

## Module C

### Standard C1

C1. Find the general solution to

$$y' + 3y = 6t + 5.$$

C1. Find the general solution to

$$y' + 4y = 4.$$

C1. Find the general solution to

$$y' + 2y = 6t - 1.$$

C1. Find the general solution to

$$y' - y = e^t.$$

C1. Find the general solution to

$$y' + y = e^t.$$

C1. Find the general solution to

$$y' - y = e^{-t}.$$

C1. Find the general solution to

$$y' + y = e^{-t}.$$

C1. Find the general solution to

$$y' + 3y = 10e^{-3t} \sin(t).$$

C1. Find the general solution to

$$y' + 2y = 10e^{-2t} \sin(t).$$

C1. Find the general solution to

$$y' + 2y = 5e^{-2t} \sin(t).$$

C1. Find the general solution to

$$y' + 3y = 10e^{-3t} \cos(t).$$

**C1.** Find the general solution to

$$y' + 2y = 10e^{-2t} \cos(t).$$

**C1.** Find the general solution to

$$y' + 2y = 5e^{-2t} \cos(t).$$

**C2.** A water droplet with a radius of  $100\ \mu\text{m}$  has a mass of about  $4 \times 10^{-12}\text{kg}$  and a terminal velocity of  $27\ \frac{\text{cm}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C2.** A water droplet with a radius of  $50\ \mu\text{m}$  has a mass of about  $5 \times 10^{-13}\text{kg}$  and a terminal velocity of  $3.5\ \frac{\text{cm}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C2.** A water droplet with a radius of  $10\ \mu\text{m}$  has a mass of about  $4 \times 10^{-15}\text{kg}$  and a terminal velocity of  $270\ \frac{\mu\text{m}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.001\ \text{s}$ ?

**C2.** A water droplet with a radius of  $5\ \mu\text{m}$  has a mass of about  $5 \times 10^{-16}\text{kg}$  and a terminal velocity of  $35\ \frac{\mu\text{m}}{\text{s}}$ . Such a droplet is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.001\ \text{s}$ ?

**C2.** A single grain of corn pollen with a radius of  $50\ \mu\text{m}$  and a mass of about  $5 \times 10^{-13}\text{kg}$  has a terminal velocity of  $27\ \frac{\text{cm}}{\text{s}}$ . Such a pollen grain is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C2.** A single grain of spruce pollen with a radius of  $25\ \mu\text{m}$  and a mass of about  $6 \times 10^{-14}\text{kg}$  has a terminal velocity of  $3\ \frac{\text{cm}}{\text{s}}$ . Such a pollen grain is dropped from rest.

- (a) Write down an Initial Value Problem (IVP) modelling the velocity.
- (b) What is its velocity after  $0.01\ \text{s}$ ?

**C3.** Find the general solution to

$$y'' + 2y' + y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' - 8y = 0.$$

**C3.** Find the general solution to

$$y'' + 4y' + 3y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' - 3y = 0.$$

**C3.** Find the general solution to

$$y'' - 2y' - 3y = 0.$$

**C3.** Find the general solution to

$$y'' + 4y' + 4y = 0.$$

**C3.** Find the general solution to

$$y'' - 4y' + 4y = 0.$$

**C3.** Find the general solution to

$$y'' + 5y' + 6y = 0.$$

**C3.** Find the general solution to

$$y'' - 2y' + 2y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' + 2y = 0.$$

**C3.** Find the general solution to

$$y'' - 6y' + 10y = 0.$$

**C3.** Find the general solution to

$$y'' + 6y' + 10y = 0.$$

**C3.** Find the general solution to

$$y'' - 2y' + 5y = 0.$$

**C3.** Find the general solution to

$$y'' + 2y' + 5y = 0.$$

**C3.** Find the general solution to

$$y'' - 4y' + 5y = 0.$$

**C3.** Find the general solution to

$$y'' + 4y' + 5y = 0.$$

**C4.** Find the solution to

$$y'' + 2y' + y = 0$$

when  $y(0) = 0$  and  $y'(0) = 2$ .

**C4.** Find the solution to

$$y'' + 2y' + y = 0$$

when  $y(0) = 2$  and  $y'(0) = 0$ .

**C4.** Find the solution to

$$y'' + 2y' - 8y = 0$$

when  $y(0) = 3$  and  $y'(0) = -6$ .

**C4.** Find the solution to

$$y'' + 4y' + 3y = 0$$

when  $y(0) = 1$  and  $y'(0) = 5$ .

**C4.** Find the solution to

$$y'' + 2y' - 3y = 0$$

when  $y(0) = 5$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' + 2y' - 3y = 0$$

when  $y(0) = 2$  and  $y'(0) = 2$ .

**C4.** Find the solution to

$$y'' - 2y' - 3y = 0$$

when  $y(0) = 2$  and  $y'(0) = 2$ .

**C4.** Find the solution to

$$y'' + 4y' + 4y = 0$$

when  $y(0) = 1$  and  $y'(0) = 3$ .

**C4.** Find the solution to

$$y'' - 4y' + 4y = 0$$

when  $y(0) = 1$  and  $y'(0) = 3$ .

**C4.** Find the solution to

$$y'' + 4y' + 4y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' - 4y' + 4y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' + 5y' + 6y = 0$$

when  $y(0) = 3$  and  $y'(0) = 1$ .

**C4.** Find the solution to

$$y'' + 5y' + 6y = 0$$

when  $y(0) = 1$  and  $y'(0) = 2$ .

C5. Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2 \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^x$$

C5. Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$



**C5.** Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

**C6.** Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?

**C6.** Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). The mass is pushed up 0.5m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) How long does it take for the mass to return to its equilibrium point?

**C6.** Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant  $1\text{kg/s}^2$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 3s?

**C6.** Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant  $1\text{kg/s}^2$ ). The mass is pushed up 0.5m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $6\text{kg/s}$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $1\text{kg/s}$ ). The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $6\text{kg/s}$ ). An external force is applied, modelled by the function  $F(t) = \sin(t)$ . The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

**C6.** Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant  $4\text{kg/s}^2$ ). A linear damper is attached to the system (with constant  $6\text{kg/s}$ ). An external force is applied, modelled by the function  $F(t) = \cos(t)$ . The mass is pulled down 1m from its equilibrium position and released from rest.

- (a) Write down an IVP modelling the position of the mass.
- (b) Where is the mass after 2s?

## Module F

### Standard F1

**F1.** Sketch a solution curve through each point marked in the slope field.



**F1.** Sketch a solution curve through each point marked in the slope field.



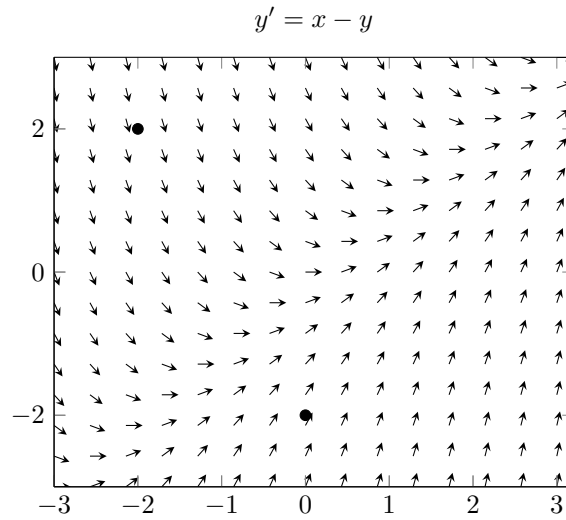
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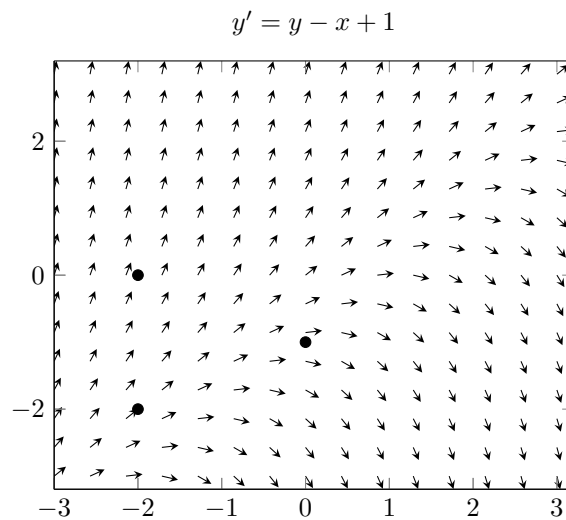
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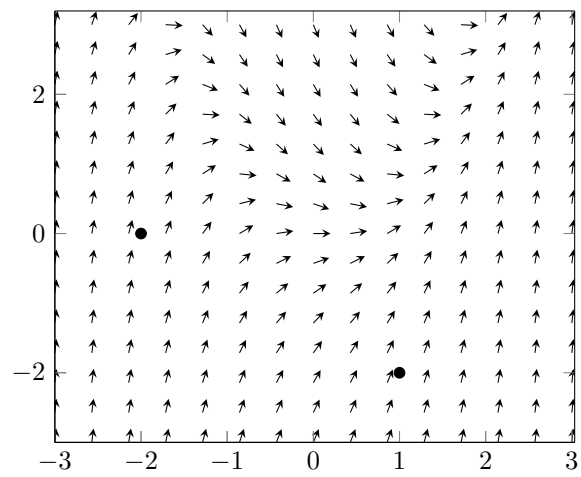


**F1.** Sketch a solution curve through each point marked in the slope field.



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$$y' = x^2 - y$$





**F2.** Find the general solution to  $\frac{dy}{dx} + 3xy = 0$ .

**F2.** Find the general solution to  $y' - y \sin(x) = 0$ .

**F2.** Find the general solution to  $y' - y^2 e^x = 0$ .

**F2.** Find the general solution to  $y' = \frac{x+2}{y^2}$ .

**F2.** Find the general solution to  $xy' = y$ .

**F2.** Find the general solution to  $y \frac{dy}{dx} = y^2 \cos(x)$ .

**F2.** Find the general solution to  $xy^2 \frac{dy}{dx} = 1$ .

**F2.** Find the general solution to  $x \cos(y) y' = 1$ .

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take the ball to travel across a tennis court, which is 24m long?

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 0.5s?

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take the ball to travel across a tennis court, which is 24m long?

**F3.** A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 0.5s?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How long does it take for the ball to reach a defender standing 30m away?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).
- (b) How far has the ball gone after 1s?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 45m/s.

- (a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How long does it take for the ball to reach a defender standing 30m away?

**F3.** A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 45m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How far has the ball gone after 1s?

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x - 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 4$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = 1 - x.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (x - 3)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(1) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (x + 4)^2.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(4) = 0$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (4 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(3) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = (5 - x)^3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 4$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 7x + 10.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(0) = 3$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(3) = 0$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(5) = 1$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F4.** Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

- (a) Find and classify the critical points.
- (b) Describe the long term behavior of the solution passing through the point  $x(2) = 2$ .

**F5.** Find the general solution to  $xy' + 4y = 2x$ .

**F5.** Find the general solution to  $xy' + 2y = x^2$ .

**F5.** Find the general solution to  $xy' + 2y = 4x^2 - 3x$ .

**F5.** Find the general solution to  $xy' + 2y = x^2 - 3x$ .

**F5.** Find the general solution to  $\cos(x)y' + \sin(x)y = x + \sin(x)\cos(x)$ .

**F5.** Find the general solution to  $\cos(x)y' + \sin(x)y = x\cos^2(x)$ .

**F5.** Find the general solution to  $(x^2 + 1)y' - 2xy = 1$ .

**F5.** Find the general solution to  $(x^2 + 1)y' - 2xy = x + 1$ .

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(x + 2y)y' + y &= 2x \\ (x + 2y)y' - y &= -2x\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(3x + 2y)y' + 3y &= 2x \\ (3x + 2y)y' - 3y &= -2x\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(x^2 + 3y^2)y' - 2xy &= -3x^2 \\ (x^2 + 3y^2)y' + 2xy &= 3x^2\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(2xy + 3y^2)y' + y^2 &= 3x^2 \\ (2xy + 3y^2)y' - y^2 &= -3x^2\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}\cos(x) \cos(y)y' &= \sin(x) \sin(y) \\ \cos(x) \cos(y)y' &= \sin(x) + \sin(y)\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}\sin(x) \sin(y)y' &= \cos(x) + \cos(y) \\ \sin(x) \sin(y)y' &= \cos(x) \cos(y)\end{aligned}$$

**F6.** One of the two ODEs below is exact. Identify which one, and solve it.

$$\begin{aligned}(y^3 e^x + x e^x)y' + 3e^x y^2 &= 3x^2 \\ (2y e^x + e^y)y' + e^x y^2 &= 3x^2\end{aligned}$$

## Module S

### Standard S1

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= x + y, \\y' &= 4x + y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= x + 2y, \\y' &= 3x + 2y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= x + 2y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 2x + y, \\y' &= 2x + 3y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 3x + y, \\y' &= x + 3y.\end{aligned}$$

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$$\begin{aligned}x' &= 3x + y, \\y' &= 2x + 2y.\end{aligned}$$



**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 4x + y, \\y' &= 2x + 3y.\end{aligned}$$

**S1.** Find the general solution of the system

$$\begin{aligned}x' &= 4x + 3y, \\y' &= x + 2y.\end{aligned}$$

**S2.** Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\begin{aligned}\frac{dG}{dt} &= 0.1G - 0.002G^2 - 0.005BG \\ \frac{dB}{dt} &= 0.1B - 0.003B^2 - 0.005BG.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a lake is stocked with , what will happen to the two populations in the long term?

**S2.** Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\begin{aligned}\frac{dG}{dt} &= 0.1G - 0.002G^2 - 0.005BG \\ \frac{dB}{dt} &= 0.1B - 0.004B^2 - 0.005BG.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a lake is stocked with , what will happen to the two populations in the long term?

**S2.** Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\begin{aligned}\frac{dG}{dt} &= 0.1G - 0.002G^2 - 0.004BG \\ \frac{dB}{dt} &= 0.1B - 0.003B^2 - 0.005BG.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If a lake is stocked with , what will happen to the two populations in the long term?

**S2.** The populations of foxes and rabbits in an ecosystem are modelled by the system

$$\begin{aligned}\frac{dF}{dt} &= -0.1F - 0.001F^2 + 0.002FR \\ \frac{dR}{dt} &= 0.2R - 0.001R^2 - 0.005FR.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If there are presently 100 foxes and 200 rabbits , what will happen to the two populations in the long term?

**S2.** The populations of foxes and rabbits in an ecosystem are modelled by the system

$$\begin{aligned}\frac{dF}{dt} &= -0.1F - 0.001F^2 + 0.002FR \\ \frac{dR}{dt} &= 0.4R - 0.001R^2 - 0.005FR.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If there are presently 100 foxes and 200 rabbits , what will happen to the two populations in the long term?

**S2.** The populations of greenflies and ladybirds in an ecosystem are modelled by the system

$$\begin{aligned}\frac{dL}{dt} &= -0.1L - 0.001L^2 + 0.002LG \\ \frac{dG}{dt} &= 0.4G - 0.001G^2 - 0.001GL.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If there are presently 1000 ladybirds and 2000 greenflies, what will happen to the two populations in the long term?

**S2.** The populations of greenflies and ladybirds in an ecosystem are modelled by the system

$$\begin{aligned}\frac{dL}{dt} &= -0.1L - 0.001L^2 + 0.002LG \\ \frac{dG}{dt} &= 0.2G - 0.001G^2 - 0.001GL.\end{aligned}$$

- (a) Identify all equilibrium points for the system.
- (b) If there are presently 1000 ladybirds and 2000 greenflies, what will happen to the two populations in the long term?

**S3.**

## Module N

### Standard N1

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x^2y + xy^2; \quad y(1) = 3$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = 2x^2 + xy + 3y^2; \quad y(1) = -1$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = x + \ln(y); \quad y(1) = 2$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt{x+y}; \quad y(1) = 1$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \sqrt[3]{x-y}; \quad y(2) = 2$$

**N1.** Determine whether existence of at least one solution of the given initial value problem is guaranteed and, if so, whether uniqueness of that solution is guaranteed.

$$y' = \frac{y}{x}; \quad y(2) = 1$$

**N2.** Consider the differential equation

$$xy'' + y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$xy'' - y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' - 4xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' - xy' - 3y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$x^2y'' + xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$e^xy'' - 2y' + 4e^{4x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N2.** Consider the differential equation

$$y'' + y' - e^{-2x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{3}{t}x + 2y, \\y' &= 2\ln(t)x + y + 1\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -\frac{2}{t}x + y, \\y' &= x + \ln(t)y + 2\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= -x + \sqrt{t}, \\y' &= 2x + ty + \sqrt[3]{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + 2y + \sqrt{t}, \\y' &= x + y + \sqrt[3]{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + y + \sqrt[3]{t}, \\y' &= x + 2y + \sqrt{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= tx + 2y + \sqrt[3]{t}, \\y' &= -y + \sqrt{t}\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned}x' &= x + \ln(t)y + 2, \\y' &= -\frac{1}{t}y + 2t\end{aligned}$$

**N3.** Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = 2 \ln(t)x + y + 1,$$

$$y' = -\frac{2}{t}x + y$$



N4.

N4.

## Module D

### Standard D1

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{u(t-5)\}(s) = \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{e^{3t}\}(s) = \frac{1}{s-3}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4) + e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t) + u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

**D1.** Demonstrate directly from the definition that

$$\mathcal{L}\{1 + e^t\}(s) = \frac{1}{s} + \frac{1}{s-1}.$$

**D2.**

**D3.**

D4.