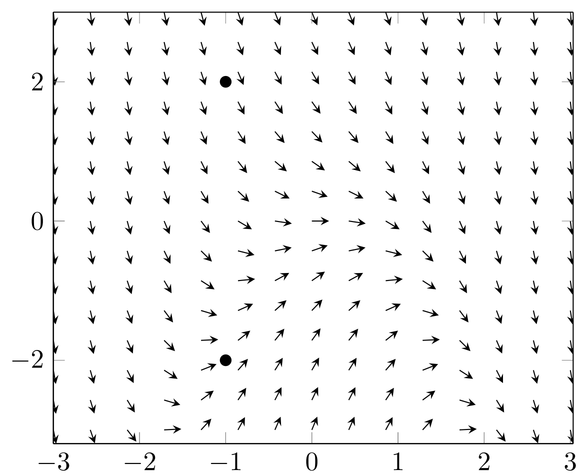
Sample Assessment Exercises

This document contains one exercise and solution for each standard. The goal is to give you an idea of what the exercises might look like, and what the expectations for a complete solution are.

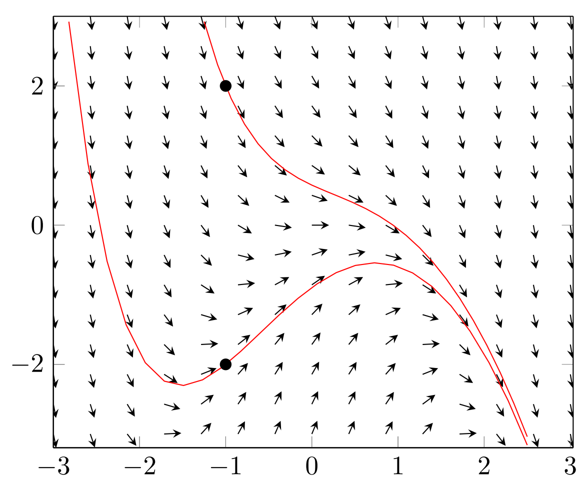
C1. Sketch a solution curve through each point marked in the slope field.

$y' = -y - x^2$



Solution:

$y' = -y - x^2$



$\square$

C2. Find the general solution to

$$y' + y = -t^2.$$

Solution: First, we find a general solution to the homogeneous equation

$$y' + y = 0.$$

This has auxilliary equation $r + 1 = 0$, which has a single root at $r = 1$, so ce^{-t} is a solution. We can find a particular solution y_p to the given equation by using undetermined coefficients; since $-t^2$ is a polynomial, we let $y_p = At^2 + Bt + D$ and determine the coefficients A, B, and D.

$$\begin{aligned} y_p' + y_p = &= (2At + B) + (At^2 + Bt + D) \\ &= At^2 + (2A + B) + (B + D) \end{aligned}$$

So if y_p is a solution, we must have $y_p' + y_p = -t^2$, giving us the system of equations

$$\begin{aligned} A &= -1 \\ 2A + B &= 0 \\ B + D &= 0 \end{aligned}$$

Thus we easily deduce that $A = -1$, $B = 2$, and $D = -2$, giving $y_p = -t^2 + 2t - 2$. Thus, the general solution is

$$y = -t^2 + 2t - 2 + ce^{-t}.$$

$\square$

C3. Find the general solution to

$$y'' + 6y' + 13y = 0.$$

Solution: We begin by writing the auxilliary equation $r^2 + 6r + 13 = 0$ and finding the roots. There are many ways to do this; here, we complete the square:

$$0 = r^2 + 6r + 13 = r^2 + 6r + 9 + 4 = (r + 3)^2 + 4.$$

Thus, we can easily solve to obtain $r = -3 \pm 2i$. Thus the general solution is

$$y = c_1 e^{-3t} \cos(2t) + c_2 e^{-3t} \sin(2t).$$

$\square$