3.4

November 26, 2024

[59]:

```python
import numpy as np

def nodes_degree(A):
    """
    Function returning the degree of a node.

    Parameters
    ==========
    A : Adjacency matrix (assumed symmetric).
    """

    degree = np.sum(A, axis=0)

    return degree
```

[60]:

```python
def path_length(A, i, j):
    """
    Function returning the minimum path length between thwo nodes.

    Parameters
    ==========
    A : Adjacency matrix (assumed symmetric).
    i, j : Nodes indices.
    """

    Lij = - 1

    if A[i, j] > 0:
        Lij = 1
    else:
        N = np.size(A[0, :])
        P = np.zeros([N, N]) + A
        n = 1
        running = True
        while running:
            P = np.matmul(P, A)
            n += 1
            running
```

```
            if P[i, j] > 0:
                Lij = n
            if (n > N) or (Lij > 0):
                running = False

    return Lij
```

[61]:
```
def matrix_path_length(A):
    """
    Function returning a matrix L of minimum path length between nodes.

    Parameters
    ==========
    A : Adjacency matrix (assumed symmetric).
    """

    N = np.size(A[0, :])
    L = np.zeros([N, N]) - 1

    for i in range(N):
        for j in range(i + 1, N):
            L[i, j] = path_length(A, i, j)
            L[j, i] = L[i, j]

    return L
```

[62]:
```
def average_path_length(A):
    """
    Calculate the average path length for an adjacency matrix A.
    """
    L = matrix_path_length(A)
    L = L[np.where(L > 0)]  # delete length <= 0

    if len(L) == 0:
        return np.inf
    return np.mean(L)
```

[63]:
```
def clustering_coefficient(A):
    """
    Function returning the clustering coefficient of a graph.

    Parameters
    ==========
    A : Adjacency matrix (assumed symmetric).
    """

    K = nodes_degree(A)
```

```
    N = np.size(K)

    C_n = np.sum(np.diagonal(np.linalg.matrix_power(A, 3)))
    C_d = np.sum(K * (K - 1))

    C = C_n / C_d

    return C
```

[64]:
```
def erdos_renyi_rg(n, p):
    """
    Function generating an Erdős-Rényi random graph

    Parameters
    ==========
    n : Number of nodes.
    p : Probability that each possible edge is present.
    """

    A = np.zeros([n, n])
    rn = np.random.rand(n, n)
    A[np.where(rn < p)] = 1

    for i in range(n):
        A[i, i] = 0

    # This below is for plotting in a circular arrangement.
    x = np.cos(np.arange(n) / n * 2 * np.pi)
    y = np.sin(np.arange(n) / n * 2 * np.pi)

    return A, x, y
```

0.1 Task 1

n = 100

[ ]:
```
from matplotlib import pyplot as plt



n = 100  # nodes
ps = [0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.2, 0.4, 0.5, 0.6,␣
↪0.7, 0.8, 0.9, 1] # different p value
average_lengths = []
clustering_coeffs = []

length_std_errors = []
clustering_std_errors = []
```

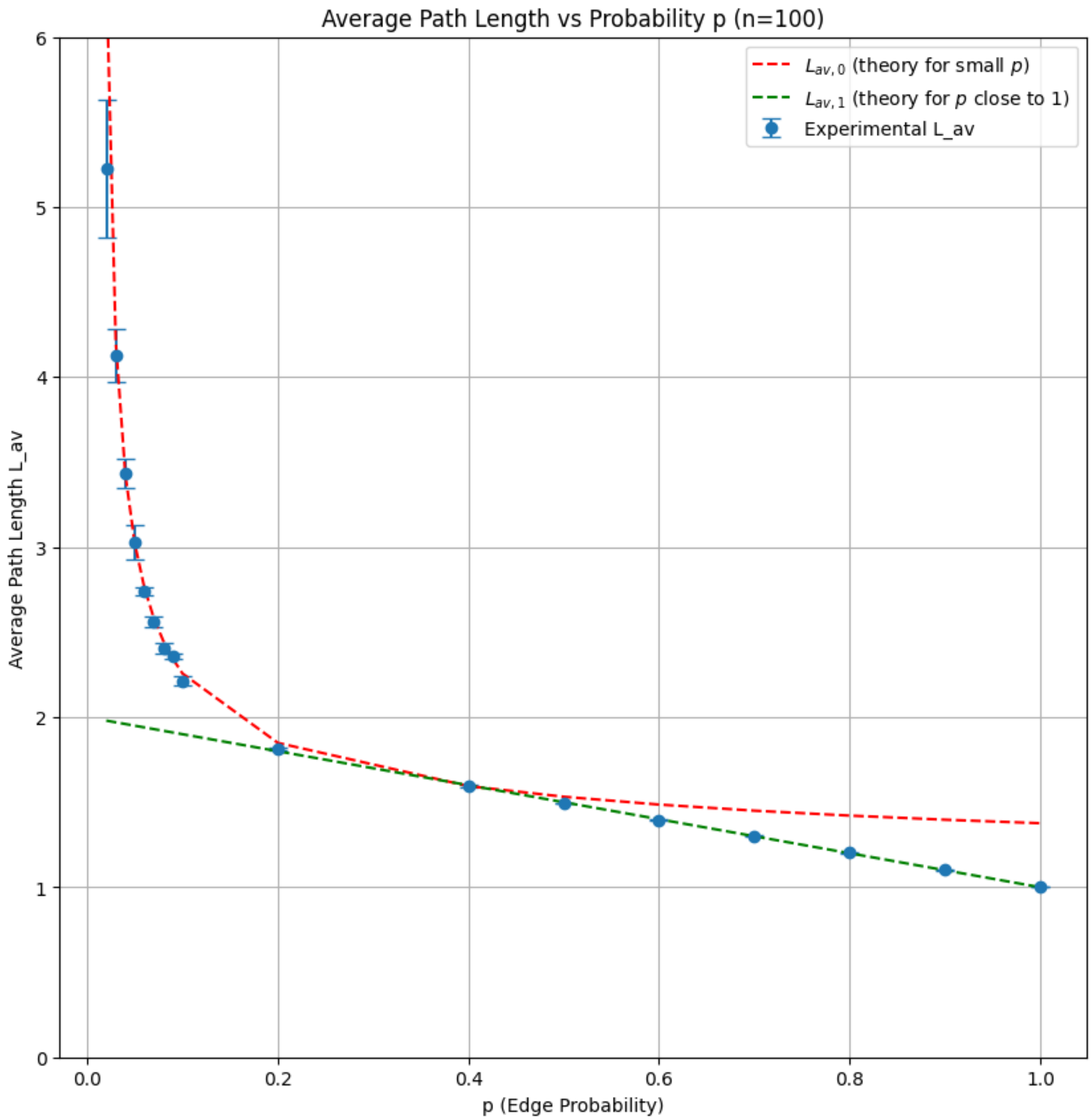
```python
gamma = 0.57722 # Euler-Mascheroni constant

for p in ps:
    lengths = []
    coeffs = []
    for _ in range(3):
        A, _, _ = erdos_renyi_rg(n, p)
        avg_len = average_path_length(A)
        lengths.append(avg_len)
        clus_coeff = clustering_coefficient(A)
        coeffs.append(clus_coeff)

    average_lengths.append(np.mean(lengths))
    length_std_errors.append(np.std(lengths))
    clustering_coeffs.append(np.mean(coeffs))
    clustering_std_errors.append(np.std(coeffs))

    print(f'p = {p}, saved.')
```

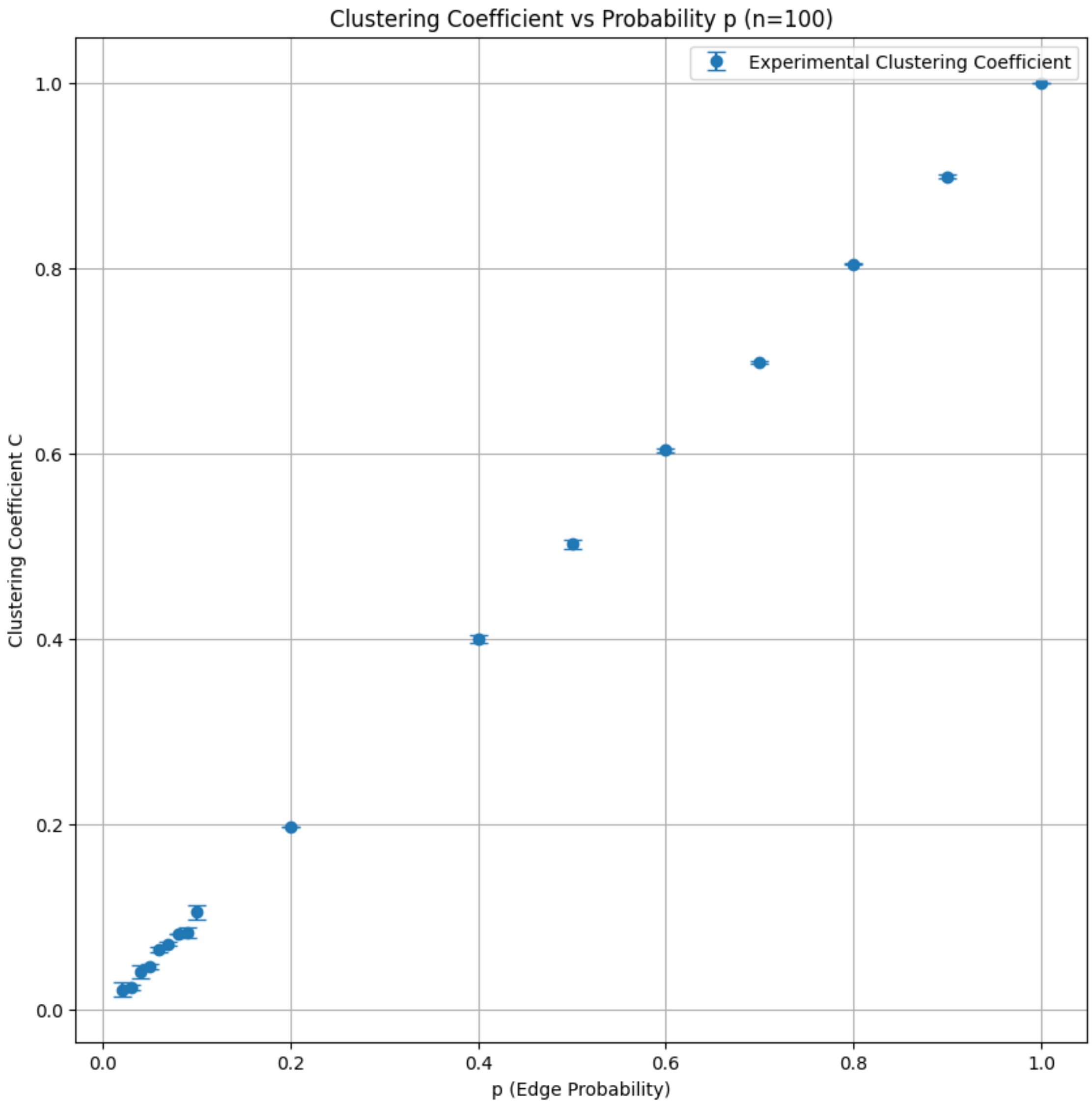
```
p = 0.02, saved.
p = 0.03, saved.
p = 0.04, saved.
p = 0.05, saved.
p = 0.06, saved.
p = 0.07, saved.
p = 0.08, saved.
p = 0.09, saved.
p = 0.1, saved.
p = 0.2, saved.
p = 0.4, saved.
p = 0.5, saved.
p = 0.6, saved.
p = 0.7, saved.
p = 0.8, saved.
p = 0.9, saved.
p = 1, saved.
```

[68]:

```python
ps = np.array(ps)
lav_0 = (np.log(n) - gamma) / np.log(ps * (n - 1)) + 0.5
lav_1 = 2 - ps

plt.figure(figsize=(10, 10))
plt.ylim(0, 6)
plt.errorbar(ps, average_lengths, yerr=length_std_errors, fmt='o',␣
 ↪label='Experimental L_av', capsize=5)
plt.plot(ps, lav_0, 'r--', label=r'$L_{av,0}$ (theory for small $p$)')
plt.plot(ps, lav_1, 'g--', label=r'$L_{av,1}$ (theory for $p$ close to 1)')
```

```
plt.xlabel('p (Edge Probability)')
plt.ylabel('Average Path Length L_av')
plt.title('Average Path Length vs Probability p (n=100)')
plt.legend()
plt.grid()
plt.savefig('3.4_Task1_P1.png')
plt.show()

plt.figure(figsize=(10, 10))

plt.errorbar(ps, clustering_coeffs, yerr=clustering_std_errors, fmt='o',␣
 ↪label='Experimental Clustering Coefficient', capsize=5)
plt.xlabel('p (Edge Probability)')
plt.ylabel('Clustering Coefficient C')
plt.title('Clustering Coefficient vs Probability p (n=100)')
plt.legend()
plt.grid()
plt.savefig('3.4_Task1_P2.png')
plt.show()
```


Average Path Length vs Probability p (n=100)
$L_{av,0}$ (theory for small p)
$L_{av,1}$ (theory for p close to 1)
Experimental L_av
Average Path Length L_av
p (Edge Probability)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
6



0.2 Task 2

n = 200

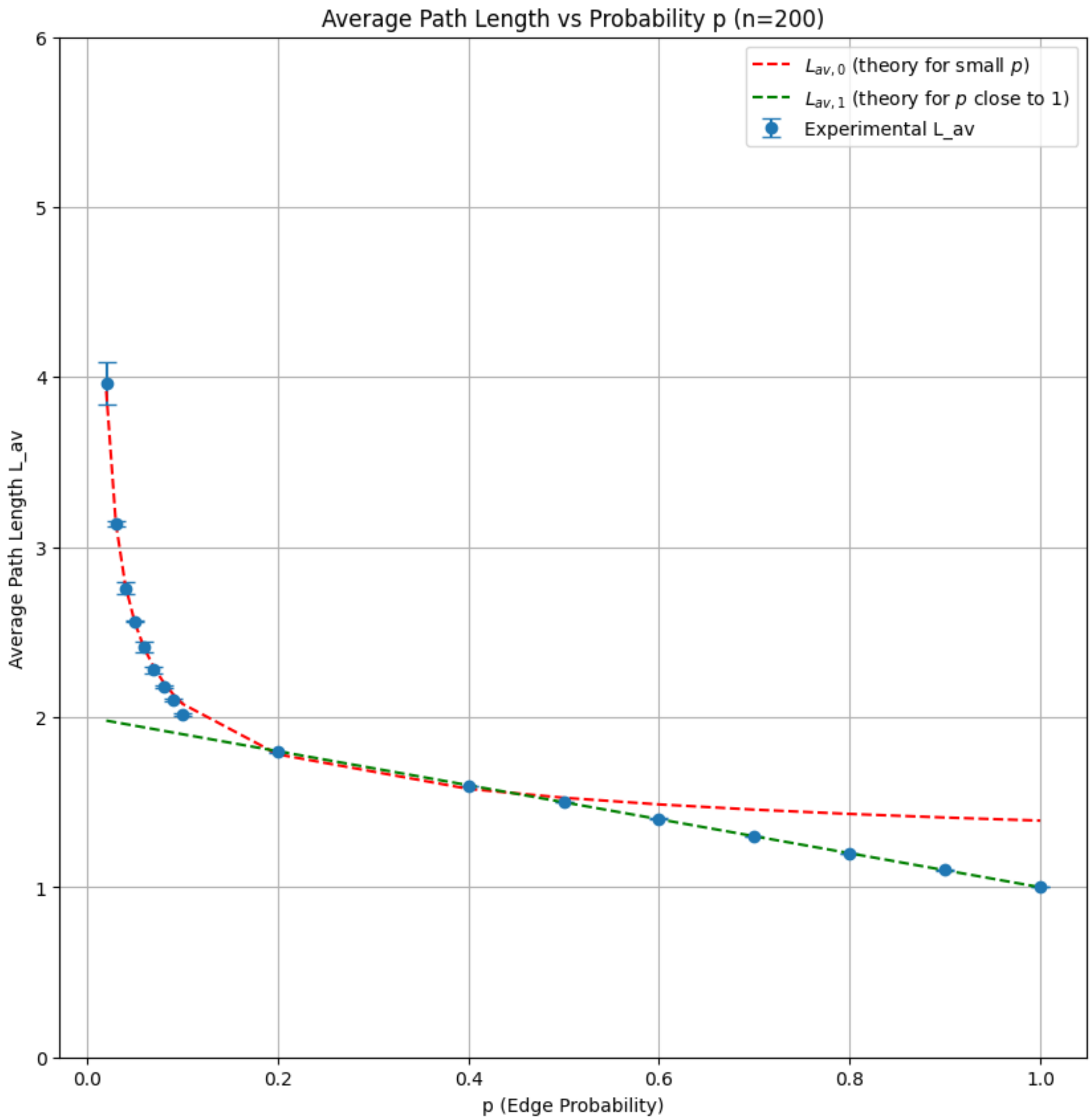
[69]:
```
from matplotlib import pyplot as plt


n = 200  # nodes
ps = [0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.2, 0.4, 0.5, 0.6,␣
 ↪0.7, 0.8, 0.9, 1] # different p value
average_lengths = []
clustering_coeffs = []
```

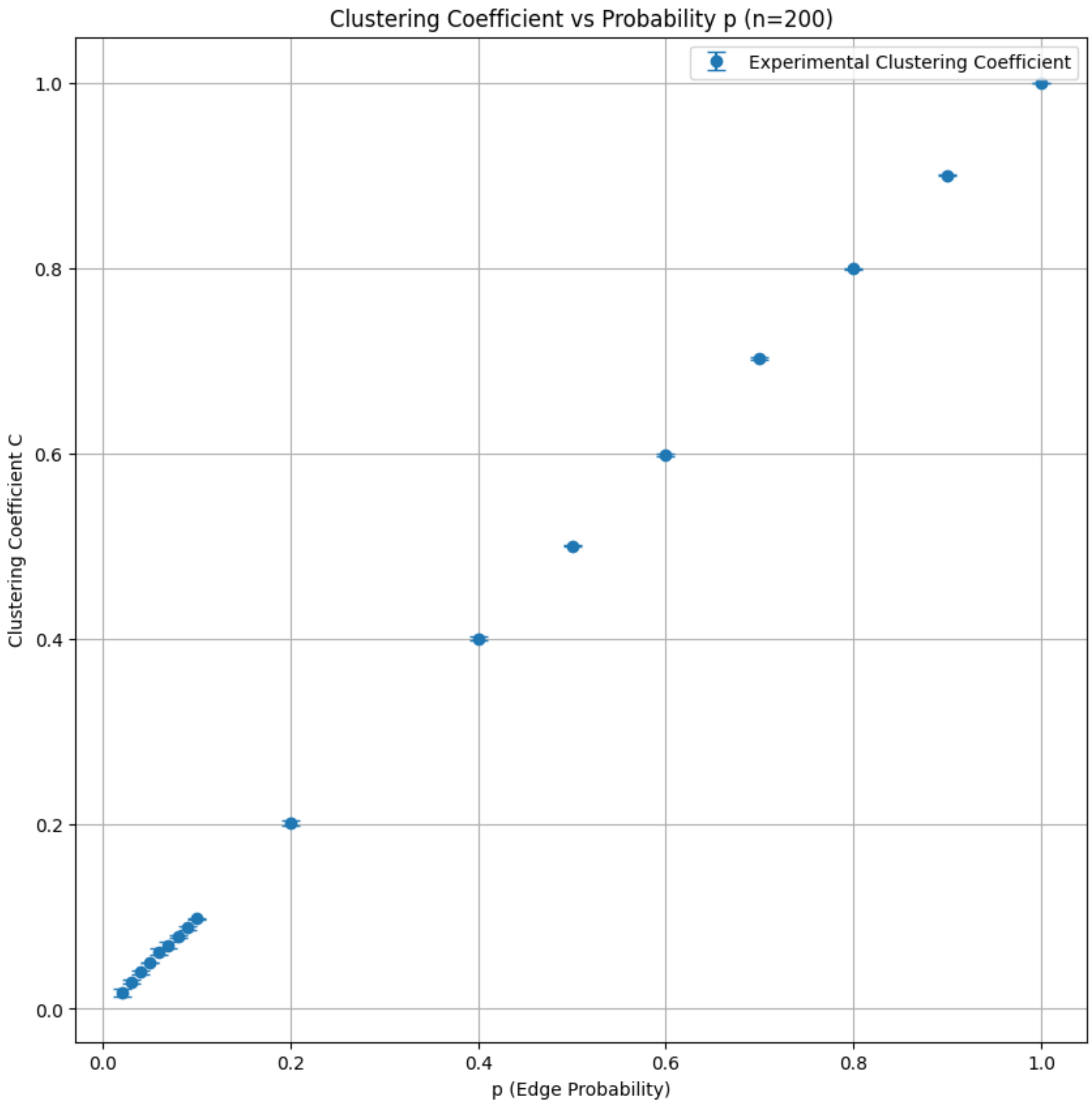
```
length_std_errors = []
clustering_std_errors = []
gamma = 0.57722 # Euler-Mascheroni constant

for p in ps:
    lengths = []
    coeffs = []
    for _ in range(3):
        A, _, _ = erdos_renyi_rg(n, p)
        avg_len = average_path_length(A)
        lengths.append(avg_len)
        clus_coeff = clustering_coefficient(A)
        coeffs.append(clus_coeff)

    average_lengths.append(np.mean(lengths))
    length_std_errors.append(np.std(lengths))
    clustering_coeffs.append(np.mean(coeffs))
    clustering_std_errors.append(np.std(coeffs))

    print(f'p = {p}, saved.')
```

```
p = 0.02, saved.
p = 0.03, saved.
p = 0.04, saved.
p = 0.05, saved.
p = 0.06, saved.
p = 0.07, saved.
p = 0.08, saved.
p = 0.09, saved.
p = 0.1, saved.
p = 0.2, saved.
p = 0.4, saved.
p = 0.5, saved.
p = 0.6, saved.
p = 0.7, saved.
p = 0.8, saved.
p = 0.9, saved.
p = 1, saved.
```

[70]:
```
ps = np.array(ps)
lav_0 = (np.log(n) - gamma) / np.log(ps * (n - 1)) + 0.5
lav_1 = 2 - ps

plt.figure(figsize=(10, 10))
plt.ylim(0, 6)
plt.errorbar(ps, average_lengths, yerr=length_std_errors, fmt='o',
 ↪label='Experimental L_av', capsize=5)
```

```
plt.plot(ps, lav_0, 'r--', label=r'$L_{av,0}$ (theory for small $p$)')
plt.plot(ps, lav_1, 'g--', label=r'$L_{av,1}$ (theory for $p$ close to 1)')
plt.xlabel('p (Edge Probability)')
plt.ylabel('Average Path Length L_av')
plt.title('Average Path Length vs Probability p (n=200)')
plt.legend()
plt.grid()
plt.savefig('3.4_Task2_P3.png')
plt.show()

plt.figure(figsize=(10, 10))
plt.errorbar(ps, clustering_coeffs, yerr=clustering_std_errors, fmt='o',
    label='Experimental Clustering Coefficient', capsize=5)
plt.xlabel('p (Edge Probability)')
plt.ylabel('Clustering Coefficient C')
plt.title('Clustering Coefficient vs Probability p (n=200)')
plt.legend()
plt.grid()
plt.savefig('3.4_Task2_P4.png')
plt.show()
```


Average Path Length vs Probability p (n=200)
$L_{av,0}$ (theory for small p)
$L_{av,1}$ (theory for p close to 1)
Experimental L_av
Average Path Length L_av
p (Edge Probability)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
6


Clustering Coefficient vs Probability p (n=200)
Experimental Clustering Coefficient
Clustering Coefficient C
p (Edge Probability)
0.0
0.2
0.4
0.6
0.8
1.0