Report of FFR120 HM3: Chapter 9-12

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1 Exercise 1 Particles with phoretic interaction.

1.1 Q1

The time step Δt must be chosen to balance simulation accuracy and computational efficiency while ensuring stability. The following factors influence this choice:

1. Particle Speed (v): To prevent particles from traveling too far (e.g., exceeding their radius R or interaction range $r_c = 10R$), the condition is:

$$\Delta t \ll \frac{r_c}{v} = \frac{10 \times 10^{-6}}{5 \times 10^{-6}} = 2\,\mathrm{s}.$$

2. Phoretic Interaction Speed (v_0): To capture interactions accurately:

$$\Delta t \ll \frac{r_c}{v_0} = \frac{10 \times 10^{-6}}{20 \times 10^{-6}} = 0.5\,\mathrm{s}.$$

3. Volume Exclusion and Particle Overlap: To prevent particle overlap, their displacement must remain smaller than the particle radius R. This gives:

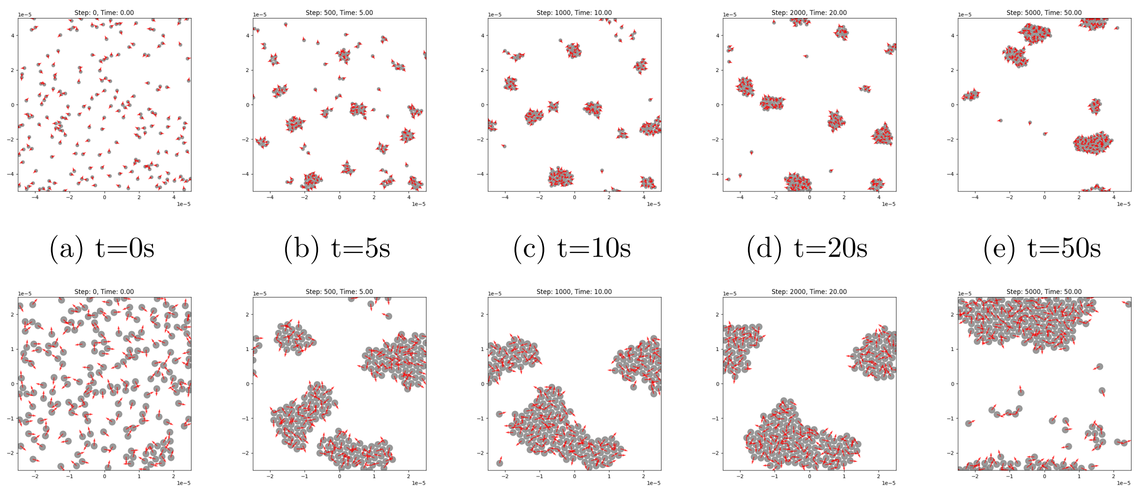
$$\Delta t \ll \frac{R}{v} = \frac{1 \times 10^{-6}}{5 \times 10^{-6}} = 0.2\,\mathrm{s}.$$

So he most restrictive condition is $\Delta t \ll 0.2\,\mathrm{s}$, one can choose a suitable time step is:

$$\Delta t = 0.01\,\mathrm{s}.$$

1.2 P1 - P2

Particles of the same color represent those belonging to the same cluster. Overall, at both time points, the number of clusters under negative delay is smaller than that under positive delay.



(a) t=0s (b) t=5s (c) t=10s (d) t=20s (e) t=50s

(f) t=0s (g) t=5s (h) t=10s (i) t=20s (j) t=50s

Figure 1: (a)- (b) is Positive delay; (c) - (d) is Negative delay

Observations for P1 $L = 100R$: Weaker interactions result in slower clustering and a more random distribution, with less order emerging over time.

1. Initial stage ($t = 0$ s): Random particle distribution similar to P1.
2. Intermediate stage ($t = 5$ to $t = 20$ s):

   - Clustering is weaker and slower compared to P1 due to reduced interactions ($L = 50R$).
   - Clusters remain small and scattered.

3. Final stage ($t = 50$ s):

   - Particles remain more dispersed, with smaller and less dominant clusters compared to P1.
   - Particle orientations show less alignment.

Observations for P2 $L = 50R$: The system exhibits strong particle interactions, leading to significant clustering and increased order over time.

1. Initial stage ($t = 0$ s): Particles are randomly distributed with random orientations.
2. Intermediate stage ($t = 5$ to $t = 20$ s):

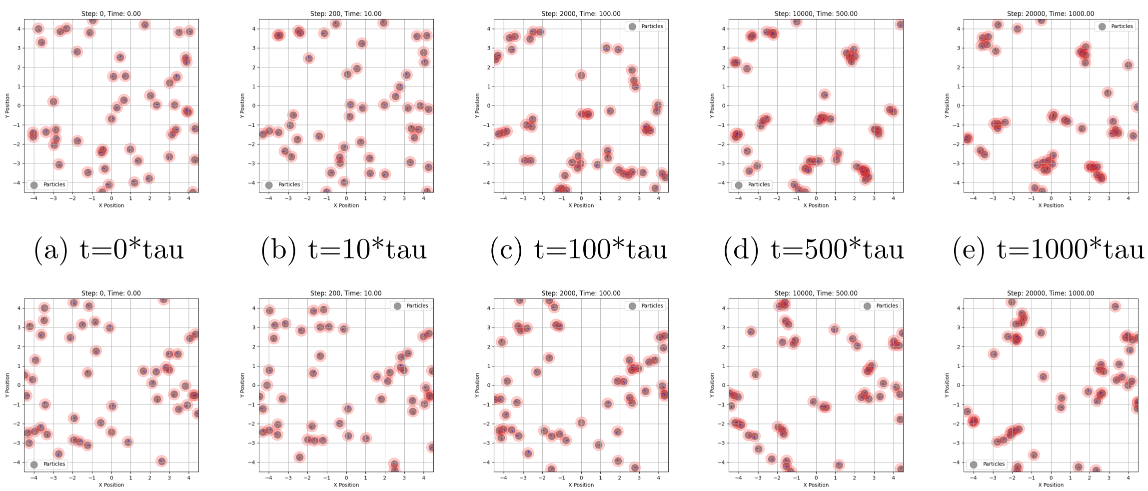
   - Small clusters begin to form, and particle orientations start aligning within clusters.
   - Clusters grow larger over time as interactions strengthen.

3. Final stage ($t = 50$ s): Large clusters dominate the system, and particles within each cluster show strong alignment.

Comparison of P1 and P2:

- P2 shows larger and fewer clusters, P1 has smaller and more dispersed clusters.
- P2 demonstrates stronger alignment of particle orientations, P1 retains more randomness.
- Spatial Distribution: P2 clusters are closer together, P1 particles remain more scattered.

2 Exercise 2 Sensory Delay

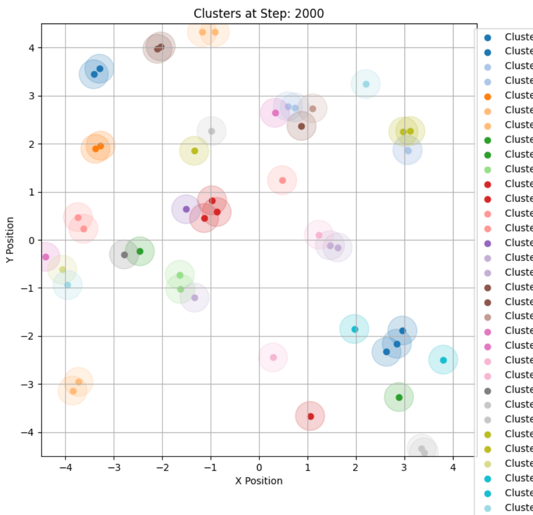


(a) t=0*tau (b) t=10*tau (c) t=100*tau (d) t=500*tau (e) t=1000*tau

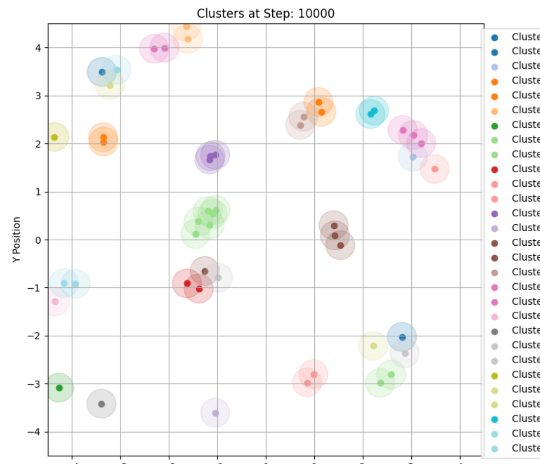
(f) t=0*tau (g) t=10*tau (h) t=100*tau (i) t=500*tau (j) t=1000*tau

Figure 2: (a)- (e) is P1 Positive Delay; (f) - (j) is P2 Negative Delay

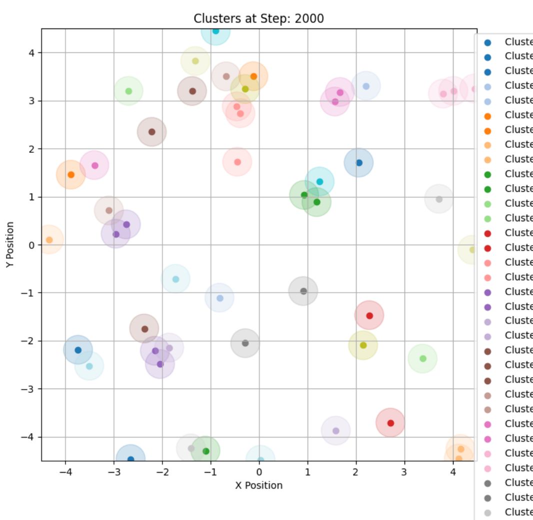
2.1 P3 Q1



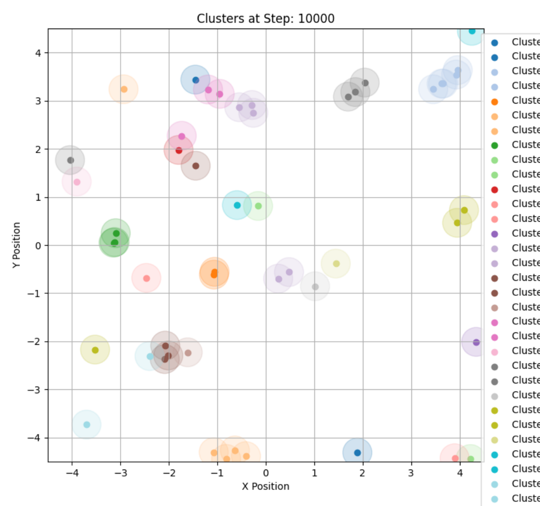
(a) t=100*tau



(b) t=500tau



(c) t=100tau



(d) t=500tau

Figure 3: (a)- (e) is P1 L=100R; (f) - (j) is P2 L = 50R

3 Exercise 3 Disease Spreading(SRI model)

Introducing probability (alpha) to represent agents become susceptible again after recovered.

3.1 P1 and Q1

I0 = 30, 5 runs, steps = 50000, alpha = 0.05

the disease dose not die out, because the number of recover below 10.

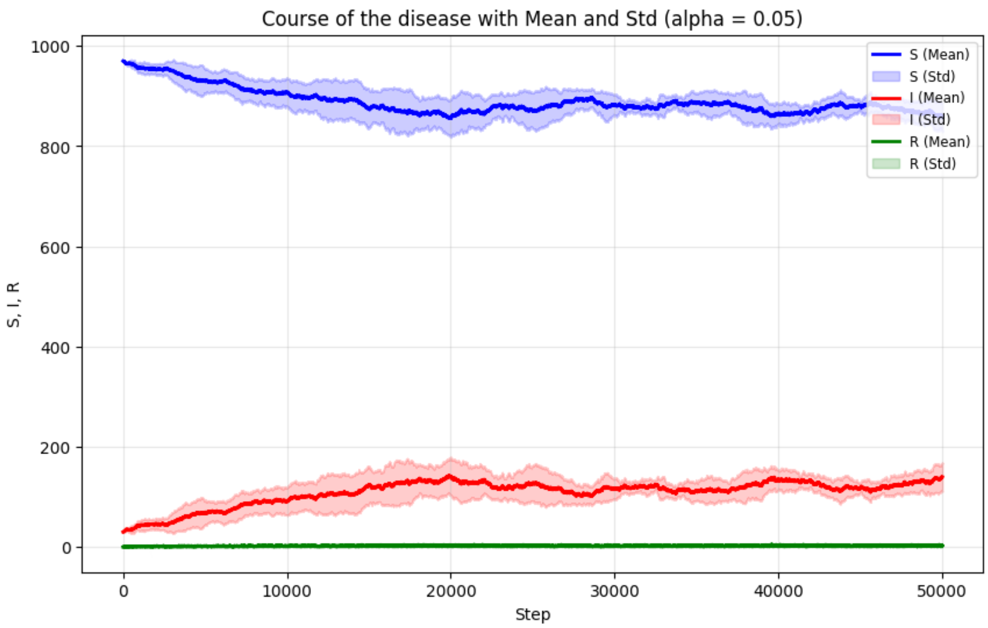
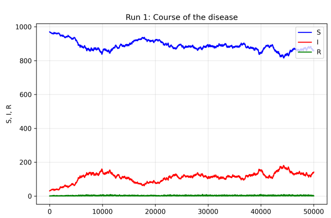
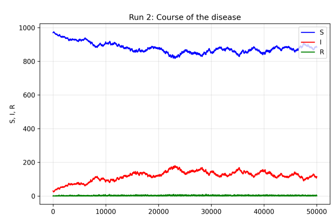


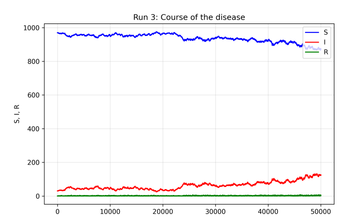
Figure 4: I0 =30, course of the disease with Mean and Std (alpha =0.05)



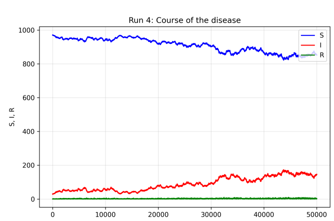
(a) t=0*tau



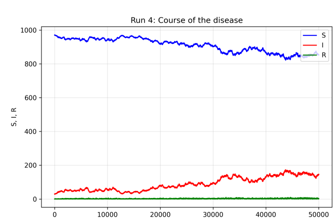
(b) t=10*tau



(c) t=100*tau



(d) t=500*tau



(e) t=1000*tau

Figure 5

3.2 P2 and Q2

I0 = 10, 5 runs, steps = 50000, alpha = 0.005

the disease sometimes die out, because the number of infected agent is zero in some runs.

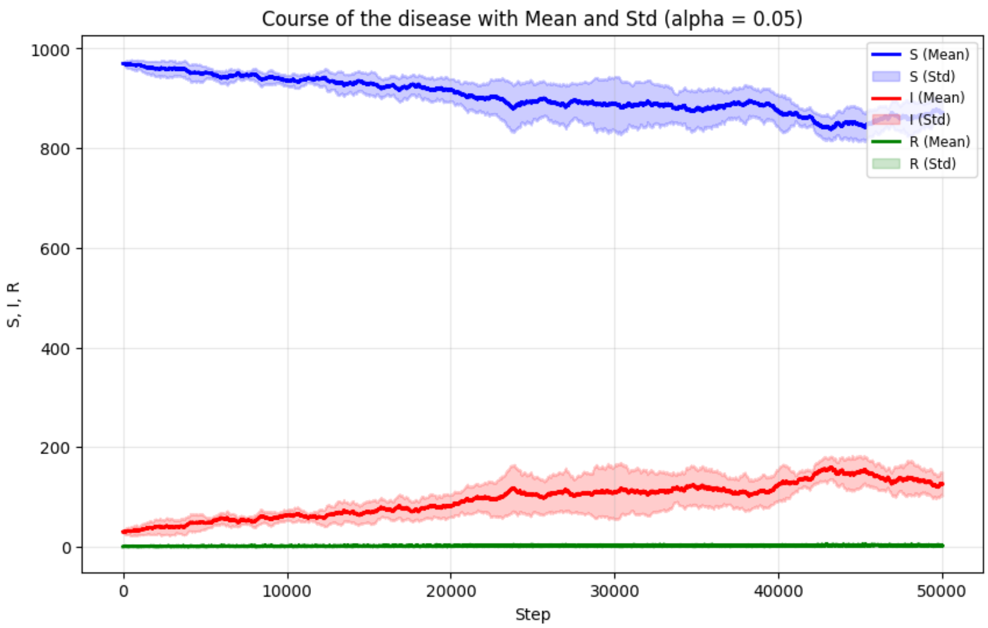
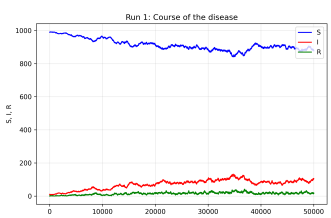
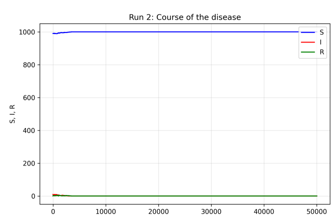


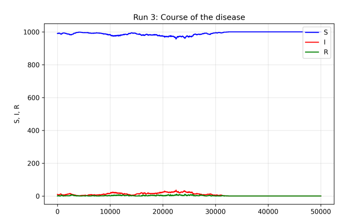
Figure 6: I0 =10, course of the disease with Mean and Std (alpha =0.05)



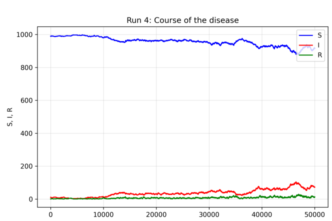
(a) t=0*tau



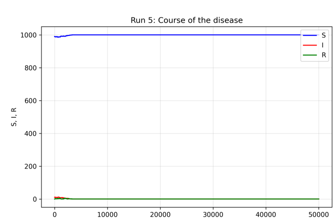
(b) t=10*tau



(c) t=100*tau



(d) t=500*tau



(e) t=1000*tau

Figure 7

4 Exercise 4 Network models

The clustering coefficient C increases linearly with p. For small p, C is close to 0 because very few triangles form. When p→1, the graph approaches a complete graph, so C approaches 1. The experimental results closely follow the theoretical linear trend C=p.

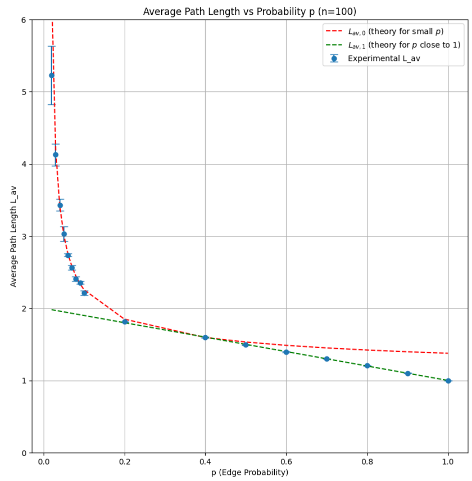
The average path length L_{av} is very high for small ($p \ll 1$) and decreases rapidly as p increases. For $p \to 1, L_{av} \to 1$. The experimental results match the theoretical predictions and the trends shown in Fig. 12.5a.

Q1: Why is $C = p$ in Erdős-Rényi graphs?

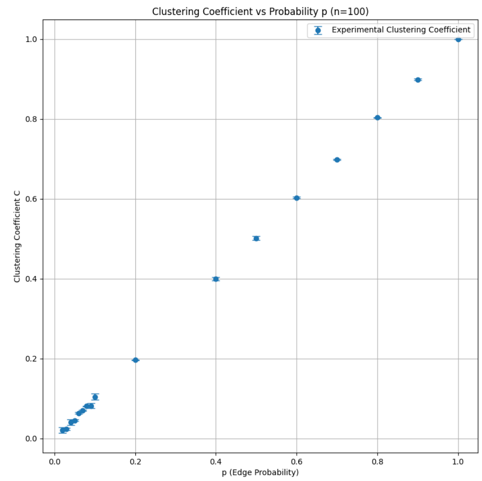
- 1. Independence of Edges: In Erdős-Rényi graphs, edges between nodes are generated independently with probability p.
- 2. Triangle Formation: For a node i, the probability that two neighbors j and k are connected is also p, leading to a direct proportionality between C and p.

n = 100 vs n = 200

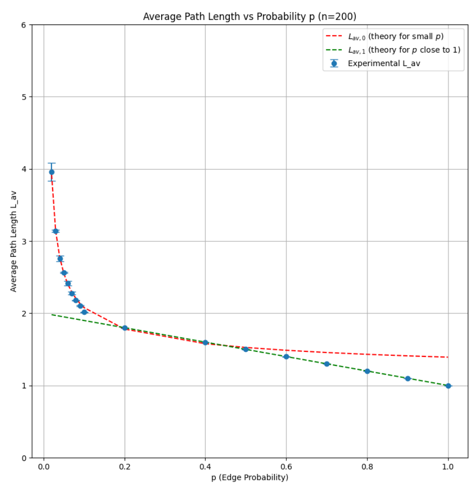
- Average Path Length: For $n = 200$, L_{av} is slightly shorter than for $n = 100$ because a larger graph allows more potential connections.
- Clustering Coefficient: C is independent of n, as it depends only on p. The trend is the same for both cases.



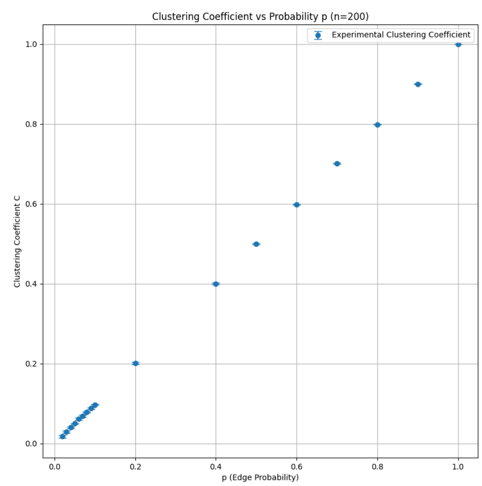
(a) Average path length n=100



(b) Clustering Coefficient n =100



(c) Average path length n=200



(d) Clustering Coefficient n =200

Figure 8: Erdos-R´enyi random graph