Module C

Standard C1

C1. Find the general solution to
$$y' + 3y = 6t + 5.$$

C1. Find the general solution to
$$y' + 4y = 4.$$

C1. Find the general solution to
$$y' + 2y = 6t - 1.$$

C1. Find the general solution to
$$y' - y = e^t.$$

C1. Find the general solution to
$$y' + y = e^t.$$

C1. Find the general solution to
$$y' - y = e^{-t}.$$

C1. Find the general solution to
$$y' + y = e^{-t}.$$

C1. Find the general solution to
$$y' + 3y = 10\sin(t).$$

C1. Find the general solution to
$$y' + 2y = 10\sin(t).$$

C1. Find the general solution to
$$y' + 2y = 5\sin(t).$$

C1. Find the general solution to
$$y' + 3y = 10\cos(t).$$

C1. Find the general solution to

$$y' + 2y = 10\cos(t).$$

C1. Find the general solution to

$$y' + 2y = 5\cos(t).$$

C2. A water droplet with a radius of 100 μm has a mass of about 4×10^{-12}kg and a terminal velocity of 27 $\frac{\text{cm}}{\text{s}}$. Such a droplet is dropped from rest.

(a) Write down an IVP modelling the velocity.

(b) What is its velocity after 0.01 s?

C2. A water droplet with a radius of 100 μm has a mass of about 4×10^{-12}kg. Such a droplet is dropped from rest.

(a) Write down an IVP modelling the velocity.

(b) After 0.02 s it has reached half of its terminal velocity. What is its terminal velocity?

C2. A water droplet with a radius of 10 μm has a mass of about 4×10^{-15}kg and a terminal velocity of 270 $\frac{\mu\text{m}}{\text{s}}$. Such a droplet is dropped from rest.

(a) Write down an IVP modelling the velocity.

(b) What is its velocity after 0.001 s?

C2. A water droplet with a radius of 10 μm has a mass of about 4×10^{-15}kg. Such a droplet is dropped from rest.

(a) Write down an IVP modelling the velocity.

(b) After 0.002 s it has reached half of its terminal velocity. What is its terminal velocity?

C2. A single grain of corn pollen with a radius of 50 μm and a mass of about 5×10^{-13}kg has a terminal velocity of 27 $\frac{\text{cm}}{\text{s}}$. Such a pollen grain is dropped from rest.

(a) Write down an IVP modelling the velocity.

(b) What is its velocity after 0.01 s?

C2. A single grain of corn pollen with a radius of 50 μm and a mass of about 5×10^{-13}kg. Such a pollen grain is dropped from rest.

(a) Write down an IVP modelling the velocity.

(b) After 0.01 s it has reached half of its terminal velocity. What is its terminal velocity?

C3. Find the general solution to
$$y'' + 2y' + y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 8y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 3y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' - 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' - 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' + 5y' + 6y = 0.$$

C3. Find the general solution to
$$y'' - 2y' + 2y = 0.$$

C3. Find the general solution to
$$y'' + 2y' + 2y = 0.$$

C3. Find the general solution to
$$y'' - 6y' + 10y = 0.$$

C3. Find the general solution to
$$y'' + 6y' + 10y = 0.$$

C3. Find the general solution to
$$y'' - 2y' + 5y = 0.$$

C3. Find the general solution to
$$y'' + 2y' + 5y = 0.$$

C3. Find the general solution to
$$y'' - 4y' + 5y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 5y = 0.$$

C4. Find the solution to
$$y'' + 2y' + y = 0$$
when $y(0) = 0$ and $y'(0) = 2$.

C4. Find the solution to
$$y'' + 2y' + y = 0$$
when $y(0) = 2$ and $y'(0) = 0$.

C4. Find the solution to
$$y'' + 2y' - 8y = 0$$
when $y(0) = 3$ and $y'(0) = -6$.

C4. Find the solution to
$$y'' + 4y' + 3y = 0$$
when $y(0) = 1$ and $y'(0) = 5$.

C4. Find the solution to
$$y'' + 2y' - 3y = 0$$
when $y(0) = 5$ and $y'(0) = 1$.

C4. Find the solution to
$$y'' + 2y' - 3y = 0$$
when $y(0) = 2$ and $y'(0) = 2$.

C4. Find the solution to
$$y'' - 2y' - 3y = 0$$
when $y(0) = 2$ and $y'(0) = 2$.

C4. Find the solution to
$$y'' + 4y' + 4y = 0$$
when $y(0) = 1$ and $y'(0) = 3$.

C4. Find the solution to
$$y'' - 4y' + 4y = 0$$
when $y(0) = 1$ and $y'(0) = 3$.

C4. Find the solution to
$$y'' + 4y' + 4y = 0$$
when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to
$$y'' - 4y' + 4y = 0$$
when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to
$$y'' + 5y' + 6y = 0$$

when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to

$$y'' + 5y' + 6y = 0$$

when $y(0) = 1$ and $y'(0) = 2$.

C5. Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2\sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^x$$

C5. Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

C6. Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant 4kg/s^2). The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) How long does it take for the mass to return to its equillibrium point?

C6. Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant 4kg/s^2). The mass is pushed up 0.5m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) How long does it take for the mass to return to its equillibrium point?

C6. Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant 1kg/s^2). The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 3s?

C6. Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant 1kg/s^2). The mass is pushed up 0.5m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 1kg/s. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s. An external force is applied, modelled by the function $F(t) = \sin(t)$. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s. An external force is applied, modelled by the function $F(t) = \cos(t)$. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

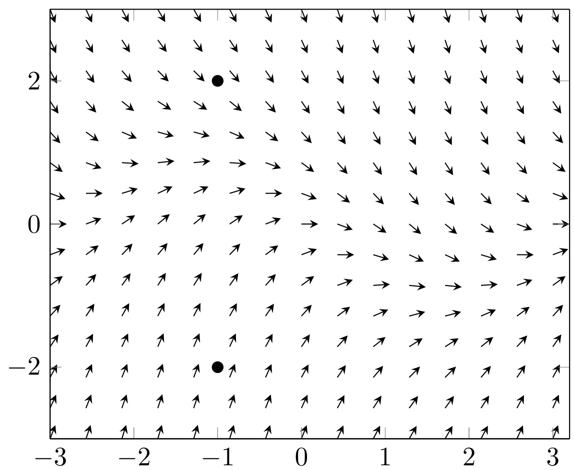
(b) Where is the mass after 2s?

Module F

Standard F1

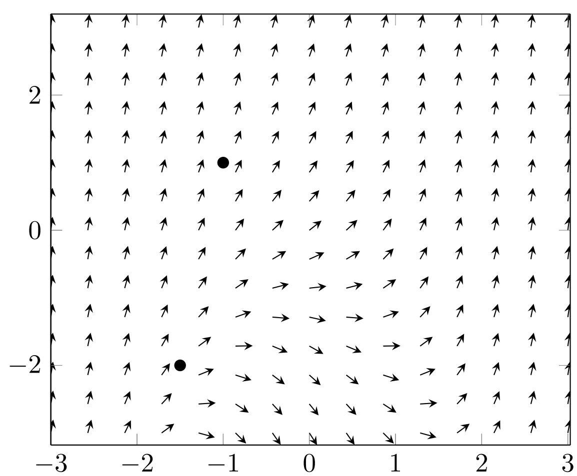
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = -y - \sin(x)$$



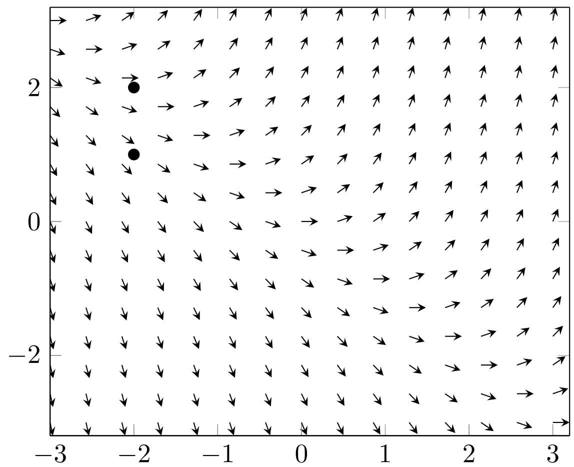
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 + y + 1$$



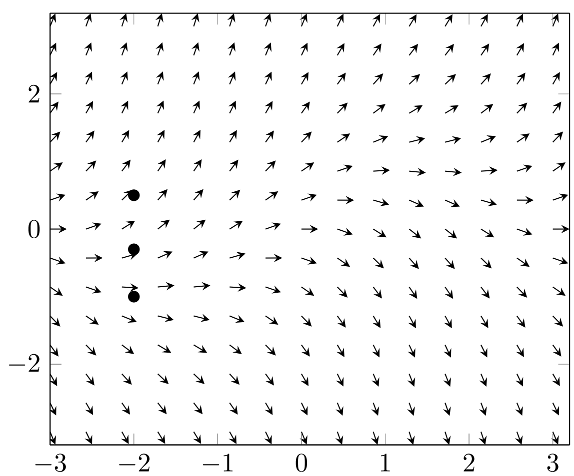
F1. Sketch a solution curve through each point marked in the slope field.

$y' = x + y$



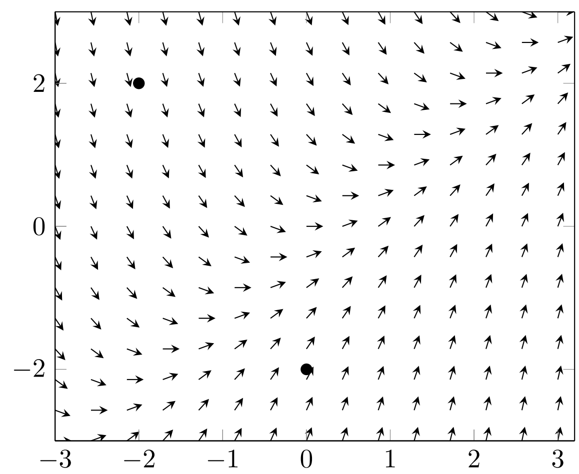
F1. Sketch a solution curve through each point marked in the slope field.

$y' = y - \sin(x)$



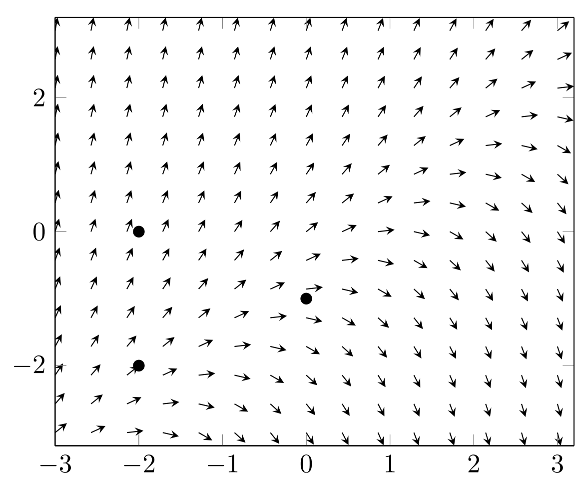
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x - y$$



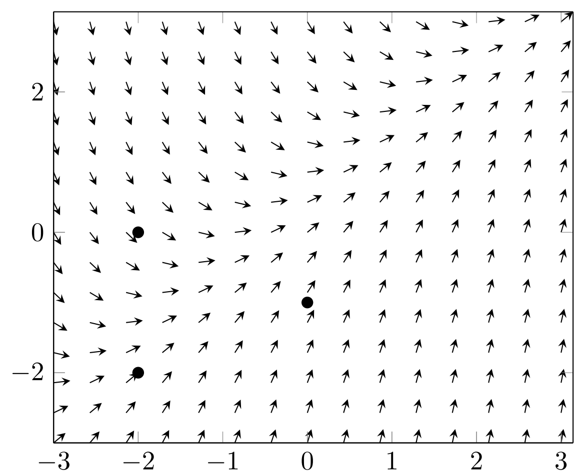
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = y - x + 1$$



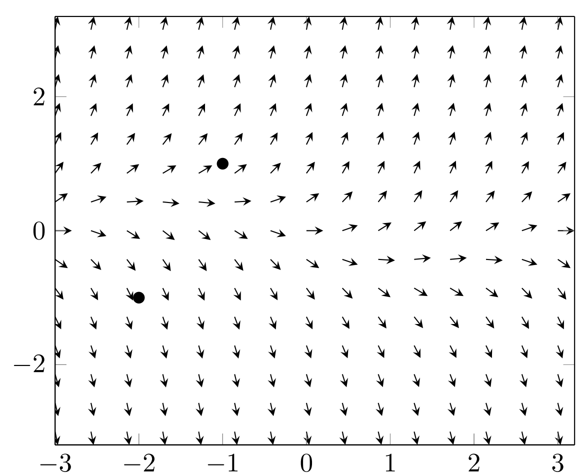
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x - y + 1$$



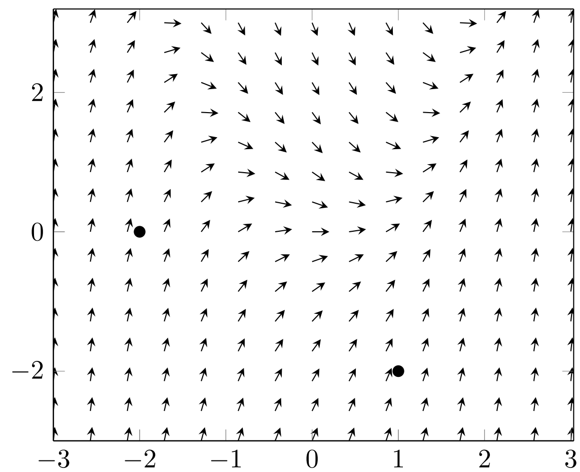
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = \sin(x) + 2y$$



F1. Sketch a solution curve through each point marked in the slope field.

$y' = x^2 - y$



F2. Find the general solution to $\frac{dy}{dx} + 3xy = 0$.

F2. Find the general solution to $y' - y\sin(x) = 0$.

F2. Find the general solution to $y' = \frac{x+2}{y}$.

F2. Find the general solution to $\frac{dy}{dx} = \frac{1+x}{1+y}$.

F2. Find the general solution to $xy' = y$.

F2. Find the general solution to $y\frac{dy}{dx} = y^2\cos(x)$.

F2. Find the general solution to $xy\frac{dy}{dx} = 1$.

F2. Find the general solution to $x\cos(y)y' = 1$.

F3.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = x - 3.$$
(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = 1 - x.$$
(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (x - 3)^2.$$
(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(1) = 2$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (x + 4)^2.$$
(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(4) = 0$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (4 - x)^3.$$
(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(3) = 2$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (5 - x)^3.$$
(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = x^2 - 7x + 10.$$
(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 3$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(3) = 0$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(5) = 1$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F5. Find the general solution to $xy' + 4y = 2x$.

F5. Find the general solution to $xy' + 4y = \sqrt{x}$ (for $x > 0$).

F5. Find the general solution to $xy' + 2y = x^2$.

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(x + 2y)y' + y = 2x$$
$$(x + 2y)y' - y = -2x$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(3x + 2y)y' + 3y = 2x$$
$$(3x + 2y)y' - 3y = -2x$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(x^2 + 3y^2)y' - 2xy = -3x^2$$
$$(x^2 + 3y^2)y' + 2xy = 3x^2$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(2xy + 3y^2)y' + y^2 = 3x^2$$
$$(2xy + 3y^2)y' - y^2 = -3x^2$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\cos(x)\cos(y)y' = \sin(x)\sin(y)$$
$$\cos(x)\cos(y)y' = \sin(x) + \sin(y)$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\sin(x)\sin(y)y' = \cos(x) + \cos(y)$$
$$\sin(x)\sin(y)y' = \cos(x)\cos(y)$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(y^3e^x + xe^x)y' + 3e^xy^2 = 3x^2$$
$$(2ye^x + e^y)y' + e^xy^2 = 3x^2$$

Module S

Standard S1

S1. Find the general solution of the system

$$x' = x + y,$$
$$y' = 4x + y.$$

S1. Find the general solution of the system

$$x' = x + 2y,$$
$$y' = 3x + 2y.$$

S1. Find the general solution of the system

$$x' = 2x + y,$$
$$y' = x + 2y.$$

S1. Find the general solution of the system

$$x' = 2x + y,$$
$$y' = 2x + 3y.$$

S1. Find the general solution of the system

$$x' = 3x + y,$$
$$y' = x + 3y.$$

S1. Find the general solution of the system

$$x' = 3x + y,$$
$$y' = 2x + 2y.$$

S1. Find the general solution of the system

$$\begin{aligned} x' &= 4x + y, \\ y' &= 2x + 3y. \end{aligned}$$

S1. Find the general solution of the system

$$\begin{aligned} x' &= 4x + 3y, \\ y' &= x + 2y. \end{aligned}$$

Module N

Standard N1

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x^2y + xy^2; \qquad y(1) = 3$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = 2x^2 + xy + 3y^2; \qquad y(1) = -1$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x + \ln(y); \qquad y(1) = 2$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt{x+y}; \qquad y(1) = 1$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt[3]{x-y}; \qquad y(2) = 2$$

N1. Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \frac{y}{x}; \qquad y(2) = 1$$

N2. Consider the differential equation
$$xy'' + y' = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$xy'' - y' = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$x^2y'' - 4xy' + 4y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$x^2y'' - xy' - 3y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$x^2y'' + xy' + 4y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$y'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$e^x y'' - 2y' + 4e^{4x}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation
$$y'' + y' - e^{-2x}y = 0.$$
Determine all intervals on which a unique solution is guaranteed to exist.

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{3}{t}x + 2y,$$
$$y' = 2\ln(t)x + y + 1$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{2}{t}x + y,$$
$$y' = x + \ln(t)y + 2$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -x + \sqrt{t},$$
$$y' = 2x + ty + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + 2y + \sqrt{t},$$
$$y' = x + y + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + y + \sqrt[3]{t},$$
$$y' = x + 2y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = tx + 2y + \sqrt[3]{t},$$
$$y' = -y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + \ln(t)y + 2,$$
$$y' = -\frac{1}{t}y + 2t$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = 2\ln(t)x + y + 1,$$
$$y' = -\frac{2}{t}x + y$$

Module D

Standard D1

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t-5)\}(s) = \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{3t}\}(s) = \frac{1}{s-3}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4) + e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t) + u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{1 + e^t\}(s) = \frac{1}{s} + \frac{1}{s-1}.$$