

# Module S: Systems of ODEs

# How can we solve and apply systems of linear ODEs?

At the end of this module, students will be able to...

- S1. Solving systems.** ...solve systems of constant coefficient ODEs
- S2. Modeling interacting populations.** ...model the populations of two interacting populations with a system of ODEs
- S3. Modeling coupled oscillators.** ...model systems of coupled mechanical oscillators using a system of ODEs

## Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Add Euclidean vectors and multiply Euclidean vectors by scalars.
- Perform basic manipulations of augmented matrices and linear systems **E1,E2,E3**.
- Apply linear combinations and spanning sets **V3,V4**.

The following resources will help you prepare for this module.

- Adding and subtracting Euclidean vectors (Khan Academy):  
<http://bit.ly/2y8A0wa>
- Linear combinations of Euclidean vectors (Khan Academy):  
<http://bit.ly/2nK3wne>
- Adding and subtracting complex numbers (Khan Academy):  
<http://bit.ly/1PE3ZMQ>
- Adding and subtracting polynomials (Khan Academy):  
<http://bit.ly/2d5SLGZ>

# Module S Section 1

**Activity S.1.1** (*~10 min*)

Consider the countries of Transia and Wakanda: each year, 8% of people living in Transia move to Wakanda, and 3% of Wakandans move to Transia.

Let  $T$  be the population of Transia, and  $W$  the population of Wakanda (both are functions of time,  $t$ ).

Write down two differential equations modelling the population changes  $\frac{dT}{dt}$  and  $\frac{dW}{dt}$ .

**Activity S.1.2** ( $\sim 5$  min)

This problem resulted in a **system of linear differential equations**, namely

$$T' = 0.03W - 0.08T$$

$$W' = 0.08T - 0.03W$$

Rewrite this system using differential operators.



**Activity S.1.3** (*~15 min*)

Solve the system

$$(D + 0.08)T - (0.03)W = 0$$

$$-0.08T + (D + 0.03)W = 0$$

**Observation S.1.4**

Because  $D$  is linear,  $a(D + b) = (D + b)a$  for constants  $a, b$ . This is not true in general!

Thus, for any **constant coefficient linear systems of differential equations**, we can use our typical elimination technique.

**Activity S.1.5** (*~15 min*)

Solve the system

$$x' = 5x - 2y$$

$$y' = 6y - 3x$$

with initial conditions  $x(0) = 2$ ,  $y(0) = -1$ .

**Activity S.1.6** (*~10 min*)

Solve the system

$$x' = x + 4y - t$$

$$y' = 2y - 3x + t$$