Module C

Standard C1

C1. Find the general solution to

$$y' + 3y = 6t + 5.$$

C1. Find the general solution to

$$y' + 4y = 4.$$

C1. Find the general solution to

$$y' + 2y = 6t - 1.$$

C1. Find the general solution to

$$y' - y = e^t.$$

C1. Find the general solution to

$$y' + y = e^t.$$

C1. Find the general solution to

$$y' - y = e^{-t}.$$

C1. Find the general solution to

$$y' + y = e^{-t}.$$

C1. Find the general solution to

$$y' + 3y = 10e^{-3t}\sin(t).$$

C1. Find the general solution to

$$y' + 2y = 10e^{-2t}\sin(t).$$

C1. Find the general solution to

$$y' + 2y = 5e^{-2t}\sin(t).$$

C1. Find the general solution to

$$y' + 3y = 10e^{-3t}\cos(t).$$

C1. Find the general solution to

$$y' + 2y = 10e^{-2t}\cos(t).$$

C1. Find the general solution to

$$y' + 2y = 5e^{-2t}\cos(t).$$

C2. A water droplet with a radius of 100 μm has a mass of about 4×10^{-12}kg and a terminal velocity of 27 $\frac{\text{cm}}{\text{s}}$. Such a droplet is dropped from rest.

(a) Write down an Initial Value Problem (IVP) modelling the velocity.

(b) What is its velocity after 0.01 s?

C2. A water droplet with a radius of 50 μm has a mass of about 5×10^{-13}kg and a terminal velocity of 3.5 $\frac{\text{cm}}{\text{s}}$. Such a droplet is dropped from rest.

(a) Write down an Initial Value Problem (IVP) modelling the velocity.

(b) What is its velocity after 0.01 s?

C2. A water droplet with a radius of 10 μm has a mass of about 4×10^{-15}kg and a terminal velocity of 270 $\frac{\mu\text{m}}{\text{s}}$. Such a droplet is dropped from rest.

(a) Write down an Initial Value Problem (IVP) modelling the velocity.

(b) What is its velocity after 0.001 s?

C2. A water droplet with a radius of 5 μm has a mass of about 5×10^{-16}kg and a terminal velocity of 35 $\frac{\mu\text{m}}{\text{s}}$. Such a droplet is dropped from rest.

(a) Write down an Initial Value Problem (IVP) modelling the velocity.

(b) What is its velocity after 0.001 s?

C2. A single grain of corn pollen with a radius of 50 μm and a mass of about 5×10^{-13}kg has a terminal velocity of 27 $\frac{\text{cm}}{\text{s}}$. Such a pollen grain is dropped from rest.

(a) Write down an Initial Value Problem (IVP) modelling the velocity.

(b) What is its velocity after 0.01 s?

C2. A single grain of spruce pollen with a radius of 25 μm and a mass of about 6×10^{-14}kg has a terminal velocity of 3 $\frac{\text{cm}}{\text{s}}$. Such a pollen grain is dropped from rest.

(a) Write down an Initial Value Problem (IVP) modelling the velocity.

(b) What is its velocity after 0.01 s?

C3. Find the general solution to
$$y'' + 2y' + y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 8y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 3y = 0.$$

C3. Find the general solution to
$$y'' + 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' - 2y' - 3y = 0.$$

C3. Find the general solution to
$$y'' + 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' - 4y' + 4y = 0.$$

C3. Find the general solution to
$$y'' + 5y' + 6y = 0.$$

C3. Find the general solution to
$$y'' - 2y' + 2y = 0.$$

C3. Find the general solution to
$$y'' + 2y' + 2y = 0.$$

C3. Find the general solution to
$$y'' - 6y' + 10y = 0.$$

C3. Find the general solution to
$$y'' + 6y' + 10y = 0.$$

C3. Find the general solution to

$$y'' - 2y' + 5y = 0.$$

C3. Find the general solution to

$$y'' + 2y' + 5y = 0.$$

C3. Find the general solution to

$$y'' - 4y' + 5y = 0.$$

C3. Find the general solution to

$$y'' + 4y' + 5y = 0.$$

C4. Find the solution to
$$y'' + 2y' + y = 0$$
when $y(0) = 0$ and $y'(0) = 2$.

C4. Find the solution to
$$y'' + 2y' + y = 0$$
when $y(0) = 2$ and $y'(0) = 0$.

C4. Find the solution to
$$y'' + 2y' - 8y = 0$$
when $y(0) = 3$ and $y'(0) = -6$.

C4. Find the solution to
$$y'' + 4y' + 3y = 0$$
when $y(0) = 1$ and $y'(0) = 5$.

C4. Find the solution to
$$y'' + 2y' - 3y = 0$$
when $y(0) = 5$ and $y'(0) = 1$.

C4. Find the solution to
$$y'' + 2y' - 3y = 0$$
when $y(0) = 2$ and $y'(0) = 2$.

C4. Find the solution to
$$y'' - 2y' - 3y = 0$$
when $y(0) = 2$ and $y'(0) = 2$.

C4. Find the solution to
$$y'' + 4y' + 4y = 0$$
when $y(0) = 1$ and $y'(0) = 3$.

C4. Find the solution to
$$y'' - 4y' + 4y = 0$$
when $y(0) = 1$ and $y'(0) = 3$.

C4. Find the solution to
$$y'' + 4y' + 4y = 0$$
when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to
$$y'' - 4y' + 4y = 0$$
when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to
$$y'' + 5y' + 6y = 0$$

when $y(0) = 3$ and $y'(0) = 1$.

C4. Find the solution to

$$y'' + 5y' + 6y = 0$$

when $y(0) = 1$ and $y'(0) = 2$.

C5. Find a general solution to the given equation.

$$y'' + 2y' + y = 3x + 4$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 3y = 2\sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' - 3y = 1 + xe^x$$

C5. Find a general solution to the given equation.

$$y'' - 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y' + 4y = e^{2x}$$

C5. Find a general solution to the given equation.

$$y'' + 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' - 4y = \cos(2x)$$

C5. Find a general solution to the given equation.

$$y'' + 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 9y = \sin(3x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 2y = \sin(x)$$

C5. Find a general solution to the given equation.

$$y'' - 2y' + 5y = 2x + 1$$

C6. Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant 4kg/s^2). The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) How long does it take for the mass to return to its equillibrium point?

C6. Consider the following scenario: A 1kg mass is suspended by a spring (with spring constant 4kg/s^2). The mass is pushed up 0.5m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) How long does it take for the mass to return to its equillibrium point?

C6. Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant 1kg/s^2). The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 3s?

C6. Consider the following scenario: A 4kg mass is suspended by a spring (with spring constant 1kg/s^2). The mass is pushed up 0.5m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 1kg/s. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s. An external force is applied, modelled by the function $F(t) = \sin(t)$. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

(b) Where is the mass after 2s?

C6. Consider the following scenario: A 2kg mass is suspended by a spring (with spring constant 4kg/s^2). A linear damper is attached to the system (with constant 6kg/s. An external force is applied, modelled by the function $F(t) = \cos(t)$. The mass is pulled down 1m from its equillibrium position and released from rest.

(a) Write down an IVP modelling the position of the mass.

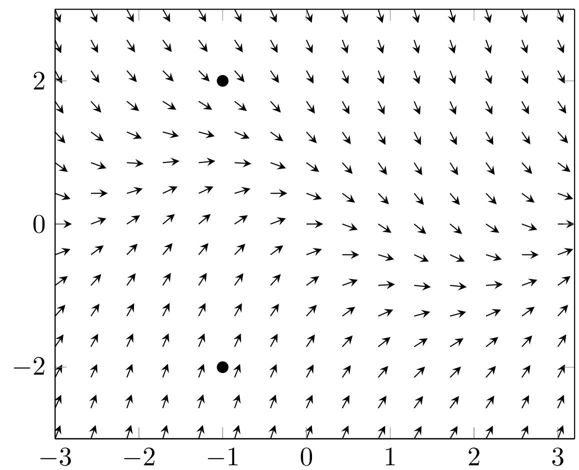
(b) Where is the mass after 2s?

Module F

Standard F1

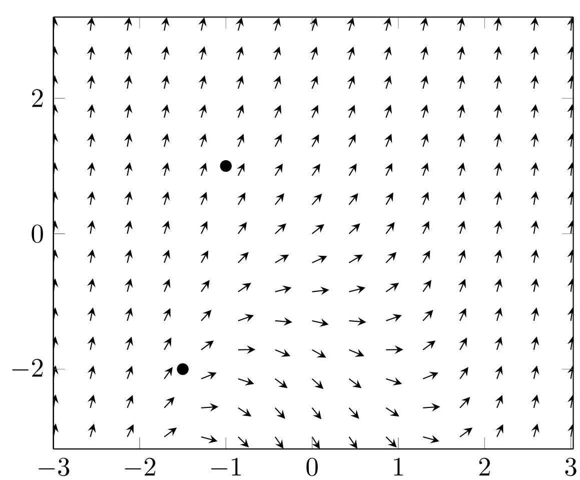
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = -y - \sin(x)$$



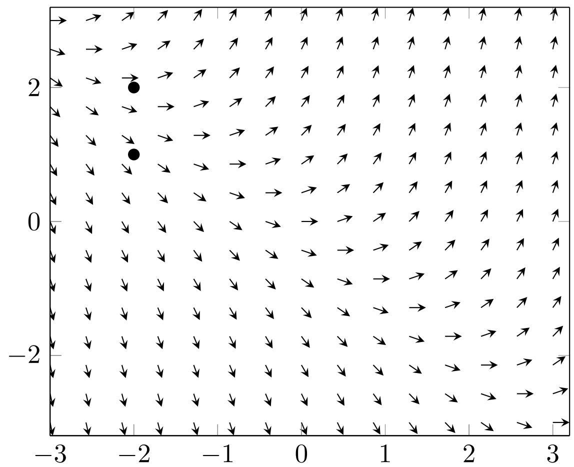
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 + y + 1$$



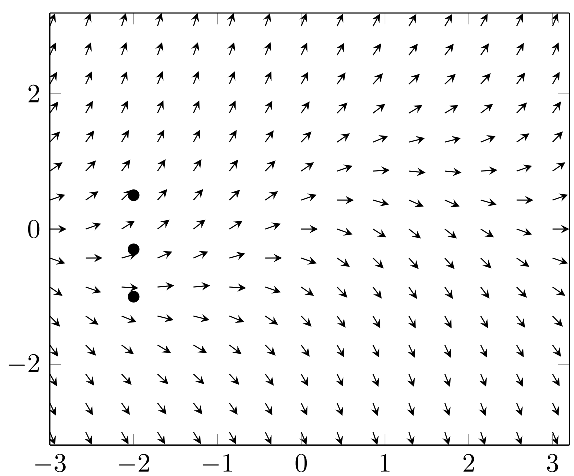
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x + y$$



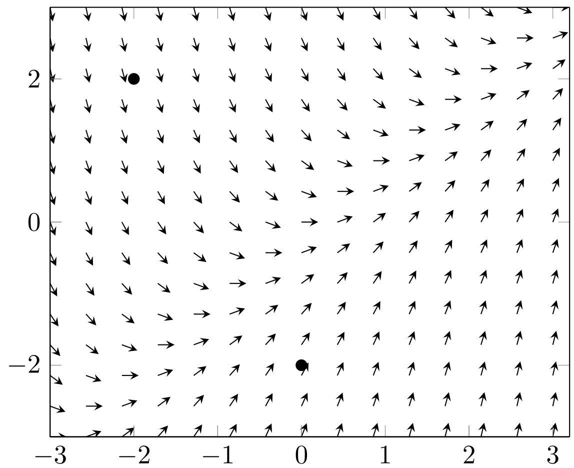
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = y - \sin(x)$$



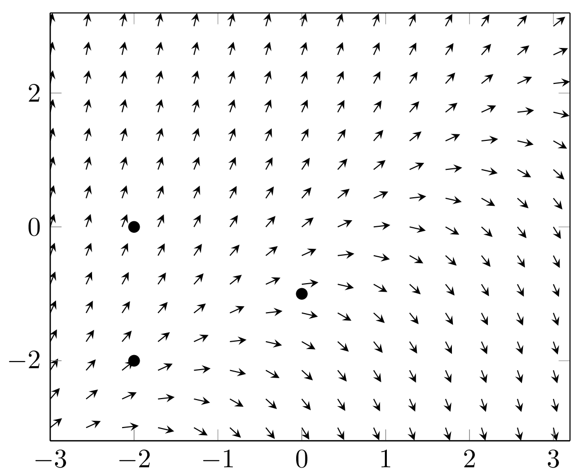
F1. Sketch a solution curve through each point marked in the slope field.

$y' = x - y$



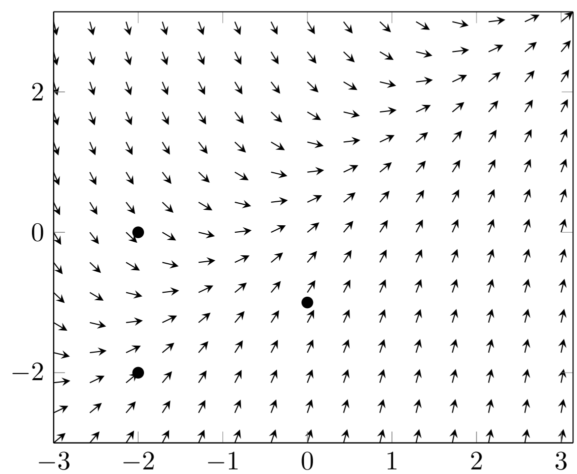
F1. Sketch a solution curve through each point marked in the slope field.

$y' = y - x + 1$



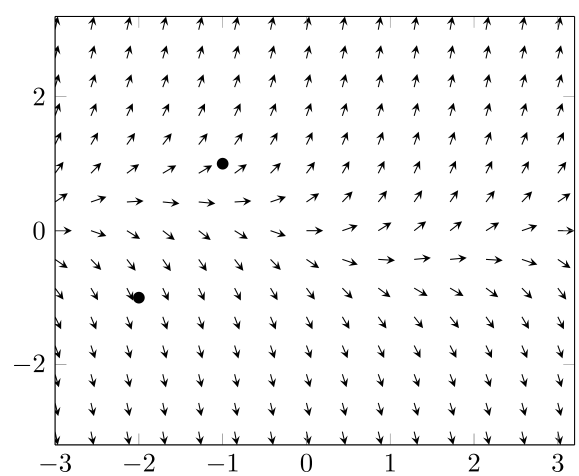
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x - y + 1$$



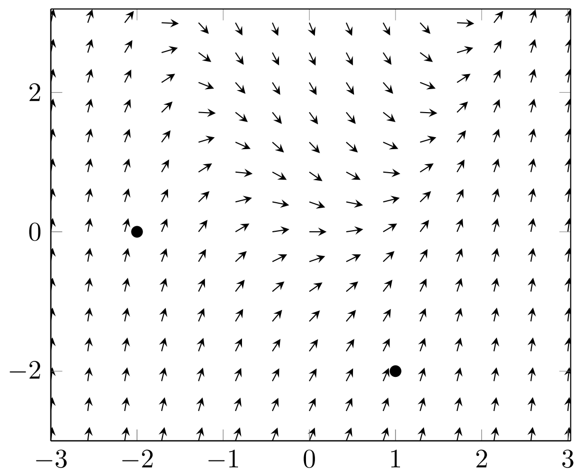
F1. Sketch a solution curve through each point marked in the slope field.

$$y' = \sin(x) + 2y$$



F1. Sketch a solution curve through each point marked in the slope field.

$$y' = x^2 - y$$



F2. Find the general solution to $\frac{dy}{dx} + 3xy = 0$.

F2. Find the general solution to $y' - y\sin(x) = 0$.

F2. Find the general solution to $y' - y^2 e^x = 0$.

F2. Find the general solution to $y' = \frac{x+2}{y^2}$.

F2. Find the general solution to $xy' = y$.

F2. Find the general solution to $y\frac{dy}{dx} = y^2\cos(x)$.

F2. Find the general solution to $xy^2\frac{dy}{dx} = 1$.

F2. Find the general solution to $x\cos(y)y' = 1$.

F3. A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How long does it take the ball to travel across a tennis court, which is 24m long?

F3. A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 60m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How far has the ball gone after 0.5s?

F3. A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How long does it take the ball to travel across a tennis court, which is 24m long?

F3. A tennis ball has a mass of 0.055kg and a drag coefficient of 0.001kg/m. It leaves the server's racket travelling 50m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How far has the ball gone after 0.5s?

F3. A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How long does it take for the ball to reach a defender standing 30m away?

F3. A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 50m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How far has the ball gone after 1s?

F3. A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 45m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How long does it take for the ball to reach a defender standing 30m away?

F3. A baseball has a mass of 0.145kg and a drag coefficient of 0.0009kg/m. A batter hits a line drive; the ball leaves the bat travelling 45m/s.

(a) Write down an IVP modelling the horizontal velocity of the ball (ignore any vertical movement in the ball).

(b) How far has the ball gone after 1s?

F4. Consider the autonomous equation
$$\frac{dx}{dt} = x - 3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = 1 - x.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (x - 3)^2.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(1) = 2$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (x + 4)^2.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(4) = 0$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (4 - x)^3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(3) = 2$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = (5 - x)^3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 4$.

F4. Consider the autonomous equation
$$\frac{dx}{dt} = x^2 - 7x + 10.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(0) = 3$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - x - 6.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(3) = 0$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2(x^2 - x - 6).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(5) = 1$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x^2 - 4x + 3.$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 4x + 3).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F4. Consider the autonomous equation

$$\frac{dx}{dt} = x(x^2 - 9x + 20).$$

(a) Find and classify the critical points.

(b) Describe the long term behavior of the solution passing through the point $x(2) = 2$.

F5. Find the general solution to $xy' + 4y = 2x$.

F5. Find the general solution to $xy' + 2y = x^2$.

F5. Find the general solution to $xy' + 2y = 4x^2 - 3x$.

F5. Find the general solution to $xy' + 2y = x^2 - 3x$.

F5. Find the general solution to $\cos(x)y' + \sin(x)y = x + \sin(x)\cos(x)$.

F5. Find the general solution to $\cos(x)y' + \sin(x)y = x\cos^2(x)$.

F5. Find the general solution to $(x^2+1)y' - 2xy = x$.

F5. Find the general solution to $(x^2+1)y' - 2xy = x^2 + 1$.

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(x + 2y)y' + y = 2x$$
$$(x + 2y)y' - y = -2x$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(3x + 2y)y' + 3y = 2x$$
$$(3x + 2y)y' - 3y = -2x$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(x^2 + 3y^2)y' - 2xy = -3x^2$$
$$(x^2 + 3y^2)y' + 2xy = 3x^2$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(2xy + 3y^2)y' + y^2 = 3x^2$$
$$(2xy + 3y^2)y' - y^2 = -3x^2$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\cos(x)\cos(y)y' = \sin(x)\sin(y)$$
$$\cos(x)\cos(y)y' = \sin(x) + \sin(y)$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$\sin(x)\sin(y)y' = \cos(x) + \cos(y)$$
$$\sin(x)\sin(y)y' = \cos(x)\cos(y)$$

F6. One of the two ODEs below is exact. Identify which one, and solve it.

$$(y^3e^x + xe^x)y' + 3e^xy^2 = 3x^2$$
$$(2ye^x + e^y)y' + e^xy^2 = 3x^2$$

Module S

Standard S1

S1. Find the general solution of the system

$$x' = x + y,$$
$$y' = 4x + y.$$

S1. Find the general solution of the system

$$x' = x + 2y,$$
$$y' = 3x + 2y.$$

S1. Find the general solution of the system

$$x' = 2x + y,$$
$$y' = x + 2y.$$

S1. Find the general solution of the system

$$x' = 2x + y,$$
$$y' = 2x + 3y.$$

S1. Find the general solution of the system

$$x' = 3x + y,$$
$$y' = x + 3y.$$

S1. Find the general solution of the system

$$x' = 3x + y,$$
$$y' = 2x + 2y.$$

S1. Find the general solution of the system

$$x' = 4x + y,$$
$$y' = 2x + 3y.$$

S1. Find the general solution of the system

$$x' = 4x + 3y,$$
$$y' = x + 2y.$$

S2. Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\frac{dG}{dt} = 0.1G - 0.002G^2 - 0.005BG$$
$$\frac{dB}{dt} = 0.2B - 0.003B^2 - 0.005BG.$$

(a) Identify all equillibrium points for the system.

(b) If a lake is stocked with 25 of each fish, what will happen to the two populations in the long term?

S2. Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\frac{dG}{dt} = 0.1G - 0.002G^2 - 0.005BG$$
$$\frac{dB}{dt} = 0.2B - 0.004B^2 - 0.005BG.$$

(a) Identify all equillibrium points for the system.

(b) If a lake is stocked with 25 bluegills and 30 greenfish, what will happen to the two populations in the long term?

S2. Two populations of competing species of fish, bluegills and greenfish, are modelled by the system

$$\frac{dG}{dt} = 0.1G - 0.005G^2 - 0.004BG$$
$$\frac{dB}{dt} = 0.2B - 0.002B^2 - 0.004BG.$$

(a) Identify all equillibrium points for the system.

(b) If a lake is stocked with 25 of each fish, what will happen to the two populations in the long term?

S2. The populations of foxes and rabbits in an ecosystem are modelled by the system

$$\frac{dF}{dt} = -0.1F - 0.001F^2 + 0.002FR$$
$$\frac{dR}{dt} = 0.2R - 0.001R^2 - 0.005FR.$$

(a) Identify all equillibrium points for the system.

(b) If there are presently 100 foxes and 200 rabbits , what will happen to the two populations in the long term?

S2. The populations of foxes and rabbits in an ecosystem are modelled by the system

$$\frac{dF}{dt} = -0.1F - 0.001F^2 + 0.002FR$$
$$\frac{dR}{dt} = 0.4R - 0.001R^2 - 0.005FR.$$

(a) Identify all equillibrium points for the system.

(b) If there are presently 100 foxes and 200 rabbits , what will happen to the two populations in the long term?

S2. The populations of greenflies and ladybirds in an ecosystem are modelled by the system

$$\begin{aligned}\frac{dL}{dt} &= -0.1L - 0.001L^2 + 0.002LG \\ \frac{dG}{dt} &= 0.4G - 0.001G^2 - 0.001GL.\end{aligned}$$

(a) Identify all equillibrium points for the system.

(b) If there are presently 1000 ladybirds and 2000 greenflies , what will happen to the two populations in the long term?

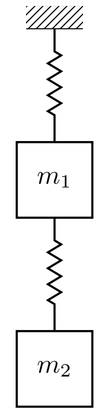
S2. The populations of greenflies and ladybirds in an ecosystem are modelled by the system

$$\begin{aligned}\frac{dL}{dt} &= -0.1L - 0.001L^2 + 0.002LG \\ \frac{dG}{dt} &= 0.2G - 0.001G^2 - 0.001GL.\end{aligned}$$

(a) Identify all equillibrium points for the system.

(b) If there are presently 1000 ladybirds and 2000 greenflies , what will happen to the two populations in the long term?

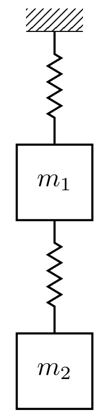
S3. Consider the dual mass-spring system at the right. Both masses are 1kg. The upper spring has a spring constant of 1N/m while the lower spring constant is 6N/m. The lower mass is pulled down 1m without disturbing the upper mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 2s?

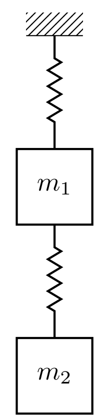
S3. Consider the dual mass-spring system at the right. Both masses are 1kg. The upper spring has a spring constant of 1N/m while the lower spring constant is 6N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 3s?

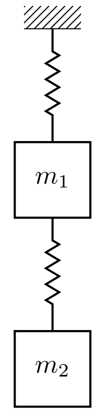
S3. Consider the dual mass-spring system at the right. Both masses are 1kg. The upper spring has a spring constant of 1N/m while the lower spring constant is 6N/m. The upper mass is pushed up 0.5m without disturbing the lower mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 4s?

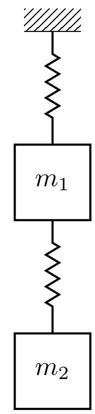
S3. Consider the dual mass-spring system at the right. Both masses are 3kg. The upper spring has a spring constant of 8N/m while the lower spring constant is 6N/m. The upper mass is pushed up 0.5m without disturbing the lower mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 2s?

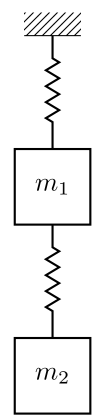
S3. Consider the dual mass-spring system at the right. Both masses are 3kg. The upper spring has a spring constant of 8N/m while the lower spring constant is 6N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 2s?

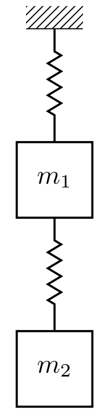
S3. Consider the dual mass-spring system at the right. The upper mass is 2kg while the lower mass is 1kg. The upper spring has a spring constant of 6N/m while the lower spring constant is 4N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 3s?

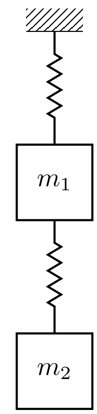
S3. Consider the dual mass-spring system at the right. The upper mass is 2kg while the lower mass is 1kg. The upper spring has a spring constant of 6N/m while the lower spring constant is 4N/m. The lower mass is pulled down 1m without disturbing the upper mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 2s?

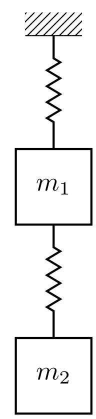
S3. Consider the dual mass-spring system at the right. The upper mass is 1kg while the lower mass is 2kg. The upper spring has a spring constant of 3N/m while the lower spring constant is 4N/m. The lower mass is pulled down 0.5m without disturbing the upper mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 3s?

S3. Consider the dual mass-spring system at the right. The upper mass is 1kg while the lower mass is 2kg. The upper spring has a spring constant of 3N/m while the lower spring constant is 4N/m. The lower mass is pulled down 1m without disturbing the upper mass, and released from rest.



(a) Write down an IVP modelling the motion of the two masses.

(b) Where will the two masses be after 2s?

Module N

Standard N1

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = x^2y + xy^2; \qquad y(1) = 3$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = 2x^2 + xy + 3y^2; \qquad y(1) = -1$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = x + \ln(y); \qquad y(1) = 2$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = x + \ln(y); \qquad y(2) = 1$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt{x + y}; \qquad y(1) = 1$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt{x + y}; \qquad y(1) = -1$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt[3]{x - y}; \qquad y(2) = 2$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \sqrt[3]{x - y}; \qquad y(2) = 3$$

N1. Determine whether a unique solution to the IVP below is guaranteed to exist. Be sure to explain your reasoning.

$$y' = \frac{y}{x}; \qquad y(2) = 1$$

N2. Consider the differential equation

$$xy'' + y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$xy'' - y' = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$x^2y'' - 4xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$x^2y'' - xy' - 3y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$x^2y'' + xy' + 4y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$y'' - \frac{1}{1+x}y' + \frac{1}{(1+x)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$xy'' + \frac{2}{x-2}y' - \frac{6}{(x-2)^2}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$e^x y'' - 2xy' + 4e^{4x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N2. Consider the differential equation

$$xy'' + y' - e^{-2x}y = 0.$$

Determine all intervals on which a unique solution is guaranteed to exist.

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{3}{t}x + 2y,$$
$$y' = 2\ln(t)x + y + 1$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -\frac{2}{t}x + y,$$
$$y' = x + \ln(t)y + 2$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = -x + \sqrt{t},$$
$$y' = 2x + ty + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + 2y + \sqrt{t},$$
$$y' = x + y + \sqrt[3]{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + y + \sqrt[3]{t},$$
$$y' = x + 2y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = tx + 2y + \sqrt[3]{t},$$
$$y' = -y + \sqrt{t}$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$x' = x + \ln(t)y + 2,$$
$$y' = -\frac{1}{t}y + 2t$$

N3. Determine all intervals on which a unique solution is guaranteed to exist.

$$\begin{aligned} x' &= 2\ln(t)x + y + 1, \\ y' &= -\frac{2}{t}x + y \end{aligned}$$

N4. Use Euler's method with stepsize $h = 0.2$ to estimate $y(1.8)$, where y is a solution to the IVP

$$y' + y^2 = x, \qquad\qquad y(1) = 1$$

.

N4. Use Euler's method with stepsize $h = 0.2$ to estimate $y(1.8)$, where y is a solution to the IVP

$$y' + y^2 = x, \qquad\qquad y(1) = 2$$

.

N4. Use Euler's method with stepsize $h = 0.1$ to estimate $y(2)$, where y is a solution to the IVP

$$y' + y^2 = x, \qquad\qquad y(1.5) = 1$$

.

N4. Use Euler's method with stepsize $h = 0.2$ to estimate $y(1.8)$, where y is a solution to the IVP

$$y' + 2y^2 = x, \qquad\qquad y(1) = 1$$

.

N4. Use Euler's method with stepsize $h = 0.1$ to estimate $y(2)$, where y is a solution to the IVP

$$y' + 2y^2 = x, \qquad\qquad y(1.5) = 1$$

.

N4. Use Euler's method with stepsize $h = 0.1$ to estimate $y(2)$, where y is a solution to the IVP

$$y' + 2y^2 = x, \qquad\qquad y(1.5) = 2$$

.

N4. Use Euler's method with stepsize $h = 0.2$ to estimate $y(1.8)$, where y is a solution to the IVP

$$y' + y^2 = x - 1, \qquad\qquad y(1) = 1$$

.

N4. Use Euler's method with stepsize $h = 0.1$ to estimate $y(2)$, where y is a solution to the IVP

$$y' + y^2 = x - 1, \qquad\qquad y(1.5) = 1$$

.

N4. Use Euler's method with stepsize $h = 0.2$ to estimate $y(1.8)$, where y is a solution to the IVP

$$y' + xy^2 = x - 1, \qquad\qquad y(1) = 1$$

.

N4. Use Euler's method with stepsize $h = 0.2$ to estimate $y(1.8)$, where y is a solution to the IVP

$$y' + xy^2 = x - 1, \qquad\qquad y(1) = 2$$

.

N4. Use Euler's method with stepsize $h = 0.1$ to estimate $y(2)$, where y is a solution to the IVP

$$y' + xy^2 = x - 1, \qquad\qquad y(1.5) = 1$$

.

N5. Use Euler's method with stepsize $h = 0.1$ to estimate $x(2.3)$ and $y(2.3)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 3x - ty & \qquad x(2) &= 1 \\ y' &= 2x - y^2 & \qquad y(2) &= 0 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.1$ to estimate $x(2.3)$ and $y(2.3)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 3x - ty & \qquad x(2) &= 1 \\ y' &= 2x - y^2 & \qquad y(2) &= 1 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.1$ to estimate $x(2.3)$ and $y(2.3)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 2x - ty & \qquad x(2) &= 1 \\ y' &= tx - y^2 & \qquad y(2) &= 1 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.2$ to estimate $x(2.6)$ and $y(2.6)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 3x - ty & \qquad x(2) &= 1 \\ y' &= 2x - y^2 & \qquad y(2) &= 0 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.2$ to estimate $x(2.6)$ and $y(2.6)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 3x - ty & \qquad x(2) &= 1 \\ y' &= 2x - y^2 & \qquad y(2) &= 1 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.2$ to estimate $x(2.6)$ and $y(2.6)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 2x - ty & \qquad x(2) &= 1 \\ y' &= tx - y^2 & \qquad y(2) &= 1 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.2$ to estimate $x(2.6)$ and $y(2.6)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 2x - ty & \qquad x(2) &= 1 \\ y' &= 2x - y^2 & \qquad y(2) &= 0 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.2$ to estimate $x(2.6)$ and $y(2.6)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 2x - 2y \qquad & x(2) &= 1 \\ y' &= 2x - y^2 \qquad & y(2) &= 1 \end{aligned}$$

N5. Use Euler's method with stepsize $h = 0.2$ to estimate $x(2.6)$ and $y(2.6)$, where x and y are solutions to the IVP

$$\begin{aligned} x' &= 2x - ty \qquad & x(2) &= 1 \\ y' &= 2tx - y^2 \qquad & y(2) &= 1 \end{aligned}$$

Module D

Standard D1

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t+1)\}(s) = \frac{e^s}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{u(t-5)\}(s) = \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+3)\}(s) = e^{3s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t-2)\}(s) = e^{-2s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{3t}\}(s) = \frac{1}{s-3}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{e^{-2t}\}(s) = \frac{1}{s+2}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t+4) + e^t\}(s) = e^{4s} + \frac{1}{s-1}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{\delta(t) + u(t-5)\}(s) = 1 + \frac{e^{-5s}}{s}.$$

D1. Demonstrate directly from the definition that

$$\mathcal{L}\{1 + e^t\}(s) = \frac{1}{s} + \frac{1}{s-1}.$$

D2.

D3.

D4.