Multivariate Dependence Risk and Portfolio Selection for International Stocks:
Data Mining and Data Driven Decision.

FINAL PROJECT

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1 Introduction

In this project we are studying the risk dependence of international stocks using risks metrics such expected returns, conditional value at risk and sharp ratio. We calculate the risk metrics using multivariate Generalised Autoregressive Conditional Hetoreskedasticity (M- GARCH) models. The models are tested using international stocks indexes, namely CAC40, BVSP, DAX, HSI and SP500. We obtained our data from yahoo finance . It is a raw daily data from 1 January 1998 to 31 December 2016. We have cleaned our data using Python, and we used R to run the models. Together with this document we include the R code that contains our models and Jupyter-notebook we used to clean our data.

2 Methodology

The aim of our project is to assess the multivariate dependence risk of international stock portfolio based on two M-GARCH models namely Dynamical Conditional Correlation GARCH (DCC-GARCH) and Generalised Orthogonal GARCH (GO-GARCH). These models are performed to model the portfolio conditional variance under time varying conditional correlation and orthogonal variance assumptions.

2.0.1 DCC-GARCH model

DCC-CARCH is therefore a generalisation of Bollerslev (1990) constant conditional correlation (CCC) model, in which the conditional covariance matrix of the returns is defined as:

$$H_t = D_t R_t D_t \tag{1}$$

Where:
H_t is the conditional variance matrix.
D_t is a diagonal matrix of time varying standard variation from univariate GARCH - processes.
R_t is the conditional correlation matrix of the standardized disturbances ε_t.
DCC-GARCH is preferred as it allows pair-wise correlations to be time-varying; hence producing consistent errors when estimating multivariate dependence across time series processes (Tse and Yu, 1999). $D_t = diag\sqrt{(h_{i,t})}$ is the diagonal matrix of conditional time varying residuals that are obtained from

the univariate GARCH models (on-diagonal elements or variance or volatility component) where h_{it} has the following specification.

$$h_{i,t} = \omega_i + \alpha_i \varepsilon^2_{i,t-1} + \beta h_{i,t-1}, \quad for \quad i = 1 \cdots m.$$

$\varepsilon^2_{i,t-1}$ is the past innovation (lagged squared residuals from the mean equation (ARMA(1,0)), R_t is the time-varying correlation matrix (off diagonal elements) The implementation of this model revolves around a two-step algorithm developed by Engle (2002). The first step computes conditional standard deviations through the univariate GARCH and the second step models the time varying correlations relying on lagged values of residuals and covariance matrices. Conditional covariance matrix is then obtained using conditional standard deviations and dynamic correlations.

2.0.2 GO-GARCH model

Let x_t be the observed p-dimensional economic process that is governed by a linear combination of uncorrelated economic components y_t. Then $x_t = Zy_t$ where Z is a linear map assumed to be constant over time and invertible. The unobserved components are normalized to have unit variance, such that

$$V = E[x_t x_t'] = ZZ'.$$

Thus, the GO-GARCH model imposes a structure on the conditional variance matrix implied by the assumption that the process satisfies the representation $\Sigma_t = var(x_t \mid F_{t-1}) = E(x_t x_t' \mid F_{t-1})$ implied by the assumption that the process $\{x_t\}_{t \geq 1}$ satisfies the representation

$$x_t = Zy_t = ZH_t^{\frac{1}{2}} \varepsilon_t \tag{2}$$

$$h_{i,t} = 1 - \alpha_i - \beta_i + \alpha_i \varepsilon y^2_{i,t-1} + \beta h_{i,t-1}, \quad for \quad i = 1 \cdots m \tag{3}$$

where $H_t = diag(h_{1,t} \cdots h_{p,t})$, Z is an non-singular $m \times m$ matrix; $\{\{h_{i,t}\}_{t \geq 0}, i = 1 \cdots m\}$ are positive, $\{F_{t-1}\}$-adapted processes with; $E(h_{i,t}) = 1$ and $\{\varepsilon_i\}_{t \geq 1}$ is a vector martingale difference sequence, with $E(\varepsilon_t \mid F_{t-1}) = 0$ and $var(\varepsilon_t \mid F_{t-1}) = I_m$.
As described in Boswijk and van der Weide (2011), the model suggests that matrix of returns x_t can be written as a non-singular transformation of a latent vector process of the same dimension, provided that $E(y_t \mid F_{t-1}) = 0$ and $var(y_t \mid F_{t-1}) = h_{i,t}$ and $cov(y_{i,t}, y_{j,t} \mid F_{t-1}) = 0$ for $i \neq j = 1 \cdots m$. In other words, the components y_t of are assumed to be conditionally uncorrelated while y_t is a covariance stationary process with mean zero and unconditional variance. This implies that x_t is covariance stationary with conditional mean 0, conditional variance $\Sigma_t = var(x_t \mid F_{t-1}) = ZH_tZ'$ and unconditional variance $\Sigma = var(x_t \mid F_{t-1}) = ZZ'$ where conditional variances are assumed to follow a GARCH type model.

2.1 Portfolio selection and risk evaluation

In the portfolio optimisation task, the main challenge consists of designing a proper model that empirically best fits the data while feasible and robust enough to generate simulation-based inference for risk evaluation. In the optimisation algorithm, investor‘s preferences are expressed through a quadratic utility function 4 that need to be maximised subject to the minimisation of a specified risk measure. Formally, the basic optimisation problem with the variance risk is

$$\min \quad \frac{1}{n}\sum_{i=1}^{n}(\sum_{i=1}^{m} w_j(r_{i,j}-\mu_j))^2 \tag{4}$$

subject to

$$\begin{aligned} \sum_{i=1}^{m} w_j\mu_j &= \mu_p \\ \sum_{i=1}^{m} w_j &= 1 \\ w_j \geqslant 0, &\quad j = 1\cdots m \end{aligned} \tag{5}$$

where μ_p is the return of portfolio, equations 5 represent the target return of the portfolio, the unity constraint on the sum of the portfolio weights w_j and the semi-definite positivity constraint on every weight. As a symmetric risk measure, the variance based optimisation algorithm relies on the normality assumption which is not, however, suitable for the tail risk characteristics of asset returns. Alternatively, the optimisation problem can be recast into a loss function-based minimax algorithm. While equations 5 are common to all portfolio optimisation problems, the minimax risk measure for portfolio optimisation in the linear programming problem, is however, more conservative due to the constraint that the difference between the maximum loss of the portfolio and the forecast target return of the portfolio be less or equal to zero (Bekiros et al., 2015). Hence, unlike the variance risk measure, the minimax optimisation problem is modified as follows:

$$\min \quad M_p$$

subject to

$$M_p - \sum_{i=1}^{m} w_j r_{i,j} \leqslant 0, \quad \forall i \in \{1\cdots m\}$$

and the three common constraints indicated in 5. Considering a coherent risk measure such as CVaR which is more appropriate to the loss function of the tail distribution, the optimisation problem becomes:

$$\min \quad \frac{1}{na}\sum_{i=1}^{n} d_i + \nu$$

subject to

$$\begin{aligned} \sum_{i=1}^{m} w_j r_{i,j} + \nu \geqslant -d_i, &\quad \forall i \in \{1\cdots m\} \\ d_i \geqslant 0, &\qquad j = 1\cdots n \end{aligned}$$

In this project we employ the CVaR based optimisation from which the Sharpe ratio for each optimal portfolio is derived using the following formula:

$$SR = \frac{E[ret]}{CVaR} \tag{6}$$

where $E[ret]$ and $CVaR$ represent the portfolio's target return and expected shortfall, respectively. In effect, it has been proven that asset returns are generally not normal and hence, the use of the standard deviation in the estimation of Sharpe ratio may be misleading. Consequently, we employ the CVaR as a measure of portfolio risk. Unlike the standard deviation which is strictly positive, the CVaR can assume a negative value. A negative CVaR implies that at a specified level of confidence, the likely worst outcome of a portfolio is a profit, rather than a loss. Practically, the optimisation is implemented using *R packages called fPortfolio* developed by and Ghalanos and Pfaf (2015). However, the optimal solution obtained from our models are based on both linear and non-linear programming algorithms embedded in *parma* function by Wuertz et al. (2009).

3 Data and preliminary analysis

The dataset is the daily returns of the five main financial markets for five countries across three continents, from January 1998 to December 2016: SP500 and BVSP (America), CAC40 and DAX (Europe) and HSI (Asia). The selected sample period allows to capture historical dramatic events including the Asian crisis in 1998, the 2001 US September attack, the 2007/2008 global financial crisis.

The skewness and kurtosis test results, point to the leptokurtic skewed type of distribution for these returns, suggesting that large fluctuations are more likely on the fat tails see histograms plots in Figure 2 and descriptive statistics in Figure 1.

Figure 1: Returns Descriptive Statistics

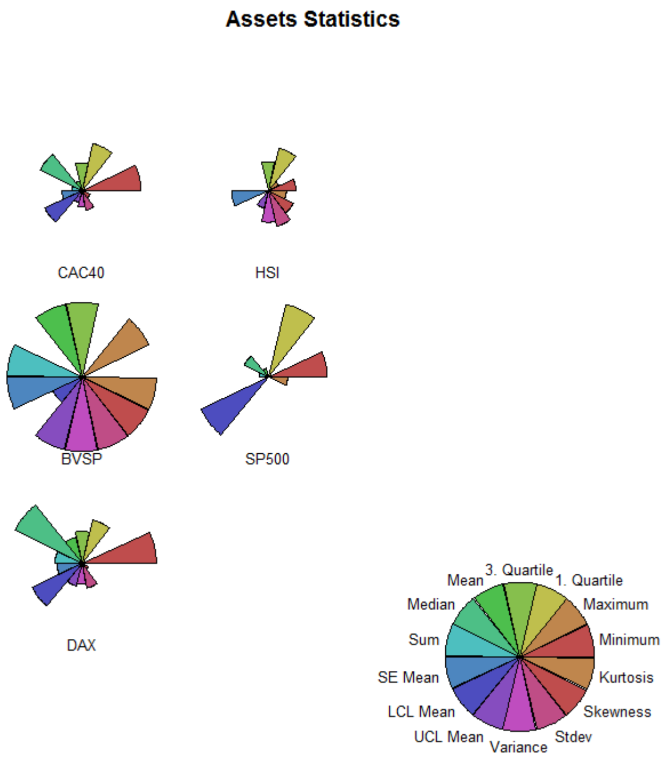
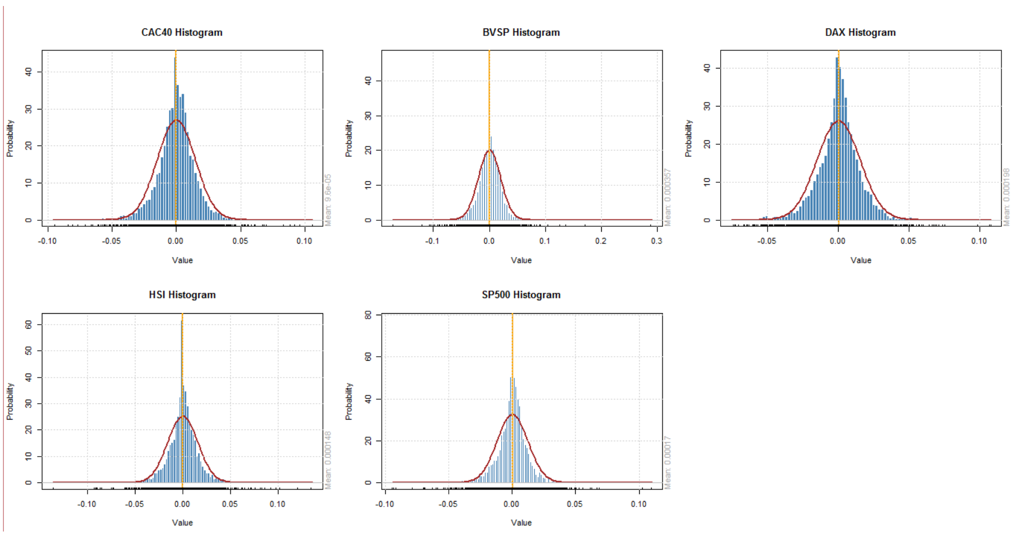


Figure 2: Returns-Histogram Plots for Each Stock



The initial inspection of the data indicates the evidence of volatility clustering see the historical plots of returns in Figures 3

Figure 3: Historical Returns Plots for Each Stock

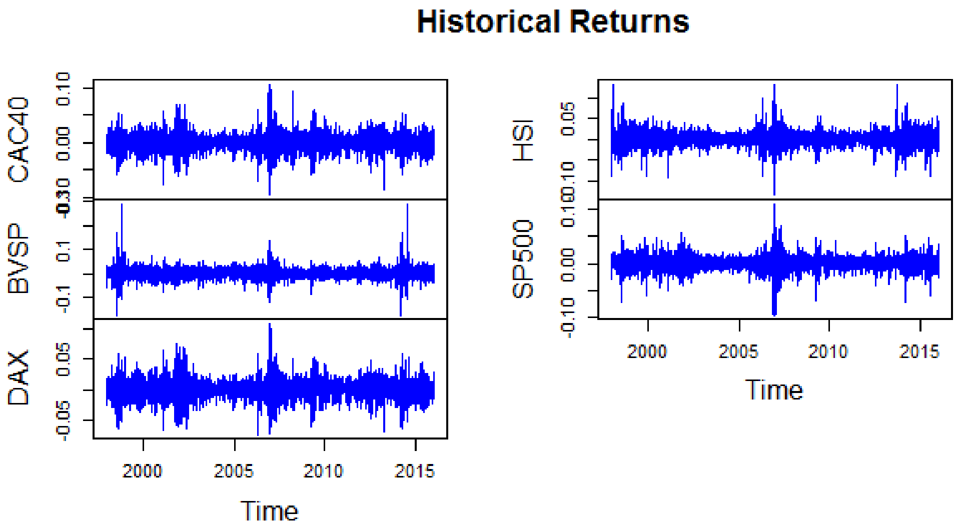
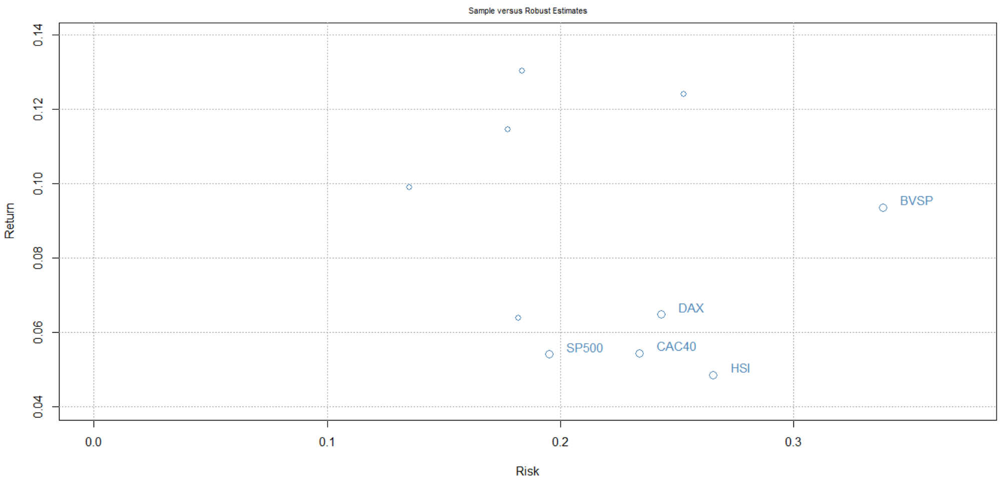


Figure 4 displays the relationship between risk and the expected returns of each stock, it is observed that BVSP is seems to be more riskier and yield higher returns compare to other stocks. And on the other side HSI has the highest risk and the lowest returns. SP500 yields same returns with CAC40, however CAC40 has the highest risk compare to SP500.

Figure 4: Returns Mean vs Variance plot

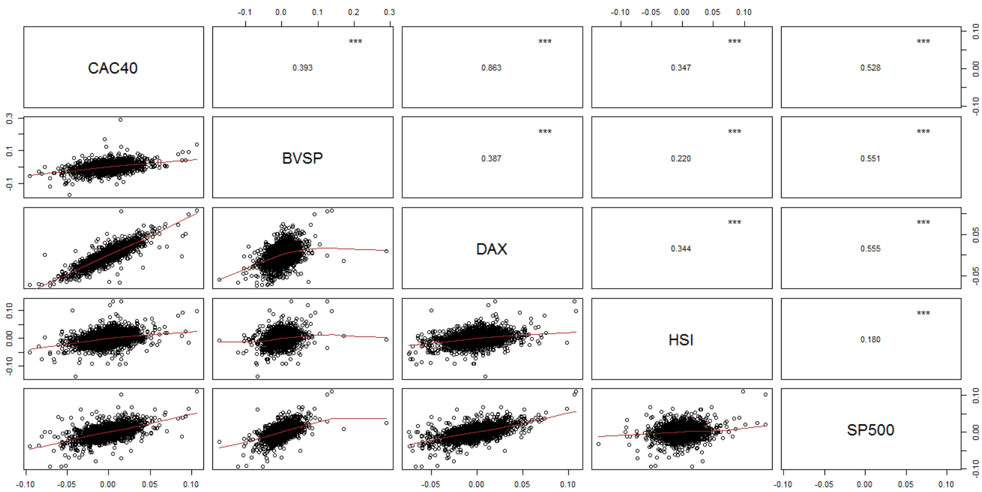


The returns of our stocks exhibit a significantly pairwise correlation, at $\alpha = 0.05$ see Figure 5

	Normality			Serial corelation	Arch effect
	Skewness test	**Kurtosis test**	**JB test**	**Portmanteau test**	**Arch test**
Chi squared	561.62	91130	91691	23.673	11103
P-value	$2.2e^{-16}$	$2.2e^{-16}$	$2.2e^{-16}$	$2.2e^{-16}$	$2.2e^{-16}$

Table 1: Multivariate Diagnotic tests

Figure 5: Returns correlation test plot

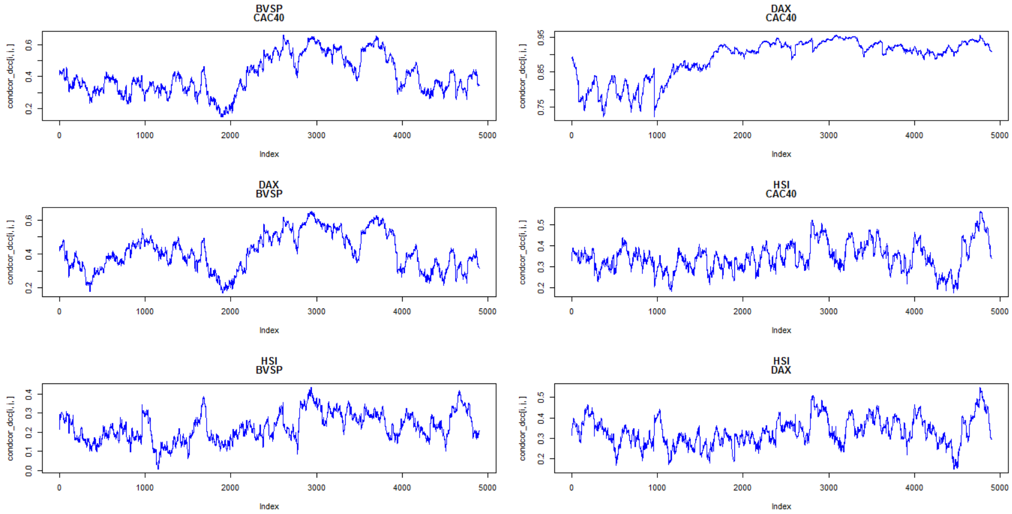


Here we test the arch effect, serial correlation and normality assumptions. The multivariate normality assumption is strongly rejected as JB statistic is significant. Similarly, there is an evidence of multivariate serial correlation and arch effect as both portmanteau and the arch tests reject the null hypotheses of absence of serial correlation and absence of arch effect, respectively. Table 1 shows the test results at $\alpha = 0.05$.

Because of the characteristics of the returns series described above, we use a multivariate t distribution to model the stocks correlations for DCC-GARCH. The evidence of the pairwise correlation using DCC-GARCH is shown in Figure 6 and in Figure 7 for GO-GARCH.

Figure 6: DCC-GARCH Estimated Time Evolutions of the Pairwise Conditional Correlations

(a)



(b)

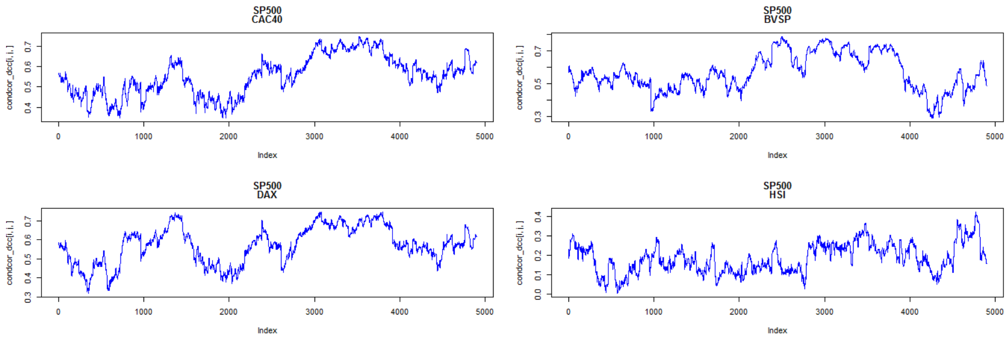
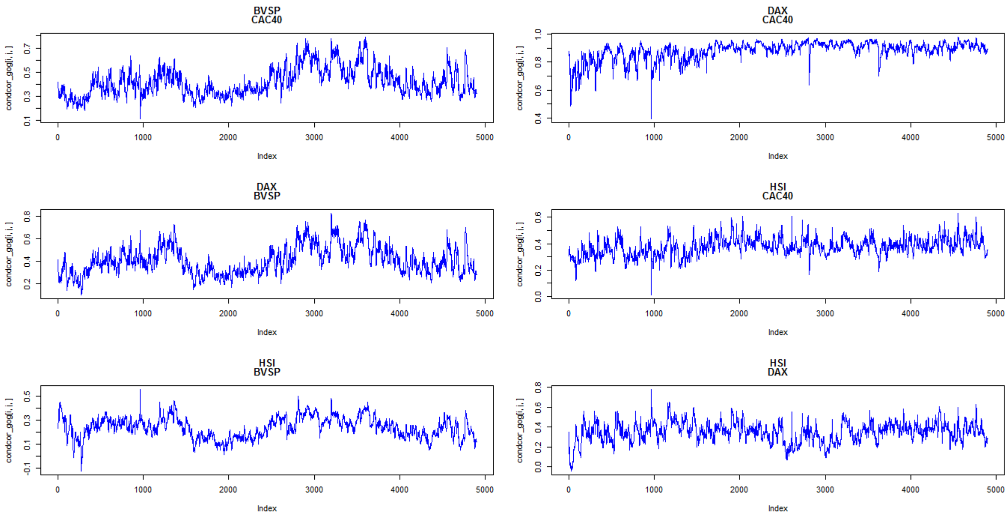
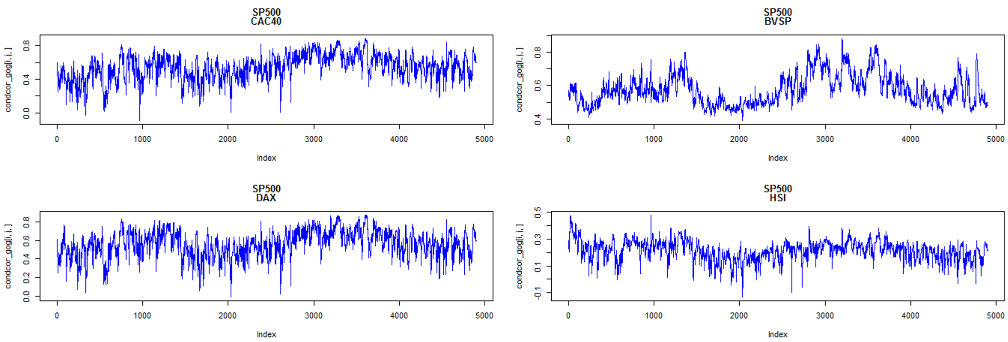


Figure 7: GOG-GARCH Estimated Time Evolutions of the Pairwise Conditional Correlations

(a)



(b)



4 Empirical findings

The dependence properties of returns matter for portfolio selection and/or risk evaluation. This section starts with the dependence estimation performed in each model. Here we discuss the dependence estimation performed using our models

4.1 Dependence estimation

4.1.1 Conditional correlation and Multivariate GARCH

In Figure 6 and 7 we show several interesting patterns of pairwise conditional correlations given by our models; the most significant observations being that the time varying correlations estimated by DCC-GARCH are more volatile than the one provided by GO-GARCH. All the stocks appear to be mostly pairwise

	DCC-GARCH	GO-GARCH
Log-Likelihood	-34016.8	-34554.9
	Diebold-Mariano Test	
DCC-GARCH = GO-GARCH	DM = 0.93755 (p-value =0.3485)	
DCC-GARCH > GO-GARCH	DM = 0.93755 (p-value = 0.1742)	
DCC-GARCH < GO-GARCH	DM = 0.93755 (p-value = 0.8258)	
	Dependence estimate under DCC-GARCH	
Joint conditional correlation	DCCb1= 0.987905 (p-value = 0.000)	
Joint conditional covariance	MSHAPE= 9.181540 (p-value = 0.000)	

Table 2: Goodness of fit of DCC-GARCH vs GO-GARCH and dependence estimate

correlated, this can be deduced from the joint correlation value (0.987905) estimated by DDC-GARCH see Table 2, it suggests a strong linkage among these stocks.

As highlighted by Boswijk and van der Weide (2011), the generality of any multivariate GARCH model can be gauged by the ability to account for the key stylized features of multivariate data while being closed under linear transformations. Besides the time-varying correlation as well as the persistence in volatility and covariation accommodated by the DCC-GARCH and GO-GARCH incorporates the spillover effects in volatility and remains closed under linear transformations. Due to these characteristics, it is not surprising that GO-GARCH model outperforms the DCC-GARCH as indicated by the log-likelihood values Table 2. However, DCC-GARCH remains more practicable as it provides not only pairwise conditional correlation but most importantly the joint dependence approximated by the joint conditional correlation and the joint conditional covariance.

Table 2 shows the performance of the DDC-GARCH against GO-GARCH. The comparison test confirm that the GO-GARCH fits our data better compared to DCC-GARCH as indicated by the log-likelihood. The joint correlation and covariance of the DCC-GARCH is therefore displayed in the lower panel of Table 2. However the Diebold Mariano test exhibit no significant differences between the models.

4.2 Portfolio optimisation

The CVaR risk measure is integrated into linear and non-linear optimisation models to estimate the minimum risk optimal portfolios. The multivariate GARCH models provide linear programing (LP) and non-linear programing (NLP) portfolio while The minimum risk optimal weights (percentage) of the international stock portfolio in Table 3 indicate that, under DCC and GO-GARCH, large weight is allocated to SP500 while DAX is assigned a marginal weight under both LP and NLP. We observe that both CAC40 has the zero

	Solver Type	CAC40	Weight BVSP	DAX	HSI	SP500	Return/Risk Metrics (Return,CVaR, SR)
DCC-	LP	0.0000	0.1541	0.0635	0.3062	0.4761	(0.0194, 3.036, 0.0064)
	NLP	0.0000	0.1464	0.1098	0.2997	0.4440	(0.0194, 2.583, 0.0075)
GO-	LP	0.0000	0.1541	0.0635	0.3062	0.4761	(0.0194, 3.036, 0.0064)
	NLP	0.0000	0.1464	0.1098	0.2997	0.4440	(0.0194, 2.583, 0.0075)

Table 3: Multivariate GARCH Portfolio Weights and Risk/Return Metrics

weights of which is expected as it displays the lowest returns compared to other stocks with the high variance, see Figure 1 and descriptive statistics of returns in Jupyter notebook.

5 Conclusion

We estimated the multivariate dependence risk using CVaR and applied mean-CVaR model to optimize the international stock portfolio, under the assumptions that all the countries have equal number of trading days. The risk/return trade off of the optimal portfolio is captured using the Sharp ratio. In Table 3 we see that the optimal portfolios given by our multivariate models using both LP and NLP yields the same expected returns, however the Models under NLP solver yields a small CVaR compared to LP solver, which gives a better sharp ratio. Thus the portfolios under NLP solver gives optimal portfolio based on sharp ratio and CVaR. This suggests that the stocks forms our portfolio are highly volatile. The portfolios are expected to be very volatile because it depends on the stocks of which they have strong pairwise conditional correlation, and the multivariate GARCH models are known to be good in handling the conditional variance of the variables. It will be good to see the results of the same data using other classes of models other than the multivariate GARCH.

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