

Statistical Field Theory

Siyang Wan (万思扬)

April 20, 2025

Contents

convention, notation, and units	2
I from spins to fields	3
1 the Ising model	4
1.1 the effective free energy	4

convention, notation, and units

- 使用 Planck units, 此时 $G, \hbar \equiv \frac{h}{2\pi}, c, k_B = 1$, 因此:

names/dimensions	expressions/values
Planck length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} \text{ m}$
Planck time (T)	$t_P = \frac{l_P}{c} = 5.391 \times 10^{-44} \text{ s}$
Planck mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}} = 2.176 \times 10^{-8} \text{ kg} \simeq 10^{19} \text{ GeV}$
Planck temperature (Θ)	$T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.417 \times 10^{32} \text{ K}$

- 时空维度用 $d = D + 1$ 表示.

- 下面是 *Statistical Field Theory*, David Tong, 中引言的一部分.

... This phenomenon is known as *universality*.
All of this makes phase transitions interesting. They involve violence, universal truths and competition between rival states. The story of phase transitions is, quite literally, the song of fire and ice.
...
... This leads us to a paradigm which now underlies huge swathes of physics, far removed from its humble origin of a pot on a stove. This paradigm revolves around two deep facts about the Universe we inhabit: **Nature is organised by symmetry. And Nature is organised by scale.**

其中 symmetry 指 Landau's approach to phase transitions, scale 是指 renormalization group.

Part I

from spins to fields

Chapter 1

the Ising model

- 考虑一个 D 维 lattice, 含有 N 个 sites, 每个 lattice site 可以处于两个状态 $s_i = \pm 1$.
- 系统的能量为

$$E\{s_i\} = -J \sum_{\langle ij \rangle} s_i s_j - B \sum_i s_i, \quad (1.0.1)$$

其中 $\langle ij \rangle$ 表示 nearest neighbor pairs.

– $J > 0$, 系统称为 ferromagnet; $J < 0$, 系统称为 anti-ferromagnet.

- 系统的 partition function 为

$$Z_C(T, B, N) = \sum_{\{s_i\}} e^{-\beta E\{s_i\}}. \quad (1.0.2)$$

- 那么, 系统的 magnetization 为

$$\langle m \rangle \equiv \frac{1}{N} \langle \sum_i s_i \rangle = \frac{1}{N\beta} \frac{\partial}{\partial B} \ln Z_C(T, B, N), \quad (1.0.3)$$

并且 $m \in [-1, 1]$.

1.1 the effective free energy

- 将 partition function 写作

$$Z_C = \sum_m \underbrace{\sum_{\{s_i\}|m} e^{-\beta E\{s_i\}}}_{:= e^{-\beta F_{\text{eff}}(m)}} \simeq \frac{1}{2/N} \int_{-1}^1 dm e^{-\beta F_{\text{eff}}(m)}, \quad (1.1.1)$$

其中 $F_{\text{eff}}(m)$ 称作 effective free energy.

Appendices