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Option Pricing Models
Mid-term Report

Submitted by :
Ninad Gajbhiye
Shubham Rathod
Siyona Bansal

What is Option PriceTheory?

Option pricing theory is a way to determine the value of an options contract. An options contract gives someone the right, but not the obligation, to buy or sell an asset (like a stock) at a specific price (known as the strike price) on or before a certain date (expiration). Option pricing theory helps us figure out how much this right is worth.

How is the Value (Premium) of an Option Calculated?

The value of an option, called the premium, is estimated by considering the likelihood that the option will be profitable when it reaches its expiration date. If the option is likely to be profitable, it will have a higher premium. If it's less likely to be profitable, the premium will be lower.

Commonly used models to calculate option values

- **Black-Scholes Model:** This model is used for European-style options (options that can only be exercised on the expiration date). It uses the factors mentioned above to calculate option premiums mathematically.

- **Binomial Option Pricing:** This model is more flexible and can be used for both European and American-style options (options that can be exercised at any time before or on the expiration date). It breaks down time into smaller steps, calculating option values at each step, taking into account the probabilities of up and down movements in the asset's price.

- **Monte Carlo Simulation:** This is a simulation-based method. It generates random scenarios of the asset's price movements over time and calculates option values based on these simulated scenarios, considering the factors mentioned earlier.

Mathematics used while coding —>

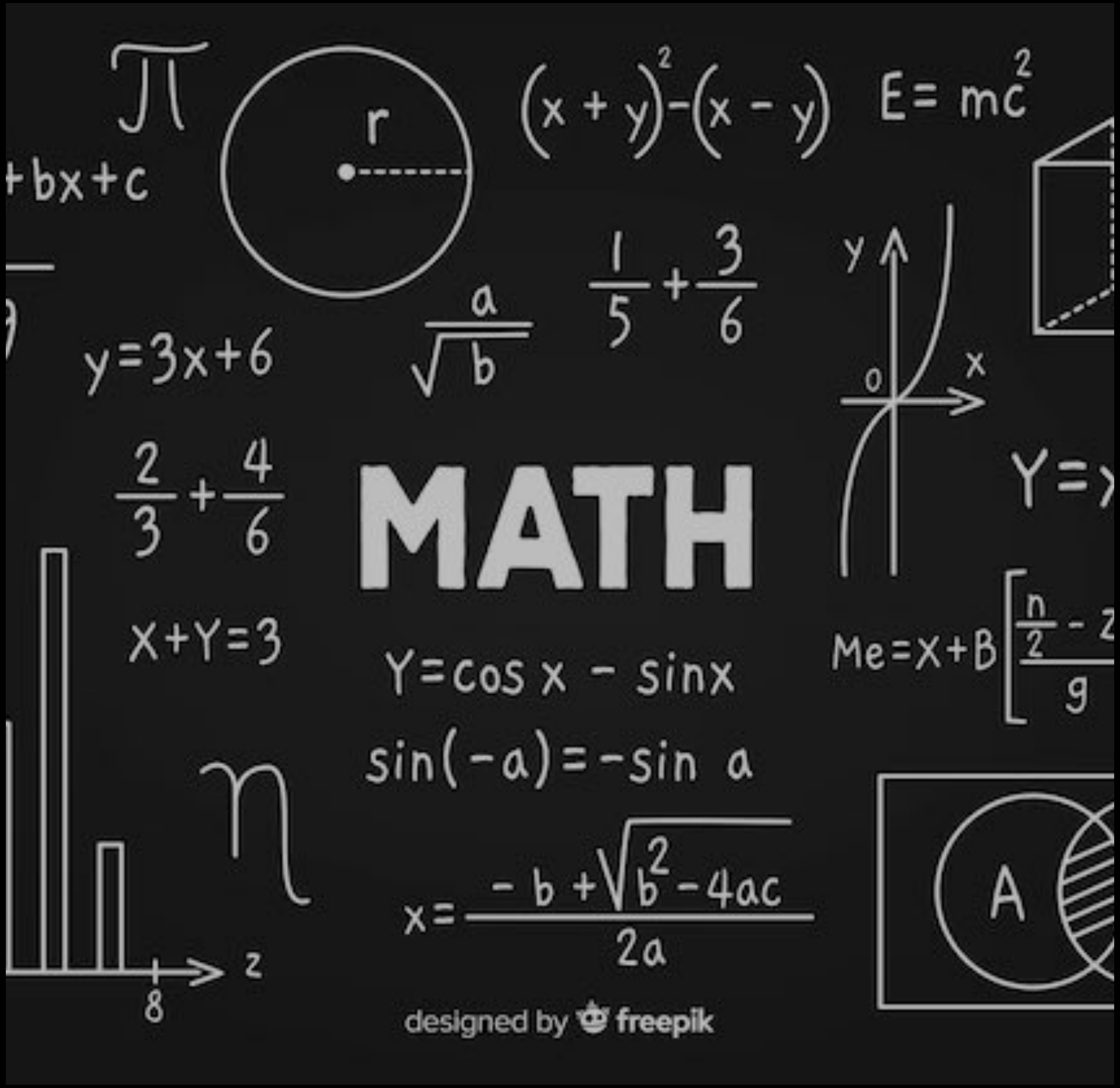
Black-Scholes Model

The partial differential equation, known as the Black-Scholes equation:

$\partial V/\partial t + 0.5\sigma^2 S^2 \partial^2 V/\partial S^2 + rS\partial V/\partial S - rV = 0$

Where:

- V: The price of the option as a function of time (t) and the underlying asset's price (S).
- t: Time to the expiration of the option.
- σ: The volatility of the underlying asset's returns.
- r: The risk-free interest rate



“The longer-term the option, the sillier the results generated by the Black-Scholes option pricing model, and the greater the opportunity for people who didnt use it”

– Michael Lewis

Binomial Option Pricing

Variables involved in this models are:
- S0: The current price of the underlying asset (stock price at time t=0).
- K: The strike price of the option (the price at which the underlying asset can be bought/sold).
- r: The risk-free interest rate (expressed as a decimal).
- T: The time to expiration (in years).
- u: The factor by which the underlying asset price increases in a one-time step.
- d: The factor by which the underlying asset price decreases in a one-time step.
- n: The number of time steps to expiration.

The formulas for u and d are as follows:
u = exp(sigma * sqrt(T / n))
d = 1 / u
Using these values, you can calculate the option prices at each node of the binomial tree using the following steps:
1. Calculate the expected return of the stock, denoted by "p":
   p = (exp(r * T / n) - d) / (u - d)
2. Calculate the stock prices at each node of the binomial tree:
   S(i,j) = S0 * u^j * d^(i-j)
 Where i represents the time step (0, 1, 2, ..., n), and j represents the number of up movements at time i.
3. Calculate the option prices at the expiration date (time T):
   For a call option, C(i,j) = max(S(i,j) - K, 0)
   For a put option, P(i,j) = max(K - S(i,j), 0)
4. Work backwards through the binomial tree to calculate the option prices at each node at previous time steps:
   C(i,j) = exp(-r * T / n) * (p * C(i+1,j+1) + (1-p) * C(i+1,j))
   P(i,j) = exp(-r * T / n) * (p * P(i+1,j+1) + (1-p) * P(i+1,j))
C(i,j) and P(i,j) represent the call and put option prices, respectively, at time i and up movements j.

Monte Carlo Simulation

Estimation Formula:

To approximate an expected value E(f) of a function f(x) with respect to a probability distribution P(x), the estimation formula is:

$E(f) \approx (1/N) * \Sigma\ f(xi)$, for i = 1 to N
Here, xi is a random sample drawn from the distribution P(x), and N is the number of samples taken.

Error Estimation Formula:

To estimate the error in the Monte Carlo approximation, the formula for the standard error (SE) is often used:

$SE \approx \mathrm{sqrt}(Var(f)/N)$
Where Var(f) is the variance of the function f(x) under the probability distribution P(x).

Integral Approximation:

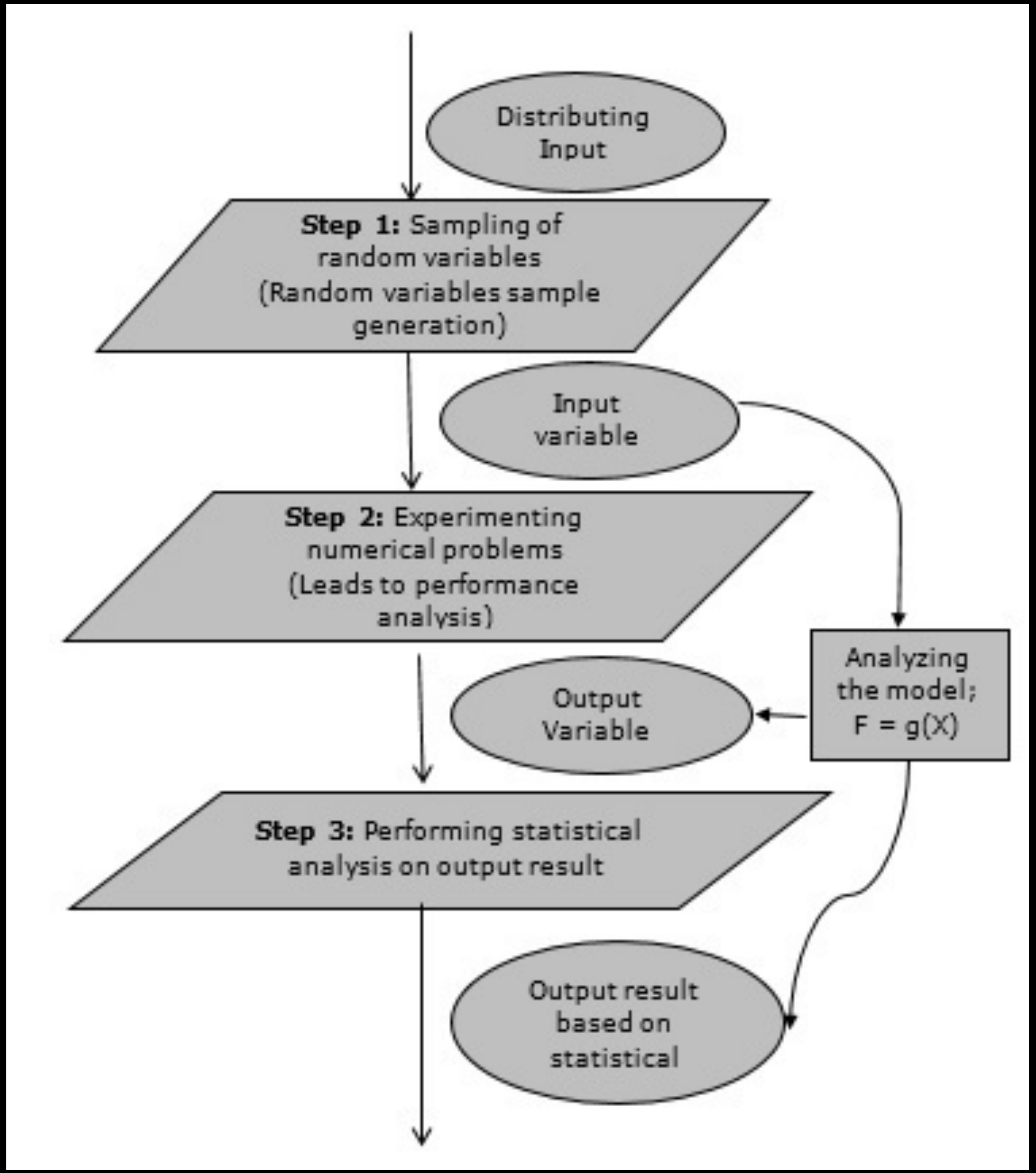
For approximating the integral of a function f(x) over a region [a, b] using Monte Carlo simulation, the formula is:

$\int [a, b]\ f(x)\ dx \approx (b - a) * (1/N) * \Sigma\ f(xi)$, for i = 1 to N

Probability Approximation:

To estimate the probability of an event A occurring under a given probability distribution P(x), the formula is:

$P(A) \approx$ (Number of favourable outcomes) / N



Advantages of each models over others

1. Black-Scholes Model:

Advantage: Analytical Solution

- The Black-Scholes model provides a closed-form, analytical solution for pricing European-style options. This means that you can calculate the option's price directly using a formula without using iterative numerical techniques.

- It is particularly useful for liquid markets and when dealing with options with no special features like dividends or early exercise.

2. Monte Carlo Simulation:

Advantage: Flexibility and Complexity

- Monte Carlo simulation allows for great flexibility in modelling complex derivatives. It can handle various instruments with multiple variables, making it suitable for pricing options with various features, such as American-style options and exotic derivatives.

- It can account for more realistic assumptions and capture the behaviour of underlying assets more accurately than some other models, as it relies on simulating random market movements.

3. Binomial Option Pricing Model:

Advantage: Versatility and Intuitive Understanding

- The Binomial model is more versatile than the Black-Scholes model as it can handle both European and American-style options and can be adapted to include dividends or other variations easily.

- It is intuitive and conceptually straightforward, making it an excellent educational tool for introducing option pricing principles.

Thank You