ising_model_final

November 11, 2024

[2]:

```python
import random
import time
from tkinter import *
import numpy as np
import matplotlib.pyplot as plt


def neighboring_spins(i_list, j_list, sl):
    """
    Function returning the position of the neighbouring spins of a list of
    spins identified by their positions in the spin lattice.

    Parameters
    ==========
    i_list : Spin position first indices.
    j_list : Spin position second indices.
    sl : Spin lattice.
    """

    Ni, Nj = sl.shape  # Shape of the spin lattice.

    # Position neighbors right.
    i_r = i_list
    j_r = list(map(lambda x:(x + 1) % Nj, j_list))

    # Position neighbors left.
    i_l = i_list
    j_l = list(map(lambda x:(x - 1) % Nj, j_list))

    # Position neighbors up.
    i_u = list(map(lambda x:(x - 1) % Ni, i_list))
    j_u = j_list

    # Position neighbors down.
    i_d = list(map(lambda x:(x + 1) % Ni, i_list))
    j_d = j_list
```

```python
    # Spin values.
    sl_u = sl[i_u, j_u]
    sl_d = sl[i_d, j_d]
    sl_l = sl[i_l, j_l]
    sl_r = sl[i_r, j_r]

    return sl_u, sl_d, sl_l, sl_r
```

[3]:
```python
def energies_spins(i_list, j_list, sl, H, J):
    """
    Function returning the energies of the states for the spins in given
    positions in the spin lattice.

    Parameters
    ==========
    i_list : Spin position first indices.
    j_list : Spin position second indices.
    sl : Spin lattice.
    """

    sl_u, sl_d, sl_l, sl_r = neighboring_spins(i_list, j_list, sl)

    sl_s = sl_u + sl_d + sl_l + sl_r

    E_u = - H - J * sl_s
    E_d =   H + J * sl_s

    return E_u, E_d
```

[4]:
```python
def probabilities_spins(i_list, j_list, sl, H, J, T):
    """
    Function returning the energies of the states for the spins in given
    positions in the spin lattice.

    Parameters
    ==========
    i_list : Spin position first indices.
    j_list : Spin position second indices.
    sl : Spin lattice.
    """

    E_u, E_d = energies_spins(i_list, j_list, sl, H, J)

    Ei = np.array([E_u, E_d])

    Z = np.sum(np.exp(- Ei / T), axis=0)  # Partition function.
    pi = 1 / np.array([Z, Z]) * np.exp(- Ei / T)  # Probability.
```

```
    return pi, Z
```

0.1 Task 1

[13]:
```
N = 200  # Size of the splin lattice.
H_values = [-5, -2, -1, -0.5, -0.2, -0.1, 0, 0.1, 0.2, 0.5, 1, 2, 5]
J = 1  # Spin-spin coupling.
T = 5  # Temperature. Temperatura critica ~2.269.

f = 0.05  # Number of randomly selected spins to flip-test..
total_steps = 500

magnetizations = []

for H in H_values:
    sl = 2 * np.random.randint(2, size=(N, N)) - 1 # Initialize N*N self-spin␣
 ↪lattice ( +1 or -1)
    Nspins = np.size(sl) # Total number of spins in the spin lattice.
    Ni, Nj = sl.shape
    S = int(np.ceil(Nspins * f))  # Number of randomly selected spins.

    step = 0
    magnetization_list = []
    running = True  # Flag to control the loop.

    while running and step < total_steps:
        ns = random.sample(range(Nspins), S)
        i_list = list(map(lambda x: x % Ni, ns))
        j_list = list(map(lambda x: x // Ni, ns))

        pi, Z = probabilities_spins(i_list, j_list, sl, H, J, T)

        rn = np.random.rand(S)

        for i in range(S):
            if rn[i] > pi[0, i]:
                sl[i_list[i], j_list[i]] = -1
            else:
                sl[i_list[i], j_list[i]] = 1

        # record magnetization
        if total_steps - 300 <= step < total_steps - 100:
            magnetization = np.sum(sl) / (N * N)
            magnetization_list.append(magnetization)

        step += 1
```

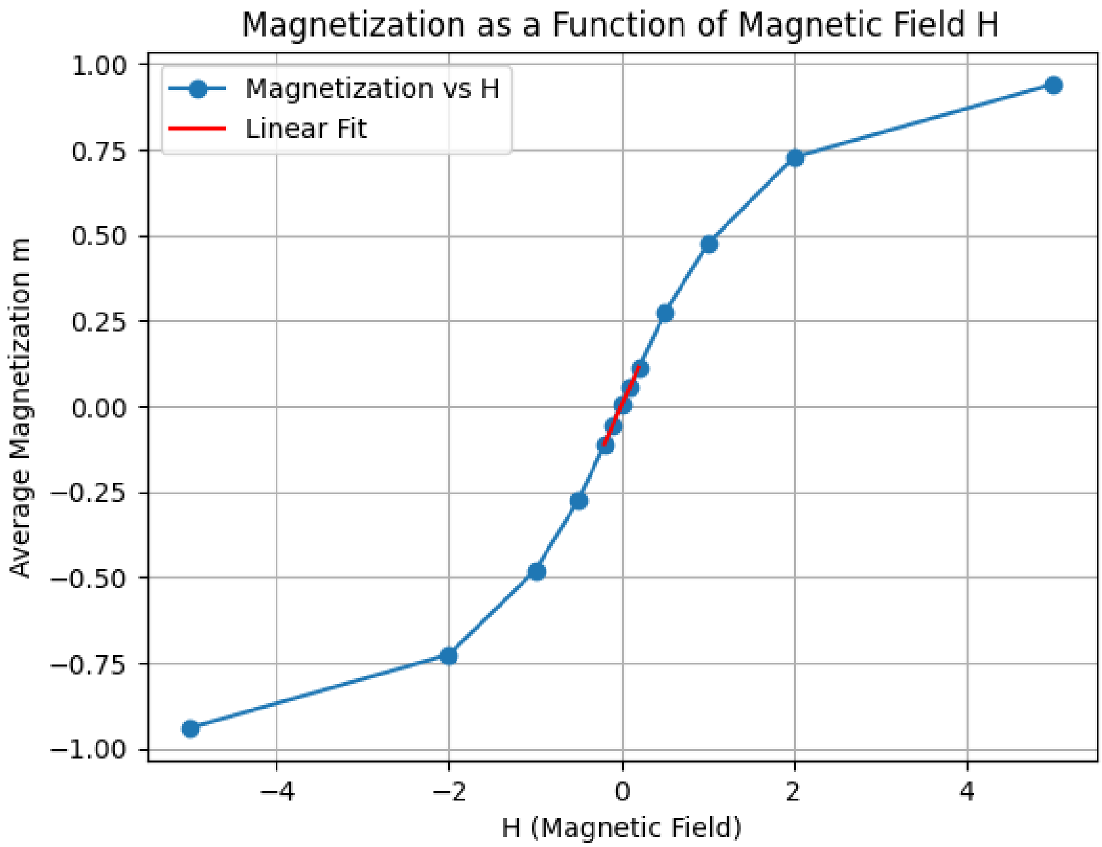
```
        if step >= total_steps:
            running = False

    print(f'H = {H}, Ising Model simulation done')

    avg_magnetization = np.mean(magnetization_list)
    magnetizations.append(avg_magnetization)
```

```
H = -5, Ising Model simulation done
H = -2, Ising Model simulation done
H = -1, Ising Model simulation done
H = -0.5, Ising Model simulation done
H = -0.2, Ising Model simulation done
H = -0.1, Ising Model simulation done
H = 0, Ising Model simulation done
H = 0.1, Ising Model simulation done
H = 0.2, Ising Model simulation done
H = 0.5, Ising Model simulation done
H = 1, Ising Model simulation done
H = 2, Ising Model simulation done
H = 5, Ising Model simulation done
```

Plot m(H) and compute linear funciton for small H values

[15]:
```
small_H_values = [-0.2, -0.1, 0, 0.1, 0.2]
small_magnetizations = [magnetizations[H_values.index(h)] for h in␣
 ↪small_H_values]
fit_params = np.polyfit(small_H_values, small_magnetizations, 1)  # linear fit
 = fit_params[0]
print(f'Calculated magnetic susceptibility  : { }')

fit_function = np.poly1d(fit_params)
fit_H_values = np.linspace(min(small_H_values), max(small_H_values), 100)
fit_magnetizations = fit_function(fit_H_values)

# Plot m(H)
plt.figure()
plt.plot(H_values, magnetizations, 'o-', label='Magnetization vs H')
plt.plot(fit_H_values, fit_magnetizations, 'r-', label='Linear Fit')
plt.xlabel('H (Magnetic Field)')
plt.ylabel('Average Magnetization m')
plt.title('Magnetization as a Function of Magnetic Field H')
plt.legend()
plt.grid(True)
plt.savefig('Magnetization_as_Function_of_Magnetic_Field_H.png', format='png',␣
 ↪dpi=300)
plt.show()
```

```
Calculated magnetic susceptibility  : 0.56006475
```



0.2 Task 2

[17]:
```
N = 200  # Size of the splin lattice.

J = 1  # Spin-spin coupling.
T_values = [0.1, 0.2, 0.5, 1, 2, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9,␣
↪3, 5] # Temperature. Temperatura critica ~2.269.

f = 0.05  # Number of randomly selected spins to flip-test.
total_steps = 5000

magnetizations = []

for T in T_values:
    sl = 2 * np.random.randint(2, size=(N, N)) - 1 # Initialize N*N self-spin␣
↪lattice ( +1 or -1)
    Nspins = np.size(sl) # Total number of spins in the spin lattice.
    Ni, Nj = sl.shape
    S = int(np.ceil(Nspins * f))  # Number of randomly selected spins.
```

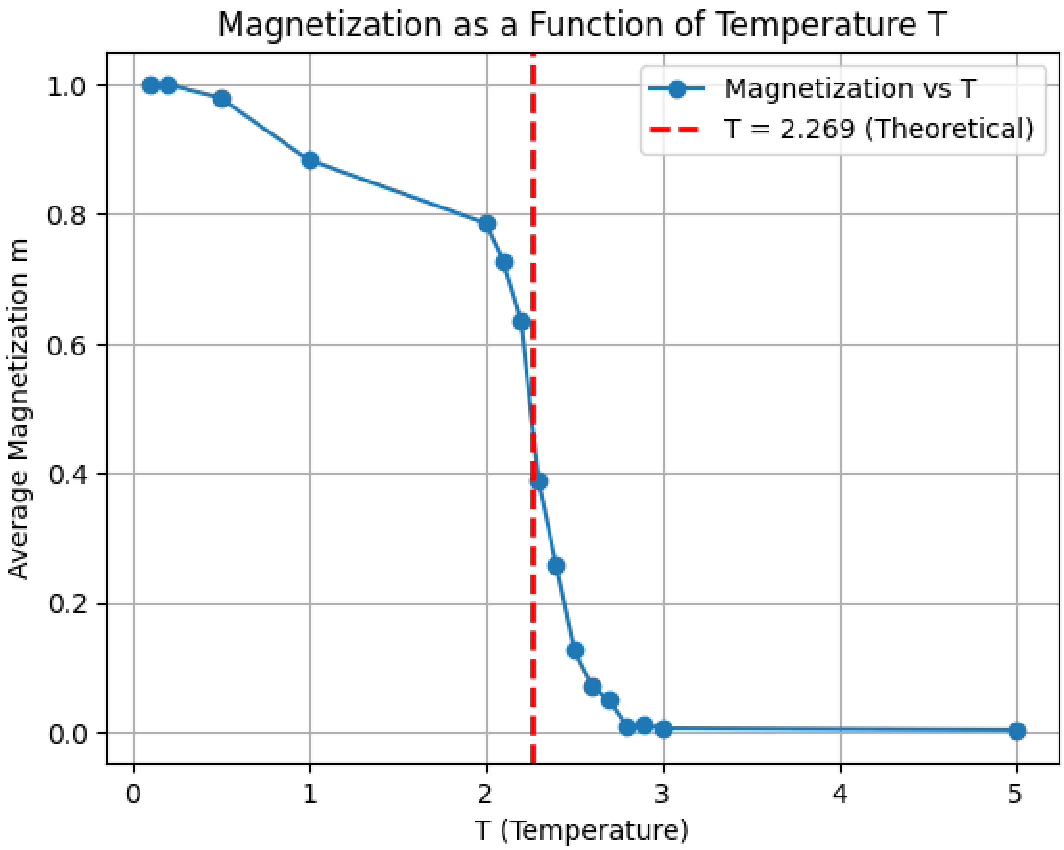
```python
    step = 0
    magnetization_list = []
    running = True  # Flag to control the loop.

    while running and step < total_steps:

        if step <= 300:
            H = 0.1
        else:
            H = 0

        ns = random.sample(range(Nspins), S)
        i_list = list(map(lambda x: x % Ni, ns))
        j_list = list(map(lambda x: x // Ni, ns))

        pi, Z = probabilities_spins(i_list, j_list, sl, H, J, T)
        rn = np.random.rand(S)

        for i in range(S):
            if rn[i] > pi[0, i]:
                sl[i_list[i], j_list[i]] = -1
            else:
                sl[i_list[i], j_list[i]] = 1

        # record magnetization

        if (total_steps - 300) <= step < (total_steps - 100):
            magnetization = np.sum(sl) / (N * N)

            if H == 0:
                magnetization = np.abs(magnetization)

            magnetization_list.append(magnetization)


        step += 1
        if step >= total_steps:
            running = False

    print(f'T = {T}, Ising Model simulation done')
    avg_magnetization = np.mean(magnetization_list)
    magnetizations.append(avg_magnetization)
```

```
T = 0.1, Ising Model simulation done
T = 0.2, Ising Model simulation done
T = 0.5, Ising Model simulation done
T = 1, Ising Model simulation done
T = 2, Ising Model simulation done
```

```
T = 2.1, Ising Model simulation done
T = 2.2, Ising Model simulation done
T = 2.3, Ising Model simulation done
T = 2.4, Ising Model simulation done
T = 2.5, Ising Model simulation done
T = 2.6, Ising Model simulation done
T = 2.7, Ising Model simulation done
T = 2.8, Ising Model simulation done
T = 2.9, Ising Model simulation done
T = 3, Ising Model simulation done
T = 5, Ising Model simulation done
```

[18]:

```
# Plot m(T)
plt.figure()
plt.plot(T_values, magnetizations, 'o-', label='Magnetization vs T')
plt.axvline(x=2.269, color='r', linestyle='--', linewidth=2, label='T = 2.269␣
↪(Theoretical)')
plt.xlabel('T (Temperature)')
plt.ylabel('Average Magnetization m')
plt.title('Magnetization as a Function of Temperature T')
plt.legend()
plt.grid(True)
plt.savefig('Magnetization_as_Function_of_Temperature_T.png', format='png',␣
↪dpi=300)
plt.show()
```


Magnetization as a Function of Temperature T
Magnetization vs T
T = 2.269 (Theoretical)
Average Magnetization m
T (Temperature)
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5