Quantum Field Theory

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Contents

convention, notation, and units

- 笔记中的**度规号差**约定为 $(+,-,-,-)$.
- 使用 natural units, 此时 $\hbar, c, k_B = 1$, 因此 $1\,\mathrm{m} = \frac{1}{1.97\times10^{-16}\,\mathrm{GeV}}$ 且:

names/dimensions	expressions/values
Planck length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616\times10^{-35}\,\mathrm{m}$
Planck time (T)	$t_P = \frac{l_P}{c} = 5.391\times10^{-44}\,\mathrm{s}$
Planck mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}} = 2.176\times10^{-8}\,\mathrm{kg} \simeq 10^{19}\,\mathrm{GeV}$
Planck temperature (Θ)	$T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.417\times10^{32}\,\mathrm{K}$

- 时空维度用 $d = D + 1$ 表示.

Part I

Field Theory

Chapter 1

cross sections and decay rates

1.1 cross sections

- cross section 定义为
$$\sigma = \frac{1}{\Phi}\frac{P}{\Delta t}, \tag{1.1.1}$$
其中 $\Phi := nv = \frac{|\vec{v}_1 - \vec{v}_2|}{V}$ 是 incoming flux, 是入射粒子数密度乘粒子速度, P 是发生散射的概率.

- 实验上定义 luminosity 为
$$L\Delta t = \frac{dN}{d\sigma}, \tag{1.1.2}$$
其中 dN 是 $d\Omega$ 内发生散射的粒子数.

- 用 S-matrix elements 来表示 cross section, 有
$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle} d\Pi, \tag{1.1.3}$$
其中 $d\Pi$ 是末态动量体元
$$d\Pi = \prod_i \delta^{(3)}(\vec{p}=0) d^3 p_{f,i} = \prod_i \frac{V}{(2\pi)^3} d^3 p_{f,i}, \tag{1.1.4}$$
这保证了无相互作用时 $\int dP = 1$.

- 对于初末态有
$$\begin{cases} \langle i|i\rangle = \langle p_1, p_2|p_1, p_2\rangle = (2\pi)^3 2\omega_{p_1}\delta^{(3)}(0)(2\pi)^3 2\omega_{p_2}\delta^{(3)}(0) = (2\omega_{p_1}V)(2\omega_{p_2}V) \\ \langle f|f\rangle = \prod_i (2\omega_{p_{f,i}}V) \end{cases}. \tag{1.1.5}$$

- 一般将 S-matrix 写为
$$S = I + i\mathcal{T}, \quad \mathcal{T} = (2\pi)^4\delta^{(4)}(\textstyle\sum_{i,f} p)\mathcal{M}, \tag{1.1.6}$$
其中 $\mathcal{T}$ 称为 transfer matrix, 而 $\mathcal{M}$ 才是 S-matrix 的 non-trivial part. 有
$$\langle f|S - I|i\rangle = i(2\pi)^4\delta^{(4)}(\textstyle\sum_{i,f} p)\langle f|\mathcal{M}|i\rangle. \tag{1.1.7}$$

- 对于 $|f\rangle \neq |i\rangle$ 的情况, 有
$$|\langle f|S|i\rangle|^2 = (2\pi)^4 TV\delta^{(4)}(\textstyle\sum_{i,f} p)|\langle f|\mathcal{M}|i\rangle|^2, \tag{1.1.8}$$
那么
$$dP = \frac{T}{V}\frac{1}{(2\omega_{p_1})(2\omega_{p_2})}|\langle f|\mathcal{M}|i\rangle|^2 d\Pi_{\text{LIPS}}, \tag{1.1.9}$$
其中 LIPS 表示 Lorentz-invariant phase space,
$$d\Pi_{\text{LIPS}} = (2\pi)^4\delta^{(4)}(\textstyle\sum_{i,f} p)\displaystyle\prod_i \frac{d^3 p_{f,i}}{(2\pi)^3 2\omega_{p_{f,i}}}. \tag{1.1.10}$$

- 最终有 (将 (1.1.1) 中的 Δt 替换为 T)
$$d\sigma = \frac{1}{|\vec{v}_1 - \vec{v}_2|(2\omega_{p_1})(2\omega_{p_2})}|\langle f|\mathcal{M}|i\rangle|^2 d\Pi_{\text{LIPS}}. \tag{1.1.11}$$

1.2 decay rates

- decay rate, Γ, 是粒子单位时间发生衰变的概率,

$$d\Gamma = \frac{dP}{T}. \tag{1.2.1}$$

- 因为 $|f\rangle \neq |i\rangle$, 有

$$d\Gamma = \frac{1}{2\omega_p} |\langle f|\mathcal{M}|i\rangle|^2 d\Pi_{\text{LIPS}}. \tag{1.2.2}$$

Chapter 2

the S-matrix and time-ordered products

2.1 the LSZ reduction formula

- S-matrix element 为

$$\begin{cases} |i\rangle = \sqrt{(2\pi)^3 2\omega_{p_1}}\sqrt{(2\pi)^3 2\omega_{p_2}}\, a^\dagger_{\vec{p}_1}(-\infty) a^\dagger_{\vec{p}_2}(-\infty) |\Omega\rangle \\ |f\rangle = \sqrt{(2\pi)^3 2\omega_{p_3}} \cdots \sqrt{(2\pi)^3 2\omega_{p_n}}\, a^\dagger_{\vec{p}_3}(+\infty) \cdots a^\dagger_{\vec{p}_n}(+\infty) |\Omega\rangle \\ \langle f|S|i\rangle = (2\pi)^{3n/2}\sqrt{2\omega_{p_1}\cdots 2\omega_{p_n}} \end{cases} \tag{2.1.1}$$

Appendices