Scintillation loss

Lexa Huang

Scintillation

- Index-of-refraction turbulence in the atmosphere induces temporal and spatial fluctuations in the intensity of the received signal
- Aim to find the fade statistics of the intensity falling below a specific threshold value
- Scintillation is expressed with the scintillation index, which gives the normalised variance of the optical intensity at a point in the detector plane:

$$\sigma_I^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2} = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1$$

Uplink vs Downlink

- Differs because of where the atmospheric turbulence layer is located
- For downlink, the turbulence layer is located only near the receiver, whereas for uplink, it is located near the transmitter
- Thus, beam wander affects the uplink case but it can be considered negligible for the downlink case

Weak vs strong fluctuation theory

- Weak fluctuation theory based on Rytov approximation is sufficient for most of the calculations required provided the zenith angle is sufficiently small (less than 60 degrees)
- Strong fluctuation theory applies for larger-diameter beams in the presence of beam wander or with larger zenith angles
- Weak or strong fluctuation theory will determine what the scintillation index looks like
- Used weak fluctuation theory for downlink

Probability Distribution of Intensity

Probability density function of intensity will help us find the fractional fade time.

Lognormal vs gamma-gamma distribution:

- Under weak turbulence (SI < 1), PDF can be said to be lognormal but values start disagreeing at tail ends, which is where we calculate fade probabilities.
- For both weak and strong turbulence, the gamma-gamma distribution holds and is more accurate, but it's more difficult to implement with the computer and is not accurate when aperture averaging occurs.
- Lognormal distribution was chosen for downlink.

Downlink model
Atmospheric Turbulence Strength, C_n^2

H-V 5/7 model:

$$C_n^2(h) = 0.00594(w/27)^2(10^{-5}\,h)^{10}\exp(-h/1000) + 2.7\times10^{-16}\exp(-h/1500) + A\exp(-h/100),$$

w = rms wind speed

A = near ground level of C_n^2

h = height

For H-V 5/7 model, $w = 21ms^{-1}$ and $A = 1.7\times10^{-14}\ m^{-2/3}$

Scintillation Index

Under weak fluctuation theory, scintillation index for downlink can be approximated by the Rytov Variance:

$$\sigma_R^2 = 2.25 k^{7/6} \sec^{11/6}(\zeta) \int_{h_0}^{H} C_n^2(h)(h - h_0)^{5/6}\, dh.$$

k = 2π / wavelength

h_0 = ground height

H = height of satellite

ς = Zenith Angle

Aperture averaging

- Whether or not aperture averaging occurs depends on the correlation width p_c, which identifies the maximum receiver aperture size that will act like a "point receiver".
- Aperture sizes larger than p_c will experience aperture averaging
- For downlink the correlation width is normally 7-10cm. Thus, aperture averaging is taken into account for our aperture
- The scintillation index is related to the aperture averaging effect through:

$$A_f(D) = \frac{\sigma_P^2(D)}{\sigma_I^2}$$

Aperture averaging

$$A_f = \left[1 + 1.062\left(\frac{D}{2\rho_I}\right)^2\right]^{-7/6}$$

$$\rho_I \approx 1.5 \cdot \sqrt{\frac{L \cdot \lambda}{2\pi}}$$

$$L \approx H_d \frac{\varepsilon/90^\circ}{\left(\varepsilon/90^\circ\right)^2 + \left(\varepsilon_{\max}/90^\circ\right)^2}$$

D = diameter of aperture

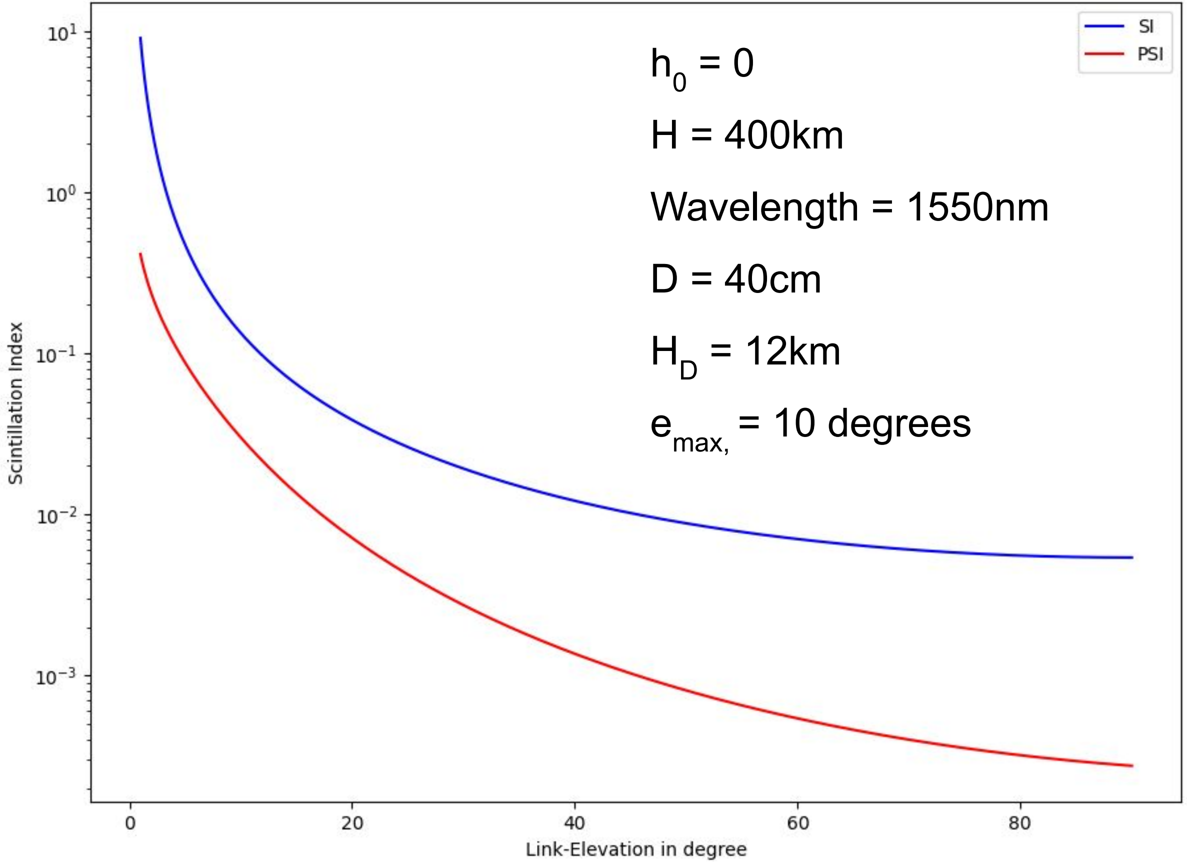
p_I = intensity structure size parameter

L = distance from a dominant turbulent layer

H_d = height of dominant turbulent layer
(assumed to be tropopause)

ε_{max} = maximum-size elevation

Graph of scintillation index against link elevation

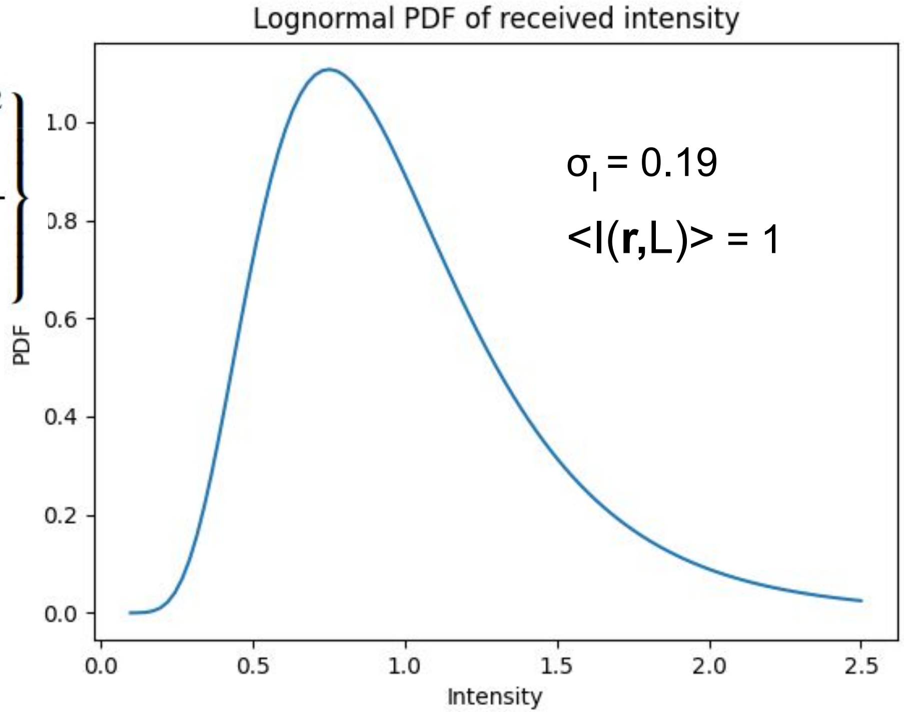


Probability density function of received intensity

$$p_I(I) = \frac{1}{I\sigma_I(\mathbf{r},L)\sqrt{2\pi}}\exp\left\{-\frac{\left[\ln\left(\frac{I}{\langle I(\mathbf{r},L)\rangle}\right)+\frac{1}{2}\sigma_I^2(\mathbf{r},L)\right]^2}{2\sigma_I^2(\mathbf{r},L)}\right\}$$

σ_I = scintillation index (SI)

<I(**r**,L)> = mean intensity

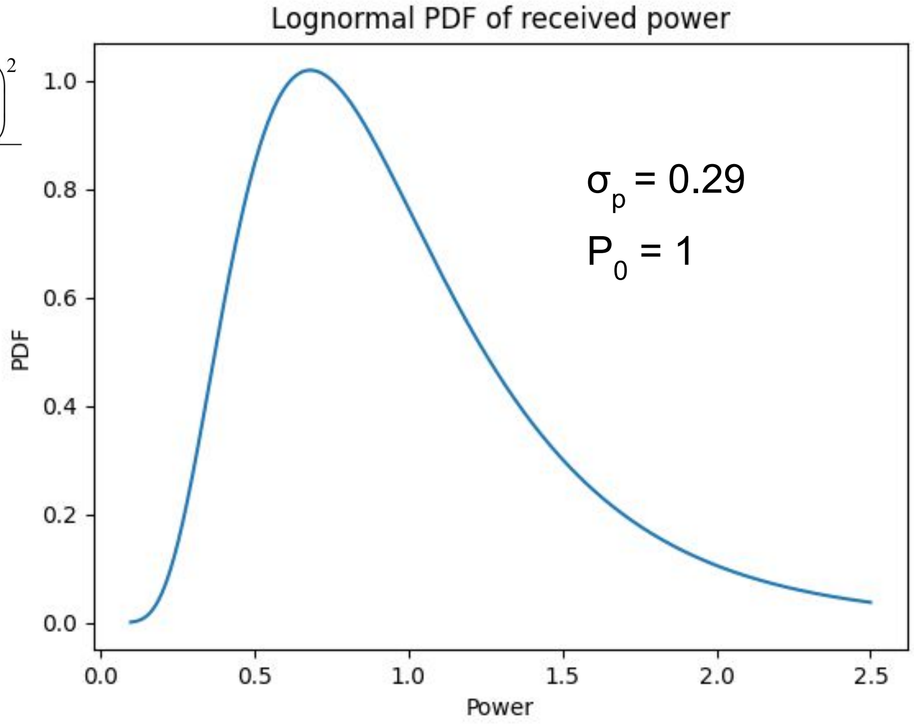


Probability density function of received power

$$p\left(P,\sigma_P^2\right) == \frac{1}{P\cdot\sqrt{2\pi\cdot\ln\left(\sigma_P^2+1\right)}}\cdot e^{-\frac{\left(\ln\frac{P}{P_0}+\frac{1}{2}\ln\left(\sigma_P^2+1\right)\right)^2}{2\cdot\ln\left(\sigma_P^2+1\right)}}$$

σ_p = power scintillation index (PSI)

P_0 = mean power



σ_p = 0.29

P_0 = 1

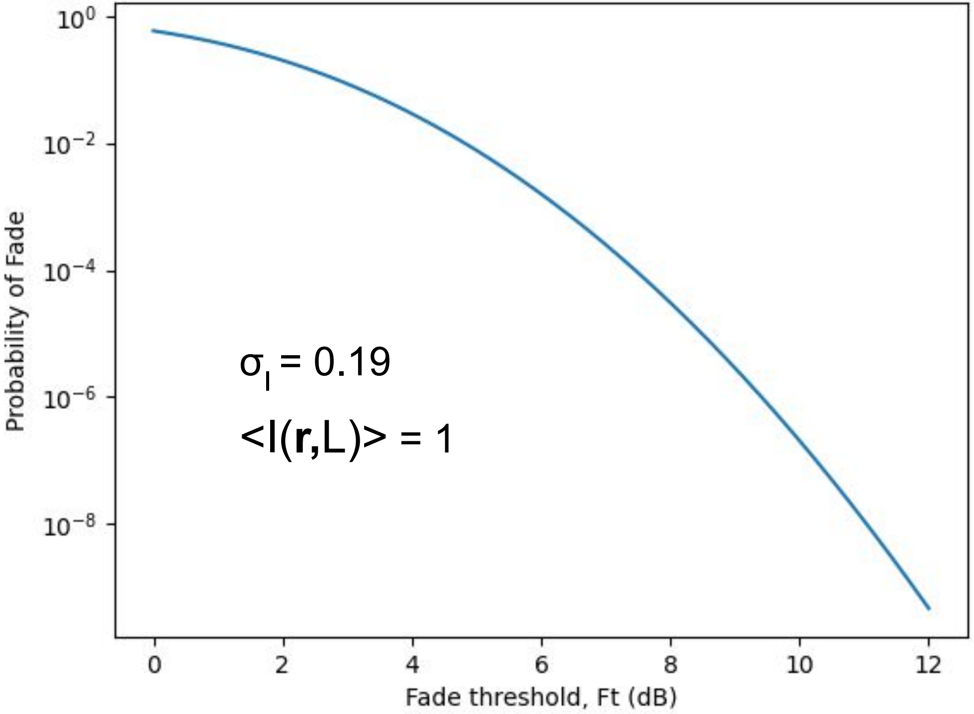
Probability of fade

- Describes the fractional fade time
- Obtained by integrating the PDF up to the intensity threshold, I_T

$$P(I \leq I_T) = \int_0^{I_T} p(I)\, dI$$

- Plotted against F_T, which expresses the intensity threshold I_T below the mean intensity in the form of decibels:

$$F_T = 10 \log_{10}\left(\frac{\langle I(0,L)\rangle}{I_T}\right). \quad [\mathrm{dB}]$$



Expected number of fades per second

$$\langle n(I_T)\rangle = \nu_0 \exp\left\{-\frac{\left[\frac{1}{2}\sigma_I^2(\mathbf{r},L)+\frac{2r^2}{W_{\mathrm{LT}}^2}-0.23F_T\right]^2}{2\sigma_I^2(\mathbf{r},L)}\right\},$$

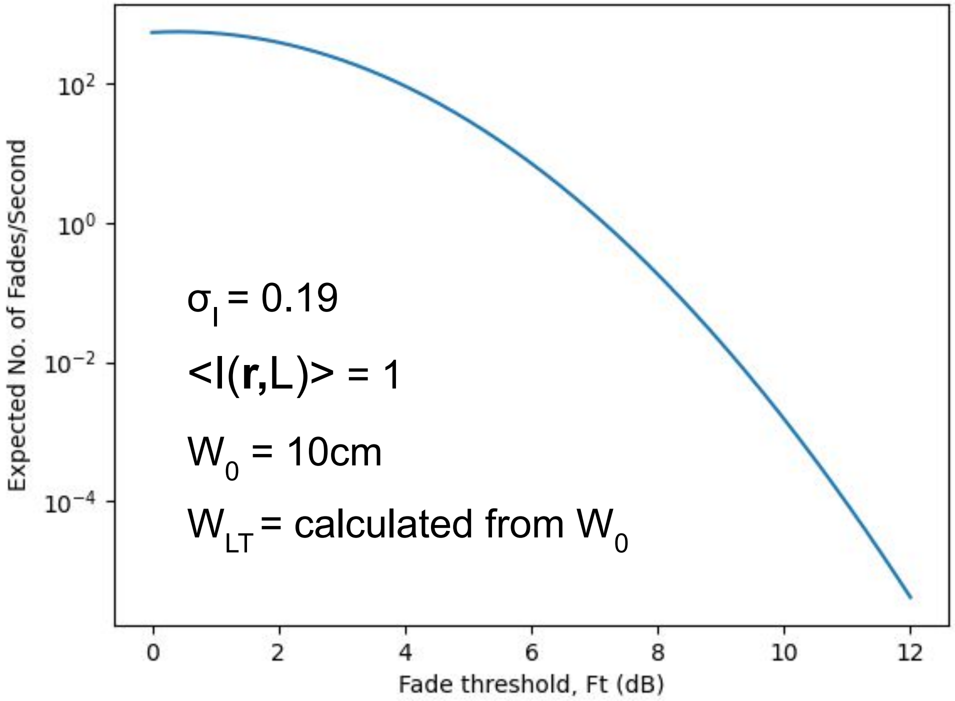
where r is related through

$$\langle I(\mathbf{r},L)\rangle = \frac{W_0^2}{W_{\mathrm{LT}}^2}\exp\left(-\frac{2r^2}{W_{\mathrm{LT}}^2}\right)$$

v_0 = 550 Hz

W_0 = beam waist at transmitter

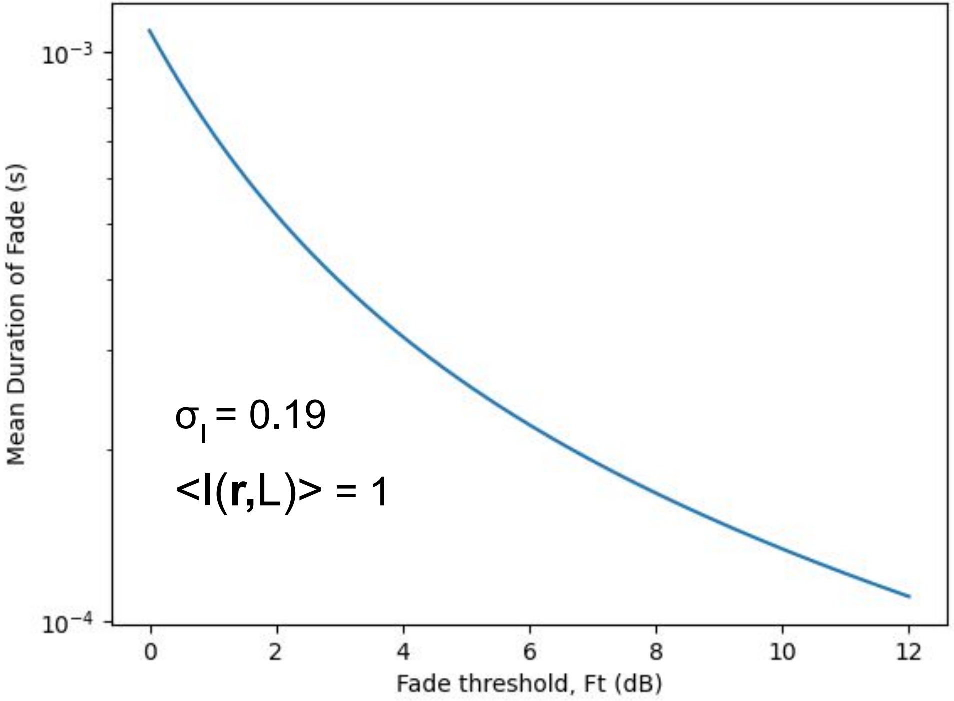
W_{LT} = long-term beam waist



Mean duration of fade

- Obtained by dividing the probability of fade with the expected number of fades per second

$$\langle t(F_T) \rangle = \frac{P_I(I \leq I_T)}{\langle n(I_T) \rangle},$$

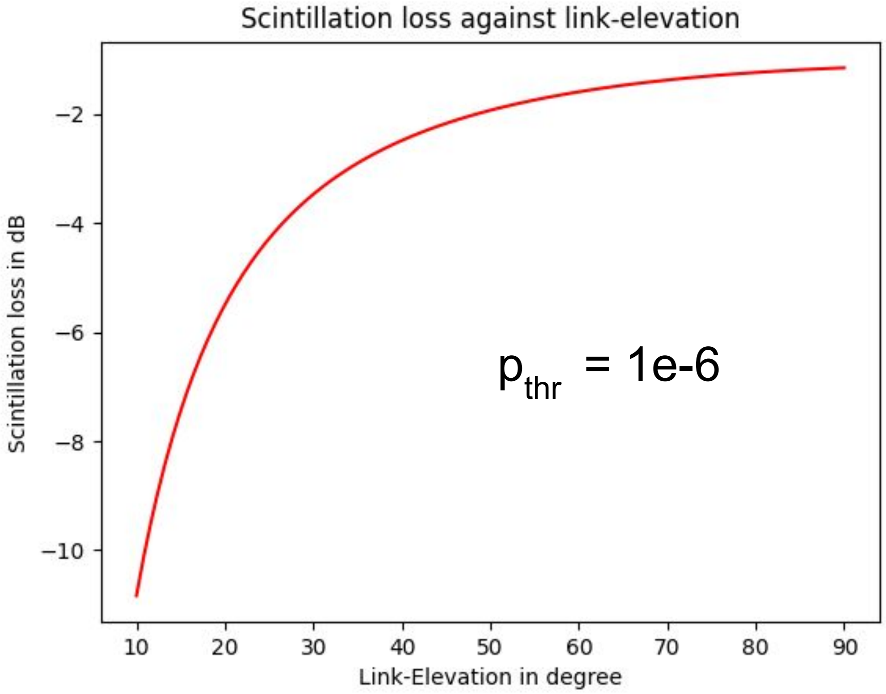


Loss expressed in dB

a_{sci} = loss associated with fade time

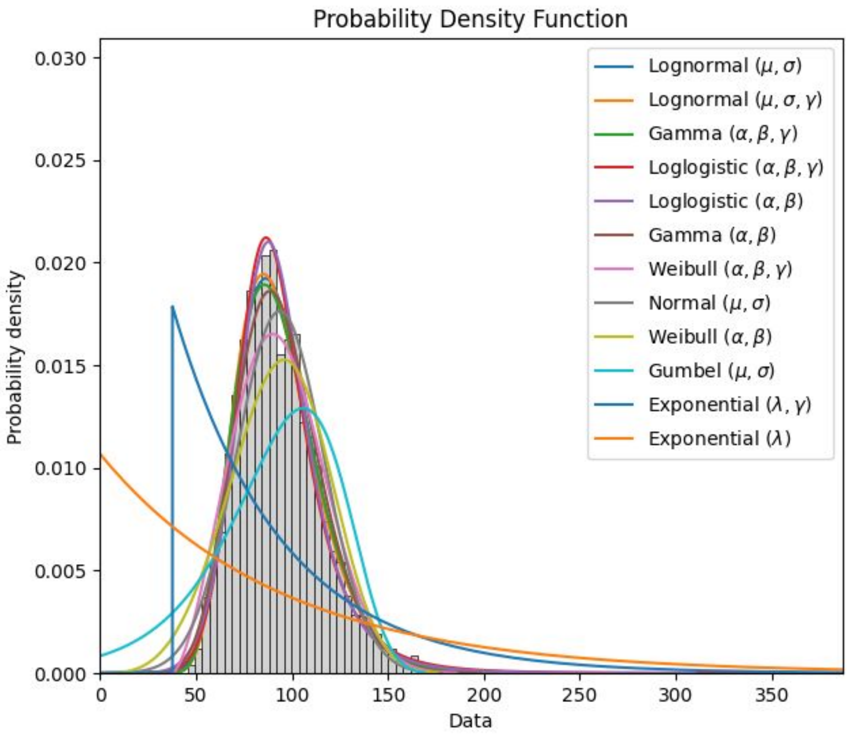
$$a_{sci} = -4.343\left[\begin{array}{l} erfinv\left(2p_{thr}-1\right)\sqrt{2\ln\left(\sigma_P{}^2+1\right)}- \\ -\dfrac{1}{2}\ln\left(\sigma_P{}^2+1\right) \end{array}\right]$$

p_{thr} = loss fraction = share where the received power falls below a given threshold from mean power (fade)



p_{thr} = 1e-6

Measurements from Archenar Star



Wavelength: 780nm

Elevation angle: 30 degrees

Scintillation index calculated from measurements: 0.12141/0.00269

Scintillation index calculated from model:

Uplink

- Calculation for scintillation index takes into account beam wander and comprises more steps:
    a. Atmospheric coherence width, r_o
    b. Pointing error variance, σ_{pe}^2
    c. Split into tracked and untracked beams
    d. Calculate off-axis and on-axis components separately
- Aperture averaging is negligible for uplink

Atmospheric coherence width

$$r_0 = \left[0.42 \sec(\zeta) k^2 \int_{h_0}^{H} C_n^2(h)\, dh\right]^{-3/5}$$

Pointing error variance

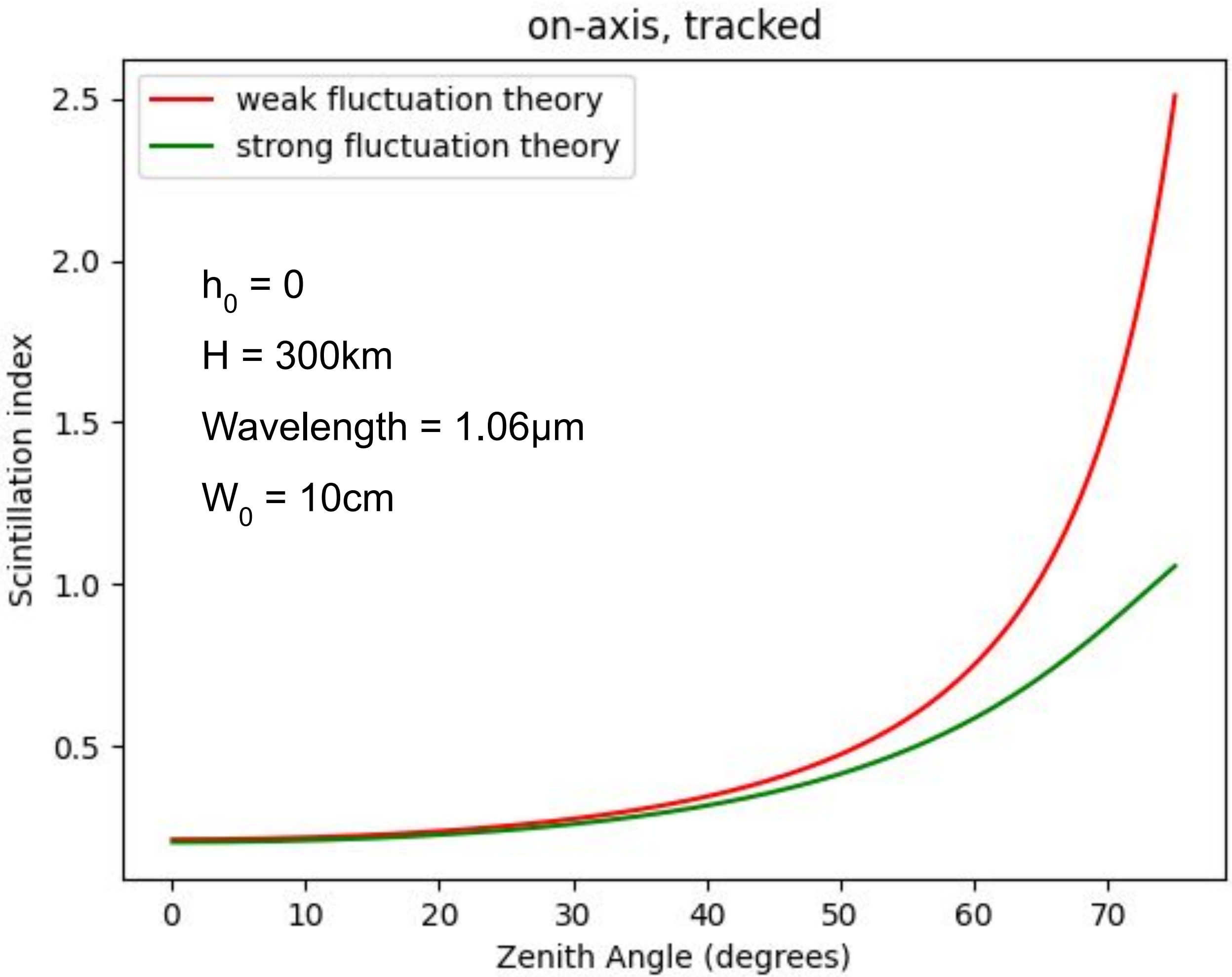
$$\begin{aligned}\sigma_{pe}^2 =& 0.54(H - h_0)^2 \mathbf{sec}^2(\zeta) \left(\frac{\pi}{kW_0}\right)^2 \left(\frac{2W_0}{r_0}\right)^{5/3} \\ & \times \left[1 - \left(\frac{C_r^2 W_0^2 / r_0^2}{1 + C_r^2 W_0^2 / r_0^2}\right)^{1/6}\right]\end{aligned}$$

SI of a spherical wave under weak fluctuation theory

$$\sigma_{Bu}^2 = 2.25k^{7/6}(H-h_0)^{5/6}\sec^{11/6}(\zeta)\int_{h_0}^{H} C_n^2(h)\left(1-\frac{h-h_0}{H-h_0}\right)^{5/6}\left(\frac{h-h_0}{H-h_0}\right)^{5/6} dh,$$

Longitudinal component of SI of a tracked beam under strong fluctuation theory

$$\sigma_{I,l}^2(L)_{\text{tracked}} = \exp\left[\frac{0.49\sigma_{Bu}^2}{\left(1+(1+\Theta)0.56\sigma_{Bu}^{12/5}\right)^{7/6}}+\frac{0.51\sigma_{Bu}^2}{\left(1+0.69\sigma_{Bu}^{12/5}\right)^{5/6}}\right]-1,$$


on-axis, tracked
weak fluctuation theory
strong fluctuation theory
h_0 = 0
H = 300km
Wavelength = 1.06μm
W_0 = 10cm
Scintillation index
2.5
2.0
1.5
1.0
0.5
0
10
20
30
40
50
60
70
Zenith Angle (degrees)

Gamma-gamma distribution

- More accurate for uplink

$$p_I(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)I}\left(\frac{I}{\langle I(\mathbf{r},L)\rangle}\right)^{(\alpha+\beta)/2} K_{\alpha-\beta}\left(2\sqrt{\frac{\alpha\beta I}{\langle I(\mathbf{r},L)\rangle}}\right),$$

- α and β are parameters for the function representing large-scale and small-scale fluctuations respectively, where $\alpha = 1/\sigma_x^2$ and $\beta = 1/\sigma_y^2$

$$\sigma_X^2 = 5.95(H-h_0)^2 \sec^2(\zeta)\left(\frac{2W_0}{r_0}\right)^{5/3}\left(\frac{\alpha_{pe}}{W}\right)^2 + \exp\left[\frac{0.49\sigma_{Bu}^2}{\left(1+(1+\Theta)0.56\sigma_{Bu}^{12/5}\right)^{7/6}}\right] - 1,$$

$$\sigma_Y^2 = \exp\left[\frac{0.51\sigma_{Bu}^2}{\left(1+0.69\sigma_{Bu}^{12/5}\right)^{5/6}}\right] - 1.$$