EECS 212 – Fall 2018

Mathematical Foundations of Computer Science

Homework 1

Instructions:

- This homework is due by 8PM on **Friday, October 12th**. Late homework will not be accepted.
- You must submit your solutions as a single PDF file on Canvas with your name at the top. Typesetting your solutions using LaTeX is encouraged.
- You are welcome to discuss the problems in groups of up to three people, but you must **write up and submit your own solutions**. You must also write the names of everyone in your group on the top of your submission.
- The primary resources for this class are the lectures, lecture slides, recitations, optional textbook, teaching staff, your (up to two) collaborators, and the course Piazza page. Use of any other resources on the homework is prohibited.
- *Scoring:* There are six problems, each of which is worth two points.
- You will receive a bonus point (+1) if you write Homework 1 in Latex. To receive this point, write `"This homework was written in \LaTeX"` in the .tex file with your solutions.

Problems:

Problem 1

Prove that in any stack of $n \geq 2$ pancakes (all of different sizes), the Bring-to-Top algorithm sorts the stack using at most $2n - 3$ flips.
Hint: The Bring-to-Top algorithm does the following: If the size of the stack is 1, we are done. Else, flip pancake n to the top, then flip it to the bottom. Repeat this algorithm on the top $n - 1$ pancakes.

Problem 2

Given an array A of n distinct integers, prove that the following algorithm *Quicksort* sorts the numbers in ascending order:

QuickSort(A)

- if $size(A) = 0$, return A.
- else select a pivot p from A
  1. Let L_1 be the list of elements of A less than pivot p.
  2. Let L_2 be the list of elements of A greater than pivot p.
  3. Return Quicksort(L_1), p, Quicksort(L_2).

Problem 3

(Do not collaborate on this question.) The Fibonacci numbers, $F_0, F_1, F_2, \dots$, are defined recursively by the equations $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$, for $n > 1$. Prove that

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$$

for all positive integers n.

Problem 4

Prove that there exist some numbers a, b, c, d such that for every natural number n, $\sum_{k=0}^{n}(3k^2 + 2k - 4) = an^3 + bn^2 + cn + d$.

Hint: $c = -\frac{5}{2}$.

Problem 5

Prove that for any $x \geq -\frac{1}{2}$ and any integer $n > 0$, $(1 + 2x)^n > 2nx$. Note that x can be negative.

Problem 6

There are two children sitting on a (very long) bench. The child on the left is a boy, the child on the right is a girl. Every minute, either two children arrive and sit down next to each other on the bench (possibly squeezing between two children who are already sitting), or two children who had been sitting next to each other get up off the bench and leave.

Furthermore, the arriving and departing pairs of children must always of the same sex (i.e., either both boys or both girls).

Is it possible, after some amount of time, that there will be only two children remaining on the bench with a girl on the left and a boy on the right?

Hint: Try to find an *invariant*, i.e., a property of the boy-girl ordering of the children that does not change with time.