Module S: Systems of ODEs

How can we solve and apply systems of linear ODEs?

At the end of this module, students will be able to...

S1. Solving systems. ...solve systems of constant coefficient ODEs

S2. Modeling interacting populations. ...model the populations of two interacting populations with a system of ODEs

S3. Modeling coupled oscillators. ...model systems of coupled mechanical oscillators using a system of ODEs

Readiness Assurance Outcomes

Before beginning this module, each student should be able to...

- Add Euclidean vectors and multiply Euclidean vectors by scalars.
- Perform basic manipulations of augmented matrices and linear systems **E1,E2,E3**.
- Apply linear combinations and spanning sets **V3,V4**.

The following resources will help you prepare for this module.

- Adding and subtracting Euclidean vectors (Khan Acaemdy): `http://bit.ly/2y8AOwa`
- Linear combinations of Euclidean vectors (Khan Academy): `http://bit.ly/2nK3wne`
- Adding and subtracting complex numbers (Khan Academy): `http://bit.ly/1PE3ZMQ`
- Adding and subtracting polynomials (Khan Academy): `http://bit.ly/2d5SLGZ`

Module S Section 1

Activity S.1.1 *(∼10 min)*
Consider the countries of Transia and Wakanda: each year, 8% of people living in Transia move to Wakanda, and 3% of Wakandans move to Transia.

Let T be the population of Transia, and W the population of Wakanda (both are functions of time, t.

Write down two differential equations modelling the population changes $\frac{dT}{dt}$ and $\frac{dW}{dt}$.

Activity S.1.2 *(∼5 min)*
This problem resulted in a **system of linear differential equations**, namely

$$T' = 0.03W - 0.08T$$
$$W' = 0.08T - 0.03W$$

Rewrite this system using differential operators.

Activity S.1.3 *(∼15 min)*
Solve the system

$$(D + 0.08)T - (0.03)W = 0$$
$$-0.08T + (D + 0.03)W = 0$$

Observation S.1.4
Because D is linear, $a(D+b) = (D+b)a$ for constants a, b. This is not true in general!

Thus, for any **constant coefficient linear systems of differential equations**, we can use our typical elimination technique.

Activity S.1.5 *(∼15 min)*
Solve the system

$$x' = 5x - 2y$$
$$y' = 6y - 3x$$

with initial conditions $x(0) = 2$, $y(0) = -1$.

Activity S.1.6 *(∼10 min)*
Solve the system

$$x' = x + 4y - t$$
$$y' = 2y - 3x + t$$