y'' - 2y' + 2y = 0.$$

C3. Find the general solution to

$$y'' + 2y' + 2y = 0.$$

C3. Find the general solution to

$$y'' - 6y' + 10y = 0.$$

C3. Find the general solution to

$$y'' + 6y' + 10y = 0.$$

C3. Find the general solution to

$$y'' - 2y' + 5y = 0.$$

C3. Find the general solution to

$$y'' + 2y' + 5y = 0.$$

C3. Find the general solution to

$$y'' - 4y' + 5y = 0.$$

C3. Find the general solution to

$$y'' + 4y' + 5y = 0.$$

C4. Find the solution to

$$y'' + 2y' + y = 0$$

when $y(0) = 0$ and $y'(0) = 2$.

C4. Find the solution to

$$y'' + 2y' + y = 0$$

when $y(0) = 2$ and $y'(0) = 0$.

C4. Find the solution to

$$y'' + 2y' - 8y = 0$$

when $y(0) = 3$ and $y'(0) = -6$.

C4. Find the solution to

$$y'' + 4y' + 3y = 0$$

when $y(0) = 1$ and $y'(0) = 5$.

C4. Find the solution to

$$y'' + 2y' - 3y = 0$$

when $y(0) = 5$ and $y'(0) = 1$.

C4. Find the solution to

$$y'' + 2y' - 3y = 0$$

when $y(0) = 2$ and $y'(0) = 2$.

C4. Find the solution to

$$y'' - 2y' - 3y = 0$$

when $y(0) = 2$ and $y'(0) = 2$.

C4. Find the solution to

$$y'' + 4y' + 4y = 0$$

when $y(0) = 1$ and $y'(0) = 3$.

C4. Find the solution to

$$y'' - 4y' + 4y = 0$$

when $y(0) = 1$ and $y'(0) = 3$.

C4. Find the solution to

$$y'' + 4y' + 4y = 0$$

when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to

$$y'' - 4y' + 4y = 0$$

when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to

$$y'' + 5y' + 6y = 0$$

when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to

$$y'' + 5y' + 6y = 0$$

when $y(0) = 1$ and $y'(0) = 2$.

C5. Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2 \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^x$$

C5. Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

C6. Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant 4kg/s^2). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?

C6. Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant 4kg/s^2). The mass is pushed up 0.5m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?

C6. Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant 1kg/s^2). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 3s?

C6. Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant 1kg/s^2). The mass is pushed up 0.5m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 1kg/s). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s). An external force is applied, modelled by the function $F(t) = \sin(t)$. The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s). An external force is applied, modelled by the function $F(t) = \cos(t)$. The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

Module F

Standard F1

F1. Sketch a solution curve through each point marked in the slope field.



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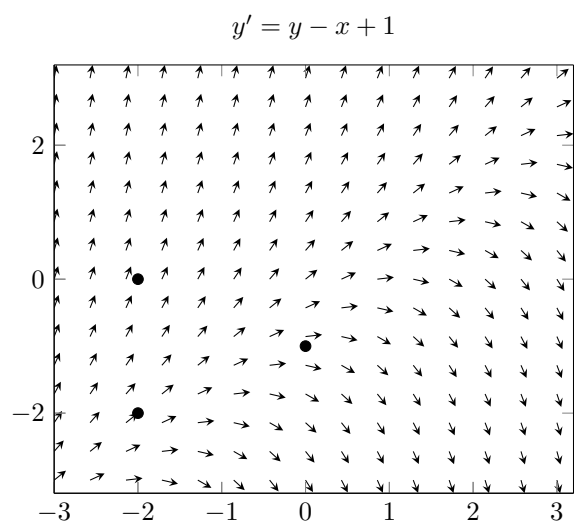
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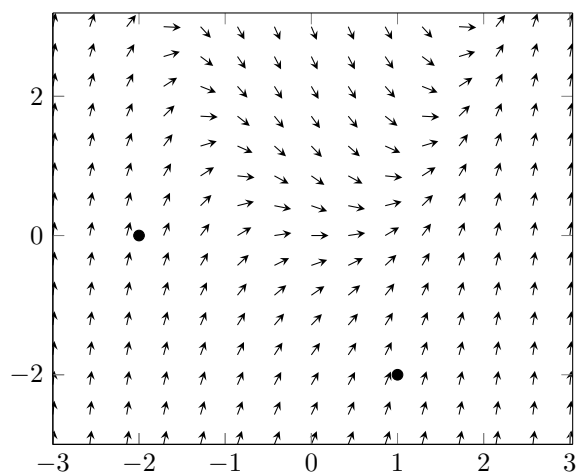


F1. Sketch a solution curve through each point marked in the slope field.



F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 - y$$



F2. Find the general solution to $\frac{dy}{dx} + 3xy = 0$.

F2. Find the general solution to $y' - y \sin(x) = 0$.

F2. Find the general solution to $y' = \frac{x+2}{y}$.

F2. Find the general solution to $\frac{dy}{dx} = \frac{1+x}{1+y}$.

F2. Find the general solution to $xy' = y$.

F2. Find the general solution to $y \frac{dy}{dx} = y^2 \cos(x)$.

F2. Find the general solution to $xy \frac{dy}{dx} = 1$.

F2. Find the general solution to $x \cos(y)y' = 1$.

F3.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x - 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = 1 - x.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = (x - 3)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(1) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = (x + 4)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(4) = 0$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = (4 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(3) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = (5 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 7x + 10.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(0) = 3$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(3) = 0$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(5) = 1$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F5. Find the general solution to $xy' + 4y = 2x$.

F5. Find the general solution to $xy' + 4y = \sqrt{x}$ (for $x > 0$).

F5. Find the general solution to $xy' + 2y = x^2$.

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(x + 2y)y' + y &= 2x \\ (x + 2y)y' - y &= -2x\end{aligned}$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(3x + 2y)y' + 3y &= 2x \\ (3x + 2y)y' - 3y &= -2x\end{aligned}$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(x^2 + 3y^2)y' - 2xy &= -3x^2 \\ (x^2 + 3y^2)y' + 2xy &= 3x^2\end{aligned}$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(2xy + 3y^2)y' + y^2 &= 3x^2 \\ (2xy + 3y^2)y' - y^2 &= -3x^2\end{aligned}$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}\cos(x) \cos(y)y' &= \sin(x) \sin(y) \\ \cos(x) \cos(y)y' &= \sin(x) + \sin(y)\end{aligned}$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}\sin(x) \sin(y)y' &= \cos(x) + \cos(y) \\ \sin(x) \sin(y)y' &= \cos(x) \cos(y)\end{aligned}$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(y^3 e^x + x e^x)y' + 3e^x y^2 &= 3x^2 \\ (2y e^x + e^y)y' + e^x y^2 &= 3x^2\end{aligned}$$

Module S

Standard S1

S1. Find the general solution of the system

$$\begin{aligned}x' &= x + y, \\y' &= 4x + y.\end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned}x' &= x + 2y, \\y' &= 3x + 2y.\end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= x + 2y.\end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= 2x + 3y.\end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned}x' &= 3x + y, \\y' &= x + 3y.\end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned}x' &= 3x + y, \\y' &= 2x + 2y.\end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned}x' &= 4x + y, \\y' &= 2x + 3y.\end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned}x' &= 4x + 3y, \\y' &= x + 2y.\end{aligned}$$

Module N

Standard N1

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x^2y + xy^2; \quad y(1) = 3$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = 2x^2 + xy + 3y^2; \quad y(1) = -1$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x + \ln(y); \quad y(1) = 2$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt{x+y}; \quad y(1) = 1$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt[3]{x-y}; \quad y(2) = 2$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \frac{y}{x}; \quad y(2) = 1$$

N2. Consider the differential equation

$$xy'' + y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$xy'' - y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$x^2y'' - 4xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$x^2y'' - xy' - 3y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$x^2y'' + xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$y'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$e^xy'' - 2y' + 4e^{4x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$y'' + y' - e^{-2x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{3}{t}x + 2y, \\y' &= 2\ln(t)x + y + 1\end{aligned}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{2}{t}x + y, \\y' &= x + \ln(t)y + 2\end{aligned}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -x + \sqrt{t}, \\y' &= 2x + ty + \sqrt[3]{t}\end{aligned}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + 2y + \sqrt{t}, \\y' &= x + y + \sqrt[3]{t}\end{aligned}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + y + \sqrt[3]{t}, \\y' &= x + 2y + \sqrt{t}\end{aligned}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= tx + 2y + \sqrt[3]{t}, \\y' &= -y + \sqrt{t}\end{aligned}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + \ln(t)y + 2, \\y' &= -\frac{1}{t}y + 2t\end{aligned}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = 2 \ln(t)x + y + 1,$$

$$y' = -\frac{2}{t}x + y$$

Module D

Standard D1

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t-5)\}(s) = \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{3t}\}(s) = \frac{1}{s-3}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4) + e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t) + u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{1 + e^t\}(s) = \frac{1}{s} + \frac{1}{s-1}.$$